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ON COEFFICIENT PROBLEM OF BI-UNIVALENT FUNCTIONS RELATED TO BERNOULLI POLYNOMIALS

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Abstract Many researchers recently investigated several interesting subclasses of the class of bi-univalent functions and found non-sharp estimates on the first two Taylor-Maclaurin coefficients using some special polynomials like Faber, Lucas, Chebyshev, Horadam, Bernoulli, Gegenbauer, Pell-Lucas, Fibonacci, and their generalizations. The main objective of the paper is to introduce a new subclass of bi-univalent functions associated with the Bernoulli polynomials and q-derivative operator. We investigate the estimates for the Taylor-Maclaurin coefficients $|a_2|$, $|a_3|$ and bounds for Fekete-Szegö inequality. The results in this paper would generalize and improve the related works of several earlier authors.

1 Introduction

Let A denote the family of functions f of the form

$$f(\varsigma) = \varsigma + \sum_{n=2}^{\infty} a_n \varsigma^n, \tag{1.1}$$

which are analytic in the open unit disc $\mathrm{E} = \{\varsigma : \varsigma \in \mathbb{C} \text{ and } |\varsigma| < 1\}$ and gratify the normalization conditions $f(0) = 0$ and $f'(0) = 1$. Any function f is said to be univalent in a domain E if it never takes the same value twice in U. Let us denote by S, the subclass of A consisting of univalent functions in E. However using Koebe-one quarter theorem [10] it is obvious that the image of E under every function $f \in \mathrm{S}$ contains a disc of radius $\frac{1}{4}$. It is well known that every function $f \in \mathrm{S}$ has an inverse f^{-1} defined by $f^{-1}(f(\varsigma)) = \varsigma, \quad (\varsigma \in \mathrm{E})$ and $f(f^{-1}(w)) = w \left(|w| < r_0(f) : r_0(f) \geq \frac{1}{4}\right)$.
A function $f \in \mathrm{A}$ is said to be bi-univalent in E, if f and f^{-1} are univalent in E. Let Σ denote the class of bi-univalent functions defined in the unit disc E. If $f \in \Sigma$ is given by [1.1], then $g = f^{-1}$ has the Taylor-Maclaurin expansion

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots. \tag{1.2}$$

The bi-univalent function class Σ was studied by Lewin [19], who showed that $|a_2| < 1.51$. Brannan and Clunie [8] then proposed that $|a_2| < \sqrt{2}$. On the other hand, Netanyahu [22] showed that $\max |a_2| = 4/3$. The problem of estimating coefficients $|a_n|, \quad n \geq 2$ is still open.
The Fekete-Szegö problem was proposed in 1933 [11]. Researchers were concern about several classes of univalent functions due to this [4, 5,7,9, 13, 16, 22, 23, 24, 29, 30].

The Fekete-Szegö problem for the coefficients of $f \in \mathrm{S}$ is

$$\left|a_3 - \epsilon a_2^2\right| \leq 1 + 2\exp\left(\frac{-2\epsilon}{1-\epsilon}\right) \quad \text{for } 0 \leq \epsilon < 1.$$

Many scholars have recently started investigating bi-univalent function related to orthogonal polynomials. In 1980, Gradshteyn and Ryzhik [12] give an expression of the Bernoulli polynomials which have important applications in number theory and classical analysis. They appear in the integral representation of differentiable periodic functions since they are employed for approximating such functions in terms of polynomials. They are also used for representing the remainder term of the composite Euler-Maclaurin quadrature rule.
The Bernoulli polynomials $\mathbb{B}_{\mathrm{n}}(x)$ are usually defined [20] using the generating function:

$$F(x,t)=\frac{te^{xt}}{e^t-1}=\sum_{n=0}^{\infty}\frac{\mathbb{B}_{\mathrm{n}}(\mathrm{x})}{n!}t^n,\quad |t|<2\pi, \tag{1.3}$$

where $\mathbb{B}_{\mathrm{n}}(\mathrm{x})$ are polynomials in x, for each non negative integer n.
The Bernoulli polynomials can be easily computed by recursion since

$$\sum_{j=0}^{n-1}(n,j)\,\mathbb{B}_{\mathrm{j}}(\mathrm{x})=nx^{n-1},\quad n=2,3,\cdots. \tag{1.4}$$

The initial few polynomials of Bernoulli are

$$\mathbb{B}_0(\mathrm{x})=1,\quad \mathbb{B}_1(\mathrm{x})=x-\frac{1}{2},\quad \mathbb{B}_2(\mathrm{x})=x^2-x+\frac{1}{6},\quad \mathbb{B}_3(\mathrm{x})=x^3-\frac{3}{2}x^2+\frac{1}{2}x,\cdots. \tag{1.5}$$

Quantum calculus can be considered as an approach to examined the calculus without using limits. Jackson [14, 15] discovered the many crucial step in q-calculus and established the useful formulas for q-integral and q-derivative operators. q-calculus is experiencing a rapid growth due to its applicable in mathematics, mechanics and physics.
The quantum calculus is an essential tool for studying diverse families of analytic function and its applications in mathematics and related fields have inspired researches, Srivastava [26] was the first person apply it in the contact of univalent functions. Numerous scholars conducted substantial work on q-calculus and examined its various applications due to the applicability of q-analysis in mathematics and other domains.
For recent investigation about q-calculus, we may refer the readers to [1, 2, 3, 11, 6, 17, 18, 25, 26, 28]. We first provide several definitions and notations of q-calculus. In 1909, Jackson [14] introduced the operator

$$\mathfrak{D}_q f(\varsigma)=\begin{cases}\frac{f(\varsigma)-f(q\varsigma)}{\varsigma(1-q)}, & \varsigma\neq 0\\ f'(0), & \varsigma=0\end{cases} \tag{1.6}$$

which is said to be q-derivative (or difference) operator of a function f. By taking q-derivative of the function f in the form (1.1) , we can see that

$$\mathfrak{D}_q f(\varsigma)=1+\sum_{n=2}^{\infty}[n]_q a_n\varsigma^{n-1},\quad \varsigma\neq 0,$$

where

$$[n]_q=\frac{1-q^n}{1-q}.$$

Note that as $q\longrightarrow 1^-$, $[n]_q\longrightarrow n$.
Using q-derivative operator and subordination now, we define new subclasses of bi-univalent functions, associated with Bernoulli polynomials.

Definition 1.1. Let f and g be two analytic functions in E, we say that f is subordinate to g, written as $f(\varsigma)\prec g(\varsigma)\quad(\varsigma\in \mathrm{E})$, if there exists a Schwarz function w, which is analytic in E with $w(0)=0$ and $|w(\varsigma)|<1,\ \ for\ \ all\ \ \varsigma\in\mathrm{E}$.
If g is a univalent function in E, then

$$f(\varsigma)\prec g(\varsigma)\quad(\varsigma\in\mathrm{E})\Longleftrightarrow f(0)=g(0)\ \ and\ \ f(\mathrm{E})\subset g(\mathrm{E}).$$

Definition 1.2. A function $f \in$ A is said to be in the class $M_{\Sigma,q}(\tau,\vartheta;F)$, for $\tau \in \mathbb{C}\backslash 0$ and $0 \leq \vartheta \leq 1$, if the following subordinations hold:

$$1+\frac{1}{\tau}\left(\mathfrak{D}_q f(\varsigma)+\vartheta\varsigma\mathfrak{D}_q(\mathfrak{D}_q f(\varsigma))-1\right) \prec F(x,\varsigma), \tag{1.7}$$

and

$$1+\frac{1}{\tau}(\mathfrak{D}_q g(\omega)+\vartheta\omega\mathfrak{D}_q(\mathfrak{D}_q g(\omega))-1) \prec F(x,\omega), \tag{1.8}$$

where $\varsigma,\omega \in$ E, $F(x,\varsigma)$ is given by (1.3), and $g=f^{-1}$ is given by (1.2).

Lemma 1.3. *(21) Let* $w(\varsigma)=\sum_{n=1}^{\infty} w_n\varsigma^n,\ \ |w(\varsigma)|<1,\ \ \varsigma \in$ E, *be an analytic function in* E. *Then*

$$|w_1| \leq 1,\ \ |w_n| \leq 1-|w_1|^2,\ \ n=2,3,\cdots.$$

2 Coefficient Estimates

In this section we determine the initial coefficients $|a_2|$ and $|a_3|$ for the functions in the subclass $M_{\Sigma,q}(\tau,\vartheta;F)$.

Theorem 2.1. *Let the function* $f \in M_{\Sigma,q}(\tau,\vartheta;F)$ *be of the form ,(1.1) then*

$$|a_2| \leq \frac{|\tau||\mathbb{B}_1(x)|\sqrt{|\mathbb{B}_1(x)|}}{\sqrt{|[3]_q\tau(1+[2]_q\vartheta)(\mathbb{B}_1(x))^2-[2]_q(1+\vartheta)^2\mathbb{B}_2(x)|}}, \tag{2.1}$$

and

$$|a_3| \leq \frac{|\tau|\mathbb{B}_1(x)}{[3]_q|1+[2]_q\vartheta|}+\frac{|\tau|^2[\mathbb{B}_1(x)]^2}{2[2]_q|1+\vartheta|^2}. \tag{2.2}$$

Proof. Let $f \in M_{\Sigma,q}(\tau,\vartheta;F)$ and $g \in f^{-1}$. From definition in (1.7) and (1.8), it follows that

$$1+\frac{1}{\tau}\left(\mathfrak{D}_q f(\varsigma)+\vartheta\mathfrak{D}_q(\varsigma\mathfrak{D}_q f(\varsigma))-1\right) \prec F(x,Y(\varsigma)), \tag{2.3}$$

and

$$1+\frac{1}{\tau}(\mathfrak{D}_q g(\omega)+\vartheta\omega\mathfrak{D}_q(\mathfrak{D}_q g(\omega))-1) \prec F(x,X(\omega)), \tag{2.4}$$

where the function Y and X are given by

$$Y(\varsigma)=r_1\varsigma+r_2\varsigma^2+\cdots, \tag{2.5}$$

$$X(\omega)=s_1\omega+s_2\omega^2+\cdots, \tag{2.6}$$

are analytic in E with $X(0)=Y(0)=0$, and $|Y(\varsigma)|<1$, $|X(\omega)|<1$, for all $\varsigma,\omega \in$ E.
As a result of Lemma 1.2,

$$|r_j| \leq 1 \ \ and \ \ |s_j| \leq 1,\ \ j \in \mathbb{N}. \tag{2.7}$$

Replacing (2.5) and (2.6) in (2.3) and (2.4) respectively, we see that

$$1+\frac{1}{\tau}\left(\mathfrak{D}_q f(\varsigma)+\vartheta\varsigma\mathfrak{D}_q(\mathfrak{D}_q f(\varsigma))-1\right)+\cdots=\mathbb{B}_0(x)+\mathbb{B}_1(x)Y(\varsigma)+\frac{\mathbb{B}_2(x)}{2!}Y^2(\varsigma)+\cdots. \tag{2.8}$$

and

$$1+\frac{1}{\tau}(\mathfrak{D}_q g(\omega)+\vartheta\omega\mathfrak{D}_q(\mathfrak{D}_q g(\omega))-1)+\cdots=\mathbb{B}_0(x)+\mathbb{B}_1(x)\chi(\omega)+\frac{\mathbb{B}_2(x)}{2!}\chi^2(\omega)+\cdots. \tag{2.9}$$

In view (1.1) and (1.2), from (2.8) and (2.9), we get

$$1+\frac{1}{\tau}\left([2]_q a_2(1+\vartheta)\varsigma+[3]_q a_3(1+[2]_q\vartheta)\varsigma^2\right)+\cdots=1+\mathbb{B}_1(x)r_1\varsigma$$
$$+\left[\mathbb{B}_1(x)r_2+\frac{\mathbb{B}_2(x)}{2!}r_1^2\right]\varsigma^2+\cdots$$

and

$$1+\frac{1}{\tau}\left(-[2]_q a_2(1+\vartheta)w+[3]_q(2a_2^2-a_3)(1+[2]_q)w^2\right)+\cdots=1+\mathbb{B}_1(x)s_1\omega + \left[\mathbb{B}_1(x)s_2+\frac{\mathbb{B}_2(x)}{2!}s_1^2\right]\omega^2+\cdots,$$

which yields the relations:

$$[2]_q a_2(1+\vartheta)=\tau\mathbb{B}_1(x)r_1, \tag{2.10}$$

$$[3]_q a_3(1+[2]_q\vartheta)=\tau\left[\mathbb{B}_1(x)r_2+\frac{\mathbb{B}_2(x)}{2!}r_1^2\right], \tag{2.11}$$

and

$$-[2]_q a_2(1+\vartheta)=\tau\mathbb{B}_1(x)s_1, \tag{2.12}$$

$$[3]_q(2a_2^2-a_3)(1+[2]_q v)=\tau\left[\mathbb{B}_1(x)s_2+\frac{\mathbb{B}_2(x)}{2!}s_1^2\right]. \tag{2.13}$$

From (2.10) and (2.12), we have

$$r_1=-s_1 \tag{2.14}$$

and

$$2[2]_q^2 a_2^2(1+\vartheta)^2=\tau^2[\mathbb{B}_1(x)]^2(r_1^2+s_1^2) \tag{2.15}$$

$$a_2^2=\frac{\tau^2[\mathbb{B}_1(x)]^2(r_1^2+s_1^2)}{2[2]_q^2(1+\vartheta)^2}. \tag{2.16}$$

Adding (2.11) and (2.13), applying (2.15), yields

$$a_2^2=\frac{\tau^2[\mathbb{B}_1(x)]^3(r_2+s_2)}{2[3]_q\tau(1+[2]_q\vartheta)[\mathbb{B}_1(x)]^2-2[2]_q(1+\vartheta)^2\mathbb{B}_2(x)}. \tag{2.17}$$

Using relation (1.6), from (2.7) for r_2 and s_2, we get (2.1).
Using (2.14) and (2.15), by subtracting (2.13) from (2.11), we get

$$a_3=\frac{\tau[\mathbb{B}_1(x)](r_2-s_2)+\frac{\mathbb{B}_2(x)}{2!}(r_1^2-s_1^2)}{2[3]_q([2]_q\vartheta+1)}+a_2^2$$

$$=\frac{\tau[\mathbb{B}_1(x)](r_2-s_2)+\frac{\mathbb{B}_2(x)}{2!}(r_1^2-s_1^2)}{2[3]_q([2]_q\vartheta+1)}+\frac{\tau^2[\mathbb{B}_1(x)]^2(r_1^2+s_1^2)}{2[2]_q^2(1+\vartheta)^2}. \tag{2.18}$$

Once again applying (2.14) and using (1.6), for the coefficients, r_1, s_1, r_2, s_2, we deduce (2.2). □

We obtain the Fekete-Szegö inequality for the class $M_{\Sigma,q}(\tau,\vartheta;F)$ due to the result of Zaprawa [29].

Theorem 2.2. *If f given by (1.1) is in the class $M_{\Sigma,q}(\tau,\vartheta;F)$ where $\epsilon\in\mathbb{R}$, then we have*

$$|a_3-\epsilon a_2^2|\leq\begin{cases}\frac{|\tau|\mathbb{B}_1(x)}{[3]_q|1+[2]_q\vartheta|}, & |h(\epsilon)|\leq\frac{1}{6(1+2\vartheta)},\\ 2|\tau|\mathbb{B}_1(x)|h(\epsilon)|, & |h(\epsilon)|\geq\frac{1}{6(1+2\vartheta)},\end{cases}$$

where

$$h(\epsilon)=\frac{(1-\epsilon)\tau[\mathbb{B}_1(x)]^2}{2[3]_q\tau(1+2\vartheta)[\mathbb{B}_1(x)]^2-[4]_q(1+\vartheta)^2\mathbb{B}_2(x)}.$$

Proof. If $f \in M_\Sigma(q,\tau,\vartheta;F)$ is given by (1.1), from (2.16) and (2.17), we have

$$
\begin{aligned}
a_3-\epsilon a_2^2 &= \frac{\tau\mathbb{B}_1(x)(r_2-s_2)}{2[3]_q\tau(1+[2]_q\vartheta)}+(1-\epsilon)a_2^2\\
&= \frac{\tau\mathbb{B}_1(x)(r_2-s_2)}{2[3]_q\tau(1+[2]_q\vartheta)}+\frac{(1-\epsilon)\tau^2[\mathbb{B}_1(x)]^3(r_2+s_2)}{2[3]_q\tau(1+[2]_q\vartheta)([\mathbb{B}_1(x)]^2-2[2]_q(1+\vartheta)^2\mathbb{B}_2(x))}\\
&= \tau\mathbb{B}_1(x)\left[\frac{r_2}{2[2]_q\tau(1+[2]_q\vartheta)}-\frac{s_2}{2[3]_q\tau(1+[2]_q\vartheta)}\right.\\
&\left.+\frac{(1-\epsilon)\tau[\mathbb{B}_1(x)]^2(r_2+s_2)}{2[3]_q\tau(1+[2]_q\vartheta)([\mathbb{B}_1(x)]^2-2[2]_q(1+\vartheta)^2\mathbb{B}_2(x))}\right]\\
&= \tau\mathbb{B}_1(x)\left[\left(h(\epsilon)+\frac{1}{2[3]_q(1+[2]_q\vartheta)\tau}\right)r_2+\left(h(\epsilon)-\frac{1}{2[3]_q\tau(1+[2]_q\vartheta)}\right)s_2\right],
\end{aligned}
$$

where

$$h(\epsilon)=\frac{\tau(1-\epsilon)[\mathbb{B}_1(x)]^2}{2[3]_q\tau(1+[2]_q\vartheta)([\mathbb{B}_1(x)]^2-2[2]_q(1+\vartheta)^2\mathbb{B}_2(x))}.$$

Now, by using (1.6)

$$a_3-\epsilon a_2^2=\tau\left(x-\frac{1}{2}\right)\left[\left(h(\epsilon)+\frac{1}{2[3]_q\tau(1+[2]_q\vartheta)}\right)r_2+\left(h(\epsilon)+\frac{1}{2[3]_q\tau(1+[2]_q\vartheta)}\right)s_2\right],$$

where

$$h(\epsilon)=\frac{\tau(1-\epsilon)\left(x-\frac{1}{2}\right)^2}{2[3]_q\tau(1+[2]_q\vartheta)\left(x-\frac{1}{2}\right)^2-2[2]_q(1+\vartheta)^2(x^2-x+\frac{1}{6})}.$$

Therefore, given (1.6) and (2.7), we conclude that the necessary inequality holds. □

For $q \longrightarrow 1^-$, we obtain the following corollaries proved in [9].

Corollary 2.3. *Let the function $f \in M_\Sigma(\tau,\vartheta;F)$ be of the form (1.1), then*

$$|a_2| \le \frac{|\tau||\mathbb{B}_1(x)|\sqrt{|\mathbb{B}_1(x)|}}{\sqrt{|[3]_q\tau(1+[2]\vartheta)(\mathbb{B}_1(x))^2-[2](1+\vartheta)^2\mathbb{B}_2(x)|}}, \tag{2.19}$$

and

$$|a_3| \le \frac{|\tau|\mathbb{B}_1(x)}{[3]|1+[2]\vartheta|}+\frac{|\tau|^2[\mathbb{B}_1(x)]^2}{2[2]|1+\vartheta|^2}. \tag{2.20}$$

Corollary 2.4. *If f given by (1.1) is in the class $M_\Sigma(\tau,\vartheta;F)$ where $\epsilon \in \mathbb{R}$, then we have*

$$|a_3-\epsilon a_2^2| \le \begin{cases} \frac{|\tau|\mathbb{B}_1(x)}{[3]|1+[2]_q\vartheta|}, & |h(\epsilon)| \le \frac{1}{6(1+2\vartheta)},\\ 2|\tau|\mathbb{B}_1(x)|h(\epsilon)|, & |h(\epsilon)| \ge \frac{1}{6(1+2\vartheta)},\end{cases}$$

where

$$h(\epsilon)=\frac{(1-\epsilon)\tau[\mathbb{B}_1(x)]^2}{2[3]\tau(1+2\vartheta)[\mathbb{B}_1(x)]^2-[4](1+\vartheta)^2\mathbb{B}_2(x)}.$$

3 Conclusion

Several researchers estimated the Taylor-Maclaurin coefficients $|a_2|$, $|a_3|$ and bounds for Fekete-Szegö inequality for functions certain subclasses of bi-univalent functions defined in the unit disc E. In our study, we introduced the subclass $M_{\Sigma,q}(\tau,\vartheta;F)$ of bi-univalent functions associated with the Bernoulli polynomials and q-derivative operator. Furthermore, we estimated the coefficients $|a_2|$, $|a_3|$ and bounds for Fekete-Szegö inequality for functions in this class. This study could inspire researchers to introduce new subclasses of bi-univalent functions associated with the Bernoulli polynomials and q-derivative operator to find new bounds of the $|a_2|$, $|a_3|$ and for Fekete-Szegö inequality for functions these subclasses.

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