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FRACTIONAL CALCULUS AND FRACTIONAL KINETIC EQUATION INVOLVING MITTAG-LEFFLER-LAGUERRE POLYNOMIALS

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Abstract. This paper is devoted to the study of the Mittag–Leffler–Laguerre polynomials, which generalize the Mittag–Leffler function and the Laguerre polynomials. We investigate their fractional calculus properties by employing Saigo operators, deriving results for both fractional integrals and derivatives. As an application, we obtain the solution to a generalized fractional reaction–kinetic equation whose kernel involves the Mittag–Leffler–Laguerre polynomials. This application highlights the practical significance and potential impact of the proposed results in modeling complex real-world processes.

1 Introduction

The Mittag–Leffler function occupies a fundamental place in fractional calculus, as it provides a natural generalization of the exponential function and arises as the fundamental solution to a wide class of fractional differential and integral equations. Its flexibility makes it an essential tool for solving problems involving fractional derivatives and integrals, particularly in modeling the memory and hereditary properties of various materials and processes. Numerous initial and boundary value problems in physics, engineering, and applied sciences can be solved in closed form using the Mittag–Leffler function. In addition, its close connections with the Laplace and Mellin transforms, as well as its asymptotic properties, make it a powerful analytic device for studying fractional dynamical systems. For further details, we refer the reader to the works of Gorenflo and Mainardi [13], Kilbas and Saigo [15], Kilbas *et al.* [16], Usman *et al.* [32], Chaib *et al.* [7], and Bin-Saad *et al.* [4, 5, 6]. The one-parameter Mittag–Leffler function, $E_\alpha(z)$, was first introduced by the Swedish mathematician Gösta Magnus Mittag-Leffler [20] and subsequently studied by many authors.by Wiman [34]. It is a special function of $x \in \mathbb{C}$ that depends on the complex parameter α and is defined by the power series

$$E_\alpha(x) = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(\alpha n + 1)}, \qquad (\Re(\alpha) > 0;\, x \in \mathbb{C})\,, \tag{1.1}$$

where, as usual, $\Gamma(\lambda)$ denotes the Gamma function. Here and in the following, let $\mathbb{C}$, $\mathbb{R}$, and $\mathbb{N}$ denote the sets of complex numbers, real numbers, and positive integers, respectively, while $\mathbb{R}^+$ denotes the set of positive real numbers, and $\mathbb{R}_0^+ = \mathbb{R}^+ \cup \{0\}$.

The two-parameter Mittag–Leffler function is defined by the series

$$E_{\alpha,\beta}(x) = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(\alpha n + \beta)}, \qquad (\Re(\alpha), \Re(\beta) > 0;\, x \in \mathbb{C})\,, \tag{1.2}$$

and represents the first generalization of the function $E_\alpha(x)$ proposed by Wiman [34]. The polynomials

$$L_n^{(\kappa)}(x) = \frac{(\kappa+1)_n}{n!}\, {}_1F_1(-n;\kappa+1;x) = \sum_{s=0}^{n} \frac{(-1)^s\, \Gamma(\kappa+n+1)\, x^s}{s!\,(n-s)!\,\Gamma(\kappa+s+1)}, \qquad (\Re(\kappa) \geq 0), \tag{1.3}$$

are called the *associated Laguerre polynomials* (see [26]), where $(\gamma)_n$ denotes the Pochhammer symbol, defined in terms of the Gamma function Γ by (see, e.g., [31]):

$$(\gamma)_n = \frac{\Gamma(\gamma+n)}{\Gamma(\gamma)} = \begin{cases} 1, & (n=0), \\ \gamma(\gamma+1)\cdots(\gamma+n-1), & (n \in \mathbb{N} := \{1,2,\ldots\}). \end{cases}$$

A new symbolic approach to studying special functions through the derivation of specific operators, known as *symbolic operators*, was introduced by Babusci *et al.* [3]. In their work, Dattoli *et al.* [10] introduced a symbolic operator denoted by $\hat{d}_{(\alpha,\beta)}$, with $\alpha, \beta \in \mathbb{R}^+$. The following equations describe how this operator acts on the vacuum function φ_0 [18]:

$$\hat{d}^{k}_{(\alpha,\beta)}\varphi_0 = \frac{\Gamma(k+1)}{\Gamma(\alpha k+\beta)}, \tag{1.4}$$

and

$$\hat{d}^{k+\delta-1}_{(\alpha,\alpha(1-\delta)+\beta)}\varphi_0 = \frac{\Gamma(k+\delta)}{\Gamma(\alpha k+\beta)}, \tag{1.5}$$

where $k \in \mathbb{R}$. Notably, when $k=0$, Eq. (1.4) yields

$$\varphi_0 = \frac{1}{\Gamma(\beta)}. \tag{1.6}$$

For this work, we recall the definition of the generalized hypergeometric series as follows (see [31], p. 42):

$${}_pF_q\left[\begin{matrix} \alpha_1,\ldots,\alpha_p; \\ \beta_1,\ldots,\beta_q; \end{matrix}\; z\right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n\cdots(\alpha_p)_n}{(\beta_1)_n\cdots(\beta_q)_n}\frac{z^n}{n!} = {}_pF_q(\alpha_1,\ldots,\alpha_p;\beta_1,\ldots,\beta_q;z). \tag{1.7}$$

Next, we recall the following fractional integral operators due to Saigo (see Mathai *et al.* [19], p. 104):

$$(I^{\mu,\nu,\eta}_{0,x}f)(x) = \frac{x^{-\mu-\nu}}{\Gamma(\mu)}\int_0^x (x-t)^{\mu-1}\, {}_2F_1\left(\mu+\nu,-\eta;\mu;1-\frac{t}{x}\right) f(t)\,dt, \tag{1.8}$$

and

$$(I^{\mu,\nu,\eta}_{x,\infty}f)(x) = \frac{1}{\Gamma(\mu)}\int_x^\infty (t-x)^{\mu-1} t^{-\mu-\nu}\, {}_2F_1\left(\mu+\nu,-\eta;\mu;1-\frac{x}{t}\right) f(t)\,dt, \tag{1.9}$$

where $x>0$ and $\mu,\nu,\eta \in \mathbb{C}$ with $\Re(\mu)>0$.

The operator $I^{\mu,\nu,\eta}_{0,x}(\cdot)$ includes both the Riemann–Liouville and the Erdélyi–Kober fractional integral operators, as seen from the following relationships [19, 24]:

$$(\mathcal{R}^{\mu}_{0,x}f)(x) = (I^{\mu,-\mu,\eta}_{0,x}f)(x) = \frac{1}{\Gamma(\mu)}\int_0^x (x-t)^{\mu-1} f(t)\,dt, \tag{1.10}$$

and

$$(\mathcal{E}^{\mu,\eta}_{0,x}f)(x) = (I^{\mu,0,\eta}_{0,x}f)(x) = \frac{x^{-\mu-\eta}}{\Gamma(\mu)}\int_0^x (x-t)^{\mu-1} t^{\eta} f(t)\,dt. \tag{1.11}$$

Similarly, the operator $I^{\mu,\nu,\eta}_{x,\infty}(\cdot)$ defined by Eq. (1.9) unifies the Weyl-type and Erdélyi–Kober fractional integral operators. Indeed, we have

$$(\mathcal{W}^{\mu}_{x,\infty}f)(x) = (I^{\mu,-\mu,\eta}_{x,\infty}f)(x) = \frac{1}{\Gamma(\mu)}\int_x^\infty (t-x)^{\mu-1} f(t)\,dt, \tag{1.12}$$

and

$$(\mathcal{K}^{\mu,\eta}_{x,\infty} f)(x) = (I^{\mu,0,\eta}_{x,\infty} f)(x) = \frac{x^{\eta}}{\Gamma(\mu)} \int_x^{\infty} (t-x)^{\mu-1} t^{-\mu-\eta} f(t)\, dt. \tag{1.13}$$

In the sequel, we shall use the following image formulas, which are consequences of the operators given by Eqs. (1.8) and (1.9) (see Mathai *et al.* [19], p. 107):

$$(I^{\mu,\nu,\eta}_{0,x}\, t^{\lambda-1})(x) = \frac{\Gamma(\lambda)\Gamma(\lambda-\nu+\eta)}{\Gamma(\lambda-\nu)\Gamma(\lambda+\mu+\eta)}\, x^{\lambda-\nu-1}, \tag{1.14}$$

and

$$(I^{\mu,\nu,\eta}_{x,\infty}\, t^{\lambda-1})(x) = \frac{\Gamma(\nu-\lambda+1)\Gamma(\eta-\lambda+1)}{\Gamma(1-\lambda)\Gamma(\nu+\mu-\lambda+\eta+1)}\, x^{\lambda-\nu-1}. \tag{1.15}$$

If we take $\nu = -\mu$ in Eqs. (1.14) and (1.15), we obtain

$$(\mathcal{R}^{\mu}_{0,x}\, t^{\lambda-1})(x) = \frac{\Gamma(\lambda)}{\Gamma(\lambda+\mu)}\, x^{\lambda+\mu-1}, \tag{1.16}$$

and

$$(\mathcal{W}^{\mu}_{x,\infty}\, t^{\lambda-1})(x) = \frac{\Gamma(1-\mu-\lambda)}{\Gamma(1-\lambda)}\, x^{\lambda+\mu-1}. \tag{1.17}$$

Similarly, for $\nu = 0$ in Eqs. (1.14) and (1.15), we have

$$(\mathcal{E}^{\mu,\eta}_{0,x}\, t^{\lambda-1})(x) = \frac{\Gamma(\lambda+\eta)}{\Gamma(\lambda+\mu+\eta)}\, x^{\lambda-1}, \tag{1.18}$$

and

$$(\mathcal{K}^{\mu,\eta}_{x,\infty}\, t^{\lambda-1})(x) = \frac{\Gamma(\eta-\lambda+1)}{\Gamma(\mu-\lambda+\eta+1)}\, x^{\lambda-1}. \tag{1.19}$$

In the present work, we introduce and investigate several properties of the *Mittag–Leffler–Laguerre polynomials* (MLLPs), denoted by ${}_{E}L_n^{(\kappa,\delta)}(x;\alpha,\beta)$, which are defined symbolically based on the Laguerre polynomials in Eq. (1.3) and the symbolic operator in Eq. (1.5) as follows:

$${}_{E}L_n^{(\kappa,\delta)}(x;\alpha,\beta) = \frac{1}{\Gamma(\delta)}\, L_n^{(\kappa)}\Big(x\hat{d}_{(\alpha,\alpha(1-\delta)+\beta)}\Big)\, \hat{d}^{\,\delta-1}_{(\alpha,\alpha(1-\delta)+\beta)}\varphi_0. \tag{1.20}$$

Using Eq. (1.3), we obtain from Eq. (1.20) the symbolic series representation:

$${}_{E}L_n^{(\kappa,\delta)}(x;\alpha,\beta) = \sum_{s=0}^{n} \frac{(-1)^s\,\Gamma(\kappa+n+1)\,x^s}{s!\,(n-s)!\,\Gamma(\kappa+s+1)\,\Gamma(\delta)}\, \hat{d}^{\,s+\delta-1}_{(\alpha,\alpha(1-\delta)+\beta)}\varphi_0. \tag{1.21}$$

According to the symbolic operator (1.5), we can express ${}_{E}L_n^{(\kappa,\delta)}(x;\alpha,\beta)$ as the following series.

Definition 1.1. Let $\kappa+s+1 \notin \{0,-1,-2,\ldots\}$ for $s = 0,\ldots,n$, and $\delta \notin \{0,-1,-2,\ldots\}$. Under these conditions, the Mittag–Leffler–Laguerre polynomials are defined by the series

$${}_{E}L_n^{(\kappa,\delta)}(x;\alpha,\beta) = \sum_{s=0}^{n} \frac{\Gamma(\kappa+n+1)\,(-1)^s\,(\delta)_s\,x^s}{s!\,(n-s)!\,\Gamma(\kappa+s+1)\,\Gamma(\alpha s+\beta)}. \tag{1.22}$$

In comparison with other hybrid polynomial families involving the Mittag–Leffler function and various classical polynomials, such as Hermite, Sheffer-type, Gegenbauer, Konhauser, and Jacobi polynomials (see, e.g., [17, 22, 29, 21]), the Mittag–Leffler–Laguerre polynomials ${}_{E}L_n^{(\kappa,\delta)}(x;\alpha,\beta)$ stand out by combining the operational simplicity of Laguerre polynomials with the fractional versatility of Mittag–Leffler functions. This dual structure provides enhanced flexibility for analytic continuation, modeling memory-dependent phenomena, and solving fractional differential equations with a wider range of parameter control.

From Eqs. (1.22) and (1.7), we can easily infer, for $\alpha \in \mathbb{N}$, the following explicit representation:

$$ {}_EL_n^{(\kappa,\delta)}(x;\alpha,\beta) = \frac{(1+\kappa)_n}{n!\,\Gamma(\beta)}\, {}_2F_{1+\alpha}\left[\begin{array}{cc} -n,\, \delta; & \dfrac{x}{\alpha^\alpha} \\ 1+\kappa,\, \Delta(\alpha;\beta); & \end{array}\right], \tag{1.23} $$

where $\Delta(\alpha;\beta)$ denotes the array

$$ \frac{\beta}{\alpha}, \frac{\beta+1}{\alpha}, \ldots, \frac{\beta+\alpha-1}{\alpha}, \qquad \alpha \geq 1. $$

In [8, 9], Dattoli and Torry introduced the Laguerre polynomials of two variables in the form

$$ L_n(x,y) = n! \sum_{s=0}^{n} \frac{(-1)^s\, y^{n-s}\, x^s}{(s!)^2 (n-s)!}, \tag{1.24} $$

together with the use of operational techniques combined with the principle of monomials [11], providing a new analytical approach for deriving solutions to large classes of partial differential equations often encountered in physical problems.

Moreover, two interesting unifications and generalizations of the Laguerre polynomials $L_n(x,y)$ were considered by Dattoli *et al.* [9] in the forms:

$$ {}_1L_{n,\rho}(x,y) = n! \sum_{s=0}^{n} \frac{y^{n-s}\, x^{s-\rho}}{s!\,(n-s)!\,\Gamma(\rho+s+1)}, \tag{1.25} $$

and

$$ L_n^{(m)}(x,y) = (m+n)! \sum_{s=0}^{n} \frac{(-1)^s\, y^{n-s}\, x^s}{s!\,(n-s)!\,(m+s)!}. \tag{1.26} $$

For the present study, we also recall the explicit expression for the Konhauser polynomials $Z_n^\alpha(x;k)$ [14, 25]:

$$ Z_n^\alpha(x;k) = \frac{\Gamma(kn+\alpha+1)}{n!} \sum_{s=0}^{n} (-1)^s \binom{n}{s} \frac{x^{ks}}{\Gamma(ks+\alpha+1)}, \tag{1.27} $$

where $\alpha > -1$ and $k = 1, 2, \ldots$.

For $k = 1$, these polynomials reduce to the Laguerre polynomials $L_n^{(\alpha)}(x)$, and their special case for $k = 2$ was encountered earlier by Spencer and Fano [30] in a calculation involving the penetration of gamma rays through matter.

It is worth noting that the series representation (1.22), in particular, yields the following relationships:

$$ {}_EL_n^{(\kappa,1)}(x;1,1) = L_n^{(\kappa)}(x), \tag{1.28} $$

$$ {}_EL_n^{(\delta-1,\delta)}(x^\alpha;\alpha,\beta+1) = \frac{(\delta)_n}{\Gamma(\alpha n+\beta+1)}\, Z_n^\beta(x;\alpha), \tag{1.29} $$

$$ {}_EL_n^{(\delta-1,\delta)}\left(\frac{x}{y};1,m+1\right) = \frac{(\delta)_n}{y^n (m+n)!}\, L_n^{(m)}(x,y), \qquad m \in \mathbb{N}, \tag{1.30} $$

$$ y^n\, {}_EL_n^{(0,1)}\left(\frac{x}{y};1,1\right) = L_n(x,y), \tag{1.31} $$

$$ {}_EL_n^{(\delta-1,\delta)}\left(-\frac{x}{y};1,\beta+1\right) = \frac{(\delta)_n\, x^\beta}{y^n\, n!}\, {}_1L_{n,\beta}(x,y). \tag{1.32} $$

2 Fractional calculus via Saigo operators

In this section, we will establish some fractional integrals and derivatives formulas for the polynomials ${}_EL_n^{(\kappa,\delta)}(t;\alpha,\beta)$.

Theorem 2.1. *Let* $\alpha\in\mathbb{N},\mu,\nu,\eta,\kappa,\beta,\delta\in\mathbb{C},x>0$ *and n a non − negative integer. Then*

$$\left(I_{0,x}^{\mu,\nu,\eta}\left[t^{\lambda-1}{}_EL_n^{(\kappa,\delta)}(t;\alpha,\beta)\right]\right)(x)=x^{\lambda-\nu-1}\frac{(1+\kappa)_n\,\Gamma(\lambda)\,\Gamma(\lambda-\nu+\eta)}{\Gamma(\beta)\,n!\,\Gamma(\lambda-\nu)\,\Gamma(\lambda+\mu+\eta)}\times{}_4F_{3+\alpha}\left[\begin{matrix}-n,\delta,\lambda,\lambda-\nu+\eta;\\ \\ 1+\kappa,\lambda-\nu,\lambda+\mu+\eta,\Delta(\alpha;\beta);\end{matrix}\;\frac{x}{\alpha^\alpha}\right].\tag{2.1}$$

Proof. Denoting the L.H.S. of equation (2.1) by Ω, using definition (1.22) and then interchanging the order of integration and summation, we obtain

$$\Omega=\sum_{s=0}^{\infty}\frac{(1+\kappa)_n\,(-n)_s\,(\delta)_s}{\Gamma(\beta)\,n!\,(\beta)_{\alpha s}\,(1+\kappa)_s\,s!}\left(I_{0,x}^{\mu,\nu,\eta}\,t^{\lambda+s-1}\right)(x).\tag{2.2}$$

Employing the relation (1.14), we arrive at

$$\begin{aligned}\Omega&=x^{\lambda-\nu-1}\sum_{s=0}^{\infty}\frac{(1+\kappa)_n\,(-n)_s\,(\delta)_s\,\Gamma(\lambda+s)\,\Gamma(\lambda-\nu+\eta+s)\,x^s}{\Gamma(\beta)\,n!\,(\beta)_{\alpha s}\,(1+\kappa)_s\,s!\,\Gamma(\lambda-\nu+s)\,\Gamma(\lambda+\mu+\eta+s)}\\&=x^{\lambda-\nu-1}\frac{(1+\kappa)_n\,\Gamma(\lambda)\,\Gamma(\lambda-\nu+\eta)}{\Gamma(\beta)\,n!\,\Gamma(\lambda-\nu)\,\Gamma(\lambda+\mu+\eta)}\sum_{s=0}^{\infty}\frac{(-n)_s\,(\delta)_s\,(\lambda)_s\,(\lambda-\nu+\eta)_s\,x^s}{(\beta)_{\alpha s}\,(1+\kappa)_s\,(\lambda-\nu)_s\,(\lambda+\mu+\eta)_s\,s!}.\end{aligned}\tag{2.3}$$

Finally, by the definition (1.7), we lead to the formula (2.1). □

Corollary 2.2. *As a consequence of (1.16) and Theorem 2.1 with* $\nu=-\mu$, *we have*

$$\left(\mathcal{R}_{0,x}^{\mu}\left[t^{\lambda-1}{}_EL_n^{(\kappa,\delta)}(t;\alpha,\beta)\right]\right)(x)=x^{\lambda+\mu-1}\frac{(1+\kappa)_n\,\Gamma(\lambda)}{\Gamma(\beta)\,n!\,\Gamma(\lambda+\mu)}\times{}_3F_{2+\alpha}\left[\begin{matrix}-n,\delta,\lambda;\\ \\ 1+\kappa,\lambda+\mu,\Delta(\alpha;\beta);\end{matrix}\;\frac{x}{\alpha^\alpha}\right].\tag{2.4}$$

If we putting $\lambda=1+\kappa$, Eq (2.4) yields

$$\left(\mathcal{R}_{0,x}^{\mu}\left[t^{\kappa}{}_EL_n^{(\kappa,\delta)}(t;\alpha,\beta)\right]\right)(x)=x^{\kappa+\mu}\frac{\Gamma(1+\kappa+n)}{\Gamma(\beta)\,n!\,\Gamma(1+\kappa+\mu)}\times{}_2F_{1+\alpha}\left[\begin{matrix}-n,\delta;\\ \\ 1+\kappa+\mu,\Delta(\alpha;\beta);\end{matrix}\;\frac{x}{\alpha^\alpha}\right].\tag{2.5}$$

Note that, given the result (1.23), the assertion (2.5) reduces to the elegant formula

$$\left(\mathcal{R}_{0,x}^{\mu}\left[t^{\kappa}{}_EL_n^{(\kappa,\delta)}(t;\alpha,\beta)\right]\right)(x)=x^{\kappa+\mu}\frac{\Gamma(\kappa+n+1)}{\Gamma(\kappa+\mu+n+1)}{}_EL_n^{(\kappa+\mu,\delta)}(x;\alpha,\beta).\tag{2.6}$$

Corollary 2.3. *As a consequence of (1.18) and Theorem 2.1 with $\nu = 0$, we have*

$$\left(\mathcal{E}_{0,x}^{\mu,\eta}\left[t^{\lambda-1}{}_{E}L_{n}^{(\kappa,\delta)}(t;\alpha,\beta)\right]\right)(x) = x^{\lambda-1}\frac{(1+\kappa)_{n}\,\Gamma(\lambda+\eta)}{\Gamma(\beta)\,n!\,\Gamma(\lambda+\mu+\eta)} \times {}_{3}F_{2+\alpha}\left[\begin{matrix} -n,\delta,\lambda+\eta; \\ 1+\kappa,\lambda+\mu+\eta,\Delta(\alpha;\beta); \end{matrix} \; \frac{x}{\alpha^{\alpha}}\right]. \tag{2.7}$$

Theorem 2.4. *Let $\alpha \in \mathbb{N}$, $\mu,\nu,\eta,\lambda,\kappa,\beta,\delta \in \mathbb{C}$, $x > 0$, and n be a non-negative integer. Then*

$$\left(I_{x,\infty}^{\mu,\nu,\eta}\left[t^{\lambda-1}{}_{E}L_{n}^{(\kappa,\delta)}\left(\frac{1}{t};\alpha,\beta\right)\right]\right)(x) = x^{\lambda-\nu-1}\frac{(1+\kappa)_{n}\,\Gamma(1-\lambda+\nu)\,\Gamma(1-\lambda+\eta)}{\Gamma(\beta)\,n!\,\Gamma(1-\lambda)\,\Gamma(1-\lambda+\mu+\nu+\eta)} \times {}_{4}F_{3+\alpha}\left[\begin{matrix} -n,\delta,1-\lambda+\nu,1-\lambda+\eta; \\ 1+\kappa,1-\lambda,1-\lambda+\mu+\nu+\eta+1,\Delta(\alpha;\beta); \end{matrix} \; \frac{1}{\alpha^{\alpha}x}\right]. \tag{2.8}$$

Proof. By considering the operator given in equation (1.15) and following the same procedure as in the proof of theorem 2.1, we can easily establish theorem 2.4. □

Corollary 2.5. *As a consequence of equation (1.17) and theorem 2.4, with $\nu = -\mu$, we obtain*

$$\left(\mathcal{W}_{x,\infty}^{\mu}\left[t^{\lambda-1}{}_{E}L_{n}^{(\kappa,\delta)}\left(\frac{1}{t};\alpha,\beta\right)\right]\right)(x) = x^{\lambda+\mu-1}\frac{(1+\kappa)_{n}\,\Gamma(1-\lambda-\mu)}{\Gamma(\beta)\,n!\,\Gamma(1-\lambda)} \times {}_{3}F_{2+\alpha}\left[\begin{matrix} -n,\delta,1-\lambda-\mu; \\ 1+\kappa,1-\lambda,\Delta(\alpha;\beta); \end{matrix} \; \frac{1}{\alpha^{\alpha}x}\right]. \tag{2.9}$$

Corollary 2.6. *As a consequence of (1.19) and theorem 2.4 with $\nu = 0$, we have*

$$\left(\mathcal{K}_{x,\infty}^{\mu,\eta}\left[t^{\lambda-1}{}_{E}L_{n}^{(\kappa,\delta)}\left(\frac{1}{t};\alpha,\beta\right)\right]\right)(x) = x^{\lambda-1}\frac{(1+\kappa)_{n}\,\Gamma(1-\lambda+\eta)}{\Gamma(\beta)\,n!\,\Gamma(1-\lambda+\mu+\eta)} \times {}_{3}F_{2+\alpha}\left[\begin{matrix} -n,\delta,1-\lambda+\eta; \\ 1+\kappa,1-\lambda+\mu+\eta,\Delta(\alpha;\beta); \end{matrix} \; \frac{1}{\alpha^{\alpha}x}\right]. \tag{2.10}$$

Another way to express the Saigo fractional integral operators (1.8) and (1.9) is as follows:

$$\left(\hat{D}_{0,x}^{\mu,\nu,\eta}f(t)\right)(x) = \left(I_{0,x}^{-\mu,-\nu,\mu+\eta}f(t)\right)(x) = \left(\frac{d}{dx}\right)^{m}\left(I_{0,x}^{-\mu+m,-\nu-m,\mu+\eta-m}f(t)\right)(x),\ Re(\mu) > 0; m = [Re(\mu)]+1, \tag{2.11}$$

$$\left(\hat{D}_{x,\infty}^{\mu,\nu,\eta}f(t)\right)(x) = \left(I_{x,\infty}^{-\mu,-\nu,\mu+\eta}f(t)\right)(x) = \left(-\frac{d}{dx}\right)^{m}\left(I_{x,\infty}^{-\mu+m,-\nu-m,\mu+\eta}f(t)\right)(x),\ Re(\mu) > 0; m = [Re(\mu)]+1. \tag{2.12}$$

In the sequel, we shall use the following image formulas, which are straightforward consequences of the operators given by equations (2.11) and (2.12):

$$\left(\hat{D}_{0,x}^{\mu,\nu,\eta}t^{\lambda-1}\right)(x) = \frac{\Gamma(\lambda)\,\Gamma(\lambda+\nu+\mu+\eta)}{\Gamma(\lambda+\nu)\,\Gamma(\lambda+\eta)}x^{\lambda+\nu-1}, \tag{2.13}$$

and

$$\left(\hat{D}_{x,\infty}^{\mu,\nu,\eta} t^{\lambda-1}\right)(x) = \frac{\Gamma(1-\lambda-\nu)\,\Gamma(\mu+\eta-\lambda+1)}{\Gamma(1-\lambda)\,\Gamma(\eta-\nu-\lambda+1)} x^{\lambda+\nu-1}. \tag{2.14}$$

The operator $\hat{D}_{0,x}^{\mu,\nu,\eta}(.)$ defined by equation (2.11) contains the Riemann-Liouville fractional derivative operator, using the following relationship:

$$\begin{aligned}\left(\hat{D}_{0,x}^{\mu} f\right)(x) &= \left(D_{0,x}^{\mu,-\mu,\eta} f(t)\right)(x) = \left(I_{0,x}^{-\mu,\mu,\mu+\eta} f(t)\right)(x) = \left(\mathcal{R}_{0,x}^{-\mu} f(t)\right)(x) \\ &= \left(\frac{d}{dx}\right)^{m} \left(\mathcal{R}_{0,x}^{m-\mu} f\right)(x), \quad (Re(\mu) > 0, m = [Re(\mu)] + 1), \end{aligned} \tag{2.15}$$

whereas the operator $\hat{D}_{x,\infty}^{\mu,\nu,\eta}(.)$ defined by equation (2.12) contains the Weyl type fractional derivative operator. Indeed, we have

$$\begin{aligned}\left(\hat{D}_{x,\infty}^{\mu} f\right)(x) &= \left(\hat{D}_{x,\infty}^{\mu,-\mu,\eta} f(t)\right)(x) = \left(I_{x,\infty}^{-\mu,\mu,\mu+\eta} f(t)\right)(x) = \left(\mathcal{W}_{x,\infty}^{-\mu} f(t)\right)(x) \\ &= \left(-\frac{d}{dx}\right)^{m} \left(\mathcal{W}_{x,\infty}^{m-\mu} f\right)(x), \quad (Re(\mu) > 0, m = [Re(\mu)] + 1). \end{aligned} \tag{2.16}$$

If we take $\nu = -\mu$ in equations (2.13) and (2.14), we have

$$\left(\hat{D}_{0,x}^{\mu} t^{\lambda-1}\right)(x) = \frac{\Gamma(\lambda)}{\Gamma(\lambda-\mu)} x^{\lambda-\mu-1}, \tag{2.17}$$

and

$$\left(\hat{D}_{x,\infty}^{\mu} t^{\lambda-1}\right)(x) = \frac{\Gamma(1+\mu-\lambda)}{\Gamma(1-\lambda)} x^{\lambda-\mu-1}. \tag{2.18}$$

Theorem 2.7. *Let* $\alpha \in \mathbb{N}, \mu, \nu, \kappa, \eta, \beta, \delta \in \mathbb{C}, x > 0$ *and* n *a non − negative integer. Then*

$$\begin{aligned}\left(\hat{D}_{0,x}^{\mu,\nu,\eta}\left[t^{\lambda-1}{}_{E}L_{n}^{(\kappa,\delta)}(t;\alpha,\beta)\right]\right)(x) &= x^{\lambda+\nu-1}\frac{(1+\kappa)_n\,\Gamma(\lambda)\,\Gamma(\lambda+\nu+\mu+\eta)}{\Gamma(\beta)\,n!\,\Gamma(\lambda+\nu)\,\Gamma(\lambda+\eta)} \\ &\times {}_4F_{3+\alpha}\left[\begin{matrix} -n, \delta, \lambda, \lambda+\nu+\mu+\eta; \\ \\ 1+\kappa, \lambda+\nu, \lambda+\eta, \Delta(\alpha;\beta); \end{matrix} \quad \frac{x}{\alpha^{\alpha}}\right]. \end{aligned} \tag{2.19}$$

Proof. Denoting the L.H.S. of equation (2.19) by Ω, using definition (1.22) and then differentiation of the order of integration and summation, we obtain

$$\Omega = \sum_{s=0}^{\infty} \frac{(1+\kappa)_n\,(-n)_s\,(\delta)_s}{\Gamma(\beta)\,n!\,(\beta)_{\alpha s}\,(1+\kappa)_s\,s!} \left(\hat{D}_{0,x}^{\mu,\nu,\eta}\, t^{\lambda+s-1}\right)(x). \tag{2.20}$$

Employing the relation (2.13), we arrive at

$$\begin{aligned}\Omega &= x^{\lambda+\nu-1} \sum_{s=0}^{\infty} \frac{(1+\kappa)_n\,(-n)_s\,(\delta)_s\,\Gamma(\lambda+s)\,\Gamma(\lambda+\nu+\eta+\mu+s)\,x^s}{\Gamma(\beta)\,n!\,(\beta)_{\alpha s}\,(1+\kappa)_s\,s!\,\Gamma(\lambda+\nu+s)\,\Gamma(\lambda+\eta+s)}, \\ &= x^{\lambda+\nu-1}\frac{(1+\kappa)_n\,\Gamma(\lambda)\,\Gamma(\lambda+\nu+\eta+\mu)}{\Gamma(\beta)\,n!\,\Gamma(\lambda+\nu)\,\Gamma(\lambda+\eta)} \sum_{s=0}^{\infty} \frac{(-n)_s\,(\delta)_s\,(\lambda)_s\,(\lambda+\nu+\eta+\mu)_s\,x^s}{(\beta)_{\alpha s}\,(1+\kappa)_s\,(\lambda+\nu)_s\,(\lambda+\eta)_s\,s!}. \end{aligned} \tag{2.21}$$

Finally, by the definition (1.7), we lead to the formula (2.19). □

Corollary 2.8. *As a consequence of (2.17) and theorem 2.7 with $\nu = -\mu$, we have*

$$\left(\hat{D}^{\mu}_{0,x}\left[t^{\lambda-1}{}_{E}L^{(\kappa,\delta)}_{n}(t;\alpha,\beta)\right]\right)(x) = x^{\lambda-\mu-1}\frac{(1+\kappa)_n\,\Gamma(\lambda)}{\Gamma(\beta)\,n!\,\Gamma(\lambda-\mu)} \times {}_3F_{2+\alpha}\left[\begin{matrix}-n,\delta,\lambda; \\ 1+\kappa,\lambda-\mu,\Delta(\alpha;\beta);\end{matrix}\ \frac{x}{\alpha^{\alpha}}\right]. \tag{2.22}$$

If we putting $\lambda = 1+\kappa$, Eq (2.22) yields

$$\left(\hat{D}^{\mu}_{0,x}\left[t^{\kappa}{}_{E}L^{(\kappa,\delta)}_{n}(t;\alpha,\beta)\right]\right)(x) = x^{\kappa-\mu}\frac{\Gamma(1+\kappa+n)}{\Gamma(\beta)\,n!\,\Gamma(1+\kappa-\mu)} \times {}_2F_{1+\alpha}\left[\begin{matrix}-n,\delta; \\ 1+\kappa-\mu,\Delta(\alpha;\beta);\end{matrix}\ \frac{x}{\alpha^{\alpha}}\right]. \tag{2.23}$$

Note that, given the result (1.23), the assertion (2.23) reduces to the elegant formula

$$\left(\hat{D}^{\mu}_{0,x}\left[t^{\kappa}{}_{E}L^{(\kappa,\delta)}_{n}(t;\alpha,\beta)\right]\right)(x) = x^{\kappa-\mu}\frac{\Gamma(\kappa+n+1)}{\Gamma(\kappa-\mu+n+1)}\,{}_{E}L^{(\kappa-\mu,\delta)}_{n}(x;\alpha,\beta). \tag{2.24}$$

Theorem 2.9. *Let* $\alpha \in \mathbb{N}, \mu,\nu,\eta,\lambda,\kappa,\beta,\delta \in \mathbb{C}, x > 0$ *and* n *a non* $-$ *negative integer. Then*

$$\left(\hat{D}^{\mu,\nu,\eta}_{x,\infty}\left[t^{\lambda-1}{}_{E}L^{(\kappa,\delta)}_{n}\left(\frac{1}{t};\alpha,\beta\right)\right]\right)(x) = x^{\lambda+\nu-1}\frac{(1+\kappa)_n\,\Gamma(1-\lambda-\nu)\,\Gamma(1-\lambda+\mu+\eta)}{\Gamma(\beta)\,n!\,\Gamma(1-\lambda)\,\Gamma(1-\lambda-\nu+\eta)} \times {}_4F_{3+\alpha}\left[\begin{matrix}-n,\delta,1-\lambda-\nu,1-\lambda+\mu+\eta; \\ 1+\kappa,1-\lambda,1-\lambda-\nu+\eta,\Delta(\alpha;\beta);\end{matrix}\ \frac{1}{\alpha^{\alpha}x}\right]. \tag{2.25}$$

Proof. By considering the operator (2.16) and proceeding in a same way as in the proof of Theorem 2.3, we can easily prove theorem 2.9. □

Corollary 2.10. *As a consequence of (2.17) and theorem 2.9 with $\nu = -\mu$, we have*

$$\left(\hat{D}^{\mu}_{x,\infty}\left[t^{\lambda-1}{}_{E}L^{(\kappa,\delta)}_{n}\left(\frac{1}{t};\alpha,\beta\right)\right]\right)(x) = x^{\lambda-\mu-1}\frac{(1+\kappa)_n\,\Gamma(1-\lambda+\mu)}{\Gamma(\beta)\,n!\,\Gamma(1-\lambda)} \times {}_3F_{2+\alpha}\left[\begin{matrix}-n,\delta,1-\lambda+\mu; \\ 1+\kappa,1-\lambda,\Delta(\alpha;\beta);\end{matrix}\ \frac{1}{\alpha^{\alpha}x}\right]. \tag{2.26}$$

Remark 2.11. The results presented in this section are of a general nature. By considering the particular cases (1.28)–(1.32), we can deduce several fractional integral and derivative formulas for the polynomials defined in (1.3) and (1.24)–(1.27).

For instance, in view of relation (1.28), theorems 2.1, 2.4, 2.7 and 2.9 yield the following fractional calculus formulas for the associated Laguerre polynomials $L^{(\kappa)}_n(x)$ defined in (1.3):

Corollary 2.12. *Let* $,\mu,\nu,\eta,\kappa\in\mathbb{C}, x>0$ *and* n *a non* $-$ *negative integer. Then*

$$\left(I_{0,x}^{\mu,\nu,\eta}\left[t^{\lambda-1}L_n^{(\kappa)}(t)\right]\right)(x)=x^{\lambda-\nu-1}\frac{(1+\kappa)_n\,\Gamma(\lambda)\,\Gamma(\lambda-\nu+\eta)}{n!\,\Gamma(\lambda-\nu)\,\Gamma(\lambda+\mu+\eta)}\times{}_3F_3\left[\begin{array}{c}-n,\lambda,\lambda-\nu+\eta;\\ \\ 1+\kappa,\lambda-\nu,\lambda+\mu+\eta;\end{array}x\right], \tag{2.27}$$

Corollary 2.13. *Let* $,\mu,\nu,\eta,\lambda,\kappa\in\mathbb{C},\ x>0$ *and* n *a non* $-$ *negative integer. Then*

$$\left(I_{x,\infty}^{\mu,\nu,\eta}\left[t^{\lambda-1}L_n^{(\kappa)}\left(\frac{1}{t}\right)\right]\right)(x)=x^{\lambda-\nu-1}\frac{(1+\kappa)_n\,\Gamma(1-\lambda+\nu)\,\Gamma(1-\lambda+\eta)}{n!\,\Gamma(1-\lambda)\,\Gamma(1-\lambda+\mu+\nu+\eta)}\times{}_3F_3\left[\begin{array}{c}-n,1-\lambda+\nu,1-\\ \\ 1+\kappa,1-\lambda,1-\lambda+\end{array}\right. \tag{2.28}$$

Corollary 2.14. *Let* $\mu,\nu,\kappa,\eta\in\mathbb{C}, x>0$ *and* n *a non* $-$ *negative integer. Then*

$$\left(\hat{D}_{0,x}^{\mu,\nu,\eta}\left[t^{\lambda-1}L_n^{(\kappa)}(t)\right]\right)(x)=x^{\lambda+\nu-1}\frac{(1+\kappa)_n\,\Gamma(\lambda)\,\Gamma(\lambda+\nu+\mu+\eta)}{n!\,\Gamma(\lambda+\nu)\,\Gamma(\lambda+\eta)}\times{}_3F_3\left[\begin{array}{c}-n,\lambda,\lambda+\nu+\mu+\eta;\\ \\ 1+\kappa,\lambda+\nu,\lambda+\eta;\end{array}x\right], \tag{2.29}$$

Corollary 2.15. *Let* $\mu,\nu,\eta,\lambda,\kappa\in\mathbb{C}, x>0$ *and* n *a non* $-$ *negative integer. Then*

$$\left(\hat{D}_{x,\infty}^{\mu,\nu,\eta}\left[t^{\lambda-1}L_n^{(\kappa)}\left(\frac{1}{t}\right)\right]\right)(x)=x^{\lambda+\nu-1}\frac{(1+\kappa)_n\,\Gamma(1-\lambda-\nu)\,\Gamma(1-\lambda+\mu+\eta)}{n!\,\Gamma(1-\lambda)\,\Gamma(1-\lambda-\nu+\eta)}\times{}_3F_3\left[\begin{array}{c}-n,1-\lambda-\nu,1-\lambda+\mu+\eta;\\ \\ 1+\kappa,1-\lambda,1-\lambda-\nu+\eta;\end{array}\frac{1}{x}\right]. \tag{2.30}$$

3 Fractional kinetic equation

Very recently, several authors have addressed the solution of generalized fractional reaction equations. For instance, in [1, 2, 23, 33], Ahmed, Pawar, Patil, and collaborators developed transform-based methods for solving fractional kinetic equations, expressing solutions in terms of special functions and orthogonal polynomials. By employing Sumudu, Laplace, and Mellin transforms, they obtained closed-form or series solutions involving Laguerre polynomials, generalized q-Bessel and k-Bessel functions, and V-functions. These works expand the toolkit of fractional kinetics by connecting special-function theory with fractional dynamics and providing new avenues for both analytic and applied studies. In this section, we highlight the relevance of the Mittag–Leffler–Laguerre polynomials ${}_EL_n^{(\kappa,\delta)}(x;\alpha,\beta)$ by formulating a fractional kinetic equation in which these polynomials appear in the kernel. Saxena and Kalla's fractional kinetic equations are defined as follows (see [27]; also [28]):

$$\mathbf{N}(\tau)-\mathbf{N}_0f(\tau)=-\varepsilon^{\nu}{}_0\hat{D}_{\tau}^{-\nu}\mathbf{N}(\tau),\quad (Re(\nu)>0), \tag{3.1}$$

where $\mathbf{N}(\tau)$ is the number density of a given species at time τ and ε is a constant. When $\tau=0$, then $\mathbf{N}_0=\mathbf{N}(0)$. Consider $f\in L(0,\infty)$ and Riemann-Liouville integral operator ${}_0\hat{D}_{\tau}^{-\nu}$ (see [27, 28] as follows

$${}_0\hat{D}_{\tau}^{-\nu}f(\tau)=\frac{1}{\Gamma(\nu)}\int_0^{\tau}(\tau-s)^{\nu-1}f(s)ds,\quad (Re(\nu)>0). \tag{3.2}$$

Theorem 3.1. *If $\mu > 0$ and $\nu > 0$ then solution of equation*

$$\mathbf{N}(\tau) - \mathbf{N}_0\, \tau^{\mu-1}{}_E L_n^{(\kappa,\delta)}\left(\tau^{\nu};\nu,\mu\right) = -\omega^{\nu}{}_0\hat{D}_{\tau}^{-\nu}\mathbf{N}(\tau), \tag{3.3}$$

is given by

$$\mathbf{N}(\tau) = \mathbf{N}_0 \sum_{s=0}^{n} \frac{(\kappa+1)_n (-1)^s (\delta)_s}{s!\,(n-s)!\,(\kappa+1)_s}\, \tau^{\mu+\nu s-1} E_{\nu,\mu+\nu s}\left(-(\omega\tau)^{\nu}\right), \tag{3.4}$$

where $E_{\nu,\mu}(x)$ is the Mittag-Leffler function (1.2).

Proof. The Laplace transform of the Riemann-Liouville fractional integral operator is given by [12]:

$$L\left[{}_0\hat{D}_{\tau}^{-\nu} f(\tau) : p\right] = p^{-\nu} F(p), \tag{3.5}$$

where

$$F(p) = \int_0^{\infty} e^{-p\tau} f(\tau)\, d\tau, \quad (Re(p) > 0). \tag{3.6}$$

Applying the Laplace transform to both sides of (3.3) gives

$$\begin{aligned} \mathbf{N}(p) &= \mathbf{N}_0 \sum_{s=0}^{n} \frac{(\kappa+1)_n (-1)^s (\delta)_s}{s!\,(n-s)!\,(\kappa+1)_s} p^{-\nu s-\mu} - \omega^{\nu} p^{-\nu}\mathbf{N}(p) \\ &= \mathbf{N}_0 \sum_{s=0}^{n} \frac{(\kappa+1)_n (-1)^s (\delta)_s}{s!\,(n-s)!\,(\kappa+1)_s\, p^{\nu s+\mu}} \left(1+\omega^{\nu} p^{-\nu}\right)^{-1}. \end{aligned} \tag{3.7}$$

Taking the Laplace inverse of (3.7) and using

$$L^{-1}\left[p^{-\nu} : \tau\right] = \frac{\tau^{\nu-1}}{\Gamma(\nu)}, \tag{3.8}$$

we find that

$$L^{-1}\{\mathbf{N}(p)\} = \mathbf{N}_0 \sum_{s=0}^{n} \frac{(\kappa+1)_n (-1)^s (\delta)_s}{s!\,(n-s)!\,(\kappa+1)_s} \sum_{r=0}^{\infty} (-1)^r \omega^{\nu r} L^{-1}\left\{p^{-(\nu r+\nu s+\mu)}\right\}, \tag{3.9}$$

which can be rewritten as

$$\begin{aligned} \mathbf{N}(\tau) &= \mathbf{N}_0 \sum_{s=0}^{n} \frac{(\kappa+1)_n (-1)^s (\delta)_s}{s!\,(n-s)!\,(\kappa+1)_s} \sum_{r=0}^{\infty} \frac{(-1)^r \omega^{\nu r} \tau^{\nu s+\nu r+\mu-1}}{\Gamma(\nu r+\nu s+\mu)} \\ &= \mathbf{N}_0 \sum_{s=0}^{n} \frac{(\kappa+1)_n (-1)^s (\delta)_s}{s!\,(n-s)!\,(\kappa+1)_s} \tau^{\nu s+\mu-1} \sum_{r=0}^{\infty} \frac{(-1)^r \omega^{\nu r} \tau^{\nu r}}{\Gamma(\nu r+\nu s+\mu)}, \end{aligned} \tag{3.10}$$

using (1.2), we get the desired result (3.4). □

4 Conclusion

In this work, we introduced and studied a new class of special functions that generalize the Mittag-Leffler function and Laguerre polynomials. By employing Saigo operators, we established several fractional calculus properties, including explicit formulas for fractional integrals and derivatives of the Mittag-Leffler-Laguerre polynomials. Furthermore, we applied these results to solve a generalized fractional reaction-kinetic equation with a kernel involving the proposed polynomials, thereby demonstrating both the theoretical depth and the practical relevance of the new family.

The novelty of our approach lies in combining the operational structure of Laguerre polynomials with the fractional adaptability of the Mittag-Leffler function, resulting in a richer analytic framework with broader applicability. These findings not only extend the existing theory of hybrid fractional polynomials but also provide new tools for modeling memory-dependent and anomalous dynamical systems. Future research may focus on developing orthogonality relations and generating functions for these polynomials, exploring their asymptotic behavior, and applying them to fractional boundary value problems, control theory, and numerical methods in applied sciences.

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