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Cosmology in Randers Space: Theory and Observations

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Abstract. In this paper, we investigate Finsler-Randers (FR) cosmological models within the framework of modified gravity theories, particularly in the context of Einstein gravity. A power-law form of the scale factor is assumed under the emergent scenario, which allows for a nonsingular cosmological evolution. Exact solutions to the corresponding field equations are derived and analyzed in detail. The behavior of cosmological parameters is thoroughly examined, and it is shown that the solutions asymptotically approach the isotropic Friedmann–Robertson–Walker (FRW) cosmological model. we explore the observational viability of these models by placing constraints on the model parameters using two independent datasets:by using 28 paints of $H(Z)$ data and 580 paints os union 2.1 compilation data and compare this results with the result of ΛCDM. At the 1-σ confidence level, the constraints obtained from the $H(Z)$ data (for the case $z <= 1$ with $\sigma >= 5$) are q =-0.15 and H_0 = 66.40 $\pm$ 2.6 $kms^{-1}Mpc^{-1}$ together with χ^2_δ= 0.94, χ^2_{min}= 26.34, while the constraints from supernova data are q =-0.35 and H_0 = 69.160 $\pm$ 3.32 $kms^{-1}Mpc^{-1}$ together with χ^2_δ = 0.4266. These results are compared with those predicted by the standard ΛCDM model.

1 Introduction

In the past years, a model called the Finsler-Rander cosmological model (CM) has gained attention after FR [1]. Metrical Reimann geometry extensions that lead to generalized gravitational field theory provide a Finslerian geometrical structure in a manifold. In the last few years, the applicability of Finsler geometry in its FR context has been discussed in the fields of astrophysics, cosmology, and general relativity. The spatially homogeneous CMs allow cosmological studies to be extended to distorted and rotating universes, evaluating the effects of anisotropy on the synthesis of primordial elements as well as the measured spectrum anisotropy known as cosmic microwave background radiation (CMBR) [2]. Anisotropic cosmologies have been studied for a variety of theoretical reasons in addition to observable ones. These include (i) a strong indication of a singularity in our history if certain reasonable criteria are met [3]. The type found in Friedman Robertson Walker (FRW) models may be very different from this one that has been considered [4]. (ii) The Chaotic Cosmology program [5] looked for a method to discuss, regardless of the initial circumstances, why the homogeneity and isotropy observed should exist [6, 7, 8].

In cosmology, anisotropic CMs have been classified into a large class, which is often studied [9]. Some theoretical justifications for the presence of an anisotropic phase that approaches an isotropic example [5] (chaotic cosmology). Also, a preferable member to exclude the theory of particular initial conditions in FRW models are anisotropic cosmological models [31]. Irregular expansion mechanisms could potentially be used to describe the early universe. It would be beneficial to look into CMs in which anisotropies that are present at the beginning of expansion fade away over time. Such models have earned the attention of many researchers [10]. The Robertson-Walker model with weak anisotropy in generalized metric spacetime has been studied by Stavrinos et al. [11]. Basilakos and Stavrinos conducted research on cosmological equivalence between the Porrati, Gabadadze, and Dvali gravity models (DGP) and the Finsler-Randers space time (FRST) [12]. Recently some authors are studied in Finsler-Randers cosmological models in different modified theory of gravity and dynamical prospectives [13, 14, 15, 16, 25, 27, 30, 51]. This encourages scientists to take into account the FRST cosmology model of the universe. Here, in our suggestion of using Finsler-Randers cosmology to research the evolution of the universe in view of cosmologists' increased interest in this topic, The Finsler-Randers cosmology's FRW model has been examined in this essay.

2 Finsler-Randers Model in Einstein Theory

The FR cosmological framework is built upon Finsler geometry, which generalizes Riemannian geometry. Notice that a Riemannian geometry appears as a special case within the broader Finslerian setting. Below we discuss only the main features of the theory for more details see [17, 18, 19, 20]. Generally, a Finsler space is derived from a generating differentiable function $F(x, y)$ on a tangent bundle $F : TM_0 \longrightarrow R$ where $TM_0 = TM \setminus \{0\}$ on a manifold M. The function F is homogeneous of degree one in the variable with respect to $y = \dot{x} = \frac{dx}{dt}$ and it remains continuous at the zero section. In other words, F introduces a structure on the space-time manifold M that is called Finsler space-time. In the case of a FR space-time, the Finsler structure is specified by a Randers-type function,

$$F(x, y) = \alpha(x, y) + b_i(x)y^i, \quad \alpha(x, y) = \sqrt{a_{ij}y^iy^j}, \tag{2.1}$$

where a_{ij} is a Riemannian metric and $b_i = (b_0, 0, 0, 0)$ is a weak primordial vector field with $|b_i| << 1$. Now the Finslerian metric tensor g_{ij} is constructed by the Hessian of F^2

$$g_{ij} = \frac{1}{2}\frac{\partial^2 F^2}{\partial y^i \partial y^j}. \tag{2.2}$$

It is interesting to mention that the Cartan tensor $C_{ijk} = \frac{1}{2}\frac{\partial g_{ij}}{\partial y^k} = \frac{1}{4}\frac{\partial^3 F^2}{\partial y^i \partial y^j \partial y^k}$ is a significant ingredient of the Finsler geometry. Indeed it has been found in [11] that $b_0 = 2C_{000}$.
The field equations of Finslerian-Rander are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -\frac{8\pi G}{c^4}T_{ij}, \tag{2.3}$$

where R_{ij} =Finslerian Ricci tensor, $g_{ij} = \frac{F}{\alpha}a_{ij}$, T_{ij}= energy-momentum tensor and T = trace of the energy-momentum tensor. Modelling the expanding Universe as a Finslerian perfect fluid with four-velocity u_i for co-moving observers, we have

$$T_{ij} = -pg_{ij} + (\rho + p)u_iu_j, \tag{2.4}$$

where ρ= total energy density and p=pressure of the cosmic fluid respectively.

Thus the energy momentum tensor becomes

$$T_{ij} = diag(\rho, -pg_{11}, -pg_{22}, -pg_{33}). \tag{2.5}$$

Following the work of Kolassis et al. [21], Chatterjee and Banerjee [22] and Bali [23], we discuss briefly week, dominant and strong energy conditions in the context of Finslerian cosmology for our model.

We have $T_0^0 = \rho$, $T_1^1 = T_2^2 = T_3^3 = -p$ in the locally Minkowskian frame. Obviously the roots of matrix equation

$$|T_{ij} - rg_{ij}| = diag[(\rho - r), (r + p), (r + p), (r + p)] = 0. \tag{2.6}$$

give the eigenvalues r for our energy momentum tensor as $r_0 = \rho$ and $r_1 = -p = r_2 = r_3$. In the context of a FRW metric

$$a_{ij} = diag\left(1, -\frac{a^2}{1 - kr^2}, -a^2r^2, -a^2r^2sin^2\theta\right), \tag{2.7}$$

where a is the function of time t only and k is the curvature parameter having values $+1, 0, -1$ for closed, flat and open models respectively. The non-zero components of the Finslerian Ricci tensors are

$$R_{00} = 3\left(\frac{\ddot{a}}{a} + \frac{3}{4}\frac{\dot{a}}{a}\dot{u_0}\right) \tag{2.8}$$

and

$$R_{ii} = -\left(\frac{a\ddot{a} + 2\dot{a}^2 + 2k + \frac{11}{4}a\dot{a}\dot{u_0}}{\triangle_{ii}}\right), \tag{2.9}$$

where $\triangle_{11} = \frac{1}{1-kr^2}$, $\triangle_{22} = r^2$ and $\triangle_{33} = r^2 sin^2\theta$.
The gravitational FR field equations (2.3), for co-moving observers, FRW Einstein Field equations are

$$\frac{\ddot{a}}{a} + \frac{3}{4}\frac{\dot{a}}{a}Z_t = -\frac{4\pi G}{3}(\rho + 3p) \tag{2.10}$$

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{k}{a^2} + \frac{11}{4}\frac{\dot{a}}{a}Z_t = 4\pi G(\rho - p), \tag{2.11}$$

where the over-dot denotes derivative with respect to the cosmic time t and $Z_t = \dot{b_0} < 0$ [11]. From Eq. (2.2) and (2.3), we get

$$H^2 + \frac{k}{a^2} + HZ_t = \frac{8\pi G}{3}\rho. \tag{2.12}$$

Obviously, the extra term $H(t)Z_t$ in the modified Friedmann equation (2.12) affects the dynamics of the Universe. If we consider $b_0 \equiv 0$ or ($C_{000} \equiv 0$, $\frac{F}{\alpha} = 1$), which implies $Z_t = 0$, then the field equations (2.10) and (2.11) reduce to the nominal Einstein's equations, a solution of which is the usual Friedman equation.

Here we discuss two different physically viable cosmologies, which have physical interests to describe the decelerating and accelerating phases of universe.

3 Power-law solution in Emergent Scenario

In this scenario, let us consider a Universe with the scale factor is given as [24]:

$$a = (\alpha t + \beta)^{\delta} \tag{3.1}$$

where α, β and δ are constants. For $\delta > 1$ it gives an accelerating Universe.
Power-law solutions play a central role in standard cosmology, as they provide a useful framework for understanding the behavior of more general cosmological models across different evolutionary stages of the Universe, including the radiation-dominated, matter-dominated, and dark-energy-dominated eras.
Now, Hubble parameter(H)

$$H = \frac{\dot{a}}{a} = \frac{\delta\alpha}{\alpha t + \beta} \tag{3.2}$$

while the present expansion rate of the Universe is obtain by

$$H_0 = \frac{\delta\alpha}{\alpha t_0 + \beta} \tag{3.3}$$

Relation between the scale factor a and redshift z are given by

$$\frac{a}{a_0} = \frac{1}{1+z} = \frac{(\alpha t + \beta)}{(\alpha t_0 + \beta)}^{\delta} \tag{3.4}$$

where a_0 is the present value of scale factor and t_0 represent the present age of the universe. The age of Universe at redshift z is given as

$$t(z) = \frac{\zeta}{H_0(1+z)^{\frac{1}{\delta}}} \tag{3.5}$$

The scale factor a and the redshift z are related by $a = \frac{a_0}{1+z}$. Accordingly, the Hubble parameter expressed in terms of the redshift takes the form

$$H = H_0(1+z)^{1+q} \tag{3.6}$$

Equation (18) shows that the expansion history of the Universe in power-law cosmology depends on the parameters H_0 and q. In this work, we examine a well-behaved power-law cosmological model with a focus on these two parameters. We also derive the observational constraints on H_0 and q using the latest 30 H(z) data points together with the 580 data points from the Union 2.1 supernova compilation.

4 Cosmological constraints using the observational H(z) Data

[32] determined nine H(z) data points in the range $0 < z < 2.3$ by using the differential ages of passively evolving galaxies determined the Gemini Deep Deep Survey along with archival data. More recently, [40] reported 11 additional H(z) measurements at different redshifts based on the differential ages of red-envelope galaxies, while [34] obtained three further H(z) data points. The newly H(z) data points have been used to constrain parameters of various cosmological models [35, 36, 42, 43, 44, 45]. Here, we use 30 observational H(z) data points given in Table 1 of the paper by [45] and the one at z = 0 estimated in the work by [29]. For this sake, we define the χ^2 as

$$\chi^2_H(q, H_0) = \sum_{i=1}^{i=28} \frac{\left(H_{exp}(z_i, q, H_0) - H_{obs}(z_i)\right)^2}{\sigma_i^2} \tag{4.1}$$

Where H_{exp} is the expected value of the Hubble parameter, H_{obs} is the observational value and σ_i is the corresponding 1 σ error. The linear varying deceleration parameter cosmological model contains two independent parameters namely q and H_0. These results are in good agreement with the recent findings of Singha & Singh (2025), Singh et al. (2025), and Kotambkar et al. (2025), who have also obtained comparable estimates of the Hubble parameter in the framework of modified cosmological models[46, 47, 48, 49, 50]. Hence we perform a grid search in the entire parametric space ($q > -1$ and $H_0 > 0$) to find the best fit model. As a result, we obtain the best fit values of the parameters as q =-0.15 and H_0 = 66.40 $\pm$ 2.6 $kms^{-1}Mpc^{-1}$ together with χ^2_δ= 0.94, χ^2_{min}= 26.34,$\chi^2_{reduced}$= 0.94 and the values of the parameters with 1 σ error are obtained as q =-0.15 and H_0 = 66.40 $\pm$ 2.6 $kms^{-1}Mpc^{-1}$ together with χ^2_δ= 0.94, χ^2_{min}= 26.34, where χ^2_δ=χ^2_{min}/(degree of freedom). The negative value of q suggests that the power-law cosmological model fitted with the newly obtained $H(z)$ data confirms the accelerating nature of the present-day Universe. By comparing the values of χ^2_δ of the models, we find that the exponential power law cosmological model fit well to the latest $H(z)$ data.

The observed value of Hubble parameter $H(z)$ at different redshift parameters (z) given in table 1 (Hubble Data) are employed to draw the curve corresponding to the curve of $H(z)$. Figure (4) shows the plot of observed $H(z)$ (vertical small lines) and calculated $H(z)$ (solid curve) versus redshift parameters (z).

5 Constraints from observational Union 2.1 SN Ia Data

We now constrain the above-mentioned parameters using Type Ia supernova observations, which serve as one of the most direct probes of cosmological expansion. SNe Ia are widely regarded as standard candles, allowing estimation of the apparent magnitude $m(z)$ at peak brightness after applying appropriate corrections. They are considered to provide some of the strongest constraints on cosmological parameters. In these observations, the apparent magnitude $m(z)$ of each supernova and its corresponding redshift$z$ are directly measured. In this investigation, we

Redshift (z)	Supernovae I_a H(z)	error σ_H	Our model (H(z))	Reference
0.070	69	19.6	72.97	[37]
0.10	69	12	75.02	[32]
0.12	68.6	16.2	76.38	[37]
0.17	83	8	79.78	[32]
0.179	75	4	80.45	[26]
0.199	75	5	81.80	[26]
0.20	72.9	29.6	81.83	[37]
0.24	79.69	2.65	84.55	[38]
0.27	77	14	86.60	[32]
0.28	88.8	36.6	87.28	[37]
0.350	76.3	5.6	92.05	[38]
0.352	83	14	92.15	[26]
0.40	95	17	95.45	[32]
0.43	86.45	3.68	97.49	[39]
0.44	82.6	7.8	98.18	[39]
0.48	97	62	100.9	[40]
0.593	104	13	108.4	[26]
0.600	87.9	6.1	109.1	[39]
0.680	92	8	114.5	[26]
0.730	97.3	7	117.9	[39]
0.781	105	12	121.3	[26]
0.875	125	17	127.5	[26]
0.88	90	40	128.1	[40]
0.90	117	23	129.5	[32]
1.037	154	20	138.4	[26]
1.30	168	17	156.7	[32]
1.43	177	18	165.6	[32]
1.53	140	14	172.4	[32]
1.75	202	40	187.4	[32]
2.3	224	08	224.8	[41]

Table 1. Hubble parameter versus redshift data (where $H(z)$ and σ_H are in km s^{-1} Mpc^{-1}).

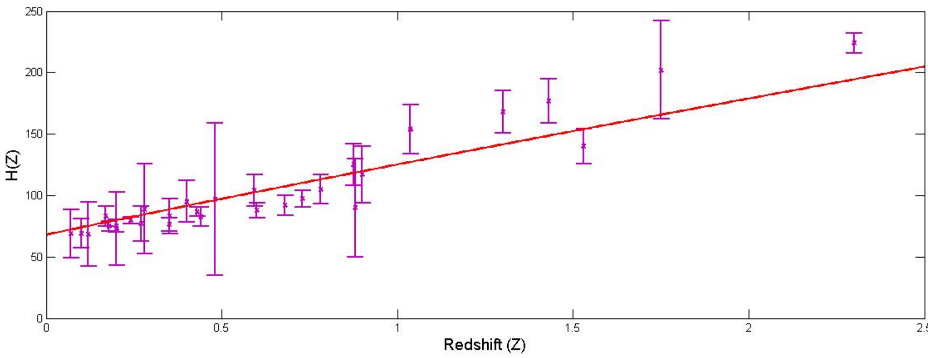

Figure 1. The plot of Hubble parameter $H(z)$ with Redshift z.

work with recently released Union2.1 SN IA Data compilation set of 580 Union2.1 SN IA data points[29]. The apparent magnitude m of the supernova is related to the luminosity distance d_L of the supernova through

$$m = M + 5\log_{10}\left(\frac{d_L}{1Mpc}\right) + 25 \tag{5.1}$$

where M is the absolute magnitude, which is believed to be constant for all SNe Ia. Equation (13) can be written as

$$m = M + 5\log_{10}(D_L(z)) - 5\log_{10} H_0 + 52.38 \tag{5.2}$$

It is convenient to work with Hubble free luminosity distance given by

$$D_L(z) = \frac{H_0}{c} d_L(z) \tag{5.3}$$

The distance modulus $\mu(z)$ is given by

$$\mu(z) = m - M = 5\log_{10}(D_L(z)) - 5\log_{10} H_0 + 52.38 \tag{5.4}$$

The Hubble free luminosity distance D_L, in the present case, can be expressed as

$$D_L(z) = (1+z)\int_0^z \frac{H_0}{H(z)} dz = \frac{1}{q}\left[(1+z) - \frac{1}{(1+z)^{q-1}}\right] \tag{5.5}$$

SNe Ia are always used as standard candles, and are believed to provide strongest constraints on the cosmological parameters. In this case, χ^2 can be defined as

$$\chi^2_{SN}(q, H_0) = \sum_{i=1}^{i=580} \frac{(\mu_{exp}(z_i, q, H_0) - \mu_{obs}(z_i))^2}{\sigma_i^2} \tag{5.6}$$

After carrying out a grid search over the entire parameter space $(q > -1)$ and $H_0 > 0$, we obtained that the best fit values of the parameters are q =-0.35 and H_0 = 69.160 together with χ^2_δ = 0.4266 . Furthermore, fitting the linear deceleration parameter cosmological model with the 580 SNe Ia data confirms the cosmic acceleration with q = -0.35. By comparing the values of χ^2_δ value shows that this model provides a good fit to the latest SN Ia observations.

The observed value of Hubble parameter $H(z)$ at different redshift parameters (z) given in table 1 (Hubble Data) are used to construct the corresponding $H(z)$ curve. Figure (4) presents the comparison, where the vertical bars represent the observed $H(z)$ values and the solid line denotes the calculated $H(z)$ as a function of redshift (z).

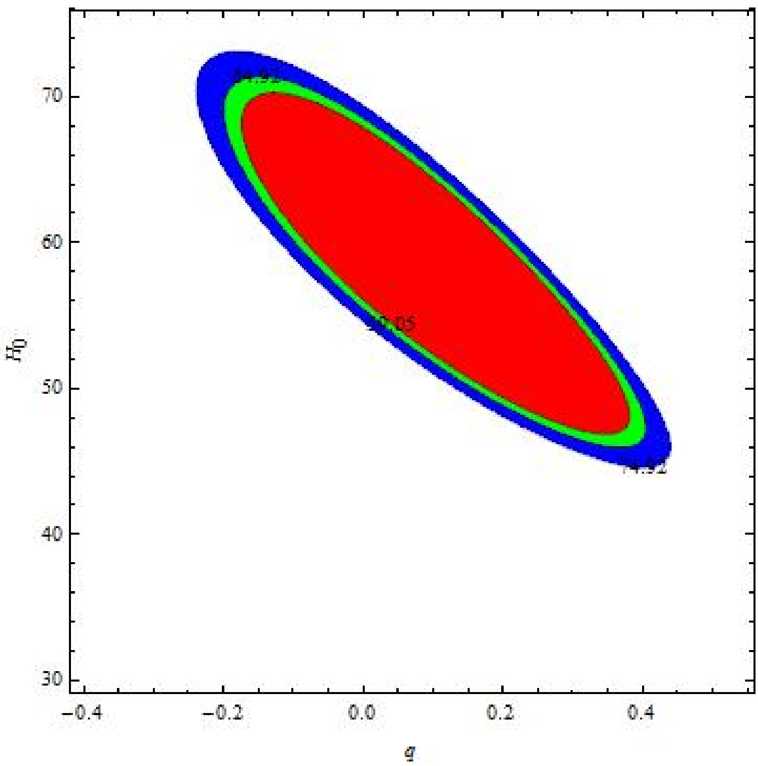

Figure 2. The likelihood contour plot of Hubble parameter (H_0) with deceleration parameter (q).

6 Cosmological Solutions

Using Eq. (2.12) and (3.2), the energy density evolving as

$$\rho = \frac{3}{8\pi G}\left[\frac{\alpha^2\delta^2}{(\alpha t+\beta)^2} + \frac{\alpha\delta}{\alpha t+\beta}Z_t + \frac{k}{(\alpha t+\beta)^{2\delta}}\right] \tag{6.1}$$

By using Eq. (2.10), (3.1) and (6.1), the pressure is given by

$$p = \frac{-3\alpha^2\delta^2+2\alpha\delta}{8\pi G(\alpha t+\beta)^2} - \frac{5\alpha\delta Z_t}{16\pi G(\alpha t+\beta)} - \frac{1}{8\pi G}\frac{k}{(\alpha t+\beta)^{2\delta}}. \tag{6.2}$$

Subcase 6a: When $Z_t = -\exp(-t)$

Substituting the value of Z_t in Eq. (6.1) and (6.2), we obtain the value ρ and p respectively as

$$\rho = \frac{3}{8\pi G}\left[\frac{\alpha^2\delta^2}{(\alpha t+\beta)^2} - \frac{\alpha\delta}{(\alpha t+\beta)\exp(t)} + \frac{k}{(\alpha t+\beta)^{2\delta}}\right]. \tag{6.3}$$

By using Eq. (3.2) and (6.2), the pressure is given by

$$p = \frac{-3\alpha^2\delta^2+2\alpha\delta}{8\pi G(\alpha t+\beta)^2} + \frac{5\alpha\delta}{16\pi G(\alpha t+\beta)\exp(t)} - \frac{1}{8\pi G}\frac{k}{(\alpha t+\beta)^{2\delta}}. \tag{6.4}$$

Subcase 6b: When $Z_t = -t^{-n}$

Substituting the value of Z_t in Eq. (6.1) and (6.2), we obtain the ρ and p respectively as

$$\rho = \frac{3}{8\pi G}\left[\frac{\alpha^2\delta^2}{(\alpha t+\beta)^2} - \frac{\alpha\delta}{\alpha t^{n+1}+\beta t^n} + \frac{k}{(\alpha t+\beta)^{2\delta}}\right]. \tag{6.5}$$

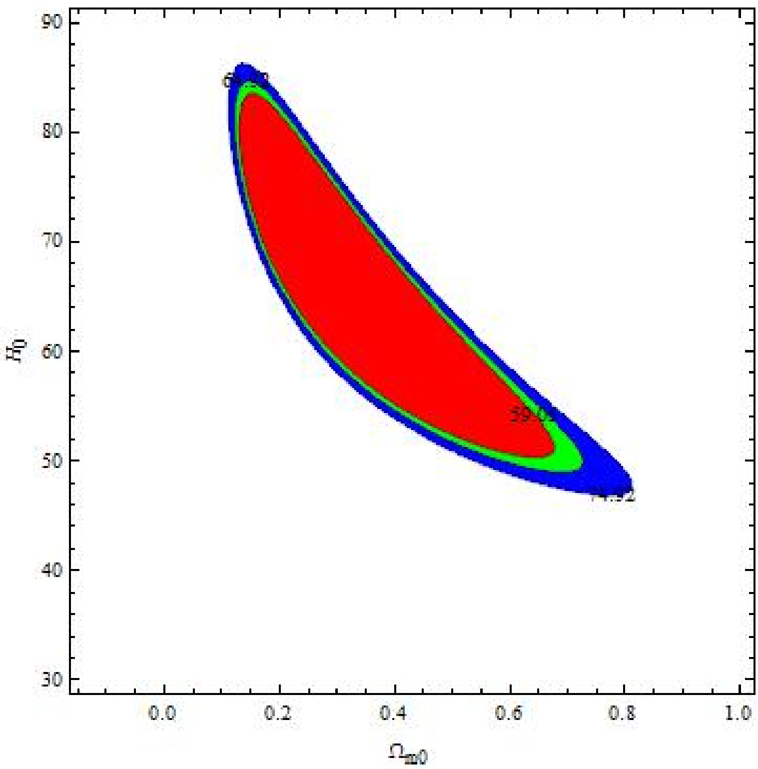

Figure 3. The likelihood contour plot of Hubble parameter (H_0) with parameter (Ω_m).

$$p = \frac{-3\alpha^2\delta^2 + 2\alpha\delta}{8\pi G(\alpha t + \beta)^2} + \frac{5\alpha\delta}{16\pi G(\alpha t^{n+1} + \beta t^n)} - \frac{1}{8\pi G}\frac{k}{(\alpha t + \beta)^{2\delta}}. \tag{6.6}$$

7 Conclusion

In this paper, we studied the Finsler–Randers cosmological model within the framework of modified gravity. Specifically, we examined its behavior in Einstein's theory by considering the cases $Z_t = -\exp(-t)$ and $-t^{-n}$. At $t \to \infty$ we obtained $Z_t \to 0$, and consequently, the Finsler–Randers cosmological model reduces to the standard Friedmann–Robertson–Walker model. We derive the observational constraints on the model parameters using 28 data points of $H(Z)$ measurements and 580 data points from the Union 2.1 supernova compilation, and then compare these results with those of the standard ΛCDM model. At the 1-σ confidence level, the constraints obtained from the $H(Z)$ data (for the case $z <= 1$ with $\sigma >= 5$) are q =-0.15 and H_0 = 66.40 ± 2.6 $kms^{-1}Mpc^{-1}$ together with χ^2_δ= 0.94, χ^2_{min}= 26.34, while the constraints from supernova data are q =-0.35 and H_0 = 69.160 ± 3.32 $kms^{-1}Mpc^{-1}$ together with χ^2_δ = 0.4266. We derive cosmological constraints on the linear deceleration parameter using the latest observational data. The analysis shows that the model describes an expanding Universe that tends toward isotropy at large values of t. Overall, the results of this study are consistent with the observed features of the Universe.

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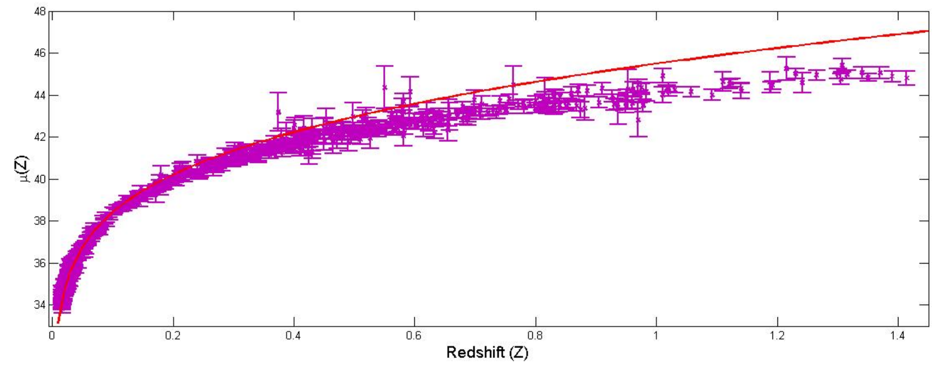

Figure 4. The plot of parameter $\mu(z)$ with Redshift z.

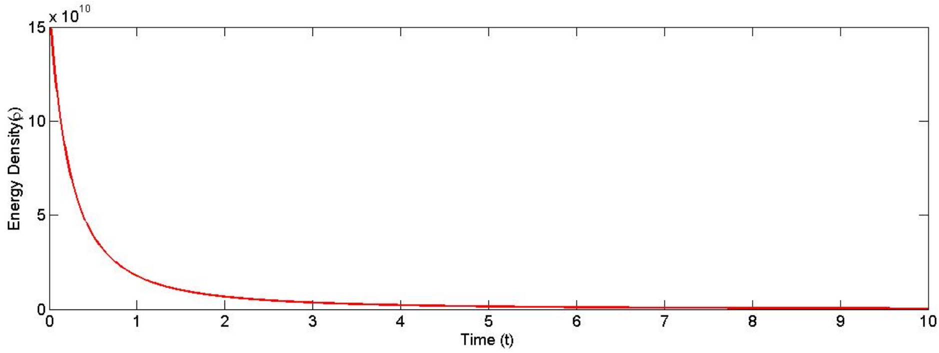

Figure 5. The plot of energy density ρ with time t.

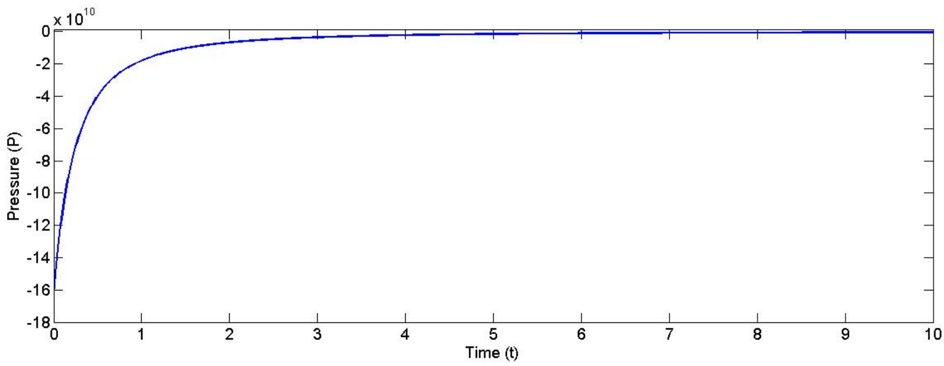

Figure 6. The plot of pressure P with time z.

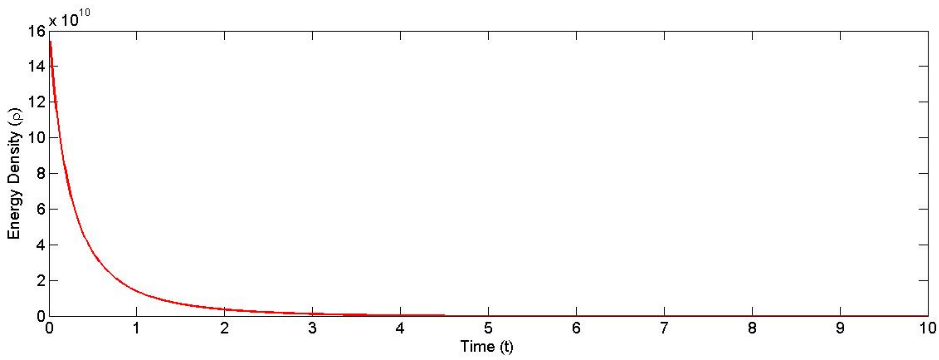

Figure 7. The plot of energy density ρ with time t.

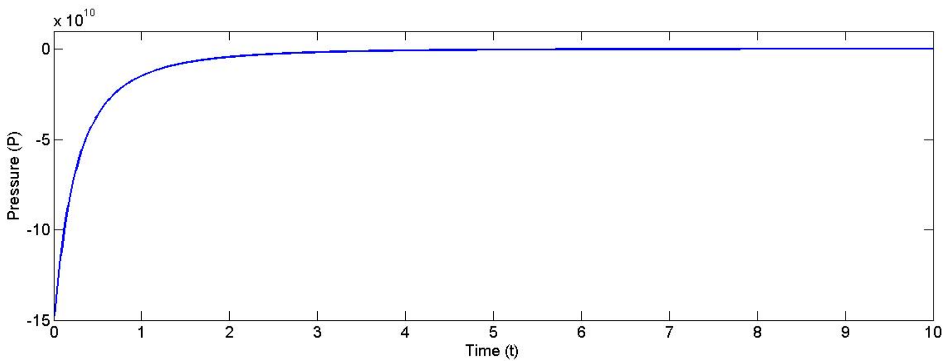

Figure 8. The plot of pressure P with time z.

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