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# Picture Fuzzy Antiideals in Semigroups

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**Abstract** Compared with classical fuzzy sets and intuitionistic fuzzy sets, picture fuzzy sets provide a more flexible framework by considering positive, neutral, and negative membership degrees independently. The main aim of this work was to extend the existing theory of picture fuzzy ideals by focusing on anti-ideal structures, which represent undesirable or incompatible elements in algebraic systems under uncertainty. In this paper, we define picture fuzzy antiideals and examine their basic properties. Their relationships with classical anti-ideals are determined using characteristic functions and level subsets. Finally, it was observed that the images, inverse images, Cartesian products, and products of these sets possess identical properties.

## 1 Introduction

Since its introduction by Zadeh [25] in 1965, the concept of fuzzy sets has evolved into a significant mathematical framework for handling uncertainty. Fuzzy set theory generalizes classical set theory by allowing each element to have a membership degree between 0 and 1. Rosenfeld [20] (1971) was the first to apply fuzzification to algebraic structures, thereby introducing fuzzy subgroups and motivating the study of fuzzy ideals, fuzzy subsemigroups, and related algebraic entities. This approach enabled the modeling of approximate reasoning and imprecise information within algebraic contexts. Semigroup theory has evolved through the study of various substructures such as ideals, filters, quasi-ideals, and antiideals, each providing insights into the internal organization of semigroups. Antiideals in semigroups were first introduced by Schwarz [21] and later developed by Iseki [13]. These structures, along with related ones such as anti-interior ideals, have been instrumental in characterizing regular, intra-regular, and semisimple semigroups. The classical theory thus provides a foundation for further generalizations under uncertainty-based frameworks. Recently, Al Tahan et al [3, 4, 5] studied anti-ideals and their fuzzification. These studies demonstrated that fuzzy antiideals preserve essential properties of classical antiideals while offering flexibility to handle vague and partially defined data. Following Atanassov [6] introduction of intuitionistic fuzzy sets, which add non-membership to Zadeh's framework, intuitionistic fuzzy antiideals were developed to provide an even more expressive modeling of uncertainty [16, 17]. These structures have further analysed fuzzy algebraic systems with degrees of membership and non-membership.

A further generalization was introduced by Cuong [9] through the concept of picture fuzzy sets (PFS), which extend intuitionistic fuzzy sets by incorporating a third parameter, the neutrality degree, alongside positive (membership) and negative (non-membership) degrees. Picture fuzzy theory allows a more balanced modeling of human judgment, where neutrality or abstention plays a significant role.

Initially, Cuong [9] constructed picture fuzzy set theory with $\leq_1$ (type-1) ordering relation on the set $\mathbb{D}^*$ as

$$(x_1, x_2, x_3) \leq_1 (y_1, y_2, y_3) \text{ iff } x_1 \leq y_1, x_2 \leq y_2, x_3 \geq y_3.$$

With this ordering relation, this theory has found many applications [15, 8, 11, 22, 7, 12, 2]. Since then, picture fuzzy sets have been applied in various algebraic structures. Panpho and Yiarayong [19] investigated picture fuzzy ideals on commutative rings, defining operations such as intersection and product, and studying their basic properties. Yiarayong [23] characterized several classes of semigroups, such as regular and intra-regular ones, via picture fuzzy bi-ideals. Adak et al. [1] studied semi-prime ideals on picture fuzzy sets. However, it was observed in these studies that, such as the intersection, product, and characteristic function of sets, in particular the PFS, may not actually be a PFS. Another problem in the use of this theory is that the set $\mathbb{D}^*$ is not a complete lattice with respect to the ordering relation $\leq_1$. We can give the following examples for these.

**Example 1.1.** Let $S = \{a, b\}$ be a semigroup and $M = (\alpha_M, \beta_M, \gamma_M)$, $N = (\alpha_M, \beta_M, \gamma_M)$ be picture fuzzy sets on $S$ with the following diagrams:

| $\cdot$ | $a$ | $b$ |
|---|---|---|
| $a$ | $a$ | $a$ |
| $b$ | $a$ | $b$ |

| $M$ | $a$ | $b$ |
|---|---|---|
| $\alpha_M$ | 0 | 0 |
| $\beta_M$ | 1 | 1 |
| $\gamma_M$ | 0 | 0 |

| $N$ | $a$ | $b$ |
|---|---|---|
| $\alpha_N$ | 0 | 0 |
| $\beta_N$ | 0 | 0 |
| $\gamma_N$ | 1 | 1 |

While $M$ and $N$ are picture fuzzy subsets of $S$, their intersection is not a picture fuzzy subset of $S$, under Yiarayong's definition of intersection. This situation causes [24, Theorem 1] to be invalid.
Also, the definition of $C^S$ given by Yiarayong [23] was changed. Because, for every $a \in S \backslash A$, $C^A = (0, 1, 1) \notin \mathbb{D}^*$ and therefore $C^A$ may not be a picture fuzzy subset of $S$. This situation causes [23, Lemma 3.1] and [23, Theorem 3.1] to be invalid.

In many real-world decision and diagnosis problems, particularly in medical contexts, certain conditions represent irreversible states that cannot be compensated for or improved by combining them with additional information or actions. Classical ideal-based fuzzy structures are designed to model desirable or admissible elements and their stability under algebraic operations. However, such approaches fail to capture situations where the presence of a single critical factor permanently excludes an element from acceptability. The concept of a picture fuzzy anti-ideal is therefore necessary to formalize persistent rejection behavior within algebraic systems. In the proposed framework, once an element reaches a maximal level of non-membership or rejection, the application of a left operation cannot reduce this effect, which reflects the clinical reality that certain pathological findings cannot be neutralized by subsequent evidence. Hence, anti-ideals provide a mathematically rigorous and semantically appropriate tool for modeling threshold-based exclusion and irreversible risk propagation, which cannot be adequately represented by classical ideal or fuzzy ideal notions. Motivated by this research gap, this paper introduces and investigates the concepts of picture fuzzy antiideals, related substructures in semigroups. We establish fundamental properties of these notions, provide illustrative examples, and explore the relationships between picture fuzzy antiideals and classical antiideals through level set analysis. The results contribute to bridging the gap between antiideal theory and picture fuzzy algebra, thus enriching the study of uncertainty in algebraic systems.

The remainder of this paper is organized as follows. Section 2 provides the fundamental concepts and preliminaries related to semigroups and picture fuzzy sets. In Section 3, the notions of picture fuzzy anti-ideal and related substructures are introduced, and their basic properties are derived. Moreover, level-set characterizations are presented to connect picture fuzzy anti-ideals with their crisp counterparts.

## 2 Preliminaries

In this section, picture fuzzy sets are considered under type-3 ordering, as type-1 ordering causes certain difficulties.

**Definition 2.1.** [25] Let $X$ be a non-empty set. A fuzzy set of $A$ is a function $\alpha_A : X \to [0, 1]$ $(F(X))$,

$$A = \{(p, \alpha_A(p)) : p \in X\}$$

**Definition 2.2.** [6] Let $X \neq \emptyset$. An intuitionistic fuzzy set ($IFS(X)$) $A$,

$$A = \{(p, \alpha_A(p), \beta_A(p)) : p \in X\}$$

Where $\alpha_A, \beta_A : X \to [0,1]$ are mappings known as membership function and non-membership function, respectively, and

$$0 \leq \alpha_A(p) + \beta_A(p) \leq 1$$

for all $p \in X$.

Cuong introduced picture fuzzy sets by adding a neutral membership degree to intuitionistic fuzzy sets. This richer structure allows for more nuanced modeling of uncertainty, particularly in contexts involving human decision-making, such as voting, where responses may include positive, neutral, or negative stances.
Cuong [9] has considered the set.

$$\mathbb{D}^* = \{((p_1, p_2, p_3) \in [0,1]^3 : p_1 + p_2 + p_3 \leq 1\}.$$

Picture fuzzy sets are specified via a membership function $A : X \to \mathbb{D}^*$ ($PFS(X)$),

$$A = \{(p, \alpha_A(p), \beta_A(p), \gamma_A(p)) : p \in X\}.$$

In which the values of membership, neutrality, and non-membership of the element $p \in X$ are $\alpha_A(p), \beta_A(p), \gamma_A(p)$, respectively, and they satisfy the condition:

$$0 \leq \alpha_A(p) + \beta_A(p) + \gamma_A(p) \leq 1$$

In fact type-3 inclusion relation is based on the following order relation on $D^*$[14]:

$$(p_1, p_2, p_3) \leq_3 (r_1, r_2, r_3) \text{ iff } p_1 \leq r_1, \quad p_1 + p_2 \leq r_1 + r_2, \quad p_3 \geq r_3, \quad (type - 3)$$

**Definition 2.3.** [18] Let $\mathcal{P} = (\alpha_\mathcal{P}, \beta_\mathcal{P}, \gamma_\mathcal{P}), \mathcal{R} = (\alpha_\mathcal{R}, \beta_\mathcal{R}, \gamma_\mathcal{R}) \in PFS(X)$. Then $\mathcal{P}$ *is a subset of* $\mathcal{R}$, denoted by $\mathcal{P} \subseteq \mathcal{R}$ iff

$$\begin{aligned}
&\alpha_\mathcal{P}(p) \leq \alpha_\mathcal{R}(p),\\
&\alpha_\mathcal{P}(p) + \beta_\mathcal{P}(p) \leq \alpha_\mathcal{R}(p) + \beta_\mathcal{R}(p),\\
&\gamma_\mathcal{P}(p) \geq \gamma_\mathcal{R}(p),
\end{aligned}$$

for all $p \in X$.

**Definition 2.4.** [18] Let $\mathcal{P} = (\alpha_\mathcal{P}, \beta_\mathcal{P}, \gamma_\mathcal{P}), \mathcal{R} = (\alpha_\mathcal{R}, \beta_\mathcal{R}, \gamma_\mathcal{R}) \in PFS(X)$. Define the $\mathcal{P} \cap \mathcal{R}, \mathcal{P} \cup \mathcal{R} \in PFS(X)$ as follows: for all $p \in X$,

$$\begin{aligned}
&\alpha_{\mathcal{P}\cap\mathcal{R}}(p) = \alpha_\mathcal{P}(p) \wedge \alpha_\mathcal{R}(p),\\
&\beta_{\mathcal{P}\cap\mathcal{R}}(p) = \alpha_\mathcal{P}(p) + \beta_\mathcal{P}(p) \wedge \alpha_\mathcal{R}(p) + \beta_\mathcal{R}(p) - (\alpha_\mathcal{P}(p) \wedge \alpha_\mathcal{R}(p)),\\
&\gamma_{\mathcal{P}\cap\mathcal{R}}(p) = \gamma_\mathcal{P}(p) \vee \gamma_\mathcal{R}(p),\\
&\alpha_{\mathcal{P}\cup\mathcal{R}}(p) = \alpha_\mathcal{P}(p) \vee \alpha_\mathcal{R}(p),\\
&\beta_{\mathcal{P}\cup\mathcal{R}}(p) = \alpha_\mathcal{P}(p) + \beta_\mathcal{P}(p) \vee \alpha_\mathcal{R}(p) + \beta_\mathcal{R}(p) - (\alpha_\mathcal{P}(p) \vee \alpha_\mathcal{R}(p)),\\
&\gamma_{\mathcal{P}\cup\mathcal{R}}(p) = \gamma_\mathcal{P}(p) \wedge \gamma_\mathcal{R}(p).
\end{aligned}$$

Then $\mathcal{P} \cap \mathcal{R}$ and $\mathcal{P} \cup \mathcal{R}$ are called the *intersection* and *union* of $\mathcal{P}$ and $\mathcal{R}$, respectively.

**Definition 2.5.** [18] Let $\{\mathcal{P}_t \mid t \in T\} \subseteq PFS(X)$. Define the $\bigcap_{t\in T} \mathcal{P}_t, \bigcup_{t\in T} M_i \in PFS(X)$ as follows: for all $p \in S$,

$$\alpha_{\bigcap_{t\in T}\mathcal{P}_t}(p) = \bigwedge_{t\in T} \alpha_{\mathcal{P}_t}(p),$$
$$\beta_{\bigcap_{t\in T}\mathcal{P}_t}(p) = \bigwedge_{t\in T} (\alpha_{\mathcal{P}_t}(p) + \beta_{\mathcal{P}_t}(p)) - \bigwedge_{t\in T} \alpha_{\mathcal{P}_t}(p),$$
$$\gamma_{\bigcap_{t\in T}\mathcal{P}_t}(p) = \bigvee_{t\in T} \gamma_{\mathcal{P}_t}(p),$$
$$\alpha_{\bigcup_{t\in T}\mathcal{P}_t}(p) = \bigvee_{t\in T} \alpha_{\mathcal{P}_t}(p),$$
$$\beta_{\bigcup_{t\in T}\mathcal{P}_t}(p) = \bigvee_{t\in T} (\alpha_{\mathcal{P}_t}(p) + \beta_{\mathcal{P}_t}(p)) - \bigvee_{t\in T} \alpha_{\mathcal{P}_t}(p),$$
$$\gamma_{\bigcup_{t\in T}\mathcal{P}_t}(p) = \bigwedge_{t\in T} \gamma_{\mathcal{P}_t}(p).$$

Then $\bigcap_{t\in T} \mathcal{P}_t$ and $\bigcup_{t\in T} \mathcal{P}_t$ are called the intersection and union of the family $\{\mathcal{P}_t \mid t \in T\}$, respectively.

**Definition 2.6.** [18] Let $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(X)$ and $\mathcal{R} = (\alpha_{\mathcal{R}}, \beta_{\mathcal{R}}, \gamma_{\mathcal{R}}) \in PFS(X')$. Define the $\mathcal{P} \times \mathcal{R} \in PFS(X \times X')$ as follows:

$$\alpha_{\mathcal{P}\times\mathcal{R}}(p, r) = \alpha_{\mathcal{P}}(p) \wedge \alpha_{\mathcal{R}}(r),$$
$$\beta_{\mathcal{P}\times\mathcal{R}}(p, r) = \alpha_{\mathcal{P}}(p) + \beta_{\mathcal{P}}(p) \wedge \alpha_{\mathcal{R}}(r) + \beta_{\mathcal{R}}(r) - \alpha_{\mathcal{P}}(p) \wedge \alpha_{\mathcal{R}}(r),$$
$$\gamma_{\mathcal{P}\times\mathcal{R}}(p, r) = \gamma_{\mathcal{P}}(p) \vee \gamma_{\mathcal{R}}(r).$$

for all $(p, r) \in X \times X'$. Then $\mathcal{P} \times \mathcal{R}$ is called the *cartesian product* of $\mathcal{P}$ and $\mathcal{R}$.

**Definition 2.7.** [18] Let $A \subseteq X$. Then the *picture characteristic function of* $A$, denoted by $C^A = (\alpha_{C^A}, \beta_{C^A}, \gamma_{C^A})$ is defined as:

$$\alpha_{C^A}(p) = \begin{cases} 1, & p \in A \\ 0, & otherwise. \end{cases}$$
$$\beta_{C^A}(p) = \begin{cases} 0, & p \in A \\ 0, & otherwise. \end{cases}$$
$$\gamma_{C^A}(p) = \begin{cases} 0, & p \in A \\ 1, & otherwise. \end{cases}$$

It is clear that $C^S = (1, 0, 0)$, $C^\emptyset = (0, 0, 1)$ for all $x \in S$ and denoted by $C^S = S$, $C^\emptyset = \emptyset$.

**Definition 2.8.** [18] Let $f$ be a mapping from $X$ into $X'$ and $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(X)$, $\mathcal{R} = (\alpha_{\mathcal{R}}, \beta_{\mathcal{R}}, \gamma_{\mathcal{R}}) \in PFS(X')$. Then *the image of* $\mathcal{P}$ and *the pre-image of* $\mathcal{R}$ under $f$ are denoted by $f(\mathcal{P})$ and $f^{-1}(\mathcal{R})$, respectively and are defined as: for all $p \in X$ and $p' \in X'$,

$$\alpha_{f(\mathcal{P})}(p') = \bigvee_{p'=f(p)} \alpha_{\mathcal{P}}(p),$$
$$\beta_{f(\mathcal{P})}(p') = \bigvee_{p'=f(p)} (\alpha_{\mathcal{P}}(p) + \beta_{\mathcal{P}}(p)) - \bigvee_{p'=f(p)} \alpha_{\mathcal{P}}(p),$$
$$\gamma_{f(\mathcal{P})}(p') = \bigwedge_{p'=f(p)} \gamma_{\mathcal{P}}(p),$$
$$\alpha_{f^{-1}(\mathcal{R})}(p) = \alpha_{\mathcal{R}}(f(p)),$$
$$\beta_{f^{-1}(\mathcal{R})}(p) = \beta_{\mathcal{R}}(f(p)),$$
$$\gamma_{f^{-1}(\mathcal{R})}(p) = \gamma_{\mathcal{R}}(f(p)).$$

**Definition 2.9.** [18] Let $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(X)$. For each $\alpha \in [0,1]$, the sets are defined $\alpha$-cut of $\mathcal{P}$, respectively,

$$PM(\mathcal{P}:\alpha) = \{p \in X \mid \alpha \leq \alpha_{\mathcal{P}}(p)\}$$
$$N(\mathcal{P}:\alpha) = \{p \in X \mid \alpha \leq \alpha_{\mathcal{P}}(p) + \beta_{\mathcal{P}}(p)\}$$
$$NM(\mathcal{P}:\alpha) = \{p \in X \mid \alpha \geq \gamma_{\mathcal{P}}(p)\}$$

**NOTE:** In a semigroup $S$, the sets $P(S)$, $F(S)$, and $IFS(S)$ can be embedded into the set $PFS(S)$ by the following proposition. The mappings here preserve the orderings on the respective sets.

**Proposition 2.10.** *[18]*

*(i) Let $P(S)$ be the set of all subsets of $S$. Then the function $\Phi : P(S) \longrightarrow PFS(S)$ defined as $\Phi(A) = C^A$ is a one-to-one order preserving lattice homomorphism.*

*(ii) Let $F(S)$ be the set of all fuzzy subsets on $S$. Then the function $\phi : F(S) \longrightarrow PFS(S)$ defined as $\phi(\alpha_M) = (\alpha_M, 0, 0)$ is a one-to-one order preserving lattice homomorphism.*

*(iii) Let $IFS(S)$ be the set of all intuitionistic fuzzy subsets on $S$. Then the function $\psi : IFS(S) \longrightarrow PFS(S)$ defined as $\psi(\alpha_M, \gamma_M) = (\alpha_M, 0, \gamma_M)$ is a one-to-one order preserving lattice homomorphism.*

A semigroup is defined as $S \neq \emptyset$ together with a binary operation that satisfies the associative law. Unless otherwise stated, the set $S$ will always be taken as a semigroup in this paper.
The *product* of two subsets $A$ and $B$ of a semigroup $S$ is defined as

$$AB = \{\, ab \mid a \in A,\ b \in B \,\}.$$

**Definition 2.11.** [13, 4] Let $A$ be any non-empty subset of $S$. Then

(i) $A$ is called a *left antiideal of $S$* if $SA \cap A = \emptyset$,

(ii) $A$ is called a *right antiideal of $S$* if $AS \cap A = \emptyset$,

(iii) $A$ is called a *antiideal of $S$* if it is both *left antiideal of $S$* and *right antiideal of $S$*,

(iv) $A$ is called a *bi-antiideal of $S$* if $ASA \cap A = \emptyset$,

The sets of all left antiideals, right antiideals, antiideals, bi-antiideals of $S$ are written as $LAI(S)$, $RAI(S)$, $AI(S)$, and $BAI(S)$, respectively.

## 3 Picture Fuzzy Antiideals in Semigroups

In this section, we introduce the notions of picture fuzzy left (right, bi) antiideals and explore their fundamental algebraic properties. In addition, the connections between picture fuzzy antiideals and their corresponding characteristic functions and level subsets are analyzed, and their relationships with classical antiideals are discussed in detail.

**Definition 3.1.** Let $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(S)$.

(i) $\mathcal{P}$ is called *picture fuzzy left antiideal* of $S$ if

$$\alpha_{\mathcal{P}}(p_2) \wedge \alpha_{\mathcal{P}}(p_1p_2) = 0,$$
$$\left[\alpha_{\mathcal{P}}(p_2) + \beta_{\mathcal{P}}(p_2)\right] \wedge \left[\alpha_{\mathcal{P}}(p_1p_2) + \beta_{\mathcal{P}}(p_1p_2)\right] = 0,$$
$$\gamma_{\mathcal{P}}(p_2) \vee \gamma_{\mathcal{P}}(p_1p_2) = 1,$$

(ii) $\mathcal{P}$ is called *picture fuzzy right antiideal* of $S$ if

$$\alpha_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_1p_2) = 0,$$
$$\left[\alpha_{\mathcal{P}}(p_1) + \beta_{\mathcal{P}}(p_1)\right] \wedge \left[\alpha_{\mathcal{P}}(p_1p_2) + \beta_{\mathcal{P}}(p_1p_2)\right] = 0,$$
$$\gamma_{\mathcal{P}}(p_1) \vee \gamma_{\mathcal{P}}(p_1p_2) = 1,$$

for all $p_1, p_2 \in S$.

A picture fuzzy set $\mathcal{P}$ on $S$ is called a *picture fuzzy antiideal* if it is both a picture fuzzy left antiideal of $S$ and a picture fuzzy right antiideal of $S$.

All picture fuzzy left antiideals, right antiideals, and antiideals of $S$ are denoted by $PFLAI(S)$, $PFRAI(S)$, and $PFAI(S)$, respectively.
**NOTE:** Let $S$ be a monoid. If $\mathcal{P}$ is a picture fuzzy anti left (right) ideal of $S$, then $\mathcal{P} = C^{\emptyset}$. However, when $S$ is a monoid, no anti left (right) ideal exists.

**Example 3.2.** Let $(\mathbb{N}^*, \circ)$, $a \circ b = a + b + 5$ be the semigroup and $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}})$ is defined as:

$$\alpha_{\mathcal{P}}(p) = \begin{cases} \frac{1}{2}, & p < 5 \\ 0, & otherwise. \end{cases}$$

$$\beta_{\mathcal{P}}(p) = \begin{cases} \frac{1}{4}, & p < 5 \\ 0, & otherwise. \end{cases}$$

$$\gamma_{\mathcal{P}}(p) = \begin{cases} 0, & p < 5 \\ 1, & otherwise. \end{cases}$$

Since $\alpha_{\mathcal{P}}(a \circ b) = 0, \beta_{\mathcal{P}}(a \circ b) = 0, \gamma_{\mathcal{P}}(a \circ b) = 1$ for all $a, b \in S$ and hence $\mathcal{P} \in PFLAI(S)$. Since $(\mathbb{N}^*, \circ)$ is a commutative semigroup, $\mathcal{P} \in PFRAI(S)$.

The classical anti-ideals of $S$ can be characterized by picture fuzzy anti-ideals with the following proposition.

**Proposition 3.3.** *Let $A \subset S$ and $A \neq \emptyset$. Then, the following statements hold:*

*(i)* $A \in LAI(S)$ *iff* $C^A \in PFLAI(S)$,

*(ii)* $A \in RAI(S)$ *iff* $C^A \in PFRAI(S)$,

*(iii)* $A \in AI(S)$ *iff* $C^A \in PFAI(S)$.

*Proof.* (i) Let $A \in LAI(S)$ and $p_1 \in S$, $p_2 \in A$. Because $A \in LAI(S)$, we have $p_1p_2 \notin A$. Thus,

$$\begin{aligned} \alpha_{C^A}(p_1p_2) \wedge \alpha_{C^A}(p_2) &= 0 \wedge 1 \\ &= 0, \end{aligned}$$

$$(\alpha_{C^A}(p_1p_2) + \beta_{C^A}(p_1p_2)) \wedge (\alpha_{C^A}(p_2) + \beta_{C^A}(p_2)) = (0+0) \wedge (1+0) = 0,$$

$$\begin{aligned} \gamma_{C^A}(p_1p_2) \vee \gamma_{C^A}(p_2) &= 1 \vee 0 \\ &= 1. \end{aligned}$$

Clearly, if $p_2 \notin A$,

$$\alpha_{C^A}(p_1p_2) \wedge \alpha_{C^A}(p_2) = 0,$$

$$(\alpha_{C^A}(p_1p_2) + \beta_{C^A}(p_1p_2)) \wedge (\alpha_{C^A}(p_2) + \beta_{C^A}(p_2)) = 0$$

$$\gamma_{C^A}(p_1p_2) \vee \gamma_{C^A}(p_2) = 1$$

Therefore $C^A \in PFLAI(S)$.
Conversely $C^A \in PFLAI(S)$. Suppose that $SA \cap A \neq \emptyset$. Then there exists $p_1 \in S, p_2 \in A$ with $a = p_1p_2 \in SA \cap A$.

$$\alpha_{C^A}(p_1p_2) \wedge \alpha_{C^A}(p_2) = 1 \neq 0$$

Therefore $A \in LAI(S)$.The proofs of assertions remainder of the theorem are similar. □

**Theorem 3.4.** *Let $\{\mathcal{P}_t \mid t \in T\} \subset PFS(S)$. Then, the following statements hold:*

*(i) If $\{\mathcal{P}_t \mid t \in T\} \subseteq PFLAI(S)$, then $\bigcap_{t\in T} \mathcal{P}_t \in PFLAI(S)$,*

*(ii) If $\{\mathcal{P}_t \mid t \in T\} \subseteq PFRAI(S)$, then $\bigcap_{t\in T} \mathcal{P}_t \in PFRAI(S)$,*

*(iii) If $\{\mathcal{P}_t \mid t \in T\} \subseteq PFAI(S)$, then $\bigcap_{t\in T} \mathcal{P}_t \in PFAI(S)$.*

*Proof.* (i) Let $\mathcal{P}_t \in PFLAI(S)$ and $p_1, p_2 \in S$.

$$\begin{aligned}\alpha_{\bigcap_{t\in T}\mathcal{P}_t}(p_1p_2) \wedge \alpha_{\bigcap_{t\in T}\mathcal{P}_t}(p_2) &= \bigwedge_{t\in T} \alpha_{\mathcal{P}_t}(p_1p_2) \wedge \bigwedge_{t\in T} \alpha_{\mathcal{P}_t}(p_2)\\ &= \bigwedge_{t\in T} (\alpha_{\mathcal{P}_t}(p_1p_2) \wedge \alpha_{\mathcal{P}_t}(p_2))\\ &= 0\end{aligned}$$

$$\begin{aligned}&\alpha_{\bigcap_{t\in T}\mathcal{P}_t}(p_1p_2) + \beta_{\bigcap_{t\in T}\mathcal{P}_t}(p_1p_2) \wedge \alpha_{\bigcap_{t\in T}\mathcal{P}_t}(p_2) + \beta_{\bigcap_{t\in T}\mathcal{P}_t}(p_2)\\ &= \bigwedge_{t\in T} (\alpha_{\mathcal{P}_t}(p_1p_2) + (\bigwedge_{t\in T} (\alpha_{\mathcal{P}_t}(p_1p_2) + \beta_{\mathcal{P}_t}(p_1p_2) - \bigwedge_{t\in T} \alpha_{\mathcal{P}_t}(p_1p_2)) \wedge\\ &\qquad \bigwedge_{t\in T} (\alpha_{\mathcal{P}_t}(p_2) + (\bigwedge_{t\in T} (\alpha_{\mathcal{P}_t}(p_2) + \beta_{\mathcal{P}_t}(p_2)) - \bigwedge_{t\in T} \alpha_{\mathcal{P}_t}(p_2))\\ &= \bigwedge_{t\in T} (\alpha_{\mathcal{P}_t}(p_1p_2) + \beta_{\mathcal{P}_t}(p_1p_2)) \wedge \bigwedge_{t\in T} (\alpha_{\mathcal{P}_t}(p_2) + \beta_{\mathcal{P}_t}(p_2)\\ &= (\bigwedge_{t\in T} [(\alpha_{\mathcal{P}_t}(p_1p_2) + \beta_{\mathcal{P}_t}(p_1p_2)) \wedge (\alpha_{\mathcal{P}_t}(p_2) + \beta_{\mathcal{P}_t}(p_2))]\\ &= 0.\end{aligned}$$

$$\begin{aligned}\gamma_{\bigcap_{i\in\gamma}\mathcal{P}_t}(p_1p_2) \vee \gamma_{\bigcap_{t\in T}\mathcal{P}_t}(p_2) &= \bigvee_{t\in T} \gamma_{\mathcal{P}_t}(p_1p_2) \vee \bigvee_{t\in T} \gamma_{\mathcal{P}_t}(p_2)\\ &= \bigvee_{t\in T} (\gamma_{\mathcal{P}_t}(p_1p_2) \vee \gamma_{\mathcal{P}_t}(p_2))\\ &= 1\end{aligned}$$

So, $\bigcap_{t\in T} \mathcal{P}_t \in PFLAI(S)$.
(ii) and (iii) are obtained similarly to (i). □

**NOTE:** The union of picture fuzzy antiideals may not be a picture fuzzy antiideal in general. We can illustrate this with the following example.

**Example 3.5.** Let $S = \{0, 1, 2\}$ be a semigroup according to the following operation and $\mathcal{P} = (\alpha_\mathcal{P}, \beta_\mathcal{P}, \gamma_\mathcal{P})$, $\mathcal{R} = (\alpha_\mathcal{P}, \beta_\mathcal{P}, \gamma_\mathcal{P})$ be picture fuzzy sets on $S$ with the following tables:
It is easily seen that $\mathcal{P}, \mathcal{R} \in PFLAI(S)$. Since

| $\cdot$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 |

| $\mathcal{P}$ | 0 | 1 | 2 |
|---|---|---|---|
| $\alpha_\mathcal{P}$ | 0 | 1 | 0 |
| $\beta_\mathcal{P}$ | 0 | 0 | 0 |
| $\gamma_\mathcal{P}$ | 1 | 0 | 1 |

| $\mathcal{R}$ | 0 | 1 | 2 |
|---|---|---|---|
| $\alpha_\mathcal{R}$ | 0 | 0 | 1 |
| $\beta_\mathcal{R}$ | 0 | 0 | 0 |
| $\gamma_\mathcal{R}$ | 1 | 1 | 0 |

$$\begin{aligned}\alpha_{\mathcal{P}\cup\mathcal{R}}(2\cdot 2) \wedge \alpha_{\mathcal{P}\cup\mathcal{R}}(2) &= (\alpha_\mathcal{P}(1) \vee \alpha_\mathcal{R}(1)) \wedge (\alpha_\mathcal{P}(2) \vee \alpha_\mathcal{R}(2))\\ &= 1\\ &\neq 0\end{aligned}$$

we have that $\mathcal{P} \cup \mathcal{R} \notin PFLAI(S)$.

Picture fuzzy sets can be characterized via their level subsets, as stated in the following theorem.

**Theorem 3.6.** *$\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(S)$. Then, the following statements hold:*

*(i) $\mathcal{P} \in PFLAI(S)$ iff PM$(\mathcal{P} : \alpha) \neq \emptyset$, N$(\mathcal{P} : \alpha) \neq \emptyset$, and NM$(\gamma_{\mathcal{P}} : \alpha) \neq \emptyset$ are left antiideals of $S$ for every $\alpha \in (0, 1)$,*

*(ii) $\mathcal{P} \in PFRAI(S)$ iff PM$(\mathcal{P} : \alpha) \neq \emptyset$, N$(\mathcal{P} : \alpha) \neq \emptyset$, and NM$(\mathcal{P} : \alpha) \neq \emptyset$ are right antiideals of $S$ for every $\alpha \in (0, 1)$,*

*(iii) $\mathcal{P} \in PFAI(S)$ iff PM$(\mathcal{P} : \alpha) \neq \emptyset$, N$(\mathcal{P} : \alpha) \neq \emptyset$, and NM$(\mathcal{P} : \alpha) \neq \emptyset$ are antiideals of $S$ for every $\alpha \in (0, 1)$.*

*Proof.* (i) Assume that $\mathcal{P} \in PFLAI(S)$. Suppose that $a \in S \cdot PM(\mathcal{P} : \alpha) \cap PM(\mathcal{P} : \alpha)$. Then, $p_1 \in S, p_2 \in PM(\mathcal{P} : \alpha), a = p_1p_2$. It is known that, $\alpha \leq \alpha_{\mathcal{P}}(p_1p_2)$ and $\alpha \leq \alpha_{\mathcal{P}}(p_2)$. Then,

$$\alpha_{\mathcal{P}}(p_1p_2) \wedge \alpha_{\mathcal{P}}(p_2) \geq \alpha \neq 0.$$

The contradiction is obtained. Therefore $S \cdot PM(\mathcal{P} : \alpha) \cap PM(\mathcal{P} : \alpha) = \emptyset$. $N(\mathcal{P} : \alpha)$ and $NM(\mathcal{P} : \alpha)$ are proved similarly. So $PM(\mathcal{P} : \alpha), N(\mathcal{P} : \alpha)$ and $NM(\mathcal{P} : \alpha)$ are left antiideals of $S$.
Conversely, assume that the non-empty sets $PM(\alpha_{\mathcal{P}} : \alpha)$, $N(\beta_{\mathcal{P}} : \alpha)$, and $NM(\gamma_{\mathcal{P}} : \alpha)$ are left antiideals of $S$ for every $\alpha \in (0, 1)$. Let $p_1, p_2 \in S$ and consider $\alpha = \alpha_{\mathcal{P}}(p_1p_2) \wedge \alpha_{\mathcal{P}}(p_2)$ and $\alpha \neq 0$. So, $\alpha \leq \alpha_{\mathcal{P}}(p_1p_2)$ and $\alpha \leq \alpha_{\mathcal{P}}(p_2)$. Having $p_1p_2, p_2 \in PM(\alpha_{\mathcal{P}} : \alpha)$ implies that $p_1p_2 \in S \cdot PM(\mathcal{P} : \alpha) \cap PM(\mathcal{P} : \alpha) = \emptyset$. Since $\alpha_{\mathcal{P}}(p_1p_2) \wedge \alpha_{\mathcal{P}}(p_2) = 0$. Similarly, it is easy to see that

$$\begin{aligned} \alpha_{\mathcal{P}}(p_1p_2) + \beta_{\mathcal{P}}(p_1p_2) \wedge \alpha_{\mathcal{P}}(p_2) + \beta_{\mathcal{P}}(p_2) &= 0 \\ \gamma_{\mathcal{P}}(p_1p_2) \vee \gamma_{\mathcal{P}}(p_2) &= 1. \end{aligned}$$

Hence, $\mathcal{P} \in PFLAI(S)$. We can prove (ii)-(iii) in a similar (i). □

**Theorem 3.7.** *Let $f : S \to S'$ be a semigroup homomorphism and $f$ is an one-to-one. $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(S)$. Then*

*(i) If $\mathcal{P} \in PFLAI(S)$, then $f(\mathcal{P}) \in PFLAI(S')$,*

*(ii) If $\mathcal{P} \in PFRAI(S)$, then $f(\mathcal{P}) \in PFRAI(S')$,*

*(iii) If $\mathcal{P} \in PFAI(S)$, then $f(\mathcal{P}) \in PFAI(S')$.*

*Proof.* (i) Let $\mathcal{P} \in PFLAI(S)$ and $p'_1, p'_2 \in S'$. Then there exists $p_1, p_2 \in S$ with $f(p_1) = p'_1, f(p_2) = p'_2$

$$\alpha_{f(\mathcal{P})}(p'_1p'_2) \wedge \alpha_{f(\mathcal{P})}(p'_2) \leq \alpha_{\mathcal{P}}(p_1p_2) \wedge \alpha_{\mathcal{P}}(p_2) = 0$$

On the other hand,

$$\begin{aligned} \alpha_{f(\mathcal{P})}(p'_1p'_2) + \beta_{f(\mathcal{P})}(p'_1p'_2) \wedge \alpha_{f(\mathcal{P})}(p'_2) + \beta_{f(\mathcal{P})}(p'_2) &= \alpha_{\mathcal{P}}(p_1p_2) + \beta_{\mathcal{P}}(p_1p_2) \wedge \alpha_{\mathcal{P}}(p_2) + \beta_{\mathcal{P}}(p_2) \\ &= 0. \end{aligned}$$

Similarly, we can prove that $\gamma_{f(\mathcal{P})}(p'_1p'_2) \vee \gamma_{f(\mathcal{P})}(p'_2) = 1$. Hence, $f(\mathcal{P}) \in PFLAI(S')$.
The proof of (ii) and (iii) is obtained similarly to (i). □

**Theorem 3.8.** *Let $f : S \to S'$ be a semigroup homomorphism and $f$ is an one-to-one. $\mathcal{R} = (\alpha_{\mathcal{R}}, \beta_{\mathcal{R}}, \gamma_{\mathcal{R}}) \in PFS(S')$. Then, the following statements hold:*

*(i) If $\mathcal{R} \in PFLAI(S')$, then $f^{-1}(\mathcal{R}) \in PFLAI(S)$,*

*(ii) If $\mathcal{R} \in PFRAI(S')$, then $f^{-1}(\mathcal{R}) \in PFRAI(S)$,*

*(iii) If $\mathcal{R} \in PFAI(S')$, then $f^{-1}(\mathcal{R}) \in PFAI(S)$.*

*Proof.* (i) Let $\mathcal{R} \in PFLAI(S')$ and $p_1, y \in S$.

$$\begin{aligned}\alpha_{f^{-1}(\mathcal{R})}(p_1p_2) \wedge \alpha_{f^{-1}(\mathcal{R})}(p_2) &= \alpha_{\mathcal{R}}(f(p_1p_2)) \wedge \alpha_{\mathcal{R}}(f(p_2)) \\ &= \alpha_{\mathcal{R}}(f(p_1)f(p_2)) \wedge \alpha_{\mathcal{R}}(f(p_2)) \\ &= 0.\end{aligned}$$

Also,

$$\begin{aligned}&\alpha_{f^{-1}(\mathcal{R})}(p_1p_2) + \beta_{f^{-1}(\mathcal{R})}(p_1p_2) \wedge \alpha_{f^{-1}(\mathcal{R})}(p_2) + \beta_{f^{-1}(\mathcal{R})}(p_2) \\ &= \alpha_{\mathcal{R}}(f(p_1p_2)) + \beta_{\mathcal{R}}(f(p_1p_2)) \wedge \alpha_{\mathcal{R}}(f(p_2)) + \beta_{\mathcal{R}}(f(p_2)) \\ &= \alpha_{\mathcal{R}}(f(p_1)f(p_2)) + \beta_{\mathcal{R}}(f(p_1)f(p_2)) \wedge \alpha_{\mathcal{R}}(f(p_2)) + \beta_{\mathcal{R}}(f(p_2)) \\ &= 0.\end{aligned}$$

Similarly, it can be seen that $\gamma_{f^{-1}(\mathcal{R})}(p_1p_2) \vee \gamma_{f^{-1}(\mathcal{R})}(p_2) = 1$. Thus, $f^{-1}(\mathcal{R}) \in PFLAI(S)$. The proofs of assertions remainder of the theorem are similar. □

**Theorem 3.9.** *Let $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(S_1)$ and $\mathcal{R} = (\alpha_{\mathcal{R}}, \beta_{\mathcal{R}}, \gamma_{\mathcal{R}}) \in PFS(S_2)$. Then, the following statements hold:*

*(i) If $\mathcal{P} \in PFLAI(S_1)$ and $\mathcal{R} \in PFLAI(S_2)$, then $\mathcal{P} \times \mathcal{R} \in PFLAI(S_1 \times S_2)$,*

*(ii) If $\mathcal{P} \in PFRAI(S_1)$ and $\mathcal{R} \in PFRAI(S_2)$, then $\mathcal{P} \times \mathcal{R} \in PFRAI(S_1 \times S_2)$,*

*(iii) If $\mathcal{P} \in PFAI(S_1)$ and $\mathcal{R} \in PFAI(S_2)$, then $\mathcal{P} \times \mathcal{R} \in PFAI(S_1 \times S_2)$.*

*Proof.* (i) Let $\mathcal{P} \in PFLAI(S_1)$, $\mathcal{R} \in PFLAI(S_2)$, and $(p_1, r_1), (p_2, r_2), \in \mathcal{P} \times \mathcal{R}$.

$$\begin{aligned}\alpha_{\mathcal{P}\times\mathcal{R}}((p_1, r_1)(p_2, r_2)) \wedge \alpha_{\mathcal{P}\times\mathcal{R}}(p_2, r_2)) &= (\alpha_{\mathcal{P}}(p_1p_2) \wedge \alpha_{\mathcal{R}}(r_1r_2)) \wedge (\alpha_{\mathcal{P}}(p_2) \wedge \alpha_{\mathcal{R}}(r_2)) \\ &= \alpha_{\mathcal{P}}(p_1p_2) \wedge \alpha_{\mathcal{P}}(p_2) \wedge \alpha_{\mathcal{R}}(r_1r_2) \wedge \alpha_{\mathcal{R}}(r_2) \\ &= 0 \wedge 0 = 0.\end{aligned}$$

Also,

$$\begin{aligned}&\alpha_{\mathcal{P}\times\mathcal{R}}((p_1, r_1)(p_2, r_2)) + \beta_{\mathcal{P}\times\mathcal{R}}((p_1, r_1)(p_2, r_2)) \wedge \alpha_{\mathcal{P}\times\mathcal{R}}(p_2, r_2) + \beta_{\mathcal{P}\times\mathcal{R}}(p_2, r_2) \\ &= (\alpha_{\mathcal{P}}(p_1p_2) \wedge \alpha_{\mathcal{R}}(r_1r_2)) + (\alpha_{\mathcal{P}}(p_1p_2) + \beta_{\mathcal{P}}(p_1p_2) \wedge \alpha_{\mathcal{R}}(r_1r_2) + \beta_{\mathcal{R}}(r_1r_2) - (\alpha_{\mathcal{P}}(p_1p_2) \\ &\wedge \alpha_{\mathcal{R}}(r_1r_2)) \wedge (\alpha_{\mathcal{P}}(p_2) \wedge \alpha_{\mathcal{R}}(r_2)) + (\alpha_{\mathcal{P}}(p_2) + \beta_{\mathcal{P}}(p_2) \wedge \alpha_{\mathcal{R}}(r_2) + \beta_{\mathcal{R}}(r_2) - (\alpha_{\mathcal{P}}(p_2) \wedge \alpha_{\mathcal{R}}(r_2)) \\ &= (\alpha_{\mathcal{P}}(p_1p_2) + \beta_{\mathcal{P}}p_1p_2) \wedge \alpha_{\mathcal{R}}(r_1r_2) + \beta_{\mathcal{R}}(r_1r_2)) \wedge (\alpha_{\mathcal{P}}(p_2) + \beta_{\mathcal{P}}(p_2) \wedge \alpha_{\mathcal{R}}(r_2) + \beta_{\mathcal{R}}(r_2)) \\ &= (\alpha_{\mathcal{P}}(p_1p_2) + \beta_{\mathcal{P}}(p_1p_2) \wedge \alpha_{\mathcal{P}}(p_2) + \beta_{\mathcal{P}}(p_2)) \wedge (\alpha_{\mathcal{R}}(r_1r_2) + \beta_{\mathcal{R}}(r_1r_2) \wedge \alpha_{\mathcal{R}}(r_2) + \beta_{\mathcal{R}}(r_2)) \\ &= 0.\end{aligned}$$

Similarly, we can prove that $\gamma_{\mathcal{P}\times\mathcal{R}}((p_1, r_1)(p_2, r_2)) \vee \gamma_{\mathcal{P}\times\mathcal{R}}(p_2, r_2) = 1$. This implies that $\mathcal{P} \times \mathcal{R} \in PFLAI(S_1 \times S_2)$.
(ii)-(iii) It can be proved similarly to (i). □

**Definition 3.10.** Let $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(S)$. $\mathcal{P}$ is called *picture fuzzy bi-antiideal* of $S$ if

$$\begin{aligned}\alpha_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_3) &= 0, \\ \alpha_{\mathcal{P}}(p_1p_2p_3) + \beta_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{P}}(p_1) + \beta_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_3) + \beta_{\mathcal{P}}(p_3) &= 0, \\ \gamma_{\mathcal{P}}(p_1p_2p_3) \vee \gamma_{\mathcal{P}}(p_1) \vee \gamma_{\mathcal{P}}(p_3) &= 1,\end{aligned}$$

for all $p_1, p_2, p_3 \in S$.

All picture fuzzy bi-antiideal is denoted by $PFBAI(S)$.

**Example 3.11.** Let $(\mathbb{N}^*, +)$ be the semigroup and $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}})$ is defined as:

$$\alpha_{\mathcal{P}}(p) = \begin{cases} \frac{2}{3}, & p \leq 2 \\ 0, & otherwise. \end{cases}$$

$$\beta_{\mathcal{P}}(p) = \begin{cases} \frac{1}{3}, & p \leq 2 \\ 0, & otherwise. \end{cases}$$

$$\gamma_{\mathcal{P}}(p) = \begin{cases} 0, & p \leq 2 \\ 1, & otherwise. \end{cases}$$

Since $\alpha_{\mathcal{P}}(p_1 + p_2 + p_3) = 0, \beta_{\mathcal{P}}(p_1 + p_2 + p_3) = 0, \gamma_{\mathcal{P}}(p_1 + p_2 + p_3) = 1$ for all $p_1, p_2, p_3 \in S$ and hence $\mathcal{P} \in PFBAI(S)$.

**Proposition 3.12.** *Let $A \subset S$ and $A \neq \emptyset$. $A \in BAI(S)$ iff $C^A \in PFBAI(S)$.*

*Proof.* (i) Let $A \in BAI(S)$ and $p_1, p_3 \in A, p_2 \in S$. $p_1p_2p_3 \notin A$ because $A \in BAI(S)$. Thus,

$$\alpha_{C^A}(p_1p_2p_3) \wedge \alpha_{C^A}(p_1) \wedge \alpha(p_3) = 0$$

$$\big(\alpha_{C^A}(p_1p_2p_3) + \beta_{C^A}(p_1p_2p_3)\big) \wedge \big(\alpha_{C^A}(p_1) + \beta_{C^A}(p_1)\big) \wedge \big(\alpha_{C^A}(p_3) + \beta_{C^A}(p_3)\big) = 0,$$

$$\gamma_{C^A}(p_1p_2p_3) \vee \gamma_{C^A}(p_1) \vee \gamma_{C^A}(p_3) = 1.$$

Clearly, if $p_1 \notin A$ or $p_3 \notin A$,

$$\alpha_{C^A}(p_1p_2p_3) \wedge \alpha_{C^A}(p_1) \wedge \alpha(p_3) = 0 \wedge 1 \wedge 1 = 0,$$

$$\begin{aligned}(\alpha_{C^A}(p_1p_2p_3) + \beta_{C^A}(p_1p_2p_3)) \wedge (\alpha_{C^A}(p_1) + \beta_{C^A}(p_1)) \wedge (\alpha_{C^A}(p_3) + \beta_{C^A}(p_3)) \\ = (0 + 0) \wedge (1 + 0) \wedge (1 + 0) = 0,\end{aligned}$$

$$\gamma_{C^A}(p_1p_2p_3) \vee \gamma_{C^A}(p_1) \vee \gamma_{C^A}(p_3) = 1 \vee 0 \vee 0 = 1.$$

Therefore $C^A \in PFBAI(S)$.
Conversely $C^A \in PFBAI(S)$. Suppose that $ASA \cap A \neq \emptyset$. Then there exists $p_2 \in S, p_1, p_3 \in A$ with $a = p_1p_2p_3 \in ASA \cap A$.

$$\alpha_{C^A}(p_1p_2p_3) \wedge \alpha_{C^A}(p_1) \wedge \alpha_{C^A}(p_3) = 1 \neq 0$$

Therefore $A \in BAI(S)$. □

**Theorem 3.13.** *Let $\{\mathcal{P}_t \mid t \in T\} \subset PFS(S)$. $\{\mathcal{P}_t \mid t \in T\} \subseteq PFBAI(S)$, then $\bigcap_{t\in T} \mathcal{P}_t \in PFBAI(S)$.*

*Proof.* (i) Let $\mathcal{P}_t \in PFBAI(S)$ and $p_1, p_2, p_3 \in S$.

$$\begin{aligned}\alpha_{\bigcap_{t\in T}\mathcal{P}_t}(p_1p_2p_3) \wedge \alpha_{\bigcap_{t\in T}\mathcal{P}_t}(p_1) \wedge \alpha_{\bigcap_{t\in T}\mathcal{P}_t}(p_3) &= \bigwedge_{t\in T} \alpha_{\mathcal{P}_t}(p_1p_2p_3) \wedge \bigwedge_{t\in T} \alpha_{\mathcal{P}_t}(p_1) \wedge \bigwedge_{t\in T} \alpha_{\mathcal{P}_t}(p_3) \\ &= \bigwedge_{t\in T} (\alpha_{\mathcal{P}_t}(p_1p_2p_3) \wedge \alpha_{\mathcal{P}_t}(p_1) \wedge \alpha_{\mathcal{P}_t}(p_3)) \\ &= 0\end{aligned}$$

$$\begin{aligned}
&\left[\alpha_{\bigcap_{t\in T}\mathcal{P}_t}(p_1p_2p_3)+\beta_{\bigcap_{t\in T}\mathcal{P}_t}(p_1p_2p_3)\right]\wedge\left[\alpha_{\bigcap_{t\in T}\mathcal{P}_t}(p_1)+\beta_{\bigcap_{t\in T}\mathcal{P}_t}(p_1)\right]\wedge\left[\alpha_{\bigcap_{t\in T}\mathcal{P}_t}(p_3)+\beta_{\bigcap_{t\in T}\mathcal{P}_t}(p_3)\right]\\
&=\Big[\bigwedge_{t\in T}\alpha_{\mathcal{P}_t}(p_1p_2p_3)+\Big(\bigwedge_{t\in T}(\alpha_{\mathcal{P}_t}(p_1p_2p_3)+\beta_{\mathcal{P}_t}(p_1p_2p_3))-\bigwedge_{t\in T}\alpha_{\mathcal{P}_t}(p_1p_2p_3)\Big)\Big]\wedge\Big[\bigwedge_{t\in T}\big(\alpha_{\mathcal{P}_t}(p_1)+\\
&\Big(\bigwedge_{t\in T}(\alpha_{\mathcal{P}_t}(p_1)+\beta_{\mathcal{P}_t}(p_1))-\bigwedge_{t\in T}\alpha_{\mathcal{P}_t}(p_1)\Big)\Big]\wedge\Big[\bigwedge_{t\in T}(\alpha_{\mathcal{P}_t}(p_3)+\Big(\bigwedge_{t\in T}(\alpha_{\mathcal{P}_t}(p_3)+\beta_{\mathcal{P}_t}(p_3))-\bigwedge_{t\in T}\alpha_{\mathcal{P}_t}(p_3)\Big)\Big]\\
&=\Big(\bigwedge_{t\in T}\alpha_{\mathcal{P}_t}(p_1p_2p_3)+\beta_{\mathcal{P}_t}(p_1p_2p_3)\Big)\wedge\Big(\bigwedge_{t\in T}(\alpha_{\mathcal{P}_t}(p_1)+\beta_{\mathcal{P}_t}(p_1))\wedge\Big(\bigwedge_{t\in T}(\alpha_{\mathcal{P}_t}(p_3)+\beta_{\mathcal{P}_t}(p_3))\Big)\\
&=\Big(\bigwedge_{t\in T}\big(\alpha_{\mathcal{P}_t}(p_1p_2p_3)+\beta_{\mathcal{P}_t}(p_1p_2p_3)\big)\wedge\big(\alpha_{\mathcal{P}_t}(p_1)+\beta_{\mathcal{P}_t}(p_1)\big)\wedge\big(\alpha_{\mathcal{P}_t}(p_3)+\beta_{\mathcal{P}_t}(p_3)\big)\Big)\\
&=0
\end{aligned}$$

$$\begin{aligned}
\gamma_{\bigcap_{t\in T}\mathcal{P}_t}(p_1p_2p_3)\vee\gamma_{\bigcap_{t\in T}\mathcal{P}_t}(p_1)\vee\gamma_{\bigcap_{t\in T}\mathcal{P}_t}(p_3)&=\bigvee_{t\in T}\gamma_{\mathcal{P}_t}(p_1p_2p_3)\vee\bigvee_{t\in T}\gamma_{\mathcal{P}_t}(p_1)\vee\bigvee_{t\in T}\gamma_{\mathcal{P}_t}(p_3)\\
&=\bigvee_{t\in T}\big(\gamma_{\mathcal{P}_t}(p_1p_2p_3)\vee\gamma_{\mathcal{P}_t}(p_1)\vee\gamma_{\mathcal{P}_t}(p_3)\big)\\
&=1
\end{aligned}$$

So, $\bigcap_{t\in T}\mathcal{P}_t\in PFBAI(S)$. □

**NOTE:** The union of picture fuzzy bi-antiideals may not be a picture fuzzy bi-antiideal in general.

**Example 3.14.** Let $S=\{0,1,2,3\}$, $x\cdot y=min(x+y,3)$. $(S,\cdot)$ be a semigroup and $\mathcal{P}=(\alpha_\mathcal{P},\beta_\mathcal{P},\gamma_\mathcal{P})$, $\mathcal{R}=(\alpha_\mathcal{P},\beta_\mathcal{P},\gamma_\mathcal{P})$ be picture fuzzy sets on $S$ with the following tables:

| $\mathcal{P}$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\alpha_\mathcal{P}$ | 0 | 1 | 0 | 0 |
| $\beta_\mathcal{P}$ | 0 | 0 | 0 | 0 |
| $\gamma_\mathcal{P}$ | 1 | 0 | 1 | 1 |

| $\mathcal{R}$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\alpha_\mathcal{R}$ | 0 | 0 | 1 | 0 |
| $\beta_\mathcal{R}$ | 0 | 0 | 0 | 0 |
| $\gamma_\mathcal{R}$ | 1 | 1 | 0 | 1 |

It is easily seen that $\mathcal{P},\mathcal{R}\in PFBAI(S)$. Since

$$\begin{aligned}
\alpha_{\mathcal{P}\cup\mathcal{R}}(1\cdot0\cdot1)\wedge\alpha_{\mathcal{P}\cup\mathcal{R}}(1)\wedge\alpha_{\mathcal{P}\cup\mathcal{R}}(1)&=(\alpha_\mathcal{P}(2)\vee\alpha_\mathcal{R}(2))\wedge(\alpha_\mathcal{P}(1)\vee\alpha_\mathcal{R}(1))\\
&=(0\vee1)\wedge(1\vee0)\\
&=1\\
&\neq0,
\end{aligned}$$

we have that $\mathcal{P}\cup\mathcal{R}\notin PFBAI(S)$.

**Theorem 3.15.** *$\mathcal{P}=(\alpha_\mathcal{P},\beta_\mathcal{P},\gamma_\mathcal{P})\in PFS(S)$. Then, the following statements hold: $\mathcal{P}\in PFBAI(S)$ iff the non-empty sets PM$(\mathcal{P}:\alpha)$, N$(\mathcal{P}:\alpha)$, and NM$(\mathcal{P}:\alpha)$ are bi-antiideals of $S$ for every $\alpha\in(0,1)$.*

*Proof.* (i) Let $\mathcal{P}\in PFBAI(S)$. Suppose that PM$(\mathcal{P}:\alpha)\cdot S\cdot$ PM$(\mathcal{P}:\alpha)\cap$ PM$(\mathcal{P}:\alpha)\neq\emptyset$ . Then, $p_2\in S, p_1,p_3\in$ PM$(\mathcal{P}:\alpha)$, $p_1p_2p_3\in$ PM$(\mathcal{P}:\alpha)$. It is known that, $\alpha\leq\alpha_\mathcal{P}(p_1p_2p_3)$, $\alpha\leq\alpha_\mathcal{P}(p_1)$, $\alpha\leq\alpha_\mathcal{P}(p_3)$ and $\alpha\neq0$. Then,

$$\alpha_\mathcal{P}(p_1p_2p_3)\wedge\alpha_\mathcal{P}(p_1)\wedge\alpha_\mathcal{P}(p_3)\geq\alpha\neq0$$

Contradiction is obtained. Therefore PM$(\mathcal{P}:\alpha)\cdot S\cdot$ PM$(\mathcal{P}:\alpha)\cap$ PM$(\mathcal{P}:\alpha)=\emptyset$. N$(\mathcal{P}:\alpha)$ and NM$(\mathcal{P}:\alpha)$ are proved similarly. So, PM$(\mathcal{P}:\alpha)$,N$(\mathcal{P}:\alpha)$ and NM$(\mathcal{P}:\alpha)$ are bi-antiiideals

of $S$.
Conversely, assume that the non-empty sets $PM(\alpha_{\mathcal{P}} : \alpha)$, $N(\mathcal{P} : \alpha)$, and $NM(\mathcal{P} : \alpha)$ are picture fuzzy bi-antiideals of $S$ for every $\alpha \in (0, 1)$. Let $p_1, p_2, p_3 \in S$ and take $\alpha = \alpha_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_3)$. So, $\alpha \leq \alpha_{\mathcal{P}}(p_1p_2p_3)$, $\alpha \leq \alpha_{\mathcal{P}}(p_1)$, and $\alpha \leq \alpha_{\mathcal{P}}(p_3)$. Then, $p_1p_2p_3, p_1, p_3 \in PM(\alpha_{\mathcal{P}} : \alpha)$. Hence $p_1p_2p_3 \in PM(\mathcal{P} : \alpha) \cdot S \cdot PM(\mathcal{P} : \alpha) \cap PM(\mathcal{P} : \alpha) = \emptyset$. Since $\alpha_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_3) = 0$. Similarly, it is easy to see that

$$\alpha_{\mathcal{P}}(p_1p_2p_3) + \beta_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{P}}(p_1) + \beta_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_3) + \beta_{\mathcal{P}}(p_3) = 0$$
$$\gamma_{\mathcal{P}}(p_1p_2p_3) \vee \gamma_{\mathcal{P}}(p_1) \vee \gamma_{\mathcal{P}}(p_3) = 1.$$

Hence, $\mathcal{P} \in PFBAI(S)$. □

**Theorem 3.16.** *Suppose that $f : S \to S'$ is a semigroup isomorphism. $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(S)$. If $\mathcal{P} \in PFBAI(S)$, then $f(\mathcal{P}) \in PFBAI(S)$.*

*Proof.* Let $\mathcal{P} \in PFBAI(S)$ and $p'_1, p'_2, p'_3 \in S'$. Then there exists $p_1, p_2, p_3 \in S$ with $f(p_1) = p'_1, f(p_2) = p'_2, f(p_3) = p'_3$

$$\alpha_{f(\mathcal{P})}(p'_1p'_2p'_3) \wedge \alpha_{f(\mathcal{P})}(p'_1) \wedge \alpha_{f(\mathcal{P})}(p'_3) \leq \alpha_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_3) = 0.$$

On the other hand,

$$\begin{aligned}
&\alpha_{f(\mathcal{P})}(p'_1p'_2p'_3) + \beta_{f(\mathcal{P})}(p'_1p'_2p'_3) \wedge \alpha_{f(\mathcal{P})}(p'_1) + \beta_{f(M)}(p'_1) \wedge \alpha_{f(\mathcal{P})}(p'_3) + \beta_{f(\mathcal{P})}(p'_3)\\
&= \alpha_{\mathcal{P}}(p_1p_2p_3) + \beta_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{P}}(p_1) + \beta_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_3) + \beta_{\mathcal{P}}(p_3)\\
&= 0.
\end{aligned}$$

Similarly, we can prove that $\gamma_{f(\mathcal{P})}(p'_1p'_2p'_3) \vee \gamma_{f(\mathcal{P})}(p'_1) \vee \gamma_{f(\mathcal{P})}(p'_3) = 1$. Hence, $f(\mathcal{P}) \in PFBAI(S')$. □

**Theorem 3.17.** *Suppose that $f : S \to S'$ is a semigroup homomorphism. $\mathcal{R} = (\alpha_{\mathcal{R}}, \beta_{\mathcal{R}}, \gamma_{\mathcal{R}}) \in PFS(S')$. If $\mathcal{R} \in PFBAI(S')$, then $f^{-1}(\mathcal{R}) \in PFBAI(S)$.*

*Proof.* Let $\mathcal{R} \in PFBAI(S')$ and $p_1, p_2, p_3 \in S$.

$$\begin{aligned}
&\alpha_{f^{-1}(\mathcal{R})}(p_1p_2p_3) \wedge \alpha_{f^{-1}(\mathcal{R})}(p_1) \wedge \alpha_{f^{-1}(\mathcal{R})}(p_3) = \alpha_{\mathcal{R}}(f(p_1p_2p_3)) \wedge \alpha_{\mathcal{R}}(f(p_1)) \wedge \alpha_{\mathcal{R}}(f(p_3)),\\
&= \alpha_{\mathcal{R}}(f(p_1)f(p_2)f(p_3)) \wedge \alpha_{\mathcal{R}}(f(p_1)) \wedge \alpha_{\mathcal{R}}(f(p_3)),\\
&= 0.
\end{aligned}$$

Also,

$$\begin{aligned}
&\alpha_{f^{-1}(\mathcal{R})}(p_1p_2p_3) + \beta_{f^{-1}(\mathcal{R})}(p_1p_2p_3) \wedge \alpha_{f^{-1}(\mathcal{R})}(p_1) + \beta_{f^{-1}(\mathcal{R})}(p_1) \wedge \alpha_{f^{-1}(\mathcal{R})}(p_3) + \beta_{f^{-1}(\mathcal{R})}(p_3)\\
&= \alpha_{\mathcal{R}}\big(f(p_1p_2p_3)\big) + \beta_{\mathcal{R}}\big(f(p_1p_2p_3)\big) \wedge \alpha_{\mathcal{R}}(f(p_1)) + \beta_{\mathcal{R}}(f(p_1)) \wedge \alpha_{\mathcal{R}}(f(p_3)) + \beta_{\mathcal{R}}(f(p_3))\\
&= \alpha_{\mathcal{R}}\big(f(p_1)f(p_2)f(p_3)\big) + \beta_{\mathcal{R}}\big(f(p_1)f(p_2)f(p_3)\big) \wedge \alpha_{\mathcal{R}}(f(p_1)) + \beta_{\mathcal{R}}(f(p_1)) \wedge \alpha_{\mathcal{R}}(f(p_3)) + \beta_{\mathcal{R}}(f(p_3))\\
&= 0.
\end{aligned}$$

Similarly, it can be seen that $\gamma_{f^{-1}(\mathcal{R})}(p_1p_2p_3) \vee \gamma_{f^{-1}(\mathcal{R})}(p_1) \vee \gamma_{f^{-1}(\mathcal{R})}(p_3) = 1$. Thus, $f^{-1}(\mathcal{R}) \in PFBAI(S)$. □

**Theorem 3.18.** *Let $\mathcal{P} = (\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}}) \in PFS(S_1)$ and $\mathcal{R} = (\alpha_{\mathcal{R}}, \beta_{\mathcal{R}}, \gamma_{\mathcal{R}}) \in PFS(S_2)$. If $\mathcal{P} \in PFBAI(S_1)$ and $\mathcal{R} \in PFBAI(S_2)$, then $\mathcal{P} \times \mathcal{R} \in PFBAI(S_1 \times S_2)$.*

*Proof.* Let $\mathcal{P} \in PFBAI(S_1)$, $\mathcal{R} \in PFBAI(S_2)$, and $(p_1, r_1), (p_2, r_2), (p_3, r_3) \in S_1 \times S_2$.

$$\begin{aligned}
&\alpha_{\mathcal{P}\times\mathcal{R}}\big((p_1, r_1)(p_2, r_2)(p_3, r_3)\big) \wedge \alpha_{\mathcal{P}\times\mathcal{R}}(p_1, r_1) \wedge \alpha_{\mathcal{P}\times\mathcal{R}}(p_3, r_3)\\
&= \big(\alpha_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{R}}(r_1r_2r_3)\big) \wedge \big(\alpha_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{R}}(r_1)\big) \wedge \big(\alpha_{\mathcal{P}}(p_3) \wedge \alpha_{\mathcal{R}}(r_3)\big)\\
&= \alpha_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_3) \wedge \alpha_{\mathcal{R}}(r_1r_2r_3) \wedge \alpha_{\mathcal{R}}(r_1) \wedge \alpha_{\mathcal{R}}(r_3)\\
&= 0 \wedge 0 = 0.
\end{aligned}$$

Also,

$$\begin{aligned}
&\big[\alpha_{\mathcal{P}\times\mathcal{R}}((p_1,r_1)(p_2,r_2)(p_3,r_3)\big) + \beta_{\mathcal{P}\times\mathcal{R}}((p_1,r_1)(p_2,r_2)(p_3,r_3))\big] \wedge \big[\alpha_{\mathcal{P}\times\mathcal{R}}(p_1,r_1) + \beta_{\mathcal{P}\times\mathcal{R}}(p_1,r_1)\big] \\
&\qquad \wedge \big[\alpha_{\mathcal{P}\times\mathcal{R}}(p_3,r_3) + \beta_{\mathcal{P}\times\mathcal{R}}(p_3,r_3)\big] \\
&= \big[(\alpha_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{R}}(r_1r_2r_3)) + \big(\alpha_{\mathcal{P}}(p_1p_2p_3) + \beta_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{R}}(r_1r_2r_3) + \beta_{\mathcal{R}}(r_1r_2r_3) - (\alpha_{\mathcal{P}}(p_1p_2p_3))\big] \\
&\wedge \alpha_{\mathcal{R}}(r_1r_2r_3)) \wedge \big[(\alpha_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{R}}(r_1)) + (\alpha_{\mathcal{P}}(p_1) + \beta_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{R}}(r_1) + \beta_{\mathcal{R}}(r_1) - (\alpha_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{R}}(r_1))\big] \\
&\wedge \big[(\alpha_{\mathcal{P}}(p_3) \wedge \alpha_{\mathcal{R}}(r_3)) + (\alpha_{\mathcal{P}}(p_3) + \beta_{\mathcal{P}}(p_3) \wedge \alpha_{\mathcal{R}}(r_3) + \beta_{\mathcal{R}}(r_3) - (\alpha_{\mathcal{P}}(p_3) \wedge \alpha_{\mathcal{R}}(r_3))\big] \\
&= \big(\alpha_{\mathcal{P}}(p_1p_2p_3) + \beta_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{R}}(r_1r_2r_3) + \beta_{\mathcal{R}}(r_1r_2r_3)\big) \wedge \big(\alpha_{\mathcal{P}}(p_1) + \beta_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{R}}(r_1) + \beta_{\mathcal{R}}(r_1)\big) \\
&\qquad \wedge \big(\alpha_{\mathcal{P}}(p_3) + \beta_{\mathcal{P}}(p_3) \wedge \alpha_{\mathcal{R}}(r_3) + \beta_{\mathcal{R}}(r_3)\big) \\
&= \big(\alpha_{\mathcal{P}}(p_1p_2p_3) + \beta_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{P}}(p_1) + \beta_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_3) + \beta_{\mathcal{P}}(p_3)\big) \wedge \big(\alpha_{\mathcal{R}}(r_1r_2r_3) + \beta_{\mathcal{R}}(r_1r_2r_3) \\
&\qquad \wedge \alpha_{\mathcal{R}}(r_1) + \beta_{\mathcal{R}}(r_1) \wedge \alpha_{\mathcal{R}}(r_3) + \beta_{\mathcal{R}}(r_3)\big) \\
&= 0.
\end{aligned}$$

Similarly, we can prove that $\gamma_{\mathcal{P}\times\mathcal{R}}((p_1,r_1)(p_2,r_2)(p_3,r_3) \vee \gamma_{\mathcal{P}\times\mathcal{R}}(p_1,r_1) \vee \gamma_{\mathcal{P}\times\mathcal{R}}(p_3,r_3) = 1$. This implies that $\mathcal{P}\times\mathcal{R} \in PFBAI(S_1\times S_2)$. □

**Theorem 3.19.** *Every picture fuzzy left antiideal (right antiideal, antiideal) is a picture fuzzy bi-antiideal.*

*Proof.* Let $\mathcal{P} \in PFLAI(S)$. For $p_1,p_2,p_3 \in S$,

$$0 \le \alpha_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_3) \le \alpha_{\mathcal{P}}((p_1p_2)p_3) \wedge \alpha_{\mathcal{P}}(p_3) = 0$$

Also,

$$\begin{aligned}
&\alpha_{\mathcal{P}}(p_1p_2p_3) + \beta_{\mathcal{P}}(p_1p_2p_3) \wedge \alpha_{\mathcal{P}}(p_1) + \beta_{\mathcal{P}}(p_1) \wedge \alpha_{\mathcal{P}}(p_3) + \beta_{\mathcal{P}}(p_3) \\
&\le \alpha_{\mathcal{P}}((p_1p_2)p_3) + \beta_{\mathcal{P}}((p_1p_2)p_3) \wedge \alpha_{\mathcal{P}}(p_3) + \beta_{\mathcal{P}}(p_3) \\
&= 0
\end{aligned}$$

Similarly, it can be seen that $1 \ge \gamma_{\mathcal{P}}((p_1p_2)p_3) \vee \gamma_{\mathcal{P}}(p_1) \vee \gamma_{\mathcal{P}}(p_3) = 1$. Thus, $\mathcal{P} \in PFBAI(S)$. □

## 4 Conclusion

Picture fuzzy sets are used in areas such as decision making, medical diagnosis, data analysis, information systems, and intelligent control. Moreover, picture fuzzy sets extend fuzzy sets and intuitionistic fuzzy sets, making them suitable for modeling real-world problems that involve uncertainty and incomplete information. In this paper, by using the type-3 ordering, we extended the concepts of anti-ideals in semigroups to picture fuzzy set structures, thereby providing a more general framework. We investigated the basic properties of picture fuzzy anti-ideals and established their connections with corresponding level subsets. The results presented in this study, picture fuzzy anti-ideals naturally generalize classical anti-ideals, fuzzy anti ideals, and intuitionistic anti ideals. Also, this study forms a foundation for further research on picture fuzzy anti-ideals. In future work, these concepts may be applied to decision-making problems and extended to other algebraic structures, where uncertainty and negative information play an important role.

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