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$\mathcal{L}-$FUZZY $1-$ABSORBING PRIME IDEALS AND FILTERS IN AN ADL

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Abstract The concept of $\mathcal{L}-$fuzzy $1-$absorbing prime ideals and $\mathcal{L}-$fuzzy $1-$absorbing prime filters has been introduced within the context of an Almost Distributive Lattice. Specifically, we explore the connections between $\mathcal{L}-$fuzzy prime ideals(filters) and $\mathcal{L}-$fuzzy $1-$absorbing prime ideals(filters), as well as the relationships between $\mathcal{L}-$fuzzy $1-$absorbing prime ideals(filters) and $\mathcal{L}-$fuzzy $2-$absorbing ideals(filters). Ultimately, we demonstrate that both the image and inverse image of $\mathcal{L}-$fuzzy $1-$absorbing prime ideals(filters) result in $\mathcal{L}-$fuzzy $1-$absorbing prime ideals(filters).

1 Introduction

Exploring the structural theory of distributive lattices, especially in the context of Boolean algebras, relies significantly on the concept of prime ideals. The notion of 2-absorbing ideals in commutative rings was initially introduced by Badawi [3] as an extension of prime ideals [7, 4]. Chuadhari [5] further extended this idea to semi-rings. Badawi and Darani [2] introduced weakly 2-absorbing ideals in commutative rings, a generalization of weakly prime ideals by Anderson and Smith [1]. Extending these concepts to lattices, Wasakidar and Gaikerad [24] introduced 2-absorbing ideals and weakly 2-absorbing ideals in lattices. Natnael [11, 12, 13] contributed to the field by introducing weakly $2-$absorbing ideals and weakly $2-$absorbing filters, along with $1-$absorbing prime filters, in an ADL. Zadeh [25] initially defined a fuzzy subset of a set X as a function mapping elements of X to real numbers in the interval $[0, 1]$. Goguen [8] extended this by replacing the valuation set $[0, 1]$ with a complete lattice L, aiming to provide a more comprehensive exploration of fuzzy set theory. Darani and Ghasemi [6], and Mandal [10] introduced the notions of fuzzy 2-absorbing ideals and 2-absorbing fuzzy ideals for commutative rings, respectively, which generalize the concept of fuzzy prime ideals in rings investigated by June [9] and Sharma [20]. Nimbhorkar and Patil [16, 17] extended these ideas to lattices, introducing the notions of fuzzy weakly 2-absorbing ideals. In our previous work [21, 22], we laid the foundation by presenting the concepts of fuzzy ideals and filters within an ADL. Natnael [14, 15] later expanded on this research by introducing the concept of fuzzy $2A-$ideals and filters in an ADL.

In this paper, we have introduced the concept of $\mathcal{L}-$fuzzy $1A-$prime ideals and filters within an ADL, with the aim of expanding upon the notion of $\mathcal{L}-$fuzzy prime ideals and filters as outlined in [18, 19]. Initially, we define $\mathcal{L}-$fuzzy $1A-$prime ideals, which are less restrictive than $\mathcal{L}-$fuzzy prime ideals. We also delve into the study of $\mathcal{L}-$fuzzy $1A-$prime filters, which exhibit a weaker nature compared to $\mathcal{L}-$fuzzy prime filters. Our primary focus is on exploring the interconnections between $\mathcal{L}-$fuzzy prime ideals and $\mathcal{L}-$fuzzy $1A-$prime ideals, as well as the relationships between $\mathcal{L}-$fuzzy $1A-$prime ideals and $\mathcal{L}-$fuzzy $2A-$ideals. Additionally, we investigate the associations between $\mathcal{L}-$fuzzy prime filters and $\mathcal{L}-$fuzzy $1A-$prime filters, as well as $\mathcal{L}-$fuzzy $1A-$prime filters and $\mathcal{L}-$fuzzy $2A-$filters. Counterexamples are provided to illustrate that the converse of these relationships does not hold. Furthermore, we demonstrate that the direct product of any two $\mathcal{L}-$fuzzy prime ideals ($\mathcal{L}-$fuzzy prime filters) yields an $\mathcal{L}-$fuzzy $1A-$prime ideal ($\mathcal{L}-$fuzzy $1A-$prime filter). However, it is crucial to note that the product of $\mathcal{L}-$fuzzy $1A-$prime ideals ($\mathcal{L}-$fuzzy $1A-$prime filters) may not necessarily result in an $\mathcal{L}-$fuzzy $1A-$prime ideal ($\mathcal{L}-$fuzzy $1A-$prime filter). Additionally, we establish that both the image and pre-image of any $\mathcal{L}-$fuzzy $1A-$prime ideals ($\mathcal{L}-$fuzzy $1A-$prime filters) are again $\mathcal{L}-$fuzzy $1A-$prime ideals ($\mathcal{L}-$fuzzy $1A-$prime filters).

In this document, R represents an ADL denoted as $(R, \wedge, \vee, 0)$, featuring a maximal element. Meanwhile, L is indicative of a complete lattice $(L, \wedge, \vee, 0, 1)$ adhering to the infinite meet distributive law, and such a lattice is specifically referred to as a frame.

2 PRELIMINARIES

In this portion, we revisit certain definitions and fundamental findings primarily sourced from[21, 18, 23].

Definition 2.1. An algebra $R = (R, \wedge, \vee, 0)$ of type $(2, 2, 0)$ is referred to as an ADL if it meets the subsequent conditions for all r, s and t in R.

(i) $0 \wedge r = 0$

(ii) $r \vee 0 = r$

(iii) $r \wedge (s \vee t) = (r \wedge s) \vee (r \wedge t)$

(iv) $r \vee (s \wedge t) = (r \vee s) \wedge (r \vee t)$

(v) $(r \vee s) \wedge t = (r \wedge t) \vee (s \wedge t)$

(vi) $(r \vee s) \wedge s = s$.

Every distributive lattice with a lower bound is categorized as an ADL.

Example 2.2. For any nonempty set A, it's possible to transform it into an ADL that doesn't constitute a lattice by selecting any element 0 from A and fixing an arbitrary element $u_0 \in R$. For every $u, v \in R$, define $\wedge$ and $\vee$ on R as follows:

$$u \wedge v = \begin{cases} v & \text{if } u \neq u_0 \\ u_0 & \text{if } u = u_0 \end{cases} \quad \text{and} \quad u \vee v = \begin{cases} u & \text{if } u \neq u_0 \\ v & \text{if } u = u_0 \end{cases}$$

Then $(A, \wedge, \vee, u_0)$ is an ADL (called the **discrete ADL**) with u_0 as its zero element.

Definition 2.3. Consider $R = (R, \wedge, \vee, 0)$ be an ADL. For any r and $s \in R$, establish $r \leq s$ if $r = r \wedge s$ (which is equivalent to $r \vee s = s$). Then $\leq$ is a partial order on R with respect to which 0 is the smallest element in R.

Theorem 2.4. *The following conditions are valid for any r, s and t in an ADL R.*

(1) $r \wedge 0 = 0 = 0 \wedge r$ *and* $r \vee 0 = r = 0 \vee r$

(2) $r \wedge r = r = r \vee r$

(3) $r \wedge s \leq s \leq s \vee r$

(4) $r \wedge s = r$ *iff* $r \vee s = s$

(5) $r \wedge s = s$ *iff* $r \vee s = r$

(6) $(r \wedge s) \wedge t = r \wedge (s \wedge t)$ *(in other words, $\wedge$ is associative)*

(7) $r \vee (s \vee r) = r \vee s$

(8) $r \leq s \Rightarrow r \wedge s = r = s \wedge r$ $\big($ *iff* $r \vee s = s = s \vee r\big)$

(9) $(r \wedge s) \wedge t = (s \wedge r) \wedge t$

(10) $(r \vee s) \wedge t = (s \vee r) \wedge t$

(11) $r \wedge s = s \wedge r$ *iff* $r \vee s = s \vee r$

(12) $r \wedge s = \inf\{r, s\}$ *iff* $r \wedge s = s \wedge r$ *iff* $r \vee s = \sup\{r, s\}$.

Definition 2.5. Let R and G be ADLs and form the set $R \times G = \{(r, g) : r \in R$ and $g \in G\}$. For all $(r_1, g_1), (r_2, g_2) \in R \times G$, define $\wedge$ and $\vee$ in $R \times G$ by $(r_1, g_1) \wedge (r_2, g_2) = (r_1 \wedge r_2, g_1 \wedge g_2)$ and $(r_1, g_1) \vee (r_2, g_2) = (r_1 \vee r_2, g_1 \vee g_2)$. Then $(R \times G, \wedge, \vee, 0)$ is an ADL under the pointwise operations and $0 = (0, 0)$ is the zero element in $R \times G$.

Definition 2.6. A non-empty subset, denoted as F in an ADL R is termed an ideal (filter) in R if it satisfies the conditions: if u and v belong to F, then $u \vee v$ $(u \wedge v)$ is also in F, and for every element r in R, the $u \wedge r$ $(r \vee u)$ is in F.

Definition 2.7. A proper ideal (filter) F in R is a prime ideal (filter) if for any u and v belongs R, $u \wedge v$ $(u \vee v)$ belongs F, then either u belongs F or v belongs F.

Definition 2.8. Let R and G be ADLs. A mapping $k : R \to G$ is called a homomorphism if the following are satisfied, for any $r, s, t \in R$.
(1). $k(r \wedge s \wedge t) = k(r) \wedge k(s) \wedge k(t)$
(2). $k(r \vee s \vee t) = k(r) \vee k(s) \vee k(t)$
(3). $k(0) = 0$.

Definition 2.9. An $\mathcal{L}-$fuzzy subset ϕ is defined as a mapping from R to a complete lattice L that adheres to the infinite meet distributive law. When the lattice L is represented by the unit interval $[0, 1]$ of real numbers, these $\mathcal{L}-$fuzzy subsets correspond to the conventional notion of $\mathcal{L}-$fuzzy subsets in R.

Definition 2.10. An $\mathcal{L}-$fuzzy subset Φ is an $\mathcal{L}-$fuzzy ideal (filter) in R, if $\Phi(0) = 1\big(\Phi(u) = 1$, for any maximal element u in $R\big)$ and $\Phi(r \vee s) = \Phi(r) \wedge \Phi(s)$ $\big(\Phi(r \wedge s) = \Phi(r) \wedge \Phi(s)\big)$, for all r and s belongs to R.

Theorem 2.11. *Let Φ be an $\mathcal{L}-$fuzzy ideal and $\emptyset \neq F \subseteq R$. Then for any r and s belongs to R, we have the following:*

(1) If $r \leq s$, then $\Phi(s) \leq \Phi(r)$

(2) If r is an associate with s, then $\Phi(r) = \Phi(s)$

(3) $\Phi(r \wedge s) = \Phi(s \wedge r)$ and $\Phi(r \vee s) = \Phi(s \vee r)$

(4) If $r \in \langle F]$, then $\bigwedge_{i=1}^{n} \Phi(x_i) \leq \Phi(r)$, for some $x_1, x_2, ..., x_n \in F$

(5) If $r \in \langle s]$, then $\Phi(s) \leq \Phi(r)$

(6) If u is maximal in R, then $\Phi(u) \leq \Phi(r)$

(7) $\Phi(u) = \Phi(v)$, for any maximal elements u and v in R.

Theorem 2.12. *Let Φ be an $\mathcal{L}-$fuzzy filter and $\emptyset \neq F \subseteq R$. Then for any $r, s \in R$, we have the following.*
(1) *If $r \leq s$, then $\Phi(r) \leq \Phi(s)$*
(2) *If r is an associate with s, then $\Phi(r) = \Phi(s)$*
(3) $\Phi(r \vee s) = \Phi(s \vee r)$
(4) *If $r \in [F\rangle$, then $\bigwedge_{i=1}^{n} \Phi(x_i) \leq \Phi(r)$, for some $x_1, x_2, ..., x_n \in F$*
(5) *If $r \in [s\rangle$, then $\Phi(s) \leq \Phi(r)$.*

Definition 2.13. A proper $\mathcal{L}-$fuzzy ideal (filter) ϕ is referred to as a prime $\mathcal{L}-$fuzzy ideal(filter) if $\psi \wedge \eta \leq \phi$ implies either $\psi \leq \phi$ or $\eta \leq \phi$, for any fuzzy ideals(filters) ψ and η in R.

Definition 2.14. A proper $\mathcal{L}-$fuzzy ideal (filter) Φ is considered an $\mathcal{L}-$fuzzy prime ideal(filter) in R if $\Phi(r \wedge s)\Big(\Phi(r \vee s)\Big)$ equals either $\Phi(r)$ or $\Phi(s)$, for any r and s in R.

3 $\mathcal{L}-$FUZZY $1A-$PRIME IDEALS

In the following, we introduce the notions of $\mathcal{L}-$fuzzy $1-$absorbing prime ideals in an ADL R and their characterizations. First let us recall from [11] that a proper ideal P in R is a $1-$absorbing prime ideal (is denoted by $1A-$prime ideal) in R if for all elements $r, s, t \in R$, the condition $r \wedge s \wedge t$ belonging to P implies either $r \wedge s$ belonging to P or t belonging to P. Here, we extend this result to the case of $\mathcal{L}-$fuzzy $1A-$prime ideals in the following.

Definition 3.1. A proper $\mathcal{L}-$fuzzy ideal Φ in R is referred to as an $\mathcal{L}-$fuzzy $1A-$prime ideal in R if for all elements r, s and t belongs to R, $\Phi(r \wedge s \wedge t)$ equals either $\Phi(r \wedge s)$ or $\Phi(t)$.

Example 3.2. Let $R = \{0, r, s, t\}$ and L be 4 elements chain $\{0, \gamma, \beta, 1\}$, where $0 < \gamma < \beta < 1$ and let $\vee$ and $\wedge$ be binary operations on A defined by:

$\vee$	0	r	s	t
0	0	r	s	t
r	r	r	r	r
s	s	s	s	s
t	t	r	s	t

$\wedge$	0	r	s	t
0	0	0	0	0
r	0	r	s	t
s	0	r	s	t
t	0	t	t	t

Define an $\mathcal{L}-$fuzzy subset Φ by $\Phi(0) = 1$, $\Phi(r) = \gamma = \Phi(s)$ and $\Phi(t) = \beta$. Clearly Φ is an $\mathcal{L}-$fuzzy ideal. Next we observe that, for any r, s and $t \in R$, $\Phi(r \wedge s \wedge t) = \Phi(s \wedge t)$ or $\Phi(t)$. Therefore, Φ is an $\mathcal{L}-$fuzzy $1A-$prime ideal.

Theorem 3.3. *Let Φ be an $\mathcal{L}-$fuzzy ideal in R. If Φ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in R, then $\Phi(r \wedge s \wedge t) \leq \Phi(r \wedge s) \vee \Phi(t)$, for all r, s and $t \in R$.*

Proof. Suppose that Φ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in R. Then, for all $r, s, t \in R$, $\Phi(r \wedge s \wedge t) = \Phi(r \wedge s)$ and clearly $\Phi(r \wedge s) \leq \Phi(r \wedge s) \vee \Phi(t)$. Hence the result. □

Next, we characterize the idea of $\mathcal{L}-$fuzzy $1A-$prime ideal in terms of β-cut.

Theorem 3.4. *Let Φ be an $\mathcal{L}-$fuzzy ideal. Then an ideal Φ_β is a $1A-$prime ideal in R, for all $\beta \in L$ iff Φ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in R.*

Proof. Suppose that Φ_β is a $1A-$prime ideal, for all $\beta \in L$. Then for all $r, s, t \in R$, either $r \wedge s \in \Phi_{\Phi(r \wedge s \wedge t)}$ or $t \in \Phi_{\Phi(r \wedge s \wedge t)}$. It follows that $\Phi(r \wedge s \wedge t) \leq \Phi(r \wedge s)$ or $\Phi(r \wedge s \wedge t) \leq \Phi(t)$. On the other inequality, by theorem 2.11(1) and (3), we have $\Phi(r \wedge s) \leq \Phi(r \wedge s \wedge t)$ and $\Phi(t) \leq \Phi(r \wedge s \wedge t)$ (since $r \wedge s \wedge t \leq t$). Thus, $\Phi(r \wedge s \wedge t) = \Phi(r \wedge s)$ or $\Phi(t)$. Hence the result. Conversely suppose Φ is an $\mathcal{L}-$fuzzy $1A-$prime ideal. Let $r, s, t \in R$ such that $r \wedge s \wedge t \in \Phi_\beta$, for all $\beta \in L$. Then $\beta \leq \Phi(r \wedge s \wedge t)$ implies $\beta \leq \Phi(r \wedge s) \vee \Phi(t)$ and hence either $\beta \leq \Phi(r \wedge s)$ or $\beta \leq \Phi(t)$. Thus either $r \wedge s \in \Phi_\beta$ or $t \in \Phi_\beta$. Therefore, Φ_β is a $1A-$prime ideal. □

Lemma 3.5. *An ideal P is a $1A-$prime ideal iff χ_P is an $\mathcal{L}-$fuzzy $1A-$prime ideal.*

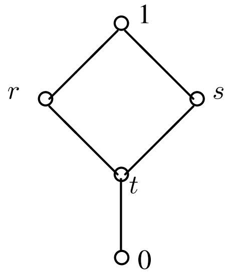
In the following theorems, we facilitate the inter-relationship between $\mathcal{L}-$fuzzy $1A-$prime ideals and both $\mathcal{L}-$fuzzy prime ideals and $\mathcal{L}-$fuzzy $2A-$ideals.

Theorem 3.6. *Let Φ be an $\mathcal{L}-$fuzzy ideal in R. Then Φ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in R only if Φ is an $\mathcal{L}-$fuzzy prime ideal in R.*

Proof. Suppose that Φ is an $\mathcal{L}-$fuzzy prime ideal. Then for $r, s, t \in R$, we have $\Phi(r \wedge s \wedge t) = \Phi(r)$ or $\Phi(r \wedge s \wedge t) = \Phi(s \wedge t)$, or $\Phi(r \wedge s \wedge t) = \Phi(r \wedge s)$ or $\Phi(r \wedge s \wedge t) = \Phi(t)$. Hence the result. □

In the following example, we show that all $\mathcal{L}-$fuzzy $1A-$prime ideals are not $\mathcal{L}-$fuzzy prime ideals.

Example 3.7. Let $D = \{0, u, v\}$ be a discrete ADL with 0 as its zero element defined in 2.2 and $L = \{0, r, s, t, 1\}$ be the lattice represented by the Hasse diagram given below:



Consider $D \times L = \{(d, e) \mid d \in D \text{ and } e \in L\}$. Then $(D \times L, \wedge, \vee, 0)$ is an ADL under the pointwise operations $\wedge$ and $\vee$ on $D \times L$ and $0 = (0, 0)$, the zero element in $D \times L$. Let $P = \{0, t\}$. Clearly P is an ideal in L. Now define $\Phi : D \times L \to [0, 1]$ by

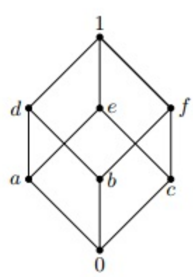
$$\Phi(d, e) = \begin{cases} 1 & \text{if } (d, e) = (0, 0) \\ 3/4 & \text{if } d \neq 0 \text{ and } e \in P \\ 0 & \text{otherwise} \end{cases}$$

for all $(d, e) \in D \times L$. Clearly Φ is an $\mathcal{L}-$fuzzy ideal. Thus Φ is an $\mathcal{L}-$fuzzy $1A-$prime ideal, while Φ is not an $\mathcal{L}-$fuzzy prime ideal, since $\Phi((u, r) \wedge (v, s)) = 3/4 \neq 0 = \Phi(u, r)$ or $\Phi(v, s)$.

Theorem 3.8. *Let Φ be an $\mathcal{L}-$fuzzy ideal in R. If Φ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in R, then Φ is an $\mathcal{L}-$fuzzy $2A-$ideal in R. The converse of this result is not true.*

Proof. Suppose Φ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in R. Then for all $r, s, t \in R$, $\Phi(r \wedge s \wedge t) \leq \Phi(r \wedge s) \vee \Phi(t)$. By theorem 2.11(1) and (3), we have $\Phi(t) \leq \Phi(t \wedge s) = \Phi(s \wedge t)$ and $\Phi(t) \leq \Phi(t \wedge r) = \Phi(r \wedge t)$. It follows that, $\Phi(t) \leq \Phi(s \wedge t) \vee \Phi(r \wedge t)$. So that, $\Phi(r \wedge s \wedge t) \leq \Phi(r \wedge s) \vee \Phi(s \wedge t) \vee \Phi(r \wedge t)$. Therefore, Φ is an $\mathcal{L}-$fuzzy $2A-$ideal in R. □

Example 3.9. Let $D = \{0, u, v\}$ be a discrete ADL with 0 as its zero element defined in 2.2 and $L = \{0, a, b, c, d, e, f, 1\}$ be a lattice whose Hasse diagram is given below:



Let $Q = \{0, b, c, f\}$. Clearly Q is an ideal in L. Define $\mathcal{L}-$fuzzy subset $\Phi : R \to [0, 1]$ by

$$\Phi(y, z) = \begin{cases} 1 & \text{if } y = 0 \text{ and } z \in Q \\ 1/3 & \text{otherwise} \end{cases}$$

for all $(y, z) \in D \times L$. Clearly Φ is an $\mathcal{L}-$fuzzy ideal. Thus Φ is an $\mathcal{L}-$fuzzy $2A-$ideal but not an $\mathcal{L}-$fuzzy $1A-$prime ideal in $D \times L$, since $\Phi((0, d) \wedge (u, e) \wedge (v, f)) = 1 \nleq 1/3 = \Phi((0, d) \wedge (u, e)) \vee \Phi(v, f)$.

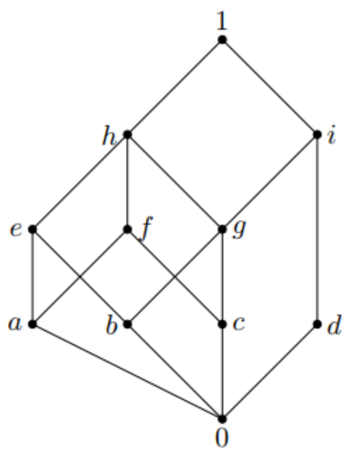
The product of $\mathcal{L}-$fuzzy subsets Φ and Ψ in R and G respectively is denoted by $\Phi \times \Psi$ and defined by $(\Phi \times \Psi)(a, b) = \Phi(a) \wedge \Psi(b)$, for all $(a, b) \in R \times G$.

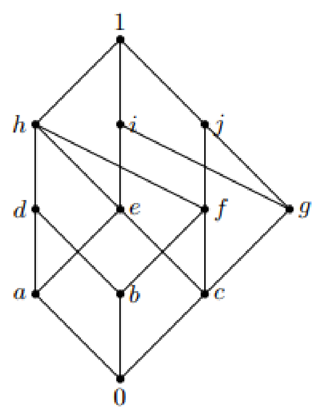
Theorem 3.10. *Let Φ and Ψ be $\mathcal{L}-$fuzzy ideals in R and G respectively. If $\Phi \times \Psi$ is an $\mathcal{L}-$fuzzy $1A-$prime ideal of $R \times G$, then Φ and Ψ are $\mathcal{L}-$fuzzy $1A-$prime ideals in R and G respectively.*

Proof. Suppose $\Phi \times \Psi$ is an $\mathcal{L}-$fuzzy $1A-$prime ideal of $R \times G$. Consider,
$\Phi(r \wedge s \wedge t) \wedge \Psi(x \wedge y \wedge z) = (\Phi \times \Psi)(r \wedge s \wedge t, x \wedge y \wedge z)$
$= (\Phi \times \Psi)\big((r, x) \wedge (s, y) \wedge (t, z)\big)$
$\leq (\Phi \times \Psi)\big((r, x) \wedge (s, y)\big) \vee (\Phi \times \Psi)(t, z)$
$= \big(\Phi(r \wedge s) \wedge \Psi(x \wedge y)\big) \vee \big(\Phi(t) \wedge \Psi(z)\big)$
$= \big(\Phi(r \wedge s) \vee \big(\Phi(t) \wedge \Psi(z)\big)\big) \wedge \big(\Psi(x \wedge y) \vee \big(\Phi(t) \wedge \Psi(z)\big)\big)$
$= \big(\Phi(r \wedge s) \vee \Phi(t)\big) \wedge \big(\Phi(r \wedge s) \vee \Psi(z)\big) \wedge \big(\Psi(x \wedge y) \vee \Phi(t)\big) \wedge \big(\Psi(x \wedge y) \vee \Psi(z)\big)$
$\leq \big(\Phi(r \wedge s) \vee \Phi(t)\big) \wedge \big(\Psi(x \wedge y) \vee \Psi(z)\big).$
Hence the result. □

The direct product of any two $\mathcal{L}-$fuzzy $1A-$prime ideals in R may not be an $\mathcal{L}-$fuzzy $1A-$prime ideal in R; consider the following example.

Example 3.11. Let $R = \{0, a, b, c, d, e, f, g, h, i, 1\}$ and $G = \{0, a, b, c, d, e, f, g, h, i, j, 1\}$ be the lattice represented by the Hasse diagram respectively given below. Define $\mathcal{L}-$fuzzy subset Φ and Ψ in R and G respectively by $\Phi(0) = \Phi(b) = \Phi(c) = \Phi(g) = 1, \Phi(a) = 0.5, \Phi(d) = \Phi(e) = \Phi(f) = \Phi(h) = \Phi(i) = \Phi(1) = 0$ and $\Psi(0) = \Psi(a) = \Psi(b) = 1, \Psi(c) = \Psi(e) = 0.75, \Psi(d) = \Psi(f) = \Psi(g) = \Psi(h) = \Psi(i) = \Psi(j) = \Psi(1) = 0$. Clearly both Φ and Ψ are $\mathcal{L}-$fuzzy $1A-$prime ideals in R and G respectively. But $\Phi \times \Psi$ is not $\mathcal{L}-$fuzzy $1A-$prime ideals of $R \times G$, since $(\Phi \times \Psi)(e \wedge f \wedge g, h \wedge i \wedge j) = (\Phi \times \Psi)(0, c) = \Phi(0) \wedge \Psi(c) = 0.75 \nleq 0.5 = (\Phi \times \Psi)(e \wedge f, h \wedge i) \vee (\Phi \times \Psi)(g, j)$.





Corollary 3.12. *Let Φ and Ψ be $\mathcal{L}-$fuzzy ideals in R and G respectively. Then Φ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in $R \times G$ iff $\Psi_\beta = \Phi_\beta \times G$ or $\Phi_\beta = R \times \Psi_\beta$, for all $\beta \in L$.*

Theorem 3.13. *Let R and G be ADLs, and $k : R \to G$ be a lattice homomorphism. If Ψ is an $\mathcal{L}-$fuzzy $1A-$prime ideal of G, then $k^{-1}(\Psi)$ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in R. Also, if k is an epimorphism and Φ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in R, then $k(\Phi)$ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in G.*

Proof. Suppose Ψ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in G and let k be a lattice homomorphism. Then, for all $r, s, t \in G$,
$k^{-1}(\Psi)(r \wedge s \wedge t) = \Psi\big(k(r \wedge s \wedge t)\big)$
$= \Psi\big(k(r) \wedge k(s) \wedge k(t)\big)$
$\leq \Psi\big(k(r) \wedge k(s)\big) \vee \Psi(k(t))$
$= \Psi\big(k(r \wedge s)\big) \vee \Psi(k(t))$
$= k^{-1}(\Psi)(r \wedge s) \vee k^{-1}(\Psi)(t).$
Thus $k^{-1}(\Psi)$ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in R. Also, let k be an isomorphism and suppose Φ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in R. Let $a, b, c \in R$. Now, consider,

$k(\Phi)(a \wedge b) \vee k(\Phi)(c) = \Big[\bigvee_{a\wedge b\in k^{-1}(x\wedge y)} \Phi(a \wedge b)\Big] \vee \Big[\bigvee_{c\in k^{-1}(z)} \Phi(c)\Big]$
$\geq \Big[\bigvee_{a\wedge b\wedge c\in k^{-1}(x\wedge y\wedge z)} \Phi(a \wedge b \wedge c)\Big]$
$= k(\Phi)(a \wedge b \wedge c).$
Therefore, $k(\Phi)$ is an $\mathcal{L}-$fuzzy $1A-$prime ideal in G. □

4 $\mathcal{L}-$FUZZY $1A-$PRIME FILTERS

In the subsequent discussion, we present the concepts of $\mathcal{L}-$fuzzy $1-$absorbing prime filters in R and their characterizations. To begin with, let's review the definition provided in [13], stating that a proper filter P is considered a $1-$absorbing prime filter (referred to as a $1A-$prime filter) if, for all elements $r, s, t \in R$, the condition $r \vee s \vee t$ belonging to P implies either $r \vee s$ belonging to P or t belonging to P. Now, we aim to extend this outcome to the realm of $\mathcal{L}-$fuzzy $1A-$prime filters as elaborated below.

Definition 4.1. A proper $\mathcal{L}-$fuzzy filter Φ in R is referred to as an $\mathcal{L}-$fuzzy $1A-$prime filter in R if for all elements r, s and t in R, $\Phi(r \vee s \vee t) \leq \Phi(r \vee s) \vee \Phi(t)$.

Example 4.2. Let $R = \{0, r, s, t\}$ be an ADL defined in example 3.2 and $L = [0, 1]$. Define an $\mathcal{L}-$fuzzy subset $\Phi : R \to L$ by $\Phi(0) = 0$, $\Phi(r) = 1$, $\Phi(s) = 3/4$ and $\Phi(t) = 1/2$. Clearly Φ is an $\mathcal{L}-$fuzzy filter. Then, for all $a, b, c \in R$, $\Phi(a \vee b \vee c) \leq \Phi(a \vee b) \vee \Phi(c)$. Therefore, Φ is an $\mathcal{L}-$fuzzy $1A-$prime filter in R.

In the following, we characterize the notion of $\mathcal{L}-$fuzzy $1A-$prime filter in terms of γ-cut.

Theorem 4.3. *Let Φ be an $\mathcal{L}-$fuzzy filter. Then a filter Φ_γ is a $1A-$prime filter in R, for all $\gamma \in L$ iff Φ is an $\mathcal{L}-$fuzzy $1A-$prime filter in R.*

Proof. Suppose Φ_γ is a $1A-$prime filter, for all $\gamma \in L$. Assume $\gamma = \Phi(r \vee s \vee t)$, for all $r, s, t \in R$. Then either $r \vee s \in \Phi_{\Phi(r\vee s\vee t)}$ or $t \in \Phi_{\Phi(r\vee s\vee t)}$. Which implies that $\Phi(r \vee s \vee t) \leq \Phi(r \vee s)$ or $\Phi(t)$. Thus, $\Phi(r \vee s \vee t) \leq \Phi(r \vee s) \vee \Phi(t)$. Hence the result. Conversely suppose Φ is an $\mathcal{L}-$fuzzy $1A-$prime filter. Let $r, s, t \in R$. If $r\vee s\vee t \in \Phi_\gamma$, then $\gamma \leq \Phi(r\vee s\vee t) \leq \Phi(r\vee s)\vee\Phi(t)$ implies that either $\gamma \leq \Phi(r \vee s)$ or $\gamma \leq \Phi(t)$. Thus either $r \vee s \in \Phi_\gamma$ or $t \in \Phi_\gamma$. Therefore, Φ_γ is a $1A-$prime filter. □

Corollary 4.4. *A filter F in R is a $1A-$prime filter iff χ_F is an $\mathcal{L}-$fuzzy $1A-$prime filter.*

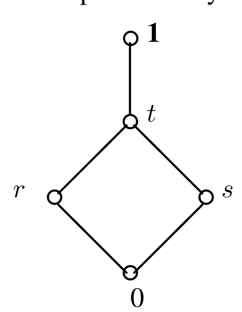
Next, we facilitate the inter-relationship between $\mathcal{L}-$fuzzy prime filters and $\mathcal{L}-$fuzzy $1A-$prime filters in an ADL.

Theorem 4.5. *Let Φ be an $\mathcal{L}-$fuzzy filter in R. If Φ is an $\mathcal{L}-$fuzzy prime filter in R, then Φ is an $\mathcal{L}-$fuzzy $1A-$prime filter in R.*

Proof. It is clear. □

We show that there are $\mathcal{L}-$fuzzy $1A-$prime filters which are not $\mathcal{L}-$fuzzy prime filters discussed in the following example.

Example 4.6. Let $D = \{0, u, v\}$ be a discrete ADL with 0 as its zero element defined in 2.2 and $L = \{0, r, s, t, 1\}$ be the lattice represented by the Hasse diagram given below:



Consider $D \times L = \{(d, e) \mid d \in D \text{ and } e \in L\}$. Then $(D \times L, \wedge, \vee, 0)$ is an ADL under the point-wise operations $\wedge$ and $\vee$ on $D \times L$ and $0 = (0, 0)$, the zero element in $D \times L$. Let $F = \{t, 1\}$. Clearly F is a filter in L. Now define $\Phi : D \times L \to [0, 1]$ by

$$\Phi(d, e) = \begin{cases} 0 & \text{if } (d, e) = (0, 0) \\ 1 & \text{if } d \neq 0 \text{ and } e \in F \\ 0.55 & \text{otherwise} \end{cases}$$

for all $(d, e) \in D \times L$. Clearly Φ is an $\mathcal{L}-$fuzzy filter in $D \times L$. Then $\Phi_1 = \{(u, t), (v, t), (u, 1), (v, 1)\}$. Thus Φ is an $\mathcal{L}-$fuzzy $1A-$prime filter in $D \times L$, while Φ is not an $\mathcal{L}-$fuzzy prime filter in $D \times L$, since Φ_1 is a $1A-$prime filter but not prime filter; for, $(u, r), (v, s) \in D \times L$, $(u, r) \vee (v, s) = (v, t)$ belongs to Φ_1 but (u, r) doesn't belongs in Φ_1 and (v, s) doesn't belongs in Φ_1.

Theorem 4.7. *Let Φ be an $\mathcal{L}-$fuzzy filter in R. Then Φ is an $\mathcal{L}-$fuzzy $2A-$filter in R only if Φ is an $\mathcal{L}-$fuzzy $1A-$prime filter in R. The converse of this result is not true.*

Proof. Suppose Φ is an $\mathcal{L}-$fuzzy $1A-$prime filter. Then $\Phi(r \vee s \vee t) \leq \Phi(r \vee s) \vee \Phi(t)$, for all $r, s, t \in R$. By Theorem 2.12(1) and (3), we have $\Phi(t) \leq \Phi(t \vee s) = \Phi(s \vee t)$ and $\Phi(t) = \Phi(t \vee r) = \Phi(r \vee t)$, since $t \leq t \vee s$ and $t \leq t \vee r$ and hence $\Phi(t) \leq \Phi(r \vee t) \vee \Phi(s \vee t)$. It follows that, $\Phi(r \vee s \vee t) \leq \Phi(r \vee s) \vee \Phi(r \vee t) \vee \Phi(s \vee t)$. Hence the result. □

Example 4.8. Let $D = \{0, u, v\}$ be a discrete ADL with 0 as its zero element defined in 2.2 and $L = \{0, a, b, c, d, e, f, 1\}$ be a lattice defined in 3.9. Define $\mathcal{L}-$fuzzy filter $\Phi : R \to [0, 1]$ by

$$\Phi(y, z) = \begin{cases} 0 & \text{if } (y, z) = (0, 0) \\ 3/4 & \text{if } y = u \text{ and } z = 1 \\ 1/2 & \text{otherwise} \end{cases}$$

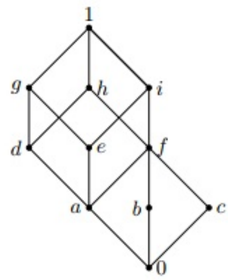
for all $(y, z) \in D \times L$. Clearly Φ is an $\mathcal{L}-$fuzzy filter. Put $H = \Phi_{3/4} = \{(u, 1)\}$. Clearly H is a filter in $D \times L$. Thus Φ is an $\mathcal{L}-$fuzzy $2A-$filter but not $\mathcal{L}-$fuzzy $1A-$prime filter, since for any $(0, a), (u, c), (v, b) \in D \times L$, $(0, a) \vee (u, c) \vee (v, b)$ belongs to H but $(0, a) \vee (u, c) = (u, e)$ doesn't belongs in H and (v, b) doesn't belongs in H.

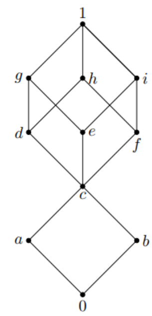
Theorem 4.9. *Let Φ and Ψ be $\mathcal{L}-$fuzzy filters in R and G respectively. If $\Phi \times \Psi$ is an $\mathcal{L}-$fuzzy $1A-$prime filter of $R \times G$, then Φ and Ψ be $\mathcal{L}-$fuzzy $1A-$prime filters in R and G respectively.*

Proof. Suppose $\Phi \times \Psi$ is an $\mathcal{L}-$fuzzy $1A-$prime filter. Let $r, s, t \in R$ and $x, y, z \in G$. Then,
$\Phi(r \vee s \vee t) \wedge \Psi(x \vee y \vee z) = (\Phi \times \Psi)(r \vee s \vee t, x \vee y \vee z)$
$= (\Phi \times \Psi)\big((r, x) \vee (s, y) \vee (t, z)\big)$
$\leq (\Phi \times \Psi)\big((r, x) \vee (s, y)\big) \vee (\Phi \times \Psi)(t, z)$
$= \Big(\Phi(r \vee s) \wedge \Psi(x \vee y)\Big) \vee \Big(\Phi(t) \wedge \Psi(z)\Big)$
$= \Big(\Phi(r \vee s) \vee \big(\Phi(t) \wedge \Psi(z)\big)\Big) \wedge \Big(\Psi(x \vee y) \vee \big(\Phi(t) \wedge \Psi(z)\big)\Big)$
$= \big(\Phi(r \vee s) \vee \Phi(t)\big) \wedge \big(\Phi(r \vee s) \vee \Psi(z)\big) \wedge \big(\Psi(x \vee y) \vee \Phi(t)\big) \wedge \big(\Psi(x \vee y) \vee \Psi(z)\big)$
$\leq \big(\Phi(r \vee s) \vee \Phi(t)\big) \wedge \big(\Psi(x \vee y) \vee \Psi(z)\big)$.
Hence the result. □

If there are $\mathcal{L}-$fuzzy $1A-$prime filters, then their direct product may not $\mathcal{L}-$fuzzy $1A-$prime filter; consider the following example.

Example 4.10. Let $R = \{0, a, b, c, d, e, f, g, h, i, 1\}$ and $G = \{0, a, b, c, d, e, f, g, h, i, 1\}$ be the lattice represented by the Hasse diagram respectively given below:
Define $\mathcal{L}-$fuzzy subset Φ and Ψ in R and G respectively by $\Phi(0) = \Phi(a) = 0$, $\Phi(b) = 0.3$, $\Phi(c) = 0$, $\Phi(d) = \Phi(e) = \Phi(g) = 0.6, \Phi(f) = 1, \Phi(h) = 0.6, \Phi(i) = 0.6, \Phi(1) = 1$ and $\Psi(0) = \Psi(a) = \Psi(b) = 0$, $\Psi(c) = \Psi(d) = \Psi(e) = \Psi(f) = 0.5, \Psi(i) = \Psi(g) = \Psi(h) = \Psi(1) = 1$. Clearly both Φ and Ψ are $\mathcal{L}-$fuzzy $1A-$prime filters in R and G respectively. But $\Phi \times \Psi$ is not $\mathcal{L}-$fuzzy $1A-$prime filter of $R \times G$, since
$(\Phi \times \Psi)(d \vee e \vee f, d \vee e \vee f) = (\Phi \times \Psi)(1, 1) = 1 \nleq 0.6 = (\Phi \times \Psi)(d \vee e, d \vee e) \vee (\Phi \times \Psi)(f, f)$.





Corollary 4.11. *Let Φ and Ψ be $\mathcal{L}-$fuzzy filters in R and G respectively. Then Φ is an $\mathcal{L}-$fuzzy $1A-$prime filter in $R \times G$ iff $\Phi_\beta = \Psi_\beta \times G$ or $\Phi_\beta = R \times \Psi_\beta$, for all $\beta \in L$.*

Finally, we discuss the homomorphism of $\mathcal{L}-$fuzzy $1A-$prime filter.

Theorem 4.12. *Let k be a lattice homomorphism from ADLs R to G. Then*
(1). $k^{-1}(\Psi)$ is an $\mathcal{L}-$fuzzy $1A-$prime filter in R only if Ψ is an $\mathcal{L}-$fuzzy $1A-$prime filter in G
(2). $k(\Phi)$ is an $\mathcal{L}-$fuzzy $1A-$prime filter in G only if k is an epimorphism and Φ is an $\mathcal{L}-$fuzzy $1A-$prime filter in R.

Proof. Let $k : R \to G$ be a lattice homomorphism.
(1). Suppose that Ψ is an $\mathcal{L}-$fuzzy $1A-$prime filter of G. For all $r, s, t \in G$. Then
$k^{-1}(\Psi)(r \vee s \vee t) = \Psi\big(k(r \vee s \vee t)\big)$
$= \Psi\big(k(r) \vee k(s) \vee k(t)\big)$
$\leq \Psi\big(k(r) \vee k(s)\big) \vee \Psi(k(t))$
$= \Psi\big(k(r \vee s)\big) \vee \Psi(k(t))$
$= k^{-1}(\Psi)(r \vee s) \vee k^{-1}(\Psi)(t)$.
Thus $k^{-1}(\Psi)$ is an $\mathcal{L}-$fuzzy $1A-$prime filter in R.
(2). Let k be an epimorphism and suppose that Φ be an $\mathcal{L}-$fuzzy $1A-$prime filter in R. For all $a, b, c \in R$. Now, consider,
$k(\Phi)(a \vee b) \vee k(\Phi)(c) = \Big[\bigvee_{a \vee b \in k^{-1}(x \wedge y)} \Phi(a \vee b)\Big] \vee \Big[\bigvee_{c \in k^{-1}(z)} \Phi(c)\Big]$
$\geq \Big[\bigvee_{a \vee b \vee c \in k^{-1}(x \wedge y \wedge z)} \Phi(a \vee b \vee c)\Big]$
$= k(\Phi)(a \vee b \vee c)$.
Thus, $g(\Phi)$ is an $\mathcal{L}-$fuzzy $1A-$prime filter of G. □

5 Conclusion

In this paper, we study on $\mathcal{L}-$fuzzy $1A-$prime ideals and filters in an Almost Distributive Lattice(ADL) which is the central part of our work and discussed properties of these. Furthermore, the relationship between $\mathcal{L}-$fuzzy weakly prime filters(ideals) and $\mathcal{L}-$fuzzy $1A-$prime filters(ideals) in ADLs are introduced and there are examples that shown that the converse of these is not true.

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