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# A Fractional Calculus Approach to Parallel RLC Circuit Dynamics via Caputo and Caputo-Fabrizio Operators

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**Abstract** This study presents an in-depth analysis of fractional-order RLC circuits, employing the Caputo derivative (CD) and Caputo-Fabrizio fractional derivative (CFD) operators. We apply two powerful mathematical techniques, the Adomian Decomposition Method (ADM) and the Picard method (PM) to effectively resolve the governing fractional differential equations. Our findings underscore the superior performance of these methods, which demonstrate rapid convergence and heightened accuracy, making them indispensable for addressing complex engineering challenges. Through rigorous graphical representations of the solutions, we validate the robustness of our approaches. This investigation not only enriches the understanding of fractional calculus applications in electrical circuit analysis, but also paves the way for innovative modelling strategies in dynamic systems exhibiting memory and heredity characteristics; it enables precise characterization of phenomena such as frequency-dependent damping and non-local charge transport, which are critical in the design and optimization of modern electrical systems.

## 1 Introduction

In contemporary scientific and engineering disciplines, the mathematical framework known as fractional calculus has emerged as a natural and essential generalization of classical calculus [1–3]. Unlike traditional calculus, which deals exclusively with integer-order derivatives and integrals, fractional calculus extends these operations to non-integer order (i.e., accurate or complex) orders. This expansion allows for modeling systems with far more intricate dynamic behaviors. It introduces an additional layer of mathematical flexibility that has proven robust and indispensable in capturing the nuances of various physical processes. Due to its generality and capacity to describe systems with memory and hereditary characteristics, fractional calculus has been increasingly adopted in various fields, such as control theory, viscoelasticity, signal processing, electrical engineering, and biomedical systems. As such, it has become a central tool for developing analytical models that closely mimic real-world systems with high fidelity [4–6].

By leveraging the tools of fractional calculus, numerous complex physical phenomena have been extensively studied and analyzed. These include anomalous diffusion, viscoelastic material behavior, dielectric relaxation, and thermal conduction. The distinguishing feature of fractional calculus lies in its ability to model memory effects where the current state of a system is influenced by its past states more rigorously and naturally than what is possible using ordinary differential equations. This capability arises from the non-local property of fractional derivatives, which inherently encode the history of the function into its current evaluation. As highlighted in [8], there are compelling theoretical and physical motivations for using non-integer order derivatives, mainly when dealing with systems whose dynamics evolve with persistent memory or long-range dependence over time. Such systems are commonplace in nature and engineering, and accurately capturing their dynamics is critical for robust modeling and control.

One of the most significant advantages of fractional-order models is their ability to provide a more detailed and accurate description of dynamic systems compared to their integer-order counterparts. Traditional integer-order differential equations are often overlooked or inadequately approximate significant dissipative and memory effects, resulting in models that fail to align with empirical observations. In contrast, fractional-order models naturally incorporate such effects, allowing for a more faithful representation of complex systems. This has led to their successful application in numerous fields, notably electrical engineering, where fractional-order elements and systems are actively investigated and utilized.

Fractional-order modeling techniques have been particularly effective in describing the behavior of electrical circuits composed of complex structures, such as tree-like topologies and domino ladders, as well as individual components like inductors, capacitors, and emerging elements such as memristors. A thorough survey of these developments is provided in [10], where various applications and realizations of fractional-order elements are reviewed. These elements exhibit non-classical dynamic responses that are better represented using fractional differential equations. For example, a conventional resistor may not adequately capture certain materials' complex interplay between charge and voltage. In contrast, a fractional resistor can incorporate frequency-dependent memory effects.

Researchers have recently proposed and studied new types of electrical circuits that integrate different classical components using fractional differential equations [11–13]. One such development involves the design of a hybrid circuit that simultaneously captures the simple harmonic motion typical of an LC circuit and the exponential decay behavior of an RC circuit, all within a single fractional-order framework [14]. This hybrid configuration provides valuable insights into how different dynamic behaviors can coexist and interact within a unified system. Furthermore, using a fractional approach, the study by [15] examined the transient behavior of a basic wire circuit connected to both direct and alternating current sources. Their findings indicated that the wire initially behaves inductively as the current begins to flow, but it transitions smoothly to a resistive state as time progresses. This phenomenon, which is challenging to model using classical integer-order techniques, is naturally explained through the lens of fractional calculus.

Complementary to these studies, Guia et al. [16] conducted an in-depth investigation into the time-domain characteristics of RC circuits by employing fractional models. Their work provided explicit expressions and analyses of critical performance metrics such as rise, delay, and settling time, vital parameters in signal transmission and control applications. The inherent complexity of real-world electrical circuits often arises from factors such as ohmic losses, internal friction, heat dissipation, nonlinear interactions, and memory-dependent behavior. These characteristics cannot be adequately represented using conventional integer-order models, which often assume ideal, memoryless components. As such, there is a growing consensus in the scientific community that fractional calculus offers a more appropriate and robust mathematical foundation for modeling such systems.

As a result of this paradigm shift, recent research has focused heavily on developing and analyzing a wide range of fractional-order electrical circuits. These include, but are not limited to, meminductors and memcapacitors [17], which possess memory-dependent inductance and capacitance, respectively, and memristors [18], which exhibit a direct relation between charge and magnetic flux with inherent memory properties. Additional studies have explored Chua's circuit [19], a nonlinear oscillator known for its chaotic behavior; De-Levie's transmission line model [20], which has been extended using fractional derivatives to account for diffusion-dominated effects; and piezoelectric actuators [21] as well as supercapacitors [22], both of which exhibit complex, memory-governed dynamic responses. These investigations have employed various definitions of fractional derivatives, offering distinct advantages regarding mathematical properties and physical interpretability. Among the most commonly used are the Caputo derivative (CD), the Caputo-Fabrizio derivative (CFD), the Atangana-Baleanu derivative, and the conformable fractional derivative, all of which are employed depending on the desired characteristics of the model.

Motivated by these developments, the current paper investigates the behavior of a parallel RLC electrical circuit within the context of fractional calculus. Specifically, the goal is to find an analytical solution to the governing fractional differential equations that describe the dynamics of this circuit. The problem is formulated using Caputo and Caputo-Fabrizio definitions of fractional derivatives, allowing for a comparative analysis of the solutions obtained under different modeling assumptions [27, 28]. To solve the fractional-order equations, we utilize the Adomian Decomposition Method (ADM) [23–26], which is known for its simplicity, rapid convergence, and reliability in handling nonlinear and fractional differential equations. The results obtained using ADM are compared with those obtained through the Picard Method (PM), a classical approach that has also been widely used for solving differential equations. Although fractional calculus has been used for electrical system modelling in previous studies, this study is the first to assess the convergence, accuracy, and effectiveness of these solution approaches in capturing intricate, memory-dependent dynamics. Furthermore, it offers thorough verification of the existence and uniqueness of the solution, in addition to numerical and graphical examples that show how fractional models provide a more accurate representation of real-world behaviors.

To summarize, the structure of this paper begins with a comprehensive overview of the basic definitions and mathematical preliminaries of fractional derivatives and integrals. This section lays the theoretical groundwork for the subsequent analysis. Following this, a detailed comparison between the Adomian Decomposition and Picard Method is presented, highlighting their respective strengths, limitations, and applicability to fractional systems. In the third section, we develop the mathematical model of a fractional-order parallel RLC circuit and establish the conditions under which solutions exist and are unique. Lastly, a series of case studies is provided, accompanied by graphical illustrations that validate the accuracy and effectiveness of the proposed analytical method in capturing the dynamic behavior of the system under study. These results demonstrate the superiority of fractional modeling techniques and underscore their relevance in modern electrical circuit analysis.

## 2 Mathematical Background

First, we present the necessary definitions and preliminaries of fractional calculus.

**A**: The Caputo's Fabrizo fractional derivative of a function $\varphi(t)$ is continuous on $(0, \infty)$ and $k-1<\gamma \leq k$ then it is defined as [6]

$$ {}^{cf}D_t^{\gamma}\,\varphi(t)=\frac{\varrho(\gamma)}{1-\gamma}\int_0^t e^{-\frac{\gamma}{1-\gamma}(t-\tau)}\varphi'(\tau)d\tau, \tag{2.1}$$

where $\varphi(t)$ is continuous and differentiable on $[0,T]$ and the function $\varrho(\gamma)>0$ is a normalized function which satisfies $\varrho(0)=\varrho(1)=0$. Its corresponding fractional integral (FI) is

$$ {}^{cf}J_t^{\gamma}\,\varphi(t)=\frac{1-\gamma}{\varrho(\gamma)}+\frac{\gamma}{\varrho(\gamma)}\int_0^t \varphi(\tau)d\tau, \tag{2.2}$$

where

$$\left({}^{cf}J_a^{\gamma}\right)\left({}^{cf}D_a^{\gamma}\right)\varphi(t)=\varphi(t)-\varphi(a)$$

The main advantage of using this definition is that there is no singularity in its definition, as shown in (2.1) and (2.2).

**B**: The Caputo's fractional derivative of a function $\varphi(x)$

$$ {}^{C}D_t^{\upsilon}\varphi(t)=\frac{1}{\Gamma(n-\upsilon)}\int_0^t \frac{\varphi^{(n)}(\tau)d\tau}{(t-\tau)^{\upsilon-n+1}},\quad n-1<\upsilon<n, \tag{2.3}$$

Its corresponding FI is

$$ {}^{C}J_t^{\upsilon}\varphi(t)=\frac{1}{\Gamma(\upsilon)}\int_0^t \varphi(\tau)\left(t-\tau\right)^{\upsilon-1}d\tau,\quad 0<\upsilon<1, \tag{2.4}$$

**Properties:**

**1**: ${}^{c}D_t^{-\gamma}k=0,\ k\in\mathbb{R}$

**2**: ${}^{c}D_t^{-\gamma}\,{}^{c}D_t^{-\delta}\,\varphi(t)={}^{c}D_t^{-\gamma-\delta}\,\varphi(t)$.

**3**: ${}^{c}D_{0,t}^{\gamma}\,t^{\eta}=\frac{\Gamma(\eta+1)}{\Gamma(\eta-\gamma+1)}t^{\eta-\gamma}$.

**4**: ${}^{c}D_{0,t}^{\gamma}\,{}^{c}D_{0,t}^{-\gamma}\,\varphi(t)=\varphi(t)$.

**5**: ${}^{c}D_t^{-\gamma}\,{}^{c}D_t^{\gamma}\,\varphi(t)=\varphi(t)-\sum_{m=0}^{k-1}\frac{\varphi^{(k)}(0)}{m!}t^{m}$.

# 3 Mathematical Model with ADM and PM

## 3.1 ADM solution algorithm

In this part, the RLC circuit shown in Figure a, which contains a resistor $R$, capacitor $C$ and inductor $L$ connected in parallel, with $i_L(0) = I_0$ corresponding to the initial inductor current, $v_c(0) = V_0$ indicating the initial capacitor, $i_s(t) = 0$ showing the current source, $i_0$ is the initial current and $u_0(t)$ is the unit step function. We want to find an expression for the voltage $v(t)$ for $t > 0$ [32].
Where $i_L$ is the inductor current, $i_R$ is the resistor current, $i_C$ is the capacitor current and $v(t)$ is the voltage source of the circuit. The mathematical equation of the (parallel RLC circuit) is

$$CD_t^{\beta} v(t) + \frac{1}{L} J^{\alpha} v(t) + \frac{1}{R} v(t) + i_0 = i_s(t), \quad t > 0. \tag{3.1}$$

$\alpha$ and $\beta$ represent the voltage and current parameters of a fractional order inductor and capacitor at which $\alpha$, $\beta \in [0,1]$, respectively. From (3.1), using the Caputo fractional operator with ADM, the voltage of the capacitor may is represented as

$$D_t^{\beta} v(t) = \frac{i_s(t)}{C} - \Upsilon\, J^{\alpha} v(t) - \psi\, v(t) - \frac{i_0}{C}, \tag{3.2}$$

where $\frac{1}{CL} = \Upsilon$ and $\frac{1}{CR} = \psi$.
Applying both sides with $I^{\beta}$ to (3.2), as defined for CD (2.3)

Using I.C., we get

$$v(t) = \sum_{i=0}^{k-1} c_i \frac{t^i}{i!} + J^{\beta} \frac{i_s(t)}{C} - J^{\beta}\left(\Upsilon\, J^{\alpha} v(t)\right) - J^{\beta}\left(\psi\, v(t)\right) - J^{\beta} \frac{i_0}{C}, \tag{3.3}$$

therefore, the relation becomes

$$v_0(t) = V_0 + J^{\beta} \frac{i_s(t)}{C} - J^{\beta} \frac{i_0}{C}, \tag{3.4}$$

$$v_{i+1}(t) = -\Upsilon\, J^{\alpha+\beta} v_i(t) - \psi\, J^{\beta}\left( v_i(t)\right). \tag{3.5}$$

where $V_0 = \sum_{i=0}^{k-1} c_i \frac{t^i}{i!}$.
In the end, the solution of (3.1) can be written as an infinite series with the formula [33, 34]

$$v(t) = \sum_{i=0}^{\infty} v_i(t). \tag{3.6}$$

If the series converges, the $n-$term partial sum $\varsigma_n = \sum_{k=0}^{n-1} v_i$ would be the approximate solution since $\lim_{n\to\infty} \varsigma_n = \sum_{i=0}^{\infty} v_i(t) = v(t)$.

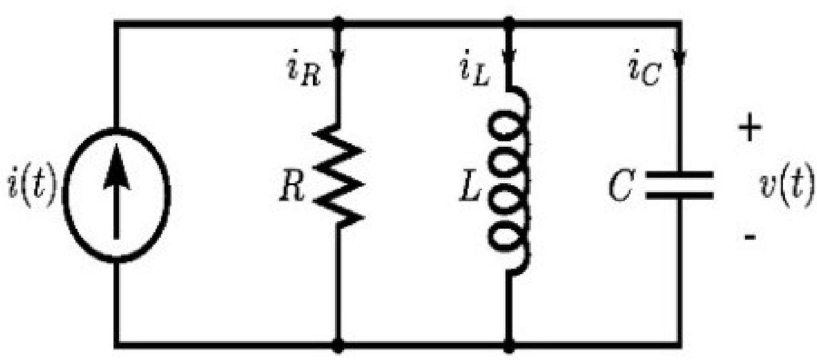


Figure 1: Schematic Figure of the circuit

### 3.2 Convergence analysis (Caputo)

**Existence and uniqueness**

Define a mapping $R : E \to E$ wherever $E = (C[\mathfrak{J}], \|\cdot\|)$ is a Banach space used for all continuous functions on $\mathfrak{J}$ with the norm $\|v\| = \max_{t\epsilon J} v(t)$.

**Theorem 3.1.** *the problem (3.1) has a unique solution if* $0 < \phi < 1$ *where* $\phi = \frac{\Upsilon T^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + \frac{\psi T^{\beta}}{\Gamma(\beta+1)}$.

*Proof.* Firstly, we define the mapping $R$ from (3.3), $R : E \to E$ as

$$R\, v(t) = \sum_{i=0}^{m} c_i \frac{t^i}{i!} - \Upsilon\, {}^{C}J^{\alpha+\beta} v(t) - \psi\, {}^{C}J^{\beta} v(t) + {}^{C}J^{\beta} g(t), \tag{3.7}$$

where $g(t) = i_s(t) - i_0$.
Assume $v$ and $z \in E$ are two different solutions of (3.7), so

$$\begin{aligned} R\, v(t) - R\, z(t) &= -\Upsilon\, {}^{C}J^{\alpha+\beta} v(t) - \psi\, {}^{C}J^{\beta} v(t) + \Upsilon\, {}^{C}J^{\alpha+\beta} z(t) + \psi\, {}^{C}J^{\beta} z(t) \\ &= -\left[\Upsilon\, {}^{C}J^{\alpha+\beta}(v(t) - z(t)) + \psi\, {}^{C}J^{\beta}(v(t) - z(t))\right]. \end{aligned}$$

Therefore

$$\begin{aligned} \|Rv(t) - Rz(t)\| &= \max_{t\epsilon J} \left|\Upsilon\, {}^{C}J^{\alpha+\beta}(v(t) - z(t)) + \psi\, {}^{C}J^{\beta}(v(t) - z(t))\right| \\ &\leq \max_{t\epsilon J} |v - z| \left[\left|\Upsilon\, {}^{C}J^{\alpha+\beta}(1)\right| + \left|\psi\, {}^{C}J^{\beta}(1)\right|\right] \\ &\leq \max_{t\epsilon J} |v - z| \left[\left|{}^{C}J^{\alpha+\beta}\left(\frac{\Upsilon}{\Gamma(\alpha+\beta)} \int_0^t (t-\tau)^{\alpha+\beta-1} d\tau\right)\right|\right. \\ &\quad \left. + \left|{}^{C}J^{\beta}\left(\frac{\psi}{\Gamma(\beta)} \int_0^t (t-\tau)^{\beta-1} d\tau\right)\right|\right] \\ &\leq \phi \|v - z\|. \end{aligned}$$

With the condition $0 < \phi < 1$, the mapping $R$ is contraction, so there is a unique solution $v \in C[\mathfrak{J}]$ for equation (3.1) and the proof is complete.

□

**Proof of convergence**

**Theorem 3.2.** *If there exists a unique solution to the problem (3.1), then the ADM series solution is converges if* $|v_1(t)| < c$ *where* $c$ *is a positive finite constant and* $v_1(t)$ *is the first term of the ADM series solution.*

*Proof.* We define a sequence $\{Q_n\}$ at which $Q_n = \sum_{i=0}^{n} v_i(t)$ is the ADM series of partial sums from the series solution $\sum_{i=0}^{n} v_i(t)$, we have Assume $Q_n, Q_m$ are two arbitrary sums with, $n \geqslant m$. Now, we prove that $\{Q_n\}$ is a Cauchy sequence in a Banach space. We get

$$Q_n - Q_m = -\Upsilon\, {}^{C}J^{\alpha+\beta} \sum_{i=0}^{n} v_i(t) - \psi\, {}^{C}J^{\beta} \sum_{i=0}^{m} v_i(t) = \sum_{i=m+1}^{n} v_i(t). \tag{3.8}$$

Let $n = m + 1$ then,

$$\begin{aligned}
\|Q_n - Q_m\| &= \max_{t\epsilon J} \left| \sum_{i=m+1}^{n} v_i(t) \right| \\
&= \max_{t\epsilon J} \left| \Upsilon\, {}^C J^{\alpha+\beta} \sum_{i=0}^{n} v_i(t) \right| + \max_{t\epsilon J} \left| \psi\, {}^C J^{\beta} \sum_{i=0}^{m} v_i(t) \right| \\
&= \max_{t\epsilon J} \left| \sum_{i=0}^{n} \frac{\Upsilon}{\Gamma(\alpha+\beta)} \int_0^t (t-\tau)^{\alpha+\beta-1} [Q_n - Q_m]\, d\tau \right| + \\
&\quad \max_{t\epsilon J} \left| \sum_{i=0}^{n} \frac{\psi}{\Gamma(\beta)} \int_0^t (t-\tau)^{\beta-1} [Q_n - Q_m]\, d\tau \right| \\
&= \max_{t\epsilon J} |Q_n - Q_m| \left[ \frac{\Upsilon T^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + \frac{\psi T^{\beta}}{\Gamma(\beta+1)} \right] \\
&\leq \phi \|Q_{n-1} - Q_{m-1}\|.
\end{aligned}$$

Then

$$\|Q_{m+1} - Q_m\| \leq \phi \|Q_m - Q_{m-1}\| \leq \phi^2 \|Q_{m-1} - Q_{m-2}\| \leq ... \leq \phi^m \|Q_1 - Q_0\|. \tag{3.9}$$

From the triangle inequality, we get

$$\begin{aligned}
\|Q_n - Q_m\| &\leq \|Q_{m+1} - Q_m\| + \|Q_{m+2} - Q_{m+1}\| + ... \|Q_n - Q_{n-1}\| \\
&\leq \left[\phi^m + \phi^{m+1} + ... + \phi^{n-1}\right] \|Q_1 - Q_0\| \\
&\leq \phi^m \left[1 + \phi + ... + \phi^{n-m+1}\right] \|Q_1 - Q_0\| \\
&\leq \phi^m \left[\frac{1 - \phi^{n-m}}{1-\delta}\right] \|v_1(t)\|.
\end{aligned}$$

If $0 < \phi < 1, n \geqslant m$ then $(1 - \phi^{n-m}) \leq 1$. Consequently

$$\|Q_n - Q_m\| \leq \frac{\phi^m}{1-\phi} \|v_1(t)\| \leq \frac{\phi^m}{1-\phi} \max_{t\epsilon J} |v_1(t)| \tag{3.10}$$

but $|v_1(t)| < \infty$ and as $m \to \infty$ then, $\|Q_n - Q_m\| \to 0$, so there is a Cauchy sequence $\{Q_n\}$ in this Banach space, and this is the complete proof. □

**Error Analysis**

**Theorem 3.3.** *The maximum absolute truncated error to (3.1) is estimated to be* $\max_{t\epsilon J} \left| v(t) - \sum_{i=0}^{m} v_i(t) \right| \leq \frac{\delta^m}{1-\delta} \max_{t\epsilon J} |v_1(t)|$.

*Proof.* From the convergence theorm 3.2 inequality (3.10) we have

$$\|Q_n - Q_m\| \leq \frac{\delta^m}{1-\delta} \max_{t\epsilon J} |v_1(t)|,$$

but, $Q_n = \sum_{i=0}^{m} v_i(t)$ as $n \to \infty$ then, $Q_n \to v(t)$ so,

$$\|v(t) - Q_m\| \leq \frac{\delta^m}{1-\delta} \max_{t\epsilon J} |v_1(t)|.$$

As a result, the maximum absolute truncated error in the interval $J$ is,

$$\max_{t\epsilon J} \left| v(t) - \sum_{i=0}^{m} v_i(t) \right| \leq \frac{\delta^m}{1-\delta} \max_{t\epsilon J} |v_1(t)|,$$

and this completes the proof. □

### 3.3 Picard method (PM)

Applying PM to (3.1), the solution is a sequence constructed by

$$\begin{cases} v_0(t) = V_0 + {}^C J^{\beta} \frac{i_s(t)}{C} - {}^C J^{\beta} \frac{i_0}{C} \\ v_{\kappa+1}(t) = v_0(t) - \Upsilon\, {}^C J^{\alpha+\beta} v_{\kappa}(t) - \psi\, {}^C J^{\beta}(v_{\kappa}(t)). \end{cases} \tag{3.11}$$

All the functions $v_{\kappa}(t)$ are continuous functions and $v_{\kappa}$ can be written as the sum of successive differences as follows

$$v_{\kappa}(t) = v_0(t) + \sum_{\kappa=1}^{m} (v_{\kappa} - v_{\kappa-1}).$$

This means that the sequence $v_{\kappa}$ convergence is equivalent to the convergence of the infinite series $\sum (v_{\kappa} - v_{\kappa-1})$. The final PM solution will take the form

$$v(t) = \lim_{k\to\infty} v_{\kappa}(t).$$

From the above relations, we can deduce that if the series $\sum (v_{\kappa} - v_{\kappa-1})$ is converging, then the sequence $v_{\kappa}(t)$ will be convergent to $v(t)$. To prove that the sequence $\{v_k(t)\}$ is informally convergent, consider the associated series

$$\sum_{\kappa=1}^{\infty} [v_{\kappa}(t) - v_{\kappa-1}(t)].$$

From (3.11) for $\kappa = 1$, we get

$$v_1(t) - v_0(t) = -\Upsilon\, {}^C J^{\alpha+\beta} v_0(t) - \psi\, {}^C J^{\beta}(v_0(t)),$$

and

$$\begin{aligned} |v_1(t) - v_0(t)| &= \left|\Upsilon\, {}^C J^{\alpha+\beta}(v_0(t)) + \psi\, {}^C J^{\beta}(v_0(t))\right| \\ &\leq \frac{\Upsilon}{\Gamma(\alpha+\beta)} \int_0^t (t-\tau)^{\alpha+\beta-1} |v_0(\tau)|\, d\tau + \\ &\quad \frac{\psi}{\Gamma(\beta)} \int_0^t (t-\tau)^{\beta-1} |v_0(\tau)|\, d\tau. \end{aligned}$$

Assume that $|v_0(\tau)| < r_0$, hence

$$\begin{aligned} |v_1(t) - v_0(t)| &\leq \frac{\Upsilon}{\Gamma(\alpha+\beta)} [r_0] \int_0^t (t-\tau)^{\alpha+\beta-1} d\tau + \frac{\psi}{\Gamma(\beta)} [r_0] \int_0^t (t-\tau)^{\beta-1} d\tau \\ &\leq [r_0] \left[\frac{\Upsilon T^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + \frac{\psi T^{\beta}}{\Gamma(\beta+1)}\right] \\ &\leq \Psi, \end{aligned} \tag{3.12}$$

where

$$\Psi = [r_0] \left[\frac{\Upsilon T^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + \frac{\psi T^{\beta}}{\Gamma(\beta+1)}\right].$$

Now, we get an estimate, for $v_{\kappa}(\tau) - v_{\kappa-1}(\tau)$ and $\kappa \geqslant 2$,

$$\begin{aligned} v_{\kappa}(t) - v_{\kappa-1}(t) &= \Upsilon\, {}^C J^{\alpha+\beta}(v_{\kappa-1}) + \psi\, {}^C J^{\beta}(v_{\kappa-1}) - \Upsilon\, {}^C J^{\alpha+\beta}(v_{\kappa-2}) - \psi\, {}^C J^{\beta}(v_{\kappa-2}) \\ &= \Upsilon\, {}^C J^{\alpha+\beta}[v_{\kappa-1} - v_{\kappa-2}] + \psi\, {}^C J^{\beta}[v_{\kappa-1} - v_{\kappa-2}], \end{aligned}$$

we find that

$$\begin{aligned} |v_{\kappa}(t) - v_{\kappa-1}(t)| &\leq \left|\Upsilon\, {}^C J^{\alpha+\beta}[v_{\kappa-1} - v_{\kappa-2}] + \psi\, {}^C J^{\beta}[v_{\kappa-1} - v_{\kappa-2}]\right| \\ &\leq \Upsilon |v_{\kappa-1}(t) - v_{\kappa-2}(t)|\, {}^C J^{\alpha+\beta}(1) + \psi |v_{\kappa-1}(t) - v_{\kappa-2}(t)|\, {}^C J^{\beta}(1) \\ &\leq |v_{\kappa-1}(t) - v_{\kappa-2}(t)| \left[\Upsilon\, {}^C J^{\alpha+\beta}(1) + \psi\, {}^C J^{\beta}(1)\right] \\ &\leq \phi |v_{\kappa-1}(t) - v_{\kappa-2}(t)|. \end{aligned}$$

In the above relation, If we put $\kappa = 2$, and use (3.12) we get

$$\begin{aligned} |v_2(t) - v_1(t)| &\leq \phi\, |v_1(t) - v_0(t)| \\ |v_2 - v_1| &\leq \phi\, \psi. \end{aligned}$$

Doing the same for $\kappa = 3, 4, \cdots$ ,

$$\begin{aligned} |v_3 - v_2| &\leq \phi\, |v_2(t) - v_1(t)| \\ &\leq \phi^2\, \psi \\ |v_4 - v_3| &\leq \phi\, |v_3(t) - v_2(t)| \\ &\leq \phi^3\, \psi \\ &\vdots \end{aligned}$$

Then the general form of this relation is

$$|v_\kappa - v_{\kappa-1}| \leq \phi^{\kappa-1}\, \psi.$$

Since $\phi \in (0, 1)$ , the series

$$\sum_{\kappa=1}^{\infty} [v_\kappa(t) - v_{\kappa-1}(t)]\,,$$

converges. Hence, the sequence $\{v_\kappa(t)\}$ will be uniformly convergent. Since $f(t, v(t))$ is continuous, we arrive at

$$\begin{aligned} v(t) &= \lim_{\kappa\to\infty} \Upsilon\, {}^{C}J^{\alpha+\beta}\, (v_{\kappa\,-1}(t)) + \psi\, {}^{C}J^{\beta}\ \ (v_{\kappa\,-1}(t)) \\ &= \Upsilon\, {}^{C}J^{\alpha+\beta}\, (v(t)) + \psi\, {}^{C}J^{\beta}\ \ (v(t))\,, \end{aligned}$$

here, we arrive at 3.11.

### 3.4 Convergence analysis (Caputo-Fabrizo)

#### Existence and uniqueness

Define a mapping $R : E \to E$ wherever $E = (C\,[\mathfrak{J}]\,, \|\cdot\|)$ is a Banach space used for all continuous functions on $\mathfrak{J}$ with the norm $\|v\| = \max_{t\epsilon J} v\,(t)\,.$

**Theorem 3.4.** *the problem (3.3) has a unique solution if* $0 < \zeta < 1$ *where*

$$\zeta = \left[\Upsilon \left(\frac{1 + (\alpha+\beta)\,(T-1)}{\varrho\,(\alpha+\beta)}\right) + \psi \left(\frac{1+\beta\,(T-1)}{\varrho\,(\beta)}\right)\right].$$

*Proof.* Firstly, we define the mapping $R$ from (3.3), $R : E \to E$ as

$$R\,v\,(t) = \sum_{i=0}^{m} c_i \frac{t^i}{i!} - \Upsilon\, {}^{CF}J^{\alpha+\beta} v\,(t) - \psi\, {}^{CF}J^{\beta} v\,(t) +\, {}^{CF}J^{\beta} g\,(t)\,. \tag{3.13}$$

Assume $i$ and $z \in E$ are two different solutions of (3.13), so

$$\begin{aligned} R\,v\,(t) - R\,z\,(t) &= -\Upsilon\, {}^{CF}J^{\alpha+\beta} v\,(t) - \psi\, {}^{CF}J^{\beta} v\,(t) + \Upsilon\, {}^{CF}J^{\alpha+\beta} z\,(t) + \psi\, {}^{CF}J^{\beta} z\,(t) \\ &= -\ \left[\Upsilon\, {}^{CF}J^{\alpha+\beta}\, (v\,(t) - z\,(t)) + \psi\, {}^{CF}J^{\beta}\, (v\,(t) - z\,(t))\right]. \end{aligned}$$

Therefore

$$\begin{aligned}
\|Rv(t)-Rz(t)\| &= \max_{t\epsilon J}\left|\Upsilon\,{}^{CF}J^{\alpha+\beta}\left(v(t)-z(t)\right)+\psi\,{}^{CF}J^{\beta}\left(v(t)-z(t)\right)\right| \\
&\le \max_{t\epsilon J}|v-z|\ \left[\left|\Upsilon\,{}^{CF}J^{\alpha+\beta}(1)\right|+\left|\psi\,{}^{CF}J^{\beta}(1)\right|\right] \\
&\le \max_{t\epsilon J}|v-z|\left[\left|\Upsilon\left(\frac{1-(\alpha+\beta)}{\varrho(\alpha+\beta)}+\frac{(\alpha+\beta)}{\varrho(\alpha+\beta)}\int_0^t d\tau\right)\,{}^{CF}J^{\alpha+\beta}(1)\right|\right. \\
&\quad\left.+\left|\psi\left(\frac{1-\beta}{\varrho(\beta)}+\frac{\beta}{\varrho(\beta)}\int_0^t d\tau\right)\,{}^{CF}J^{\beta}(1)\right|\right] \\
&\le \max_{t\epsilon J}|v-z|\left[\left|\Upsilon\left(\frac{1-(\alpha+\beta)}{\varrho(\alpha+\beta)}+\frac{(\alpha+\beta)T}{\varrho(\alpha+\beta)}\right)\right|+\left|\psi\ \left(\frac{1-\beta}{\varrho(\beta)}+\frac{\beta T}{\varrho(\beta)}\right)\right|\right] \\
&\le \zeta\|v-z\|.
\end{aligned}$$

With the condition $0<\zeta<1$, the mapping $R$ is contraction, so there is a unique solution $v\in C[\mathfrak{J}]$ for equation (3.1) and the proof is complete.

□

**Proof of convergence**

**Theorem 3.5.** *If there exists a unique solution to the problem (3.1), then the ADM series solution converges if $|v_1(t)|<c$ where $c$ is a positive finite constant.*

*Proof.* From (3.8), if we use Caputo-Fabrizio definition, it would be

$$G_n-G_m=-\Upsilon\,{}^{CF}J^{\alpha+\beta}\sum_{i=0}^{n}v_i(t)-\psi\,{}^{CF}J^{\beta}\sum_{i=0}^{m}v_i(t)=\sum_{i=m+1}^{n}v_i(t).$$

Let $n=m+1$ then,

$$\begin{aligned}
\|G_n-G_m\| &= \max_{t\epsilon J}\left|\sum_{i=m+1}^{n}v_i(t)\right| \\
&= \max_{t\epsilon J}\left|\Upsilon\,{}^{CF}J^{\alpha+\beta}\sum_{i=0}^{n}v_i(t)\right|+\max_{t\epsilon J}\left|\psi\,{}^{CF}J^{\beta}\sum_{i=0}^{m}v_i(t)\right| \\
&= \max_{t\epsilon J}\left|\ \Upsilon[G_n-G_m]\int_0^t(1)\,d\tau\right|+\max_{t\epsilon J}\left|\ \psi[G_n-G_m]\int_0^t(1)\,d\tau\right| \\
&= \max_{t\epsilon J}|G_n-G_m|\left[\Upsilon\left(\frac{1-(\alpha+\beta)}{\varrho(\alpha+\beta)}+\frac{(\alpha+\beta)}{\varrho(\alpha+\beta)}\int_0^t d\tau\right)\right. \\
&\quad\left.+\psi\left(\frac{1-\beta}{\varrho(\beta)}+\frac{\beta}{\varrho(\beta)}\int_0^t d\tau\right)\right] \\
&\le \|G_n-G_m\|\left[\Upsilon\left(\frac{1-(\alpha+\beta)}{\varrho(\alpha+\beta)}+\frac{(\alpha+\beta)T}{\varrho(\alpha+\beta)}\right)+\psi\left(\frac{1-\beta}{\varrho(\beta)}+\frac{\beta T}{\varrho(\beta)}\right)\right] \\
&\le \zeta\|G_{n-1}-G_{m-1}\|.
\end{aligned}$$

Then

$$\|G_{m+1}-G_m\|\le\zeta\|G_m-G_{m-1}\|\le\zeta^2\|G_{m-1}-G_{m-2}\|\le...\le\zeta^m\|G_1-G_0\|.$$

From the triangle inequality, we get

$$\begin{aligned}
\|G_n-G_m\| &\le \|G_{m+1}-G_m\|+\|G_{m+2}-G_{m+1}\|+...\|G_n-G_{n-1}\| \\
&\le \zeta^m\left[\frac{1-\zeta^{n-m}}{1-\zeta}\right]\|v_1(t)\|.
\end{aligned}$$

If $0<\zeta<1, n\geqslant m$ then $(1-\zeta^{n-m})\leq 1$. Consequently

$$\|G_n - G_m\| \leq \frac{\zeta^m}{1-\zeta}\|v_1(t)\| \leq \frac{\zeta^m}{1-\zeta}\max_{t\epsilon J}|v_1(t)| \tag{3.14}$$

but $|v_1(t)|<\infty$ and as $m\to\infty$ then, $\|G_n-G_m\|\to 0$, so there is a Cauchy sequence $\{G_n\}$ in this Banach space, and this is the complete proof. □

**Error Analysis**

**Theorem 3.6.** *The maximum absolute truncated error to (3.1) is estimated to be* $\max_{t\epsilon J}\left|v(t)-\sum_{k=0}^m v_k(t)\right| \leq \frac{\zeta^m}{1-\zeta}\max_{t\epsilon J}|v_1(t)|$.

*Proof.* From the convergence theorm 3.5 inequality (3.14) we have

$$\|G_n - G_m\| \leq \frac{\zeta^m}{1-\zeta}\max_{t\epsilon J}|v_1(t)|,$$

but, $G_n=\sum_{k=0}^m v_k(t)$ as $n\to\infty$ then, $G_n\to v(t)$ so,

$$\|v(t) - G_m\| \leq \frac{\zeta^m}{1-\zeta}\max_{t\epsilon J}|v_1(t)|.$$

As a result, the maximum absolute truncated error in the interval $J$ is,

$$\max_{t\epsilon J}\left|v(t)-\sum_{k=0}^m v_k(t)\right| \leq \frac{\zeta^m}{1-\zeta}\max_{t\epsilon J}|v_1(t)|,$$

and this completes the proof. □

## 3.5 Picard method (PM)

Applying PM to (3.1), the solution is a sequence constructed by

$$\begin{cases} v_0(t) = V_0 + {}^{CF}J^{\beta}\frac{i_s(t)}{C} - {}^{CF}J^{\beta}\frac{i_0}{C} \\ v_{\kappa+1}(t) = v_0(t) - \Upsilon\, {}^{CF}J^{\alpha+\beta}v_\kappa(t) - \psi\, {}^{CF}J^{\beta}\left(v_\kappa(t)\right). \end{cases} \tag{3.15}$$

All the functions $v_\kappa(t)$ are continuous functions and $v_\kappa$ can be written as the sum of successive differences as follows

$$v_\kappa(t) = v_0(t) + \sum_{\kappa=1}^{m}(v_\kappa - v_{\kappa-1}).$$

This means that the sequence $v_\kappa$ convergence is equivalent to the convergence of the infinite series $\sum(v_\kappa - v_{\kappa-1})$. The final PM solution will take the form

$$v(t) = \lim_{k\to\infty} v_\kappa(t).$$

From the above relations, we can deduce that if the series $\sum(v_\kappa - v_{\kappa-1})$ is converging, then the sequence $v_\kappa(t)$ will be convergent to $v(t)$. To prove that the sequence $\{v_k(t)\}$ is informally convergent, consider the associated series

$$\sum_{\kappa=1}^{\infty}[v_\kappa(t) - v_{\kappa-1}(t)].$$

From (3.11) for $\kappa=1$, we get

$$\begin{aligned} |v_1(t) - v_0(t)| &= \left|\Upsilon\, {}^{CF}J^{\alpha+\beta}(v_0(t)) + \psi\, {}^{CF}J^{\beta}\ (v_0(t))\right| \\ &\leq \Upsilon\left(\frac{1-(\alpha+\beta)}{\varrho(\alpha+\beta)} + \frac{(\alpha+\beta)}{\varrho(\alpha+\beta)}\int_0^t |v_0(\tau)|\,d\tau\right) \\ &\quad +\psi\left(\frac{1-\beta}{\varrho(\beta)} + \frac{\beta}{\varrho(\beta)}\int_0^t |v_0(\tau)|\,d\tau\right). \end{aligned}$$

Assume that $|v_0(\tau)| < \check{r}_0$, henceHence

$$\begin{aligned} |v_1(t) - v_0(t)| &\leq \Upsilon[\check{r}_0]\int_0^t (1)d\tau + \psi[\check{r}_0]\int_0^t (1)d\tau \\ &\leq \Upsilon[\check{r}_0]\left(\frac{1-(\alpha+\beta)}{\varrho(\alpha+\beta)} + \frac{(\alpha+\beta)T}{\varrho(\alpha+\beta)}\right) + \psi[\check{r}_0]\left(\frac{1-\beta}{\varrho(\beta)} + \frac{\beta T}{\varrho(\beta)}\right) \\ &\leq \Psi, \end{aligned}$$

where

$$\Psi = [\check{r}_0]\left[\Upsilon\left(\frac{1-(\alpha+\beta)}{\varrho(\alpha+\beta)} + \frac{(\alpha+\beta)T}{\varrho(\alpha+\beta)}\right) + \psi\left(\frac{1-\beta}{\varrho(\beta)} + \frac{\beta T}{\varrho(\beta)}\right)\right].$$

Now, we get an estimate, for $v_\kappa(\tau) - v_{\kappa-1}(\tau)$ and $\kappa \geqslant 2$,

$$v_\kappa(t) - v_{\kappa-1}(t) = \Upsilon\, {}^{CF}J^{\alpha+\beta}[v_{\kappa-1} - v_{\kappa-2}] + \psi\, {}^{CF}J^{\beta}\,[v_{\kappa-1} - v_{\kappa-2}].$$

So, we find

$$\begin{aligned} |v_\kappa(t) - v_{\kappa-1}(t)| &\leq \left|\Upsilon\, {}^{CF}J^{\alpha+\beta}[v_{\kappa-1} - v_{\kappa-2}] + \psi\, {}^{CF}J^{\beta}\,[v_{\kappa-1} - v_{\kappa-2}]\right| \\ &\leq |v_{\kappa-1}(t) - v_{\kappa-2}(t)|\,\left[\Upsilon\, {}^{CF}J^{\alpha+\beta}(1) + \psi\, {}^{CF}J^{\beta}(1)\right] \\ &\leq \left[\Upsilon\left(\frac{1-(\alpha+\beta)}{\varrho(\alpha+\beta)} + \frac{(\alpha+\beta)T}{\varrho(\alpha+\beta)}\right)\right. \\ &\quad \left. +\psi\left(\frac{1-\beta}{\varrho(\beta)} + \frac{\beta T}{\varrho(\beta)}\right)\right]|v_{\kappa-1}(t) - v_{\kappa-2}(t)| \\ &\leq \varpi\,|v_{\kappa-1}(t) - v_{\kappa-2}(t)|. \end{aligned}$$

In the above relation, If we put $\kappa = 2$, and use (3.12) we get

$$\begin{aligned} |v_2(t) - v_1(t)| &\leq \varpi\,|v_1(t) - v_0(t)| \\ |v_2 - v_1| &\leq \varpi\,\xi. \end{aligned}$$

Doing the same for $\kappa = 3, 4, \cdots$,

$$\begin{aligned} |v_3 - v_2| &\leq \varpi\,|v_2(t) - v_1(t)| \\ &\leq \varpi^2\,\xi \\ |v_4 - v_3| &\leq \varpi\,|v_3(t) - v_2(t)| \\ &\leq \varpi^3\,\xi \\ &\vdots \end{aligned}$$

Then the general form of this relation is

$$|v_\kappa - v_{\kappa-1}| \leq \varpi^{\kappa-1}\,\xi.$$

Since $\varpi \in (0,1)$, the series

$$\sum_{\kappa=1}^{\infty}[v_\kappa(t) - v_{\kappa-1}(t)],$$

converges. Hence, the sequence $\{v_\kappa(t)\}$ will be uniformly convergent. Since $f(t, v(t))$ is continuous, we arrive at

$$\begin{aligned} v(t) &= \lim_{\kappa\to\infty}\Upsilon\, {}^{CF}J^{\alpha+\beta}[v_{\kappa-1} - v_{\kappa-2}] + \psi\, {}^{CF}J^{\beta}\,[v_{\kappa-1} - v_{\kappa-2}] \\ &= \Upsilon\, {}^{CF}J^{\alpha+\beta}\,v(t) + \psi\, {}^{CF}J^{\beta}\,v(t). \end{aligned}$$

Hence, we arrive at 3.15.

## 4 Case Study

In this part, we show and determine the effect of the RLC circuit.

**Example 1.** In this example, describe the damping response of a parallel RLC circuit. The model's parameters were chosen as:

$$I_s(t) = 4u_0(t)\,A,\ i_L(0) = 0,\ v_c(0) = 0,\ R = 60\Omega,\ L = 200mH,\ C = 120mF.$$

**ADM solution:** Applying ADM to equation (3.1), we get

$$\begin{aligned} v_0(t) &= J^{\beta}\left(\frac{240}{7.2}\right), \\ v_{i+1}(t) &= -\frac{300}{7.2}J^{\alpha+\beta}v_i(t) - J^{\beta}\left(\frac{1}{7.2}v_i(t)\right), \quad i \geq 0. \end{aligned}$$

**PM solution:** Using PM to equation (3.1), the solution algorithm is

$$\begin{aligned} v_0(t) &= J^{\beta}\left(\frac{240}{7.2}\right), \\ v_{i+1}(t) &= v_0(t) - \frac{300}{7.2}J^{\alpha+\beta}v_i(t) - J^{\beta}\left(\frac{1}{7.2}v_i(t)\right), \quad i \geq 0. \end{aligned}$$

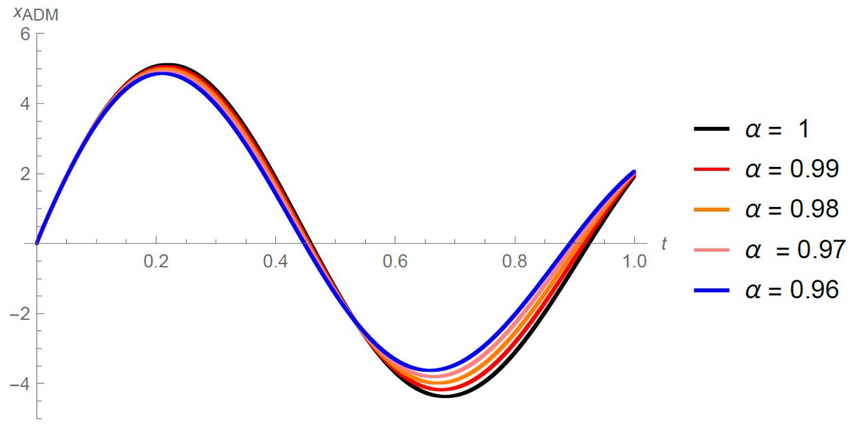


Figure 2: ADM Solution by CD

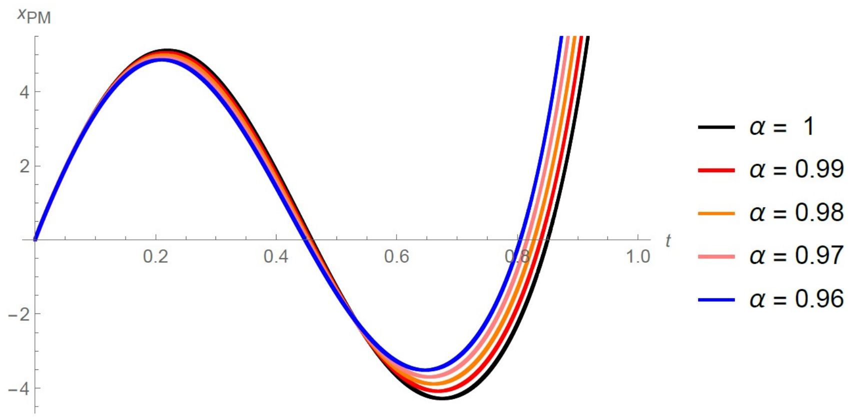


Figure 3: PM Solution by CD

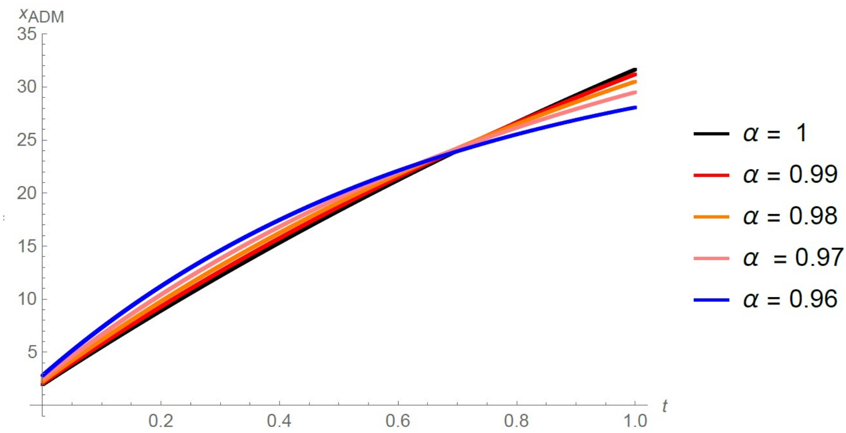


Figure 4: ADM Solution by CFD

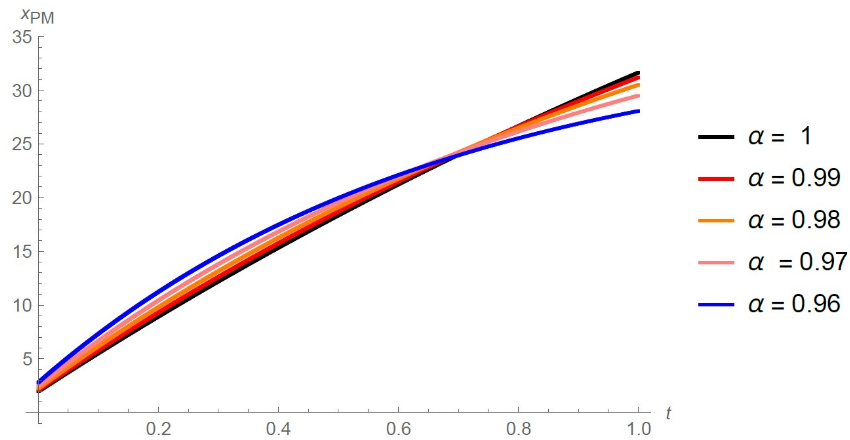


Figure 5: PM Solution by CFD

Figures 1 and 2 represent a comparison between the two used techniques by CD and CF at $\beta = 0.95$ and different cases of $\alpha$ ($\alpha = 1,\ 0.99,\ 0.98,\ 0.97,\ 0.96$). Tables 1 and 2 provide a detailed comparison of the absolute error (AE) between the solutions obtained using the PM and the ADM for a specific case where $\alpha = 1$ and $\beta = 0.95$ by CD and CF. This table highlights how closely each method approximates each others at many values of $t$. On the other hand, Tables 3, 4, 5, and 6 compare the time response for the RLC circuit based on $\alpha = 0.98$ by using Mathematica code. Due to CF, there is no singularity compared with CD, so, it makes the calculation easier, and that is clear that the time consumed by CF is less than CD. The results from these tables indicate not only the accuracy of the solutions but also the efficiency of these methods.

Table 1: AD between ADM and PM by CD

| $t$ | $\lvert \varkappa_{ADM} - \varkappa_{Picard} \rvert$ |
|---|---|
| 0.1 | $5.72783 \times 10^{-13}$ |
| 0.2 | $4.44169 \times 10^{-9}$ |
| 0.3 | $1.04654 \times 10^{-6}$ |
| 0.4 | $5.42107 \times 10^{-5}$ |
| 0.5 | $1.19331 \times 10^{-3}$ |

Table 2: AD between ADM and PM by CFD

| $t$ | $\lvert\varkappa_{ADM} - \varkappa_{Picard}\rvert$ |
|---|---|
| 0.1 | $8.65976 \times 10^{-18}$ |
| 0.2 | $3.37514 \times 10^{-17}$ |
| 0.3 | $7.39453 \times 10^{-17}$ |
| 0.4 | $1.27917 \times 10^{-16}$ |
| 0.5 | $1.94348 \times 10^{-16}$ |

Table 3: Time response by CD at $\alpha = 0.95$

| $m$ | $PM\ time\ in\ sec$ |
|---|---|
| 3 | 22.438 |
| 6 | 80.484 |
| 9 | 124.00 |

Table 4: Time response by CD at $\alpha = 0.95$

| $m$ | $ADM\ time\ in\ sec$ |
|---|---|
| 3 | 25.046 |
| 6 | 125.656 |
| 9 | 151.031 |

Table 5: Time response by CD at $\alpha = 0.95$

| $m$ | $PM\ time\ in\ sec$ |
|---|---|
| 5 | 0.453 |
| 10 | 0.500 |
| 15 | 0.547 |

Table 6: Time response by CF at $\alpha = 0.95$

| $m$ | $ADM\ time\ in\ sec$ |
|---|---|
| 5 | 1.062 |
| 10 | 1.421 |
| 15 | 1.625 |

**Example 2.** The model's parameters were chosen as:

$$I_s(t) = 10u_0(t)\,A,\ i_L(0) = 2,\ v_c(0) = 5,\ R = 32\Omega,\ L = 10H,\ C = \frac{1}{640}F.$$

**ADM solution:** Applying ADM to equation (3.1), we get

$$\begin{aligned} v_0(t) &= 5 + J^{\beta}(5120), \\ v_{i+1}(t) &= -64J^{\alpha+\beta}v_i(t) - 20\,J^{\beta}(v_i(t)), \quad i \geq 0. \end{aligned}$$

**PM solution:** Using PM to equation (3.1), the solution algorithm is

$$\begin{aligned} v_0(t) &= 5 + J^{\beta}(5120), \\ v_{i+1}(t) &= v_0(t) - 64\, J^{\alpha+\beta} v_i(t) - 20\, J^{\beta}(v_i(t)), \quad i \geq 0. \end{aligned}$$

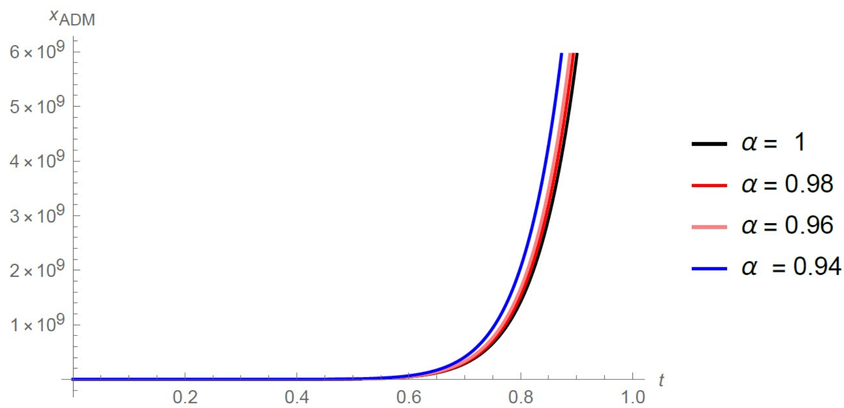


Figure 6: ADM Solution by CD

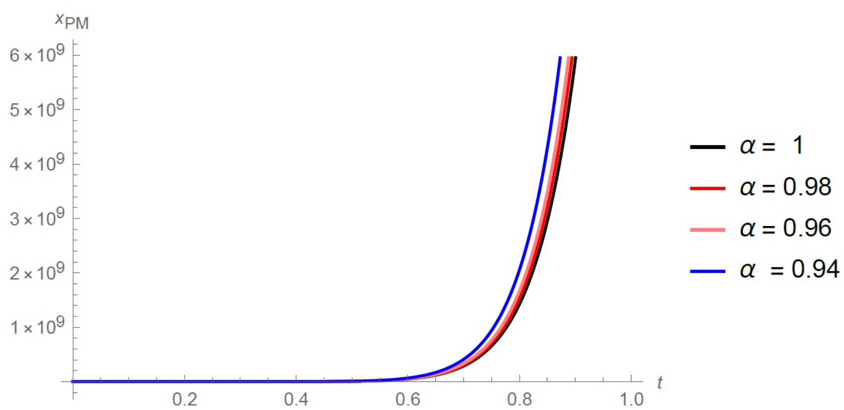


Figure 7: PM Solution by CD

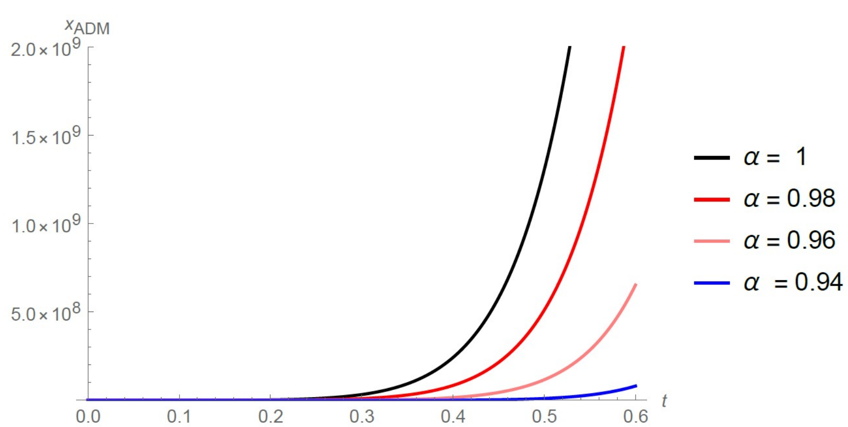


Figure 8: ADM Solution by CFD

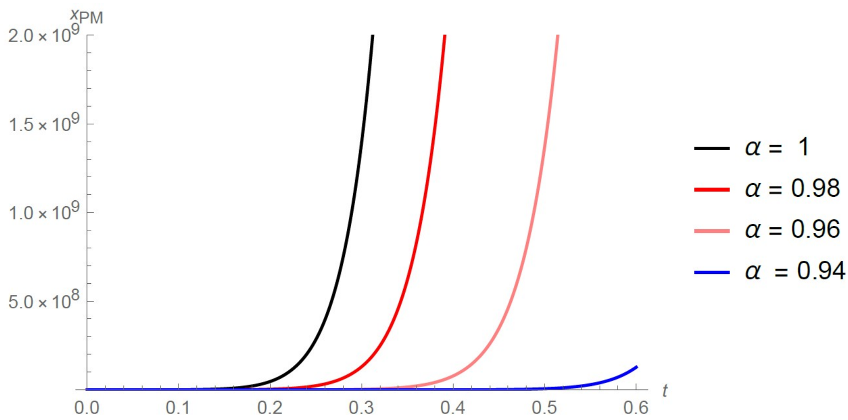


Figure 9: PM Solution by CFD

Figures 3 and 4 represent a comparison between the two used techniques by CD and CF at $\beta = 0.95$ and different cases of $\alpha$ $(\alpha = 1,\ 0.98,\ 0.96$ and $0.94)$. Tables 7, 8, 9, and 10 compare the time response for the RLC circuit based on $\alpha = 0.98$ by using Mathematica code. The results from these tables indicate not only the accuracy of the solutions but also the efficiency of these methods.

Table 7: Time response by CD at $\alpha = 0.95$

| $m$ | $PM\ time\ in\ sec$ |
|---|---|
| 3 | 52.033 |
| 6 | 216.422 |
| 9 | 370.141 |

Table 8: Time response by CD at $\alpha = 0.95$

| $m$ | $ADM\ time\ in\ sec$ |
|---|---|
| 3 | 45.657 |
| 6 | 190.531 |
| 9 | 207.422 |

Table 9: Time response by CF at $\alpha = 0.95$

| $m$ | $PM\ time\ in\ sec$ |
|---|---|
| 5 | 0.516 |
| 10 | 0.672 |
| 15 | 0.781 |

Table 10: Time response by CF at $\alpha = 0.95$

| $m$ | $ADM\ time\ in\ sec$ |
|---|---|
| 5 | 0.327 |
| 10 | 0.344 |
| 15 | 0.374 |

## 5 Conclusion

Modelling real-world phenomena with fractional calculus has been approved as an important mathematical tool. As a final step, this study generated a fractional model of a parallel RLC circuit by solving it with the ADM and the PM, which are denoted by CD and CF.

Furthermore, numerical examples have been provided to demonstrate the effectiveness of the two techniques according to fractional operator ranges in solving various cases of this model using the MATHEMATICA 11.3 software. These two methods are also theoretically investigated, and it is reasonable to conclude that they are each convergent based on the theorems mentioned above. The strategies have been carefully mathematically tested, confirming not only their existence and uniqueness but also their reliability. The outcomes show that PM needs less time than ADM. At the same interval and number of terms, ADM provides greater precision than Picard.

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