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EXTENSION OF VERTEX N-MAGIC TOTAL GRAPHS WITH DUALITY APPROACH

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Abstract Consider a finite simple undirected graph G with p vertices and q edges, where N represents how many distinct degree values occur among the vertices.A Vertex N-magic total labeling is a function ψ that establishes a one-to-one correspondence between the elements of $V(G) \cup E(G)$ and the consecutive integers $1, 2, ..., p + q$. There are N unique magic constants k_i where $i \in \{1, 2, ..., N\}$, and these constants k_i must satisfy the strictly increasing condition $k_1 < k_2, ..., k_i$ for $i \in \{1, 2, ..., N\}$. Furthermore, two additional types of vertex N-magic total labeling exist: Strong vertex N-magic total labeling and Weak vertex N-magic total labeling, both of which have been investigated.

The duality problem of relational structures, including magic constants and weak vertex N-magic total graphs, is examined. In this manuscript, we introduce the general constructions of strong vertex N-magic total labeling in some families of graphs with distinct vertices and edges. In addition, we discuss the existence of weak VNMT labelings for the Lobster and Actinia graphs. It presents a fresh perspective on this topic and unveils novel insights into its properties and practical applications. As a last point, we pose some questions that need further investigation.

1 Introduction

The labeling of discrete structures holds considerable significance because of its wide array of practical uses. The systematic organization of labeling surveys is covered in [8], while thorough information concerning vertex magic labeling is presented in [2, 3, 4, 5, 6, 7, 20]. Building upon A. Rosa's foundational contribution [19], significant research has been pursued on graph labeling, while thorough details concerning magic graphs are presented in [1, 10]. The progress in communication and information technology has improved the effectiveness of graph labeling, along with the ability to efficiently tackle common coloring problem techniques. This study concentrates on a technique called vertex N-magic total labeling.

The exploration of vertex magic labeling in graphs has identified upper and lower bounds that vary according to the number of vertices or edges in [9, 18, 21, 22]. Various independent upper limits for N-vertex magic graphs have been identified, where such limits are completely characterized by the vertex and edge totals (p and q).The weighted strength of any vertex in a graph corresponds to the aggregate of its assigned label and all edges that connect to it, with Kumar providing comprehensive details about the fundamental aspects of V-Super vertex magic labeling and E-Super vertex magic labeling in [12]. Research on the super edge-magicness of graphs with a particular degree sequence was carried out by Ichishima et al. [23], and the authors delved into the characteristics of these graphs and studied the families of graphs with especially impressive edge-magic features. The concept of vertex N-magic total labeling was initially explored by Marimuthu and Kumar in [11]. Additionally, [15, 17] present valid properties of magic constants in graphs, while [14] establishes the sharp bounds on Vertex N-magic total labeling graphs.

In contemporary research, vertex N-magic total labeling in graph theory has captured significant scholarly attention, predominantly because of its wide-ranging applicability across various sectors. This technique involves attributing labels from the domain $\{1, 2, ..., p\}$ to every node in a graph, so long as the graph conforms to the standards of a Strong vertex N-magic total labeling. In a similar manner, every edge within the graph is labeled using elements from the set $\{1, 2, ..., q\}$ when the graph adheres to the requirements of a Weak vertex N-magic total labeling. A graph G admits a Non-Strong Vertex N-magic total labeling if no vertex is assigned a label belonging to the set $\{1, 2, ..., p\}$. In the same way, a graph G has a Non-Weak vertex N-magic total labeling provided that all of its edges avoid being labeled with elements from $\{1, 2, ..., q\}$.

Various families of Strong Vertex N-magic total graphs are constructed in detail within Section 2, along with the corresponding Strong Vertex N-magic constants k_i for every $i \in \{1, 2, ..., N\}$. Section 3 presents a discussion of the vertex N-magic total labeling of vertices and its dual. The dualizability problem in graph theory has been studied for over thirty years, and the natural dualities for graph theory have recently been extended to include a wider range of structures. Specifically, in Section 4, the duality problem of relational structures, including magic constants and weak vertex N-magic total graphs, is examined. In section 5, we illustrated the application of vertex N-magic total labeling graphs. Finally, in Section 6, several open problems in various areas of Strong vertex N-magic total graphs are presented. The strong vertex N-magic for the helm graph is shown in Figure 1.

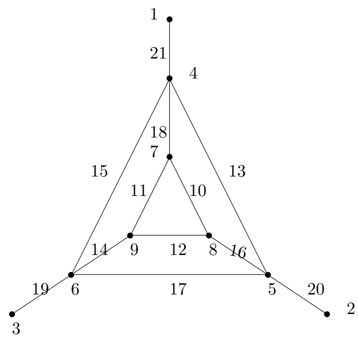

Figure 1. Strong vertex N-magic for Helm graph

To proceed further, we make the following definitions:

Definition 1.1. Attach the four pendant vertices to all vertices of the cycle C_m of length m, resulting in the Actinia graph, which has $5m$ vertices and $5m$ edges, which is denoted as $Act_{m,4}$. Let the vertices of $Act_{m,4}$ be $\{c_i, u_i, v_i, w_i, s_i; 1 \leq i \leq m\}$ and the edge set be, $\{c_iu_i, c_iv_i, c_iw_i, c_is_i; 1 \leq i \leq m\} \cup \{c_ic_{i+1}; 1 \leq i \leq m-1\} \cup \{c_1c_m\}$ respectively.

Definition 1.2. A lobster graph is defined as follows: For each $1 \leq i \leq n$, $1 \leq j \leq 6$ and $k \leq 3$, let the vertex set and the edge sets be $V(Lob_3^{2,n}) = \{c_j, c_k^*, u_i, v_i, w_i, s_i, t_i, ..., a_n\}$ and, $E(Lob_3^{2,n}) = \{c_ju_i, c_jv_i, c_jw_i, c_js_i, ..., \} \cup\{c_jc_k^*\} \cup \{c_k^*c_{k+1}^*; \}$ respectively. The two hand vertices are connected to three vertices on the backbone path, and each hand vertex is connected to n finger vertices, yielding a graph of $6n + 9$ vertices and $6n + 8$ edges.

2 Features of Strong Vertex N-magic Graphs And The Impact of Edge Removal

An investigation is in progress to analyze the features of durable VNMT graphs and the consequences of edge deletion on such elements. Here, we provide the theoretical magic constants k_i for each $i \in \{1, 2, ..., N\}$. The following findings describe the circumstances prior to determining magic constants.

Theorem 2.1. *Let G be a strong VNMT graph except the path graph. Then, removing one edge from G also has strong VNMT if G has the vertices* $p \geq 4$.

Proof. Consider the strong VNMT graph G, labeled by f. If weremove the edge q from $E(G)$ with the label $p + 1$, we have a new strong vertex N-magic total labeling g of $G - q$, defined as follows: For every $x \in V(G) \cup E(G)$, we have $| g(x) |=| f(x) - 1 |$ for $i \in \{1, 2, ..., N + 1\}$.
Case(i): Suppose G is a pendant-edged strong vertex N-magic total graph. Allow G to have a magical constant of $k_1 = p + q + 1$. The strong VNMT results from the removal of a single connection, with the magic constants $k_1^{'} = 1$ and $k_2^{'} = k_1$, and the remaining magic constants depending on the number of incident edges.
Case(ii). For simplicity, we'll refer to G as any strong VNMT graph without pendant edges. The strong vertex N-magic, with magic constants $k_1 \geq k_1^{'}$ and $k_2^{'} \leq k_1$, is obtained by cutting off a single edge, while the magic constants for the remaining edges depend on the total number of incident edges.
On the other hand, If G has the number of points $p < 4$ except a path graph. If $p = 1$, then K_1 which is non-strong VNMT. If $p = 2$, the possible choice is $2K_2$, non-strong VNMT. If $p = 3$, then the possible choices are $K_2 \cup K_1$ and $3K_1$ both are non-strong VNMT. □

Example 2.2. If we take the Sun graph and remove the edge labeled u_2, we get the Wounded Sun graph $C_n^+ - e$ for $n \geq 3$. This graph has the following set of vertices, edges, and labels. Let $V(C_n^+ - e) = \{u_1, u_3, ..., u_n, v_1, v_2, ..., v_n\}$ and the edges $E(C_n^+ - e) = u_1v_1 \cup \{u_iv_i; 3 \leq i \leq n\} \cup \{v_iv_{i+1}, 1 \leq i \leq n - 1\} \cup v_nv_1$.

Starting with v_iv_{i+1}, label the edges with $3n - i$ for $1 \leq i \leq n - 1$ and v_nv_1 by $2n$. Next, provide the label $3n+i-3$ to the edges u_iv_i for $i \neq 2, 3 \leq i \leq n$. This time, use a label of $4n-2$ for the u_1v_1 edge. The number 1 should be assigned to the unique vertex u_1. Labels $n - i + 2$ and $n + i - 1$ are assigned to the vertices u_i for $i \neq 2, 3 \leq i \leq n$ and v_i for $i = 1, 2, ..., n$. As a result, the total of k_1, k_2, and k_3 is assigned to each vertex of $V(C_n^+ - e) \cup E(C_n^+ - e)$. With magic constants of $4n - 1$, $7n - 2$, and $10n - 3$, strong VNMT labeling is possible for $n \geq 3$ in the wounded sun graph.

Corollary 2.3. *Let G be either a strong or non-strong VNMT path graph. Then the removal of one pendant edge from G also has a strong VNMT graph if the path of length either* $n \in \{4, 5\}$ *(Or)* $n \in \{6, 8, 10, ..., 2t\}$ *where* $t \geq 3$.

Proof. **Case (i):** Let's pretend G has the label f and is a strong VNMT graph. Assuming $n = 4$, with $G \cong P_4$, we may produce $P_3 \cup K_1$, which is likewise a strong VNMT graph by cutting off a single pendant edge. $P_4 \cup K_1$ is also a strong VNMT graph, as seen when looking at it in P_5.

However, if n is odd and $n \geq 7$, the set gets labelled completely using strong vertex N-magic. Thus, the following options are available for strong VNMT graphs after the removal of the pendant edge: $P_{n-1} \cup K_1$, $P_{n-2} \cup P_2$, $P_{n-3} \cup P_3$,..., $P_{n-(n-1)} \cup P_{n-1} \approx K_1 \cup P_{n-1}$. The first special situation occurs when $n \geq 7$, $P_{n-1} \cup K_1$, revealing a contradiction in the form of a non-strong VNMT graph.
For $n \geq 7$, $P_{n-2} \cup P_2$ clearly demonstrates the contradiction of a non-strong VNMT graph. In the third notable instance, with $n \geq 7$, the $P_{n-1} \cup P_3$ graph cannot permit the strong VNMT graph because any other choice of (k_1, k_2) is also a contradiction.

By continuing in this manner, we discover that every single graph we could have picked is an N-magic total graph with weak vertices, which is false.
Case(ii). Let's assume G is an N-magic total graph with non-strong vertices and the label g. Suppose $n \in \{6, 8, 10, ..., 2t\}$. When $t \geq 3$, the vertex and edge labels do not coincide in P_{2t}.

If t is less than three, then P_2 is the only viable option for the non-Strong VNMT graph. By eliminating edge e from G, we get $G'(G - e) \approx 2K_1$, which is a Non-Strong magic total graph, and this is how we update the vertex labels for P_2. One pendant edge of P_{2t}, labeled g', can be removed, and the resulting graph can be labeled as follows. It is correct to write 1 on the label for the pendant vertex v and $2t$ for $g'(v_1)$. The remaining vertices are $2, 3, ..., 2t-1$. P_{2t-1}'s first edge is labeled $3t - 1$ and $4t - (i + 1)$ for the even positions of edges at $i = 1, 2, ..., t - 1$, and $3t - i - 1$ for the odd places of edges at $i = 1, 2, ..., t - 2$. □

2.1 Strong Vertex N-magic Labeling On Some Families Of Graphs

In this subsection, we establish the general constructions of Strong Vertex N-magic total labeling on some families of graphs with distinct magic constants.

Proposition 2.4. *Necessary and sufficient condition for a graph $Act_{m,4}$ admits strong VNMT for m is odd and $m \geq 3$ iff $\frac{95m+7}{2}$.*

Proof. Assume that graph $Act_{m,4}$ allows for strong VNMT labeling ψ. Edges are labeled with the set $\{5m+1, ..., 10m-1, 10m\}$, while vertices are just labeled with the set $\{1, 2, ..., 5m\}$. For some $v \in G$, $k_i(G)$ is equal to $wgt_\psi(v) = \psi(v) + \sum_{u \in N(v)} \psi(vu)$, $\sum_{v_i \in V} |\psi(v_i) + \sum_{v_j \in N(v_i)} \psi(v_i v_j)| = 10mk_i$, where $q = 5m$. Since Actinia graph $Act_{m,4}$ has two distinct magic constants.

Clearly, $\sum_{v_i \in V} |\psi(v_i) + \sum_{v_j \in N(v_i)} \psi(v_i v_j)| = mk_2$. Here,

$$\begin{aligned} k_2 &= 4m + 1 + 10m + 8m + 1 + 8m + 6m + 1 + \frac{11m+1}{2} + 6m \\ &= \frac{95m+7}{2} \end{aligned}$$

Similarly, $\sum_{v_i \in V} |\psi(v_i) + \sum_{v_j \in N(v_i)} \psi(v_i v_j)| = 4mk_1$. Thus if m is odd $k_1 = 10m + 1$ and if m is even, $k_2 = \frac{95m+7}{2}$, which is not an integer. As a result, m must be odd, and $m \geq 3$. □

Theorem 2.5. *The Actinia graph $Act_{m,4}$ admits strong VNMT with two distinct magic constants for $m \geq 3$ and m is odd.*

Proof. For every $1 \leq i \leq m$, where m is an odd number, the central vertices of $Act_{m,4}$ are denoted by c_i, the first pendant vertices connecting with each c_i are denoted by u_i, c_i is surrounded by the vertices v_i, w_i, and s_i.

Define the function $\psi : V(Act_{m,4}) \cup E(Act_{m,4}) \to \{1, 2, ..., 10m\}$ as follows:

First, assign clockwise labels to the first m vertices u_i with $1, 2, ..., m$ and v_i with $2m + 1 - i$ in a clockwise direction for each $1 \leq i \leq m$.

In the second stage, designate the vertices c_i as $4m + i$ and the edges w_i as $2m + i$ as well as $4m + i - 1$ to the vertices s_i for each $1 \leq i \leq m$.

In Step 3, allocate the edges $c_i u_i$ by $10m + 1 - i$ and $8m + i$ for $c_i v_i$ as well as $8m + 1 - i$ for $c_i w_i$ and the edges $c_i s_i$ by $6m + i$ and the edge $c_1 c_m$ by $\frac{11m+1}{2}$.

Finally, if i is odd, the sequence $\left\{6m, 6m-1, ..., \frac{11m+3}{2}\right\}$ yields the edge labeling $c_i c_{i+1}$ for $1 \leq i \leq m-1$. In addition to $\left\{\frac{11m-1}{2}, \frac{11m-3}{2}, \frac{11m-5}{2}, ..., 5m+1\right\}$ if i is even.

The magic constants are $10m + 1$ and $\frac{95m+7}{2}$ respectively. □

Theorem 2.6. *For all $n \geq 2$ and n is even, the Lobster graph $Lob_3^{2,n}$ admits strong VNMT with four distinct magic constants.*

Proof. Let us consider a tree where each vertex i is connected to pendant vertices u_i, v_i, w_i, $s_i, t_i, ..., a_i$, and so on. The vertices are the endpoints of these pendant vertices, while c_j represents the central vertices connected to the pendant vertices for each value of j ranging from 1 to 6. The perfect central vertices of the tree are denoted by c_k^*, where k is less than or equal to 3.

Define the function $\psi : V(Lob_3^{2,n}) \cup E(Lob_3^{2,n}) \to \{1, 2, ..., 12n + 17\}$ as follows:

To commence, assign labels to the first n vertices, denoted as u_i, for each $1 \leq i \leq 6$. The labeling starts with 1 and continues in ascending order up to n. Additionally, the vertices v_i are labeled clockwise, starting with 12 and descending to 7.

The second step is to designate the numerals $13, 14, ..., 18$ as the values for the vertices w_i, and similarly, assign the integers $24, 23, ..., 19$ to the corresponding pendant vertices s_i for each $1 \leq i \leq 6$. Additionally, allocate the integers $25, 26, ..., 30$ to the vertices t_i for each value of i ranging from 1 to 6. In clockwise order, the vertices a_i of the n-th pendant should be given as follows: $6n, 6n-1, ..., 6n-5$.

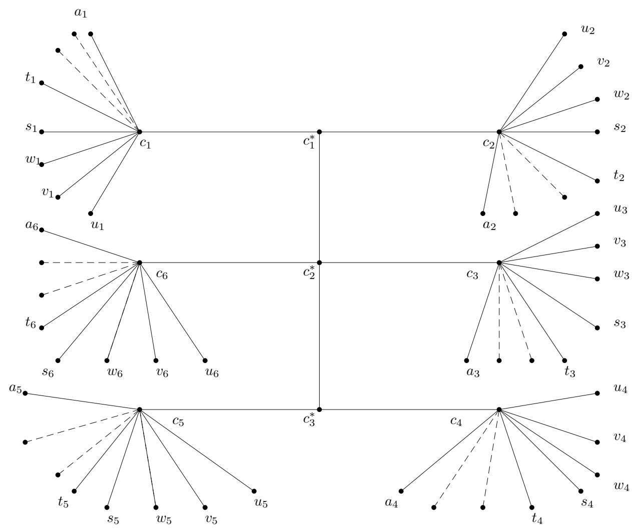

Figure 2. Lobster graph $Lob_3^{2,n}$

In Step 3, for $j \leq 3$, the central vertices c_j are marked with $6n - 1 + 2j$, and for $4 \leq j \leq 6$, the remaining vertices c_j are labeled with $6n - 2j + 14$.

The vertices c_k^*, $k \leq 2$ by $6n + 9 - k$ and c_3^* are labeled by $6n + 9$ respectively.
Assemble the remaining labels in this manner: To each $1 \leq i \leq 6$, we have

$$\begin{aligned}
\psi(c_k^* c_{k+1}^*) &= 6n + 9 + k & if\ k \leq 2 \\
\psi(c_j c_1^*) &= 6n + 19 - 2j & if\ j \leq 2 \\
\psi(c_j c_2^*) &= 6n + 16 - j & if\ 3 \leq j \leq 4 \\
\psi(c_5 c_3^*) &= 6n + 16 \\
\psi(c_6 c_3^*) &= 6n + 14 \\
\psi(c_j u_i) &= 12n + 18 - i \\
\psi(c_j v_i) &= 12n + 5 + i \\
\psi(c_j w_i) &= 12n + 6 - i \\
\psi(c_j s_i) &= 12n - 7 + i \\
\psi(c_j t_i) &= 12n - 6 - i \\
\vdots \quad & \vdots \quad \vdots \\
\vdots \quad & \vdots \quad \vdots \\
\vdots \quad & \vdots \quad \vdots \\
\psi(c_j a_i) &= 6n + 17 + i;\ if\ n\ is\ even
\end{aligned}$$

The Lobster graph $Lob_3^{2,n}$ is depicted in Figure 2. The magic constants are $12n+18$ and $24n+50$ and $30n + 53$ and $\frac{18n^2+59n+36}{2}$. □

3 Duality Between Strong and Weak Vertex N-magic Total Graphs

This section demonstrates the conversion of a graph with strong VNMT labeling into one with weak VNMT labeling. The map g on $V \cup E$ for any graph G with a strong vertex N- magic labeling f may be written as $g(v_i) = p + q + 1 - f(v_i)$, where p, q, and 1 are integers for any vertex v_i. Any edge uv is equal to $p + q + 1 - f(uv)$, where p, q, and 1 are constants. Define

g as the dual of f, as it is apparent that g is also an injective function from the set $V \cup E$ to $\{1, 2, ..., (p + q)\}$.

Theorem 3.1. *Suppose a finite undirected non-regular graph, denoted as G, with several vertices p, where p is greater than or equal to* 3*. Let f be a strong vertex N-magic labeling with distinct magic constants. Then the dual of strong vertex N-magic total labeling of any graph has weak vertex N-magic total labeling g.[10]*

Proof. Let us have a graph G that has a strong vertex N-magic labeling function f, such that the weight of vertex v, denoted as $wt_f(v)$, is equal to k_i for i ranging from 1 to N, where N is the total number of distinct magic constants in G. This property holds v in G for all vertices. The value d_j, where j is an integer more than or equal to 1, indicates the number of edges that connect a particular vertex. In summary, the expression $(d_j + 1)(p + q + 1) - k_i$ represents the weighted value of a vertex in the context of Weak vertex N-magic labeling g, where i belongs to the set $\{1, 2, ..., N\}$. □

The following observation is by the theorem mentioned above and theorem 2.1.

Observation:

If an edge is removed from a weak vertex N-magic total graph, a weak vertex N still exists a total labeling for $p \geq 3$.

4 Weak Vertex N -magic Total Labeling Graphs

Every edge of a graph labeled by $\{1, 2, ..., q\}$ if the graph admits Weak Vertex N-magic total labeling. In this section, we discuss the existence of weak vertex N-magic total labeling of the Lobster graph and the Actinia graph.

Theorem 4.1. *For all even $n \geq 2$, the Lobster graph $Lob_3^{2,n}$ has a weak VNMT with four distinct magic constants.*

Proof. The theorem 3.1 shows that a pendant edge has a dual and its central vertex c_j, is labelled with $6n + 19 - 2j$ for $j \leq 3$, and if its other vertices, c_j for $4 \leq j \leq 6$, are labelled with $6n + 2j + 4$. The vertices c_k^*, $k \leq 2$ by $6n + k + 9$ and c_3^* is labeled by $6n + 9$ respectively. Here is the remaining part of the edge labeling:

$$
\begin{aligned}
\psi(c_k^* c_{k+1}^*) &= 6n + 9 - k && k \leq 2 \\
\psi(c_j c_1^*) &= 6n + 2j - 1 && j \leq 2 \\
\psi(c_j c_2^*) &= 6n + j + 2 && 3 \leq j \leq 4 \\
\psi(c_5 c_3^*) &= 6n + 2 \\
\psi(c_6 c_3^*) &= 6n + 4
\end{aligned}
$$

The magic constants are $24n + 22$ and $30n + 37$ and $\frac{6n^2+25n+36}{2}$. □

Theorem 4.2. *For an odd $m \geq 3$, the Actinia graph $Act_{m,4}$ admits weak vertex N-magic with two distinct magic constants.*

Proof. Based on Theorem 3.1, it is established that pendant edges possess a dual property. To initiate the labeling process, for any integer i within the range of 1 to m, the vertices c_i are assigned labels of $6m + 1 - i$. The edge $c_1 c_m$ is allocated by $9m + 1$. If i is an odd number, the edge labeling $c_i c_{i+1}$ for $1 \leq i \leq m - 1$ is derived from the sequence $4m + 1, 4m + 2, 4m + 3, ..., \frac{9m-3}{2}, \frac{9m-1}{2}$.

On the other hand, if i is an even number, the edge labeling is derived from the sequence $\frac{9m+3}{2}, \frac{9m+5}{2}, \frac{9m+7}{2}, \frac{9m+9}{2}, ..., 5m$. The mathematical expression denoted as the magic constant can be represented as $\frac{45m+7}{2}$. As shown in Figure 3, the Actinia graph is an $Act_{7,4}$.

□

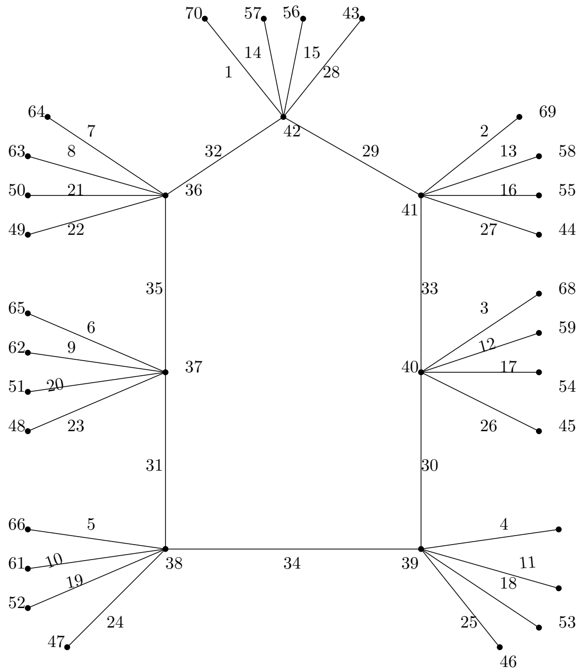

Figure 3. Actinia graph $Act_{7,4}$

5 Application of Vertex N-magic Total graphs with Duality Approach

The Network is a testbed for examining the integration of wireless access and multimedia networked computing within indoor environments. This is achieved by utilising portable multimedia endpoints and pico-cells, which are comparable in size to a typical room. The network enables users to seamlessly access multimedia data stored within a backbone wired network using multimedia endpoints, such as personal computers, laptops, and portable multimedia terminals, while allowing for mobility. The value of data access and utilisation in granting individuals and organizations greater freedom, mobility, and effectiveness has been recognised in modern times.

Our inventory consists of odd $m \geq 3$ wired adapters, each equipped with four processors. These adapters possess distinct server and sharing capacities, while the processors within them also exhibit unique server and sharing capacities.

(i) Can each adapter possess unique frequencies, while each processor has its own set of frequencies simultaneously?

(ii) Is it feasible to designate the adapter with distinct strong and weak frequencies?

The resolution of this inquiry was achieved through the utilization of vertex N magic labeling, which provided the requisite proof. By employing strong vertex N magic total labeling and weak vertex magic labeling with duality, to determine the unique frequencies $10m + 1$ possessed by the processors and the distinct frequencies associated with the wired adapter in the labeling process. Under the Strong vertex N-magic labeling, each adapter achieved unique frequencies of $\frac{95m+7}{2}$. Following the weak vertex N magic labeling, each adapter achieved frequencies of $\frac{45m+7}{2}$.

6 Open Problems on Strong Vertex N-magic Total Graphs

We have observed multiple families of graphs where the determination of those that possess Strong Vertex N-magic constants is insufficient. The development of vertex N-magic graphs for a given graph encompasses a range of captivating and widely relevant inquiries. For the purpose of further discourse, designate three of these.

Question 1: Determine the values of m and n that result in the full bipartite graph $K_{m,n}$ exhibiting the property of being Strong Vertex N-magic total.
Question 2: Assess the set of positive numbers n for the Double Fan graph $DF_{1,n}$ that allows for a Strong Vertex N-magic total. According to Theorem 2.4, the Actinia graph with an odd value of m possesses a strong Vertex N-magic total. This leads to the emergence of the following question.
Question 3: Provide an assessment of the even positive integers m for the Actinia graph $Act_{m,4}$ that allow for the Strong Vertex N-magic total. Next, evaluate the potential odd integers n for which the Lobster graph can admit a strong vertex N-magic labeling.

7 Conclusion

This manuscript provides the theoretical magic constants corresponding to each in a graph with particular reference to the strong vertex N-magic total. In addition, we developed generic constructions of strong vertex N-magic total labeling in some families of graphs with distinct magic constants. Some work on strong and Weak vertex N-magic total graphs, for instance, using a strong vertex N-magic labeling in a graph to be transformed into a weak vertex N-magic labeling. The topic of this discussion is some weak vertex N-magic labelings for the Lobster and Actinia graphs. The development of vertex N-magic graphs for a given graph encompasses a range of captivating and widely relevant inquiries.

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