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Labeling of Power Graph of Some Finite Groups

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Abstract In this paper, the power graph of a group $\mathcal{G}$ is a graph $\mathcal{P}(\mathcal{G})$ with a vertex set $\mathcal{G}$ and two vertices x and y, $x \neq y$ are adjacent if there exists some integer k such that $x = y^k$ or $y = x^k$. We study magic, supermagic, graceful, edge-magic, super edge-magic and antimagic labeling of the power graph of some finite groups.

1 Introduction

The directed power graph of a group $\mathcal{G}$ is a digraph whose vertices are elements of $\mathcal{G}$ and $a, b \in \mathcal{G}$ are adjacent in $\mathcal{P}(\mathcal{G})$ if and only if $a^n = b$ for some positive integer n [1]. Further the concept of an undirected power graph of a group $\mathcal{G}$ was introduced by [2] as a graph $\mathcal{P}(\mathcal{G})$ with vertex set $\mathcal{G}$ and two vertices $x \neq y$ are adjacent if and only if there exists some positive integer m such that $x = y^m$ or $y = x^m$. Some related studies are in [3], [4]. A group $\mathcal{G}$ is cyclic if there is an element a in $\mathcal{G}$ such that $\mathcal{G}= \{a^n | n \in \mathbb{Z}\}$ [5]. The concept of graph labelings was introduced by Alex Rosa [6] in late 1960's. Hartsfield and Ringel [7] introduced the concept of an antimagic graph and conjectured that every connected graph except K_2 is antimagic. All graphs in this paper are finite, undirected and simple. For a graph G, the set of vertices and edges is denoted by $V(G)$ and $E(G)$, respectively. A simple graph $G = (V, E)$ is a graph without loops and the parallel edges. An element of the edge set is a two-element subset of the vertex set. We denote an edge e connecting vertex u to a vertex v by $e = \{u, v\}$, which is an unordered pair, so $\{u, v\} = \{v, u\}$ [8]. A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge. A star graph is a complete bipartite graph $K_{1,n}$ with partitions where one partition has a single vertex and the other has n vertices. A cycle graph C_n, where $n \geq 3$ is a graph consisting of n vertices connected in a single closed loop, where each vertex has a degree 2. A Dutch windmill graph denoted by D_n^m or $Wd(n, m)$,$m \geq 2, n \geq 5$ is a graph obtained by taking m copies of C_n with a vertex in common [9]. A friendship graph T_n consists of n triangles with exactly one common vertex called center [10]. Thus the dutch windmill graph D_3^m is nothing but a friendship graph T_m. A graph is Eulerian if it contains an Eulerian cycle, which is a cycle visiting every edge of a graph exactly once. A labeling of a graph is a mapping that sends some set of the graph elements to a set of positive integers. If the domain is the vertex-set or the edge-set, the labelings are called vertex labelings or edge labelings. Moreover, if the domain is $V \cup E$, then the labelings are called total labelings. An edge labeling is a function $f : E \to \mathbb{Z}^+$, where $\mathbb{Z}^+ \subset \mathbb{Z}$. In other words, it is a labeling of all the edges by integers. A graph with such a labeling is an edge labeled graph [8]. A function f from a set of vertices in a graph G to the set of integers $\{0, 1, 2, \cdots, q\}$, where q is the number of edges in G, so that each edge $\{x, y\}$ is assigned the label $|f(x) - f(y)|$, with all the labels distinct is known as $\beta - valuation$ [6]. A graceful labeling of a graph with m edges is a labeling of its vertices with some subset of the integers from 0 to m inclusive, such that no two vertices share a label, and each edge is uniquely identified by the absolute difference between its endpoints, such that this magnitude lies between 1 and m inclusive. A graph that admits a graceful labeling is said to be a graceful graph [11]. An edge-magic total labeling sometimes called edge-magic labeling is a bijection $f : V \cup E \to \{1, 2, \cdots, |V \cup E|\}$ with the property that there is a constant k such that at any edge $\{x, y\}$, $f(x) + f(xy) + f(y) = k$ [12]. In other words, A (p, q) graph G is edge-magic if there exists a bijective function $f : V(G) \cup E(G) \to \{1, 2, \cdots, p + q\}$ such that $f(x) + f(xy) + f(y) = k$ is a constant called the valence of f, for any edge $\{x, y\}$ of G. Furthermore, f is a super edge-magic labeling if $f(V(G)) = \{1, 2, \cdots, p\}$. Thus, a super edge-magic graph is a graph that admits a super edge-magic labeling [13]. Given an edge-magic labeling f of a (p, q) graph G, it is always possible to find a complementary edge-magic labeling $\bar{f}$ such that $\bar{f}(x) = p + q + 1 - f(x)$, for every $x \in V(G) \cup E(G)$ [13]. A magic labeling is a one-to-one mapping $f : E \to \mathbb{R}^+$ with the property that there is a constant k such that $\sum\limits_{y \in N(x)} f(x, y) = k$, where $N(x)$ is the set of vertices adjacent to x. A magic graph is a graph that admits a magic labeling [14]. A supermagic labeling is a one-to-one mapping $f : E \to \{a, a + 1, a + 2, \cdots, a + |E|\}$, for a positive integer a with the property that there is a constant k such that at any vertex x, $\sum\limits_{y \in N(x)} f(x, y) = k$, where $N(x)$ is the set of vertices adjacent to x. Further, a supermagic graph is a graph that admits a supermagic labeling [15]. A graph G is called Antimagic if the n edges of G can be distinctly labeled 1 through n in such a way that while taking the sum of the edge labels incident to each vertex, the sums will all be different [8].

Theorem 1.1. *[15] A complete graph K_n is magic for $n = 2$ and for all $n \geq 5$.*

Theorem 1.2. *[16] K_n is supermagic for $n \geq 5$ if and only if $n > 5$ and $n \not\equiv 0(mod4)$.*

Theorem 1.3. *[17] For a finite group $\mathcal{G}$, $\mathcal{P}(\mathcal{G})$ is complete if and only if $\mathcal{G}$ is a cyclic group of order 1 or p^m, for some prime p and a positive integer m.*

Theorem 1.4. *[18] A complete graph K_n is graceful only if $n \leq 4$.*

Theorem 1.5. *[19] The Dutch windmill graph $Wd(3, m)$ is graceful if and only if $m \equiv 0(mod4)$ or $m \equiv 1(mod4)$.*

Theorem 1.6. *[6] An Eulerian graph with number of edges $m \equiv 1(mod4)$ or $m \equiv 2(mod4)$ cannot be graceful.*

Theorem 1.7. *[13] A (p, q) graph G is super edge-magic if and only if there exists a bijective function $f : V(G) \to \{1, 2, \cdots, p\}$ such that the set $S = \{f(u) + f(v) | uv \in E(G)\}$ consists of q consecutive integers. In such a case, f extends to a super edge-magic labeling of G with valence $k = p + q + s$, where $s = min(S)$ and $S = \{f(u) + f(v) | uv \in E(G)\} = \{k - (p + 1), k - (p + 2), \cdots, k - (p + q)\}$.*

Theorem 1.8. *[20] Given an edge-magic labeling f of a (p, q) graph G, it is always possible to find a complementary edge-magic labeling $\bar{f}$ such that $\bar{f}(x) = p + q + 1 - f(x)$ for every $x \in V(G) \cup E(G)$. Note that this operation does not preserve super edge-magic labeling unless $G \cong \bar{K}_n$.*

Theorem 1.9. *[13] Every star $K_{1,n}$ is super edge-magic. Moreover, there are exactly $3 \cdot 2^n$ distinct edge-magic labelings of $K_{1,n}$ of which only two are super edge-magic labelings up to isomorphism.*

2 Magic, Supermagic and Graceful labeling of Power graphs

Proposition 2.1. *Let $\mathcal{G}$ be a finite cyclic group of order p^m. Then $\mathcal{P}(\mathcal{G})$ is magic for $p = 2, m = 1$, $m \geq 3$; $p = 3, m \geq 2$, and for all primes $p \geq 5, m \in \mathbb{Z}^+$.*

Proof. Let $\mathcal{G}$ be a finite cyclic group of order p^m. Then $\mathcal{P}(\mathcal{G})$ is isomorphic to $\mathcal{P}(\mathbb{Z}_{p^m})$. Since $\mathbb{Z}_{p^m}$ is a finite cyclic group of order p^m, $\mathcal{P}(\mathbb{Z}_{p^m})$ is isomorphic to a complete graph K_{p^m}, by Theorem 1.3. Hence the result, by Theorem 1.1. □

Proposition 2.2. *Let $\mathcal{G}$ be a finite cyclic group of order p^m. Then $\mathcal{P}(\mathcal{G})$ is supermagic for $p = 1, m = 2$; $p = 3, 5, m \geq 2$, and for all primes $p \geq 7, m \in \mathbb{Z}^+$.*

Proof. Let $\mathcal{G}$ be a finite cyclic group of order p^m. Then $\mathcal{P}(\mathcal{G})$ is isomorphic to $\mathcal{P}(\mathbb{Z}_{p^m})$. Since $\mathbb{Z}_{p^m}$ is a finite cyclic group of order p^m, $\mathcal{P}(\mathbb{Z}_{p^m})$ is isomorphic to a complete graph K_{p^m}, by Theorem 1.3. Also, K_2 is a supermagic graph. Therefore we can conclude the result, by Theorem 1.2. □

Proposition 2.3. *Let $\mathcal{G}$ be a finite cyclic group of order p^m, where p is a prime and $m \in \mathbb{N}$. Then $\mathcal{P}(\mathcal{G})$ is not graceful for $p = 2, m \geq 3$; $p = 3, m \geq 2$, and for all primes $p > 5, m \in \mathbb{Z}^+$.*

Proof. Let $\mathcal{G}$ be a finite cyclic group of order p^m, where p is a prime and $m \in \mathbb{N}$. Then $\mathcal{P}(\mathcal{G})$ is isomorphic to $\mathcal{P}(\mathbb{Z}_{p^m})$. By Theorem 1.3, $\mathcal{P}(\mathbb{Z}_{p^m})$ is isomorphic to a complete graph K_{p^m}. Also, K_n is graceful only if $n \leq 4$, by Theorem 1.4. Hence the result. □

Proposition 2.4. *The power graphs $\mathcal{P}(S_3)$ and $\mathcal{P}(A_3)$ are graceful but not magic, where S_3 and A_3 are symmetric and alternating groups of degree 3, respectively.*

Proof. Consider a symmetric group $S_3 = \{(1), (12), (13), (23), (123), (132)\}$ and an alternating group $A_3 = \{(1), (123), (132)\}$. The number of edges in $\mathcal{P}(S_3)$ and $\mathcal{P}(A_3)$ are 6 and 3, respectively. Define an injective function $f : V(\mathcal{P}(S_3)) \to \{0, 1, 2, 3, 4, 5, 6\}$ as follows:

$$f((1)) = 0, f((12)) = 4, f((13)) = 5, f((23)) = 6, f((123)) = 1, f((132)) = 3.$$

Then the function f induces an injective function $f^* : E(\mathcal{P}(S_3)) \to \{1, 2, 3, 4, 5, 6\}$ defined as follows:

$$f^*((1), (12)) = 4, f^*((1), (13)) = 5, f^*((1), (23)) = 6, f^*((1), (123)) = 1$$

,

$$f^*((1), (132)) = 3, f^*((123), (132)) = 2.$$

Therefore $\mathcal{P}(S_3)$ admits a graceful labeling, see Figure 2. Hence $\mathcal{P}(S_3)$ is a graceful graph. By restriciting the same function to $V(\mathcal{P}(A_3))$, we can obtain a graceful labeling of $\mathcal{P}(A_3)$, see Figure 4. Also, $\mathcal{P}(A_3)$ is isomorphic to a complete graph K_3, which is not magic. Moreover, if we try to label the edges of $\mathcal{P}(S_3)$ by distinct positive integers, then the sum of the labels of the edges incident of vertex (1) is always greater than that of the vertices (12), (13) and (23), since (12), (13) and (23) are pendent vertices. Hence $\mathcal{P}(S_3)$ is not a magic graph. □

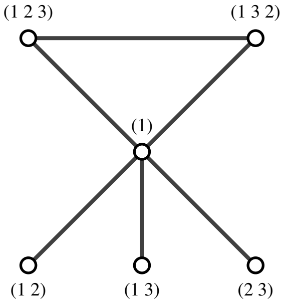

Figure 1. $\mathcal{P}(S_3)$

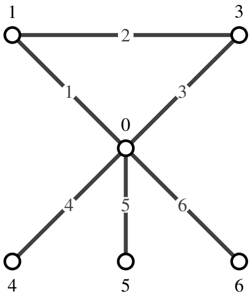

Figure 2. Graceful labeling of $\mathcal{P}(S_3)$

Proposition 2.5. *A power graph $\mathcal{P}(\mathbb{Z}_2^n)$ is graceful for all $n \in \mathbb{N}$.*

Proof. Let $V(\mathcal{P}(\mathbb{Z}_2^n)) = \{\bar{0}, x_1, x_2, \cdots, x_{2^n-1}\}$. In $\mathcal{P}(\mathbb{Z}_2^n)$, $o(x) = 2$, for all $x \neq \bar{0}$ implies every non-identity element is adjacent to only $\bar{0}$. The number of edges in $\mathcal{P}(\mathbb{Z}_2^n)$ is 2^n. We define a one to one mapping $f : V(\mathcal{P}(\mathbb{Z}_2^n)) \to \{0, 1, 2, \cdots, 2^n - 1\}$ by $f(\bar{0}) = 0, f(x_i) = i$, for all $1 \leq i \leq 2^n - 1$, which induces a one to one mapping $f^* : E(\mathcal{P}(\mathbb{Z}_2^n)) \to \{1, 2, \cdots, 2^n - 1\}$ defined by $f^*(\{\bar{0}, x_i\}) = i$, for all $1 \leq i \leq 2^n - 1$. Thus $\mathcal{P}(\mathbb{Z}_2^n)$ has a graceful labeling, see Figure 5 . □

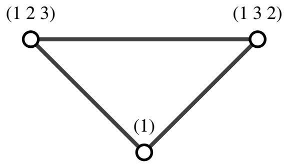

Figure 3. $\mathcal{P}(A_3)$

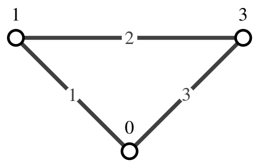

Figure 4. Graceful labeling of $\mathcal{P}(A_3)$

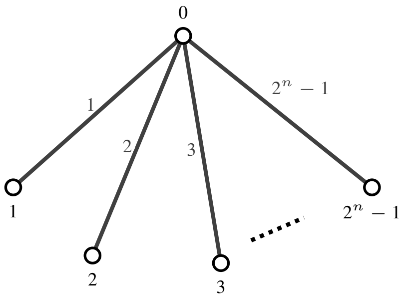

Figure 5. Graceful labeling of $\mathcal{P}(\mathbb{Z}_2^n)$

Proposition 2.6. *A power graph* $\mathcal{P}(\mathbb{Z}_3 \times \mathbb{Z}_3)$ *is graceful.*

Proof. Consider $\mathbb{Z}_3 \times \mathbb{Z}_3 = \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)\}$. The number of edges in $\mathcal{P}(\mathbb{Z}_3 \times \mathbb{Z}_3)$ is 12. Define an injective function $f : V(\mathcal{P}(\mathbb{Z}_3 \times \mathbb{Z}_3)) \to \{0,1,2,\cdots,12\}$ by

$$f(0,0) = 12, f(0,1) = 4, f(0,2) = 6, f(1,2) = 0, f(2,1) = 1,$$

$$f(1,1) = 5, f(2,2) = 2, f(1,0) = 7, f(2,0) = 3$$

which induces an injective function $f^* : E(\mathcal{P}(\mathbb{Z}_3 \times \mathbb{Z}_3)) \to \{1,2,\cdots,12\}$ defined as follows:

$$f^*((0,0),(0,1)) = 8, f^*((0,0),(0,2)) = 6, f^*((0,1),(0,2)) = 2, f^*((0,0),(1,2)) = 12,$$

$$f^*((0,0),(2,1)) = 11, f^*((1,2),(2,1)) = 1, f^*((0,0),(1,1)) = 7, f^*((0,0),(2,2)) = 10,$$

$$f^*((1,1),(2,2)) = 3, f^*((0,0),(1,0)) = 5, f^*((0,0),(2,0)) = 9, f^*((1,0),(2,0)) = 4.$$

Therefore $\mathcal{P}(\mathbb{Z}_3 \times \mathbb{Z}_3)$ admits a graceful labeling, see Figure 7. □

Proposition 2.7. *A power graph* $\mathcal{P}(\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3)$ *is graceful.*

Proof. Since $\mathcal{P}(\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3)$ contains 13 copies of C_3, it is isomorphic to $Wd(3,13)$, a dutch 13-windmill graph, which is graceful by Theorem 1.5. □

Proposition 2.8. *A power graph* $\mathcal{P}(\mathbb{Z}_3^n)$ *is graceful for all* $n \geq 2$.

Proof. We have already discussed the case for $n = 2, 3$ in Propositions 2.6, 2.7, respectively. Let $n \geq 4$. Since $\mathcal{P}(\mathbb{Z}_3^n)$ contains $\frac{3^n-1}{2}$ copies of C_3, $\mathcal{P}(\mathbb{Z}_3^n)$ is isomorphic to a dutch windmill graph $Wd(3, \frac{3^n-1}{2})$. By using binomial theorem, we can have the following expression:

$$\begin{aligned}\frac{3^n-1}{2} &= 2^{n-1} + \binom{n}{1}2^{n-2} + \binom{n}{2}2^{n-3} + \cdots + \binom{n}{n-3}2^2 + \binom{n}{n-2}2^1 + \binom{n}{n-1} \\ &= 2^{n-1} + \binom{n}{1}2^{n-2} + \binom{n}{2}2^{n-3} + \cdots + \binom{n}{n-3}2^2 + n^2\end{aligned}$$

which gives $\frac{3^n-1}{2} \equiv \begin{cases} 0(mod 4) & \text{,if n is even,} \\ 1(mod 4) & \text{,if n is odd.} \end{cases}$ Thus $\mathcal{P}(\mathbb{Z}_3^n)$ is graceful, by Theorem 1.5. □

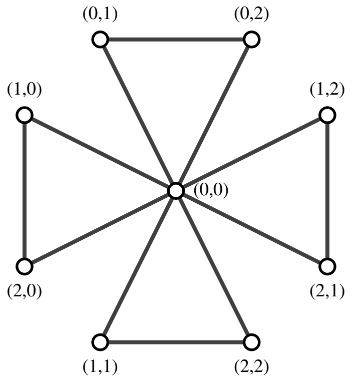

Figure 6. $\mathcal{P}(\mathbb{Z}_3 \times \mathbb{Z}_3)$

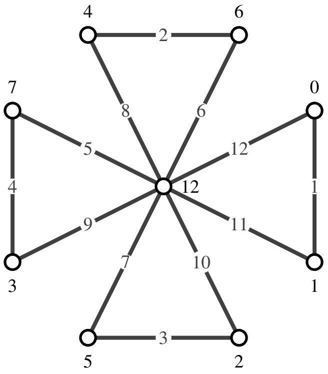

Figure 7. Graceful labeling of $\mathcal{P}(\mathbb{Z}_3 \times \mathbb{Z}_3)$

3 Edge-magic and Super edge-magic labeling of Power graphs

Proposition 3.1. *A power graph* $\mathcal{P}(\mathbb{Z}_2^n)$ *is edge-magic with the valence* $2^{n+2} - 2$.

Proof. Let $V(\mathcal{P}(\mathbb{Z}_2^n)) = \{\bar{0}, x_1, x_2, \cdots, x_{2^n-1}\}$. The number of vertices and edges in $\mathcal{P}(\mathbb{Z}_2^n)$ are 2^n and $2^n - 1$, respectively. Define a bijective function $f : V(\mathcal{P}(\mathbb{Z}_2^n)) \bigcup E(\mathcal{P}(\mathbb{Z}_2^n)) \to \{1, 2, \cdots, 2^{n+1} - 1\}$ by $f(\bar{0}) = 2^{n+1} - 1$, $f(x_i) = i$, for all $i = 1, 2, \cdots, 2^n - 1$ and $f(\{\bar{0}, x_i\}) = 2^{n+1} - 1 - i$, for all $i = 1, 2, \cdots, 2^n - 1$, where $\{\bar{0}, x_i\}$ denotes an edge in $\mathcal{P}(\mathbb{Z}_2^n)$. Then for any edge $\{x, y\}$ in $\mathcal{P}(\mathbb{Z}_2^n)$, $f(x) + f(xy) + f(y) = 2^{n+2} - 2$. Therefore $\mathcal{P}(\mathbb{Z}_2^n)$ is edge-magic graph with valence $2^{n+2} - 2$, see Figure 8. □

Proposition 3.2. *A power graph* $\mathcal{P}(\mathbb{Z}_2^n)$ *is super edge-magic with the valence* $3 \cdot 2^n$.

Proof. Let $V(\mathcal{P}(\mathbb{Z}_2^n)) = \{\bar{0}, x_1, x_2, \cdots, x_{2^n-1}\}$. Define a bijective function $f : V(\mathcal{P}(\mathbb{Z}_2^n)) \to \{1, 2, \cdots, 2^n\}$ by $f(\bar{0}) = 2^n$ and $f(x_i) = i$, for all $i = 1, 2, \cdots, 2^n - 1$. Then the set $S = \{f(u) + f(v) | \{u, v\} \in E(\mathcal{P}(\mathbb{Z}_2^n))\} = \{2^n + 1, 2^n + 2, \cdots, 2^{n+1} - 1\}$ consists of consecutive integers with $s = minS = 2^n + 1$. Therefore f extends to a super edge-magic labeling of $\mathcal{P}(\mathbb{Z}_2^n)$ with valence $k = 3 \cdot 2^n$, by lemma 1.1 [13]. Hence the result. □

Proposition 3.3. *A power graph* $\mathcal{P}(\mathbb{Z}_2^n)$ *is a super edge-magic graph with exactly* $3 \cdot 2^{2^n-1}$ *distinct edge-magic labelings out of which there are only two super edge-magic labelings upto isomorphism.*

Proof. $\mathcal{P}(\mathbb{Z}_2^n)$ is isomorphic to a star graph $K_{1,2^n-1}$. Thus there are exactly $3 \cdot 2^{2^n-1}$ distinct edge-magic labelings of $\mathcal{P}(\mathbb{Z}_2^n)$ of which only two are super edge-magic labelings upto isomorphism, by Theorem 3.1 [13]. Moreover, if f is a edge-magic labeling of $\mathcal{P}(\mathbb{Z}_2^n)$ with valence k, then $k = 2^{n+1} + 2$, $3 \cdot 2^n$ or $2^{n+2} - 2$, which corresponds to $f(\bar{0}) = 1, 2^n, 2^{n+1} - 1$, respectively. Let f_1, f_2 and f_3 be the edge-magic labelings of $\mathcal{P}(\mathbb{Z}_2^n)$ with valences $2^{n+1} + 2, 3 \cdot 2^n$ and $2^{n+2} - 2$, respectively. Then f_1, f_2 and f_3 are defined as follows:

$$f_1(\bar{0}) = 1, f_1(x_i) = i + 1, f_1(\{\bar{0}, x_i\}) = 2^{n+1} - i,$$

$$f_2(\bar{0}) = 2^n, f_2(x_i) = i, f_2(\{\bar{0}, x_i\}) = 2^{n+1} - i,$$

$$f_3(\bar{0}) = 2^{n+1} - 1, f_3(x_i) = i, f_3(\{\bar{0}, x_i\}) = 2^{n+1} - 1 - i,$$

where $1 \le i \le 2^n - 1$. All other edge-magic labelings of $\mathcal{P}(\mathbb{Z}_2^n)$ can be obtained by permuting the labels of edge $\{\bar{0}, x_i\}$ and x_i, for any i with $1 \le i \le 2^n - 1$, and that gives $3 \cdot 2^{2^n-1}$ possible permutations, amongst them only f_1 and f_2 are super edge-magic labelings of $\mathcal{P}(\mathbb{Z}_2^n)$. □

Using Theorem 1.8, the complementary edge-magic labeling of a power graph $\mathcal{P}(\mathbb{Z}_2^n)$ with respect to the edge-magic labelings f_1, f_2 and f_3 defined in Proposition 3.3 are obtained as $\bar{f}_1(x) = 2^{n+1} - f_1(x)$, $\bar{f}_2(x) = 2^{n+1} - f_2(x)$ and $\bar{f}_3(x) = 2^{n+1} - f_3(x)$, for all $x \in V(\mathbb{Z}_2^n) \cup E(\mathbb{Z}_2^n)$ with valences $2^{n+2} - 2, 3 \cdot 2^n, 2 + 2^{n+1}$, respectively.

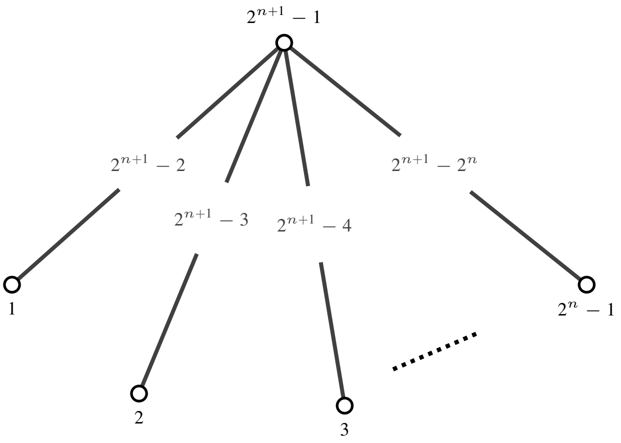

Figure 8. Edge-magic labeling of $\mathcal{P}(\mathbb{Z}_2^n)$

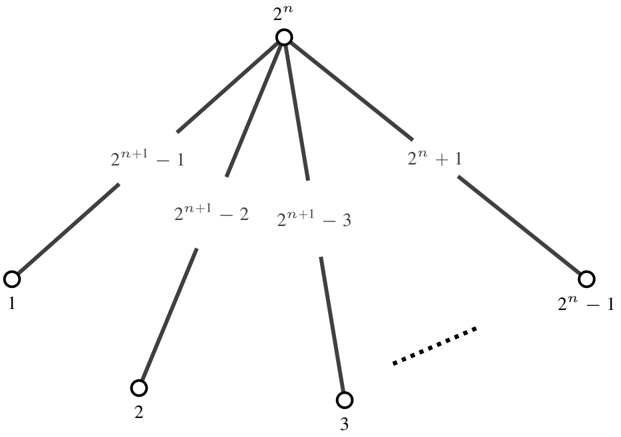

Figure 9. Super edge-magic labeling of $\mathcal{P}(\mathbb{Z}_2^n)$

Proposition 3.4. *A power graph $\mathcal{P}(\mathbb{Z}_3^n)$ is not super edge-magic for all $n \geq 3$.*

Proof. For $n = 1, \mathcal{P}(\mathbb{Z}_3) \cong K_3$, which is super edge-magic. Consider $n \geq 2$. Then $\mathcal{P}(\mathbb{Z}_3^n)$ is isomorphic to a friendship graph $T_{\frac{3^n-1}{2}}$, , which is super edge-magic if and only if $3 \leq \frac{3^n-1}{2} \leq 5$ and $\frac{3^n-1}{2} = 7$, by Theorem 3 in [10]. Therefore for all $n \geq 3$, $\mathcal{P}(\mathbb{Z}_3^n)$ is not super edge-magic. □

Proposition 3.5. *Let $\mathcal{G}$ be a finite cyclic group of order p^m, where p is a prime and $m \in \mathbb{Z}^+$. Then the power graph $\mathcal{P}(\mathcal{G})$ is not super edge-magic for $p = 2, 3, m \geq 2$, and for all primes $p > 5, m \in \mathbb{Z}^+$.*

Proof. Initially, we show that K_n is not super edge-magic for all $n \geq 4$. Clearly, K_2 and K_3 are super edge-magic graphs. K_4 is a 3−regular graph with 6(even) edges, which implies K_4 is not super edge-magic, by lemma 1.2 [13]. For $n \geq 5$, the minimum degree of K_n is ≥ 4. Therefore K_n is not super edge-magic for all $n \geq 5$, by lemma 1.3 [13]. Let $\mathcal{G}$ be a finite cyclic group of order p^m, where p is a prime and $m \in \mathbb{Z}^+$. Then $\mathcal{P}(\mathcal{G})$ is isomorphic to K_{p^m}, which is not super edge-magic for $p = 2, 3, m \geq 2$, and for all primes $p > 5, m \in \mathbb{Z}^+$. □

Example 3.6. Let $\mathcal{G}_1 = \mathbb{Z}_2$ and $\mathcal{G}_2 = \mathbb{Z}_3$. Then $\mathcal{P}(\mathcal{G}_1)$ and $\mathcal{P}(\mathcal{G}_2)$ are isomorphic to K_2 and K_3, respectively. Hence $\mathcal{P}(\mathcal{G}_1)$ and $\mathcal{P}(\mathcal{G}_2)$ are super edge-magic graphs. But $\mathcal{P}(\mathcal{G}_1 \times \mathcal{G}_2)$ is a graph on 6 vertices and 13 edges, which implies that $\mathcal{P}(\mathcal{G}_1 \times \mathcal{G}_2)$ is not a super edge-magic graph, by lemma 1.3 [13].

Example 3.7. Consider a Quaternion group $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$ with the property $i^2 = j^2 = k^2 = -1, i \cdot j = k, j \cdot k = i, k \cdot i = j, j \cdot i = -k, k \cdot j = -i, i \cdot k = -j$. Then the power graph $\mathcal{P}(Q_8)$ is $(8, 16)$ graph. Therefore it is not super edge-magic graph, by lemma 1.3 [13].

4 Antimagic labeling of Power graphs

Proposition 4.1. *A power graph $\mathcal{P}(\mathbb{Z}_2^n)$ is an antimagic graph.*

Proof. Let $V(\mathcal{P}(\mathbb{Z}_2^n)) = \{\bar{0}, x_1, x_2, \cdots, x_{2^n-1}\}$. Since every $x_i \neq \bar{0} \in V(\mathcal{P}(\mathbb{Z}_2^n))$ is adjacent to only $\bar{0}$ in $\mathcal{P}(\mathbb{Z}_2^n)$, the number of edges in $\mathcal{P}(\mathbb{Z}_2^n)$ is $2^n - 1$. Define a bijective function $f : E(\mathcal{P}(\mathbb{Z}_2^n)) \to \{1, 2, \ldots, 2^n - 1\}$ by $f(\{\bar{0}, x_i\}) = i$, where $i = 1, 2, \ldots, 2^n - 1$. Then the sum of the edge labels incident on each vertex is distinct, see table 1. Therefore f defines an antimagic labeling of $\mathcal{P}(\mathbb{Z}_2^n)$, see Figure 10. Hence the result. □

Vertex(v)	sum of labels of edges incident on v
$\bar{0}$	$2^{n-1}(2^n - 1)$
x_1	1
x_2	2
$\vdots$	$\vdots$
$x_i \neq \bar{0}$	i
$\vdots$	$\vdots$
x_{2^n-1}	$2^n - 1$

Table 1. Sum of edge labels incident on vertices of $\mathcal{P}(\mathbb{Z}_2^n)$

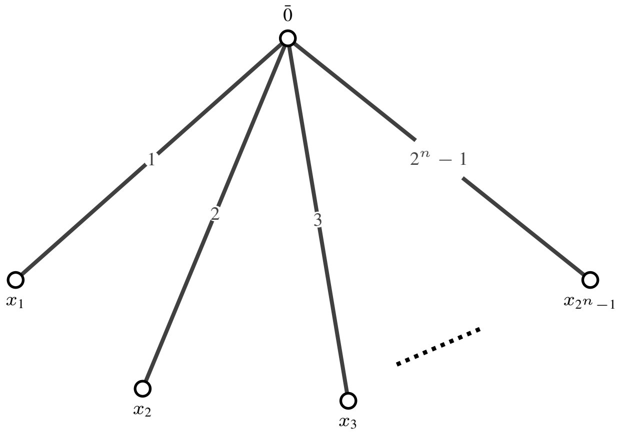

Figure 10. Antimagic labeling of $\mathcal{P}(\mathbb{Z}_2^n)$

Proposition 4.2. *A power graph $\mathcal{P}(\mathbb{Z}_3^n)$ is an antimagic graph.*

Proof. A power graph $\mathcal{P}(\mathbb{Z}_3^n)$ is isomorphic to the friendship graph $T_{\frac{3^n-1}{2}}$. Thus the number of edges in $\mathcal{P}(\mathbb{Z}_3^n)$ is $\frac{3^{n+1}-3}{2}$. Define a bijective function $f : E(\mathcal{P}(\mathbb{Z}_3^n)) \to \{1, 2, \ldots, \frac{3^{n+1}-3}{2}\}$ by

$$f(\{\bar{0}, x_1\}) = 1, f(\{x_1, y_1\}) = 2, f(\{\bar{0}, y_1\}) = 3$$

,

$$f(\{\bar{0}, x_{i+1}\}) = f(\{\bar{0}, x_i\}) + 3,$$
$$f(\{\bar{0}, y_{i+1}\}) = f(\{\bar{0}, y_i\}) + 3,$$
$$f(\{x_{i+1}, y_{i+1}\}) = f(\{x_i, y_i\}) + 3,$$

where $i = 1, 2, \ldots, \frac{3^n-3}{2}$, $x_i, y_i \in V^*(T_i)$, and T_i represents a triangle in $\mathcal{P}(\mathbb{Z}_3^n)$. The sum of the labels of the edges incident on each vertex is distinct, see Table 2. Therefore $\mathcal{P}(\mathbb{Z}_3^n)$ is an antimagic graph. □

Vertex(v)	sum of labels of edges incident on v
$\bar{0}$	$\frac{(3^n-1)(3^{n+1}-1)}{4}$
x_1	3
y_1	5
x_2	9
y_2	11
$\vdots$	$\vdots$
$\vdots$	$\vdots$
$x_{\frac{3^n-1}{2}}$	$3^{n+1}-6$
$y_{\frac{3^n-1}{2}}$	$3^{n+1}-4$

Table 2. Sum of edge labels incident on vertices of $\mathcal{P}(\mathbb{Z}_3^n)$

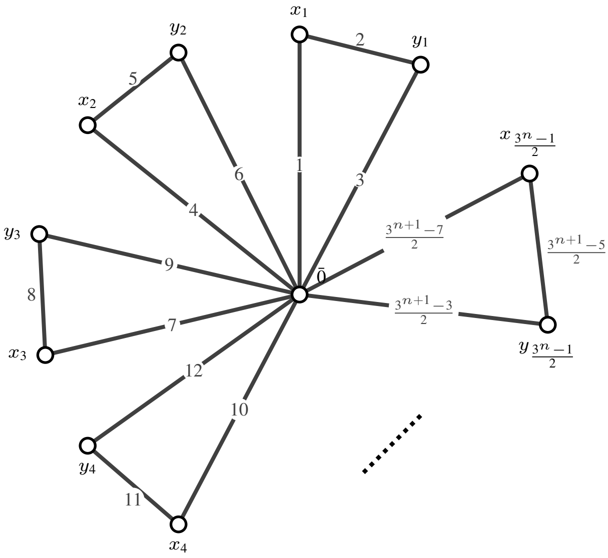

Figure 11. Antimagic labeling of $\mathcal{P}(\mathbb{Z}_3^n)$

Example 4.3. Consider a symmetric group $S_3 = \{(1), (123), (132), (12), (13), (23)\}$. We define a bijective function $f : E(\mathcal{P}(S_3)) \to \{1, 2, \ldots, |E(\mathcal{P}(S_3))|\}$ by $f(\{(123), (132)\}) = 1, f(\{(1), (123)\}) = 2, f(\{(1), (132)\}) = 3, f(\{(1), (12)\}) = 4, f(\{(1), (13)\}) = 5, f(\{(1), (23)\}) = 6$, so that the sum of the labels of the edges incident on each vertex of $\mathcal{P}(S_3)$ is distinct. Therefore $\mathcal{P}(S_3)$ is an antimagic graph. By resticting the same bijective function to $E(\mathcal{P}(A_3))$, where $A_3 = \{(1), (123), (132)\}$ is an alternating group, gives the antimagic labeling $\mathcal{P}(A_3)$. Hence $\mathcal{P}(A_3)$ is also an antimagic graph.

Example 4.4. Consider a power graph $\mathcal{P}(Q_8)$, where $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$ with $i^2 = j^2 = k^2 = -1, i \cdot j = k, j \cdot k = i, k \cdot i = j, j \cdot i = -k, k \cdot j = -i, i \cdot k = -j$ is a Quaternion group. We can define an antimagic labeling f on $\mathcal{P}(Q_8)$ as follows: $f : E(\mathcal{P}(Q_8)) \to \{1, 2, \ldots, 16\}$ defined by $f(\{1. -1\}) = 1, f(\{1, i\}) = 2, f(\{i, -i\}) = 3, f(\{-1, -i\}) = 4, f(\{1, -i\}) = 12, f(\{-1, i\}) = 11, f(\{1, j\}) = 5, f(\{j, -j\}) = 6, f(\{-1, -j\}) = 7, f(\{-1, j\}) = 13, f(\{1, -j\}) = 14, f(\{1, k\}) = 8, f(\{k, -k\}) = 9, f(\{-1, -k\}) = 10, f(\{-1, k\}) = 15, f(\{1, -k\}) = 16$. Therefore $\mathcal{P}(Q_8)$ is an antimagic graph.

Proposition 4.5. *Let $\mathcal{G}$ be a finite cyclic group of order p^m, where p is a prime and $m \in \mathbb{Z}^+$. Then the power graph $\mathcal{P}(\mathcal{G})$ of a group $\mathcal{G}$ is antimagic for $p = 2, m \geq 2$, and for all primes $p \geq 3, m \in \mathbb{Z}^+$.*

Proof. Initially, we prove that the complete graph K_n is antimagic for $n \geq 3$. We use the concept of representation table to obtain the antimagic labeling of graphs given by [21]. The representation tables for the antimagic labeling of K_n for odd and even values of n are given in Table 3 and 4, respectively. Label the vertices of a complete graph K_n as $v_1, v_2, \cdots, v_n$ and from the representation table, assign the $(i.j)^{th}$ entry to the edge between vertices v_i and v_j to get the antimagic labeling of K_n. Let $\mathcal{G}$ be a finite cyclic group of order p^m, where p is a prime and $m \in \mathbb{Z}^+$. Then $\mathcal{P}(\mathcal{G})$ is isomorphic to K_{p^m} [17]. Hence the result. □

We present an outline to construct a $n \times n$ representation table for an antimagic labeling of K_n for odd and even values of n. The technique of assigning labels to the edges of a complete graph K_n using the representation tables 3, 4 is already given in Proposition 4.5.

Algorithm to obtain representation table for Antimagic labeling of K_n, where n is odd:

Step 1: Write 0 at all entries in the principal diagonal of the table.

Step 2: Write entries $1, 2, \cdots, n-1$ in the vacant positions which are just above and diagonally parallel to the principal diagonal of table, starting from top and going towards the bottom of the table.

Step 3: Continue to write the sequence of entries $n, n+1, \cdots, \sum\limits_{i=1}^{2}(n-i)$ along the next diagonally parallel vacant positions which are above the principal diagonal of a matrix, , starting from top and going towards the bottom of the table.

Step 4: Continue this process of writing the entries in the vacant positions of the table which are above and diagonally parallel to the principal diagonal till we reach the last entry at $(1, n)^{th}$ place as $\sum\limits_{i=1}^{n-1}(n-i) = \frac{n(n-1)}{2}$.

Step 5: Since the graph is undirected, the representation table is a symmetric. So we can complete the table by using the existing entries in the table.

0	1	n	$2n-2$	$\cdots$	$\cdots$	$\sum_{i=1}^{n-1}(n-i)$
1	0	2	$n+1$	$\cdots$	$\cdots$	$\sum_{i=1}^{n-2}(n-i)$
n	2	0	3	$\cdots$	$\cdots$	$\sum_{i=1}^{n-3}(n-i)$
$2n-2$	$n+1$	3	0	$\cdots$	$\cdots$	$\sum_{i=1}^{n-4}(n-i)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0	$n-1$
$\sum_{i=1}^{n-1}(n-i)$	$\sum_{i=1}^{n-2}(n-i)$	$\sum_{i=1}^{n-3}(n-i)$	$\sum_{i=1}^{n-4}(n-i)$	$\cdots$	$n-1$	0

Table 3. Representation table for Antimagic labeling of K_n, where n is odd

Algorithm to obtain representation table for Antimagic labeling of K_n, where n is even:

Step 1: Write 0 at all entries in the principal diagonal of the table.

Step 2: Write entries $1, 2, \cdots, n-1$ in the 1^{st} row of the table starting from right to left.

Step 3: Continue to write the sequence of entries $n, n+1, \cdots, \sum\limits_{i=1}^{2}(n-i)$ in the vacant positions of the table which are just above and diagonally parallel to the principal diagonal of a table, starting from top and going towards the bottom of the table.

Step 4: Continue to write the sequence of entries $\sum\limits_{i=1}^{2}(n-i)+1, \sum\limits_{i=1}^{2}(n-i)+2, \cdots, \sum\limits_{i=1}^{3}(n-i)$ in the next diagonally parallel vacant positions above the principal diagonal of the table, , starting from top and going towards the bottom of the table.

Step 5: Continue this process of writing the entries in the vacant positions of the table which are above and diagonally parallel to the principal diagonal till we reach the last entry at $(2,n)^{th}$ place as $\sum\limits_{i=1}^{n-1}(n-i) = \frac{n(n-1)}{2}$.

Step 6: Since the graph is undirected, the representation table is a symmetric. So we can complete the table by using the existing entries in the table.

0	$n-1$	$n-2$	$n-3$	$\cdots$	2	1
$n-1$	0	n	$2n-2$	$\cdots$	$\cdots$	$\sum_{i=1}^{n-1}(n-i)$
$n-2$	n	0	$n+1$	$\cdots$	$\cdots$	$\sum_{i=1}^{n-2}(n-i)$
$n-3$	$2n-2$	$n+1$	0	$\cdots$	$\cdots$	$\sum_{i=1}^{n-3}(n-i)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
2	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0	$\sum_{i=1}^{2}(n-i)$
1	$\sum_{i=1}^{n-1}(n-i)$	$\sum_{i=1}^{n-2}(n-i)$	$\sum_{i=1}^{n-3}(n-i)$	$\cdots$	$\sum_{i=1}^{2}(n-i)$	0

Table 4. Representation table for Antimagic labeling of K_n, where n is even

Proposition 4.6. *A power graph $\mathcal{P}(D_{p^m})$ of a dihedral group D_{p^m} is antimagic for $p = 2, m \geq 2$, and $p \geq 3, m \in \mathbb{Z}^+$.*

Proof. Consider a dihedral group $D_{p^m} =< a, b | a^{p^m} = e = b^2, ba = a^{p^m-1}b >$, where $|D_{p^m}| = 2p^m$. Let $H =< a >$. Then $\mathcal{P}(H)$ is isomorphic to K_{p^m}, which is antimagic for $p = 2, m \geq 2$, and for $p \geq 3, m \in \mathbb{Z}^+$, see proposition 4.5. Therefore $\mathcal{P}(H)$ has antimagic labeling say $f : E(\mathcal{P}(H)) \to \{1, 2, \ldots, \frac{p^m(p^m-1)}{2}\}$. We can extend f to an antimagic labeling $f' : E(\mathcal{P}(D_{p^m})) \to \{1, 2, \ldots, |E(\mathcal{P}(D_{p^m}))|\}$ of $\mathcal{P}(D_{p^m})$ defined by

$$f'(\{e, b\}) = 1, f'(\{e, ab\}) = 2, \ldots, f'(\{e, a^{p^m-1}b\}) = p^m,$$

$$f'(\{x, y\}) = f(\{x, y\}) + p^m, \text{ where}\{x, y\} \in E(\mathcal{P}(H))$$

. Therefore $\mathcal{P}(D_{p^m})$ is antimagic for $p = 2, m \geq 2$ and $p \geq 3, m \in \mathbb{Z}^+$. □

Example 4.7. Consider $\mathcal{P}(D_4)$. Then the antimagic labeling of $\mathcal{P}(D_4)$ is obtained by using antimagic labeling of K_4. Table 5 is the representation table for antimagic labeling of K_4. Thus the antimagic labeling of $\mathcal{P}(H)$, where $H =< a >$ is a bijective function $f : E(\mathcal{P}(H)) \to \{1, 2, \ldots, 6\}$ defined by $f(\{v_1, v_2\}) = 3, f(\{v_1, v_3\}) = 2, f(\{v_1, v_4\}) = 1, f(\{v_2, v_3\}) = 4, f(\{v_2, v_4\}) = 6, f(\{v_3, v_4\}) = 5$. We can extend f to an antimagic labeling f' of $\mathcal{P}(D_4)$ as follows: $f' : E(\mathcal{P}(D_4)) \to \{1, 2, \ldots, 10\}$ defined by $f'(\{e, b\}) = 1, f'(\{e, ab\}) = 2, f'(\{e, a^2b\}) = 3, f'(\{e, a^3b\}) = 4, f'(\{e, a\}) = 7, f'(\{e, a^2\}) = 6, f'(\{e, a^3\}) = 5, f'(\{a, a^2\}) = 8, f'(\{a^2, a^3\}) = 9, f'(\{a, a^3\}) = 10$.

0	3	2	1
3	0	4	6
2	4	0	5
1	6	5	0

Table 5. Representation table for Antimagic labeling of K_4

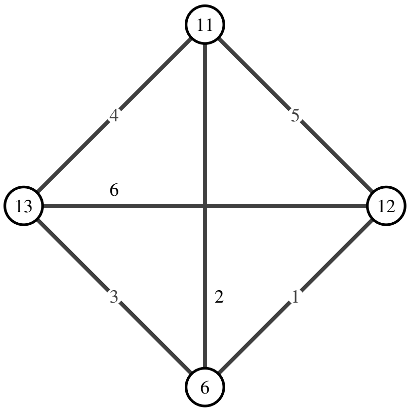

Figure 12. Antimagic labeling of K_4

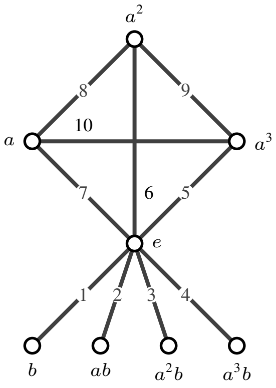

Figure 13. Antimagic labeling of $\mathcal{P}(D_4)$

5 Summary and Conclusion

In this article, we have studied magic, supermagic, graceful, edge-magic, super edge-magic and antimagic labeling of the power graph of some finite groups. The summary is given in Appendix A as table 6.

6 Statements and Declarations

The authors report that there are no competing interests to declare.

Appendix A

$\mathcal{P}(G)$, where G is	Order Parameter of a group G	Labeling type hold for $\mathcal{P}(G)$	Labeling Type do not hold for $\mathcal{P}(G)$	Labeling type open to study for $\mathcal{P}(G)$
Finite cyclic group of order p^m	$p = 2, m = 1, m \geq 3$; $p = 3, m \geq 2$; $p \geq 5, m \in \mathbb{Z}^+$	magic		edge-magic
	$p = 1, m = 2$; $p = 3, m \geq 2$; $p \geq 7, m \in \mathbb{Z}^+$	supermagic		
	$p = 2, m \geq 3$; $p = 3, m \geq 2$; $p > 5, m \in \mathbb{Z}^+$		graceful	
	$p = 2, m = 1, 2$	graceful		
	$p = 2, 3, m \geq 2$; $p > 5, m \in \mathbb{Z}^+$		super edge-magic	
	$p = 2, m \geq 2$; $p \geq 3, m \in \mathbb{Z}^+$	antimagic		
Dihedral group D_{p^m}	$p = 2, m \geq 2$; $p \geq 3, m \in \mathbb{Z}^+$	antimagic		magic, supermagic, graceful, edge-magic, super edge-magic
Symmetric group S_3		antimagic; graceful	magic	supermagic, edge-magic, super edge-magic
Alternating group A_3		antimagic; graceful	magic	supermagic, edge-magic, super edge-magic
Quaternion group Q_8		antimagic	super edge-magic	supermagic, graceful, edge-magic
$\mathbb{Z}_2^n$	$n \in \mathbb{N}$	graceful, edge-magic, super edge-magic antimagic		magic, supermagic
$\mathbb{Z}_3^n$	$n \geq 2$	graceful, antimagic		
$\mathbb{Z}_3^n$	$n \geq 3$		super edge-magic	magic, supermagic, graceful, edge-magic, antimagic

Table 6. Summary Table for Labeling of $\mathcal{P}(G)$

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