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Anisotropic Compact Stars in Rastall Gravity: Exact Solutions, Physical Viability, and Observational Constraints

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Abstract This study explores the structure and physical viability of anisotropic compact stars in the framework of Rastall gravity—a modified theory that generalizes the energy-momentum conservation law through a non-minimal coupling between geometry and matter, characterized by the higher-derivative coupling parameters α and β. Using a static spherically symmetric metric ansatz, exact analytical expressions are obtained for the effective energy density, radial and tangential pressures, and the mass function, without imposing any particular equation of state. To test the model against real astrophysical data, six well-known compact stars are considered: **EXO 1785-248**, **PSR J1614–2230**, **Cen X-3**, **Vela X-1**, **4U 1608-52**, and **KS 1731-260**. The Darmois-Israel junction conditions are used to fix the boundary, and all model parameters are obtained directly from observable stellar quantities. A detailed check of physical acceptability is then carried out, covering regularity at the centre, positivity of the energy density and pressures, the standard energy conditions, subluminal sound speeds, Herrera's cracking stability criterion, hydrostatic equilibrium via the generalized TOV equation, and adiabatic stability through the condition $\Gamma_r > 4/3$. The effect of the coupling parameters (α, β) on the overall stellar structure is also studied and compared with the corresponding General Relativity results. Overall, the findings show that Rastall gravity can support massive compact stars without invoking an unusually stiff equation of state, pointing to it as a physically reasonable alternative to General Relativity in strong-field regimes

1 Introduction

Compact astrophysical objects such as neutron stars provide a powerful laboratory for studying gravity under extreme conditions [1, 2]. Advances in observational astronomy, including X-ray binaries and radio pulsars, have significantly constrained neutron star properties such as mass and radius [3, 4, 5, 6, 7, 8].

In this study we analyse compact stars arranged according to the graphical legend used in our model. The first object considered is **EXO 1785-248**, a well-known neutron star in a globular cluster used for radius estimation [9]. This is followed by the massive pulsar **PSR J1614–2230**, whose accurately measured mass ($\sim 2\,M_\odot$) provides a strong constraint on theoretical models [4]. Next, we include X-ray binary systems **Cen X-3** and **Vela X-1**, which are among the earliest observed neutron star systems and provide important mass measurements [10, 11]. Additional canonical neutron stars considered in this work include **4U 1608-52** and **KS 1731-260**, which are widely used in compact star modelling due to their well-constrained observational data [12, 6].

These observational inputs are used to test the viability of anisotropic compact star models in the framework of Rastall gravity [13, 14]. Unlike General Relativity [15], Rastall gravity modifies the conservation law of the energy–momentum tensor, allowing a non-minimal coupling between geometry and matter [13, 16]. This modification becomes particularly significant in high-density regimes such as neutron stars [17, 18].

The gravitational collapse of massive stars, the formation of spacetime singularities, and the role of junction conditions in matching interior and exterior geometries are intimately related to the physics of compact star formation; recent investigations of these phenomena within the framework of general relativity have appeared in [19]. The differential-geometric structures underlying the modified Einstein field equations employed here—including the Einstein tensor $G_{\mu\nu}$ and the Ricci curvature tensor $R_{\mu\nu}$ that appear in Sections 2–3—have been analysed on pseudo-Riemannian and related manifolds in [20, 21].

The geometric framework of the Rastall field equations rests on the theory of curvature tensors on Lorentzian manifolds. Conformal Ricci solitons and their deformations on Sasakian manifolds have been examined in [22]. The behaviour of curvature tensors on Lorentzian–Sasakian space forms has been investigated in [23]. Biharmonic maps on Riemannian manifolds, whose associated Laplacian operators appear in the higher-derivative correction tensors $H^{(1)}_{\mu\nu}$ and $H^{(2)}_{\mu\nu}$ (Section 2), have been studied in [24]. The variational geometry of harmonic maps relevant to the derivation of the Rastall field equations is further developed in [25]. Geodesic and F-geodesic properties of tangent bundles with modified Riemannian metrics have been studied in [26].

Additionally, anisotropy—defined as the difference between radial and tangential pressures—plays a crucial role in determining the structure and stability of compact stars [27, 28, 29]. It can arise due to various physical mechanisms such as strong magnetic fields, phase transitions, or superfluidity [30, 31, 32]. In modified gravity theories, anisotropy effects can be further enhanced [33, 34].

In this work we construct an exact anisotropic model using a suitable metric ansatz [35, 36] and derive all physical quantities directly from the field equations without assuming a specific equation of state. The model is tested against the selected compact stars to ensure consistency with observational constraints [37, 6].

We further analyse physical acceptability conditions such as energy conditions [38, 39], causality and cracking stability [40, 41], equilibrium via the generalised Tolman–Oppenheimer–Volkoff equation [1, 2], and adiabatic index [42, 43]. The mass–radius relation and other physical properties are also examined [3, 6].

The paper is organised as follows. Section 2 presents the field equations of Rastall gravity. Section 3 introduces the metric ansatz and derives the physical variables. The complete field equations are given in Section 4. Boundary conditions are discussed in Section 5. Section 6 deals with physical viability. Stability and equilibrium are analysed in Section 7. The generalised TOV equation is discussed in Section 8. The adiabatic index is examined in Section 9. Section 10 provides concluding remarks.

2 Field Equations of Rastall Gravity

Rastall gravity is a modified theory of gravity proposed by P. Rastall in 1972 [13], which generalises Einstein's theory [44] by relaxing the usual conservation law of the energy–momentum tensor. In this framework, the covariant divergence ∇^μ of $T_{\mu\nu}$ is not necessarily zero but is instead proportional to the gradient of the Ricci scalar R [13, 14]:

$$\nabla^\mu T_{\mu\nu} \propto \nabla_\nu R. \tag{2.1}$$

This leads to a non-minimal coupling between matter and geometry [13, 16], especially relevant in strong gravitational fields such as those inside compact stars [17].

The field equations represent a modification of the Einstein Field Equations (EFE) [44] with in the framwork of Rastall gravity by adding geometric correction terms $H^{(1)}_{\mu\nu}$ and $H^{(2)}_{\mu\nu}$, parameterised by the higher-derivative coupling parameters α and β [45, 46]:

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} + \alpha H^{(1)}_{\mu\nu} + \beta H^{(2)}_{\mu\nu}, \tag{2.2}$$

where α controls the Ricci-squared correction and β the Weyl (conformal) correction [45]and $\mathcal{H}^{(k)}_{\mu\nu} = -H^{(k)}_{\mu\nu}$ for $k = 1, 2$.

Equivalently:

$$G_{\mu\nu} + \alpha\mathcal{H}^{(1)}_{\mu\nu} + \beta\mathcal{H}^{(2)}_{\mu\nu} = 8\pi T_{\mu\nu}, \tag{2.3}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$. These field equations differ from Einstein's field equations (in GR) by the presence of the additional term proportional to $g_{\mu\nu}R$. The action with quadratic curvature terms is [45, 46]:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + \alpha\mathcal{L}_1 + \beta\mathcal{L}_2 + \mathcal{L}_m\right], \tag{2.4}$$

or equivalently,

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + \alpha\big(R_{\mu\nu}R^{\mu\nu} - \tfrac{1}{3}R^2\big) + \beta C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right], \tag{2.5}$$

where $\mathcal{L}_1 = R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2$ is the quadratic Ricci correction [45], $\mathcal{L}_2 = C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$ is the Weyl tensor squared [47], and $\mathcal{L}_m$ is the matter Lagrangian. The variational derivatives yield [45, 46]:

$$H^{(1)}_{\mu\nu} = 2R_{\mu\alpha\nu\beta}R^{\alpha\beta} - \tfrac{2}{3}RR_{\mu\nu} + \tfrac{1}{6}g_{\mu\nu}R^2 - \tfrac{2}{3}\nabla_\mu\nabla_\nu R + \tfrac{1}{3}g_{\mu\nu}\Box R + \Box R_{\mu\nu} - 2\nabla^\alpha\nabla_{(\mu}R_{\nu)\alpha}, \tag{2.6}$$

$$H^{(2)}_{\mu\nu} = -4\nabla^\alpha\nabla^\beta C_{\mu\alpha\nu\beta} + 2R^{\alpha\beta}C_{\mu\alpha\nu\beta} + \tfrac{1}{2}g_{\mu\nu}C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}. \tag{2.7}$$

Taking the covariant divergence and using the Bianchi identity [15]:

$$\nabla^\mu T_{\mu\nu} = -\frac{1}{8\pi G}\left(\alpha\nabla^\mu H^{(1)}_{\mu\nu} + \beta\nabla^\mu H^{(2)}_{\mu\nu}\right), \tag{2.8}$$

which explicitly shows the non-conservation of matter energy–momentum and its coupling to curvature [13, 16].

2.1 Anisotropic Energy–Momentum Tensor

The interior is modelled by an anisotropic fluid [28, 29] whose energy-momentum tensor (EMT) in comoving coordinates is [15]:

$$T^\mu_\nu = \text{diag}(-\rho,\, P_r,\, P_t,\, P_t), \tag{2.9}$$

where ρ is the energy density, P_r the radial pressure, and P_t the tangential pressure respectively.

The energy momentum tensor for anisotropic matter distribution in covariant form [29] is considered as

$$T_{\mu\nu} = (\rho + P_t)u_\mu u_\nu + P_t g_{\mu\nu} + (P_r - P_t)\chi_\mu\chi_\nu, \tag{2.10}$$

with $u^\mu u_\mu = -1$, $\chi^\mu\chi_\mu = 1$, $u^\mu\chi_\mu = 0$. For the static spherically symmetric metric, $u^\mu = e^{-\nu/2}\delta^\mu_t$ and $\chi^\mu = e^{-\lambda/2}\delta^\mu_r$, the non-vanishing EMT components are [15, 29]:

$$T_{tt} = \rho\, e^\nu, \quad T_{rr} = P_r\, e^\lambda, \quad T_{\theta\theta} = P_t\, r^2, \quad T_{\phi\phi} = P_t\, r^2\sin^2\theta. \tag{2.11}$$

Pressure anisotropy ($P_t \neq P_r$) is crucial for realistic modelling of ultra-dense matter [27, 28], arising from superfluid vortices [30], strong magnetic fields [32], phase transitions [31], and solid crusts [48].

3 Anisotropic Compact Stellar Configurations in Rastall Gravity

We consider the static spherically symmetric line element:

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{3.1}$$

where $\nu(r)$ and $\lambda(r)$ are the gravitational potentials. The non-zero Christoffel symbols are:

$$\begin{aligned}
&\Gamma^t_{tr} = \tfrac{\nu'}{2}, && \Gamma^r_{tt} = \tfrac{\nu'}{2}e^{\nu-\lambda}, && \Gamma^r_{rr} = \tfrac{\lambda'}{2},\\
&\Gamma^r_{\theta\theta} = -re^{-\lambda}, && \Gamma^r_{\phi\phi} = -r\sin^2\theta\, e^{-\lambda}, &&\\
&\Gamma^\theta_{r\theta} = \Gamma^\phi_{r\phi} = \tfrac{1}{r}, && \Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta, && \Gamma^\phi_{\theta\phi} = \cot\theta.
\end{aligned} \tag{3.2}$$

The independent non-zero Riemann tensor components are:

$$\begin{aligned}
&R^t{}_{rtr} = -\tfrac{1}{2}\nu'' - \tfrac{1}{4}\nu'^2 + \tfrac{1}{4}\nu'\lambda',\\
&R^t{}_{\theta t\theta} = -\tfrac{r\nu'}{2}e^{-\lambda}, \quad R^t{}_{\phi t\phi} = -\tfrac{r\nu'}{2}e^{-\lambda}\sin^2\theta,\\
&R^r{}_{\theta r\theta} = \tfrac{r\lambda'}{2}e^{-\lambda}, \quad R^r{}_{\phi r\phi} = \tfrac{r\lambda'}{2}e^{-\lambda}\sin^2\theta,\\
&R^\theta{}_{\phi\theta\phi} = (1-e^{-\lambda})\sin^2\theta.
\end{aligned} \tag{3.3}$$

The Ricci tensor components are:

$$\begin{aligned}
&R_{tt} = e^{\nu-\lambda}\left(\tfrac{\nu''}{2} + \tfrac{\nu'^2}{4} - \tfrac{\nu'\lambda'}{4} + \tfrac{\nu'}{r}\right),\\
&R_{rr} = -\tfrac{\nu''}{2} - \tfrac{\nu'^2}{4} + \tfrac{\nu'\lambda'}{4} + \tfrac{\lambda'}{r},\\
&R_{\theta\theta} = e^{-\lambda}\left[1 + \tfrac{r}{2}(\nu' - \lambda')\right] - 1, \quad R_{\phi\phi} = R_{\theta\theta}\sin^2\theta,
\end{aligned} \tag{3.4}$$

and the Ricci scalar is:

$$R = -e^{-\lambda}\left(\nu'' + \tfrac{\nu'^2}{2} - \tfrac{\nu'\lambda'}{2} + \tfrac{2\nu'}{r} - \tfrac{2\lambda'}{r} + \tfrac{2}{r^2}\right) + \tfrac{2}{r^2}. \tag{3.5}$$

3.1 Rastall Field Equation Components

On imposing Eqs. (2.11) in the field equation (2.2), the Rastall field equation for the metric (3.1) obtained as

$$\frac{e^{-\lambda}}{r^2}\left(r\lambda' - 1 + e^{\lambda}\right) = 8\pi\rho + \alpha\mathcal{H}_1(r) + \beta\mathcal{H}_2(r), \tag{3.6}$$

$$\frac{e^{-\lambda}}{r^2}\left(r\nu' + 1 - e^{\lambda}\right) = 8\pi P_r + \alpha\mathcal{H}_3(r) + \beta\mathcal{H}_4(r), \tag{3.7}$$

$$\frac{e^{-\lambda}}{4}\left[2\nu'' + \nu'^2 - \lambda'\nu' + \tfrac{2}{r}(\nu' - \lambda')\right] = 8\pi P_t + \alpha\mathcal{H}_5(r) + \beta\mathcal{H}_6(r), \tag{3.8}$$

where:

$$\mathcal{H}_1(r) = \frac{1}{r^2}\left[(1-e^{-\lambda}) - \tfrac{r}{2}(\nu' + \lambda')e^{-\lambda}\right], \tag{3.9}$$

$$\mathcal{H}_2(r) = \frac{e^{-\lambda}}{2}\left[\nu'' + \tfrac{\nu'^2}{2} - \tfrac{\nu'\lambda'}{2} + \tfrac{\nu'-\lambda'}{r}\right]. \tag{3.10}$$

$$\mathcal{H}_3(r) = \frac{e^{-\lambda}}{r^2}\left[1 - e^{\lambda} + \tfrac{r}{2}(\nu' + \lambda')\right], \tag{3.11}$$

$$\mathcal{H}_4(r) = \frac{e^{-\lambda}}{2}\left[\nu'' + \tfrac{\nu'^2}{2} - \tfrac{\nu'\lambda'}{2} - \tfrac{\nu'+\lambda'}{r}\right]. \tag{3.12}$$

$$\mathcal{H}_5(r) = \frac{e^{-\lambda}}{4r}\left[2(\nu' - \lambda') + r(\nu'^2 - \nu'\lambda')\right], \tag{3.13}$$

$$\mathcal{H}_6(r) = \frac{e^{-\lambda}}{4}\left[2\nu'' + \nu'^2 - \lambda'\nu' - \tfrac{2}{r}(\nu' + \lambda')\right]. \tag{3.14}$$

In order to obtain a physicall viable model for compact stars,we adopt the well known metric ansatz [35, 36]:

$$e^{-\lambda(r)} = 1 + ar^2, \qquad e^{\nu(r)} = C(1 + ar^2)^n. \tag{3.15}$$

By substituting (3.15) into equation (3.9)-(3.14) we obtained the following expression:

$$\mathcal{H}_1(r) = -\frac{na}{1 + ar^2}, \tag{3.16}$$

$$\mathcal{H}_2(r) = \frac{na}{2(1 + ar^2)^2}\left[na^2r^2 + 3na - 2a\right], \tag{3.17}$$

$$\mathcal{H}_3(r) = \frac{2a(n-1)}{(1 + ar^2)^2}, \tag{3.18}$$

$$\mathcal{H}_4(r) = \frac{a}{2(1 + ar^2)^2}\left[2n(1 - ar^2) + n^2a^2r^2 - 2\right], \tag{3.19}$$

$$\mathcal{H}_5(r) = \frac{a(n+1)}{(1 + ar^2)^2}\left[1 + \tfrac{nar^2}{2}\right], \tag{3.20}$$

$$\mathcal{H}_6(r) = \frac{a}{4(1 + ar^2)^2}\left[2n(1 - ar^2) + n^2a^2r^2 - 4\right]. \tag{3.21}$$

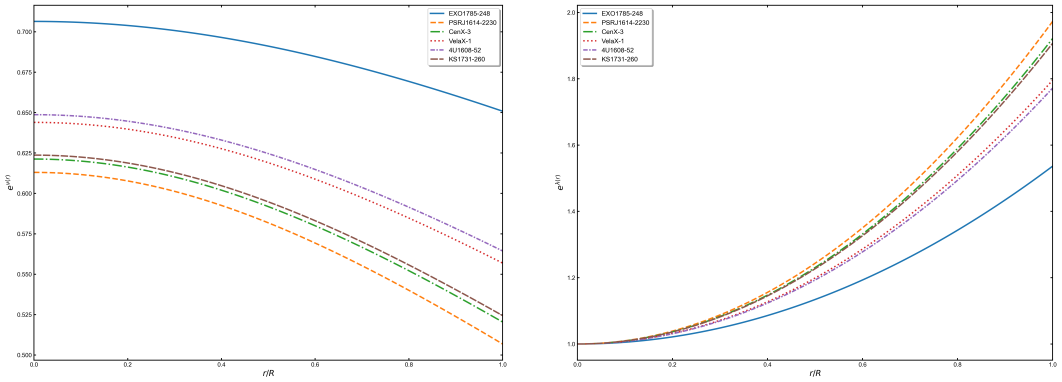


Figure 1. Variation of the metric potentials $e^{\nu(r)}$ (left) and $e^{\lambda(r)}$ (right) as a function of r/R for the six neutron star candidates under Rastall gravity with $\alpha = 1.80$ and $\beta = 0.47$.

4 Complete Field Equations for the Metric Ansatz

To solve the system of Eqs.(3.6)-(3.8) for the final version of ρ, p_r ,p_t , and Δ, using Eqs.(3.16)-(3.21), we obtained the following final expression:

$$8\pi\rho(r) = -3a + \frac{\alpha na}{1 + ar^2} - \frac{\beta na}{2(1 + ar^2)^2}\left[na^2r^2 + 3na - 2a\right], \tag{4.1}$$

$$8\pi P_r(r) = \frac{2anr^2}{1 + ar^2} - 3a - \frac{2\alpha a(n-1)}{(1 + ar^2)^2} - \frac{\beta a}{2(1 + ar^2)^2}\left[2n(1 - ar^2) + n^2a^2r^2 - 2\right], \tag{4.2}$$

$$8\pi P_t(r) = \frac{a\left[n(2+n)ar^2 - (n+1)\right]}{(1 + ar^2)^2} - \frac{\alpha a(n+1)}{(1 + ar^2)^2}\left[1 + \tfrac{nar^2}{2}\right] - \frac{\beta a}{4(1 + ar^2)^2}\left[2n(1 - ar^2) + n^2a^2r^2 - 4\right], \tag{4.3}$$

$$8\pi\Delta(r) = \frac{a\left[2a^2n^2r^4 + (4n^2 - 2n)ar^2 - (4n-2)\right]}{(1+ar^2)^3} - \frac{\alpha a\left[(n+1)\left(1+\frac{nar^2}{2}\right) - 2(n-1)\right]}{(1+ar^2)^2} - \frac{\beta a\left[n^2a^2r^2 - 2na^2r^2\right]}{4(1+ar^2)^2}. \tag{4.4}$$

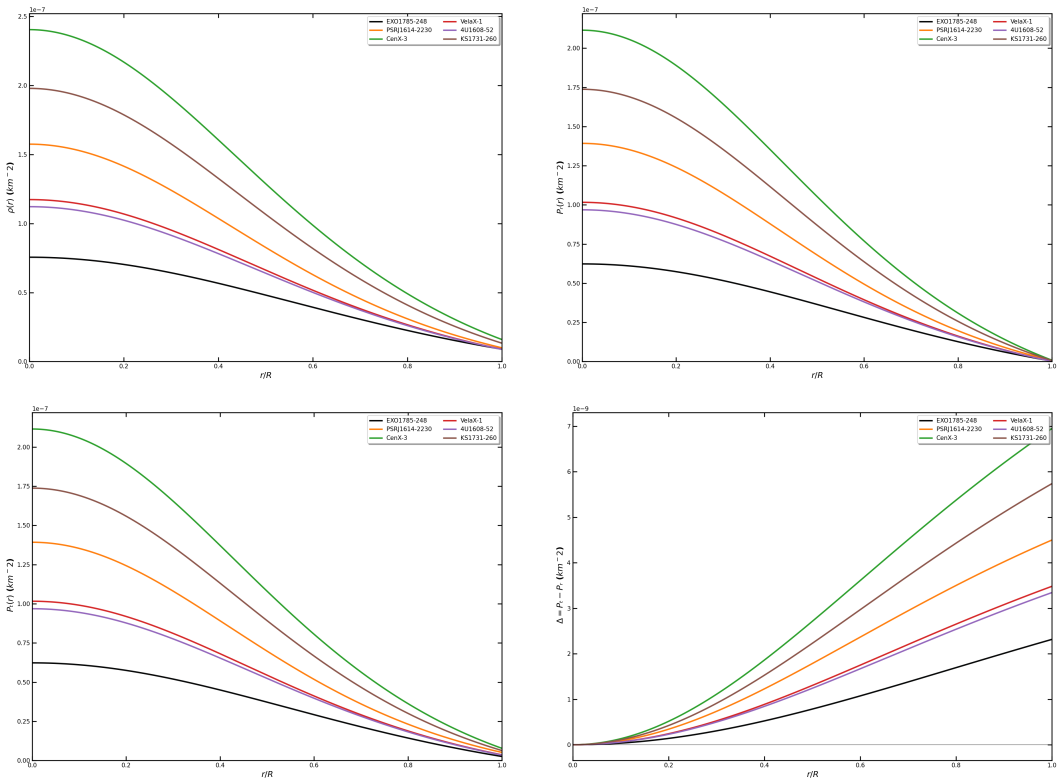


Figure 2. Physical parameters: density ρ (upper left), radial pressure p_r (upper right), tangential pressure p_t (lower left), and anisotropy Δ (lower right).

5 Boundary Conditions and Model Parameter Estimation

To match the interior geometry to the exterior Schwarzschild spacetime [49], we impose the Darmois–Israel junction conditions [50, 51] at the stellar surface $\Sigma : r = R$. The exterior metric reads:

$$ds_+^2 = -(1-2\xi)^{n+1}\left[1 + \frac{2\xi}{1-2\xi}\left(\frac{r}{R}\right)^2\right]^n dt^2 + \frac{dr^2}{1 + \frac{2\xi}{1-2\xi}\left(\frac{r}{R}\right)^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{5.1}$$

where M is the stellar mass and $\xi = M/R$ is the compactness parameter. Continuity of the first fundamental form [50, 51]:

$$e^{\nu(R)} = 1 - \frac{2M}{R}, \tag{5.2}$$

$$e^{-\lambda(R)} = 1 - \frac{2M}{R}. \tag{5.3}$$

Vanishing radial pressure at the boundary [52]:

$$P_r(R) = 0. \tag{5.4}$$

Substituting the ansatz (3.15) into (5.2)–(5.3):

$$a = -\frac{2\xi}{R^2}, \tag{5.5}$$

$$C = (1-2\xi)^{1-n}. \tag{5.6}$$

Applying (5.4) to (4.2) and solving for n:

$$n = -\frac{\beta(3R-4M)}{2R(2-\beta)}. \tag{5.7}$$

The auxiliary metric parameter B is defined as:

$$B \equiv a\,(1-2\xi), \tag{5.8}$$

where $a = -2\xi/R^2$ and $\xi = M/R$. Equations (5.5)–(5.7) together with (5.8) uniquely fix the metric parameters (a, C, n, B) for given (M, R, α, β).

Table 1. Model parameters for compact stars with $\alpha = 1.80$ and $\beta = 0.47$. The compactness $\xi = GM/(c^2R)$; parameters a and $B \equiv a(1-2\xi)$ are in km^{-2}.

Star	$M\ (M_\odot)$	R (km)	ξ	a (km^{-2})	n	B (km^{-2})
EXO 1785−248 [9]	1.30	8.850	0.2170	0.009788	0.8756	0.007266
PSR J1614−2230 [4]	1.97	10.300	0.2825	0.012244	1.1397	0.007847
Cen X−3 [10]	1.49	9.178	0.2398	0.010940	0.9600	0.007754
Vela X−1 [10]	1.77	10.560	0.2476	0.008795	0.9904	0.006129
4U 1608−52 [53]	1.74	11.750	0.2187	0.005633	0.8818	0.004167
KS 1731−260 [54]	1.61	10.000	0.2378	0.009070	0.9524	0.006455

Table 2. Physical parameters of neutron stars in Rastall gravity ($\alpha = 1.80$, $\beta = 0.47$): central energy density ρ_c, surface energy density ρ_s, central radial pressure p_c, surface compactness μ_s, and surface gravitational redshift $Z_s = (1-2\mu_s)^{-1/2} - 1$.

Star	ρ_c (10^{15} g cm^{-3})	ρ_s (10^{14} g cm^{-3})	p_c (10^{36} dyne cm^{-2})	μ_s	Z_s
EXO 1785−248 [9]	5.9637	7.9952	4.0397	0.2170	0.3291
PSR J1614−2230 [4]	9.1300	7.4691	6.7798	0.2825	0.5162
Cen X−3 [10]	7.1599	8.0017	5.0260	0.2398	0.3862
Vela X−1 [10]	5.8963	6.2074	4.1855	0.2476	0.4074
4U 1608−52 [53]	3.4512	4.5608	2.3446	0.2187	0.3333
KS 1731−260 [54]	5.8994	6.6957	4.1291	0.2378	0.3809

6 Physical Viability of the Stellar Model

6.1 Energy Density and Pressures

Figure 2 illustrates the radial variation of ρ, P_r, and P_t as functions of r/R. Each quantity attains its maximum at the stellar centre and decreases monotonically toward the boundary [55], ensuring regularity and physical admissibility.

6.2 Complete System of Equations

The equilibrium configuration is determined by:

$$\frac{dm}{dr} = 4\pi r^2\rho(r) + \tfrac{\alpha}{2}r^2\mathcal{H}_1(r) + \tfrac{\beta}{2}r^2\mathcal{H}_2(r), \tag{6.1}$$

$$\frac{dP_r}{dr} = -\frac{(\rho + P_r)(m + 4\pi r^3 P_r)}{r^2(1 - 2m/r)} + \frac{2}{r}(P_t - P_r) + \mathcal{F}(r), \tag{6.2}$$

$$P_t(r) = P_r(r) + \Delta(r), \tag{6.3}$$

with anisotropy [28, 29]:

$$\Delta(r) = \frac{1}{1-\beta}\Big[-H(r) - C_2 P_r(r) + \tfrac{\beta}{2}\rho(r)\Big] - P_r(r). \tag{6.4}$$

6.3 Mass Function in Rastall Gravity

For the metric ansatz [18, 36]:

$$\frac{dm}{dr} = 4\pi r^2\rho(r) - \frac{\alpha}{2}\frac{nar^2}{1+ar^2} + \frac{\beta}{4}\frac{nar^2}{(1+ar^2)^2}\left[na^2r^2 + 3na - 2a\right]. \tag{6.5}$$

7 Stability and Equilibrium

7.1 Energy Conditions

The standard energy conditions must be satisfied:

$$\begin{aligned}
\text{NEC:}&\quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0,\\
\text{WEC:}&\quad \rho \geq 0, \quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0,\\
\text{SEC:}&\quad \rho + p_r \geq 0, \quad \rho + p_r + 2p_t \geq 0,\\
\text{DEC:}&\quad \rho \geq 0, \quad \rho - |p_r| \geq 0, \quad \rho - |p_t| \geq 0.
\end{aligned} \tag{7.1}$$

here NEC stands for null energy condition, WEC for weak energy condition, SEC for strong energy condition and DEC for dominant energy condition As shown in Fig. 3, all energy conditions are satisfied throughout the stellar interior [38], confirming the absence of exotic matter [29, 33].

7.2 Causality Condition

Following Herrera [40, 41], the causality criterion relates local stability to the dynamical response of anisotropic matter under perturbations, and requires that both the radial and tangential sound speeds satisfy the subluminal constraints $0 < v_r^2 < c^2, 0 < v_t^2 < c^2$, where

$$v_r^2 = \frac{dp_r}{d\rho}, \qquad v_t^2 = \frac{dp_t}{d\rho}, \tag{7.2}$$

represent the radial and tangential velocities of sound within the stellar system, and c is the speed of light.

To evaluate the physical viability and robustness of the model, we first impose the relativistic causality conditions on the sound speeds, namely $0 < v_r^2 < 1$ and $0 < v_t^2 < 1$, in geometrized units $c = 1$. As shown in Figs. 4(a, b), both the radial and tangential sound speeds remain everywhere subluminal within the stellar interior, thereby satisfying the causality constraints and lending support to the dynamical stability of the configuration. Furthermore, the stability of the anisotropic matter distribution is examined through Herrera's cracking criterion, which assesses the system's response to local anisotropic perturbations. According to this criterion, stability is guaranteed if the difference in squared sound speeds satisfies $0 < v_r^2 - v_t^2 < 1$ and exhibits no sign change throughout the radial profile, as illustrated in Fig. 4(c). Our analysis confirms that this condition is fulfilled across the entire interior for all compact star candidates considered, ensuring consistency with Herrera's stability criterion.

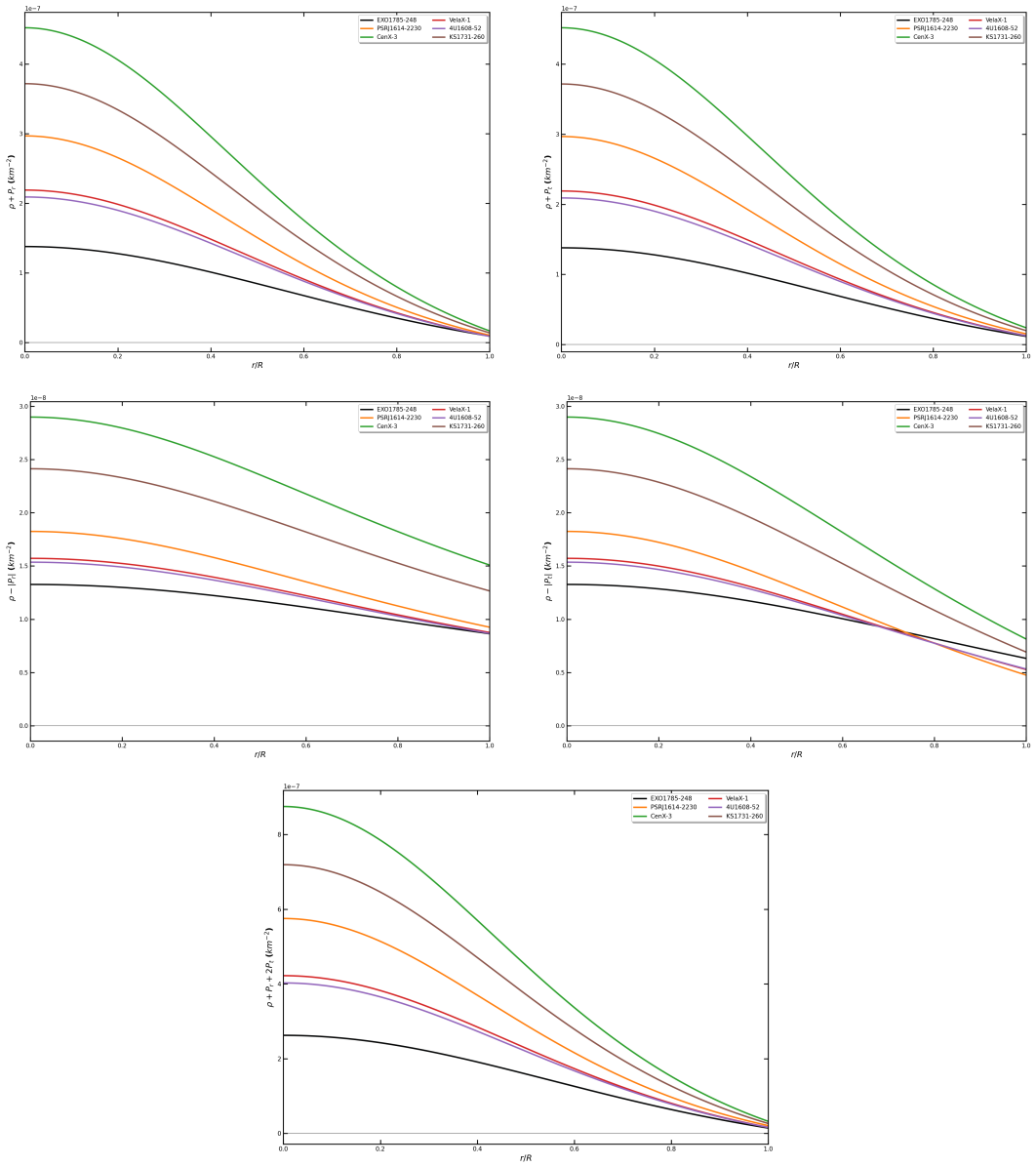


Figure 3. Profile of energy conditions versus the radial coordinate r.

8 Generalised Tolman–Oppenheimer–Volkoff Equation

For anisotropic matter ($P_t \neq P_r$), the TOV equation [1, 2] is modified following Bowers and Liang [28]:

$$\frac{dP_r}{dr} = -\frac{[\rho(r) + P_r(r)]\,[m(r) + 4\pi r^3 P_r(r)]}{r^2\left[1 - \dfrac{2m(r)}{r}\right]} + \frac{2}{r}(P_t - P_r), \tag{8.1}$$

where $\frac{2}{r}\Delta(r)$ with $\Delta(r) = P_t - P_r$ represents the anisotropic force [28, 29].

In Rastall gravity , the modified conservation law leads to the modified TOV equation:

$$\frac{dP_r}{dr} = -\frac{[\rho(r) + P_r(r)]\,[m(r) + 4\pi r^3 P_r(r)]}{r^2\left[1 - \dfrac{2m(r)}{r}\right]} + \frac{2}{r}(P_t - P_r) + \mathcal{F}(r), \tag{8.2}$$

where:

$$\mathcal{F}(r) = \frac{\alpha}{8\pi}\nabla^\mu H^{(1)}_{r\mu} + \frac{\beta}{8\pi}\nabla^\mu H^{(2)}_{r\mu}. \tag{8.3}$$

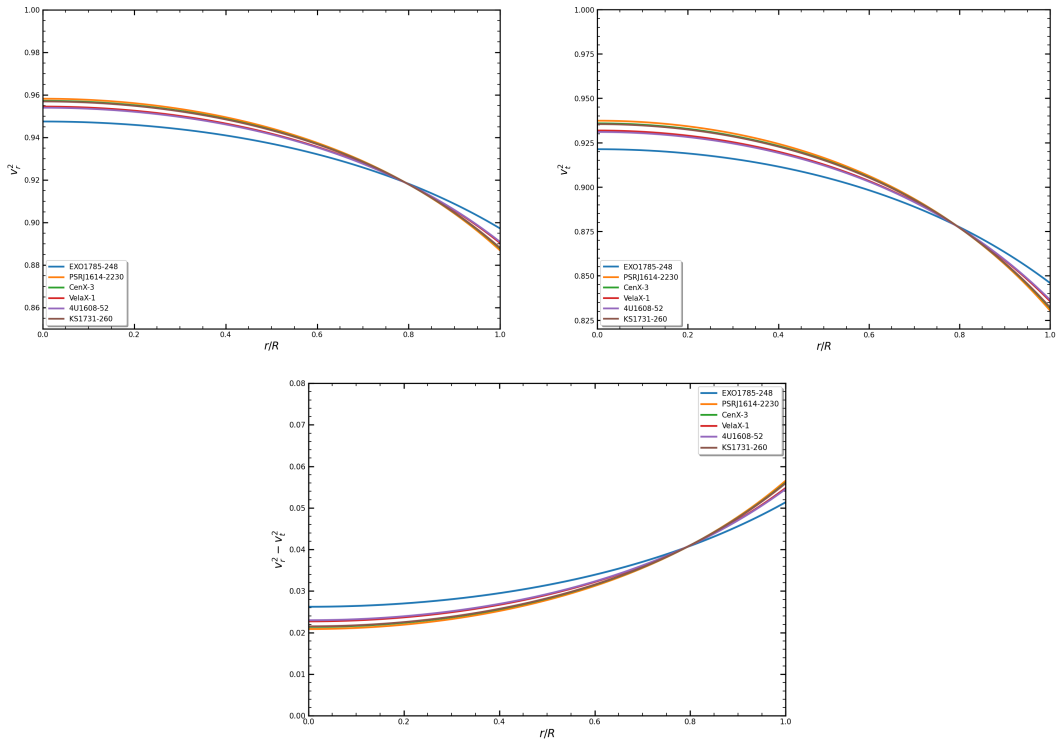


Figure 4. Stability criteria: radial sound speed v_r^2 (left), tangential sound speed v_t^2 (right), and difference $v_r^2 - v_t^2$ (bottom).

$$\mathcal{F}(r) = \frac{\alpha}{8\pi}\left[-\frac{2a^2r(3n-1)}{(1+ar^2)^2}\right] + \frac{\beta}{8\pi}\left[\frac{a^2r}{2(1+ar^2)^3}\left(2n(2-3ar^2) + n^2a^2r^2(5+ar^2) - 4\right)\right]. \tag{8.4}$$

The equilibrium condition decomposes into four force components :

$$F_g = -\frac{\nu'}{2}(\rho + P_r), \qquad \text{(gravitational force)}, \tag{8.5}$$

$$F_h = -\frac{dP_r}{dr}, \qquad \text{(hydrostatic gradient force)}, \tag{8.6}$$

$$F_a = \frac{2}{r}(P_t - P_r), \qquad \text{(anisotropic force)}, \tag{8.7}$$

$$F_r = \mathcal{F}(r), \qquad \text{(Rastall correction force)}, \tag{8.8}$$

satisfying $F_g + F_h + F_a + F_r = 0$. Figure 5 illustrates the force-balance decomposition of the modified TOV equation. The gravitational force F_g remains negligibly small and essentially flat at zero throughout the stellar interior, indicating that its contribution to the overall equilibrium is comparatively minor for this configuration. The hydrostatic force F_h is positive (directed outward) throughout, rising sharply from the center and gradually decreasing toward the surface. The anisotropic force F_a is also positive throughout, increasing monotonically from zero at the centre to its maximum value at the surface, thereby reinforcing the outward push. In contrast, the Rastall correction force F_r remains negative (directed inward) over the entire stellar interior before decreasing toward the boundary.

8.1 Mass, Compactness, and Redshift

The mass function: In this subsection ,we will discuss some important physical aspects of the mass funtion ,compactness and redshift of the celestial object. The mass function of the stellar

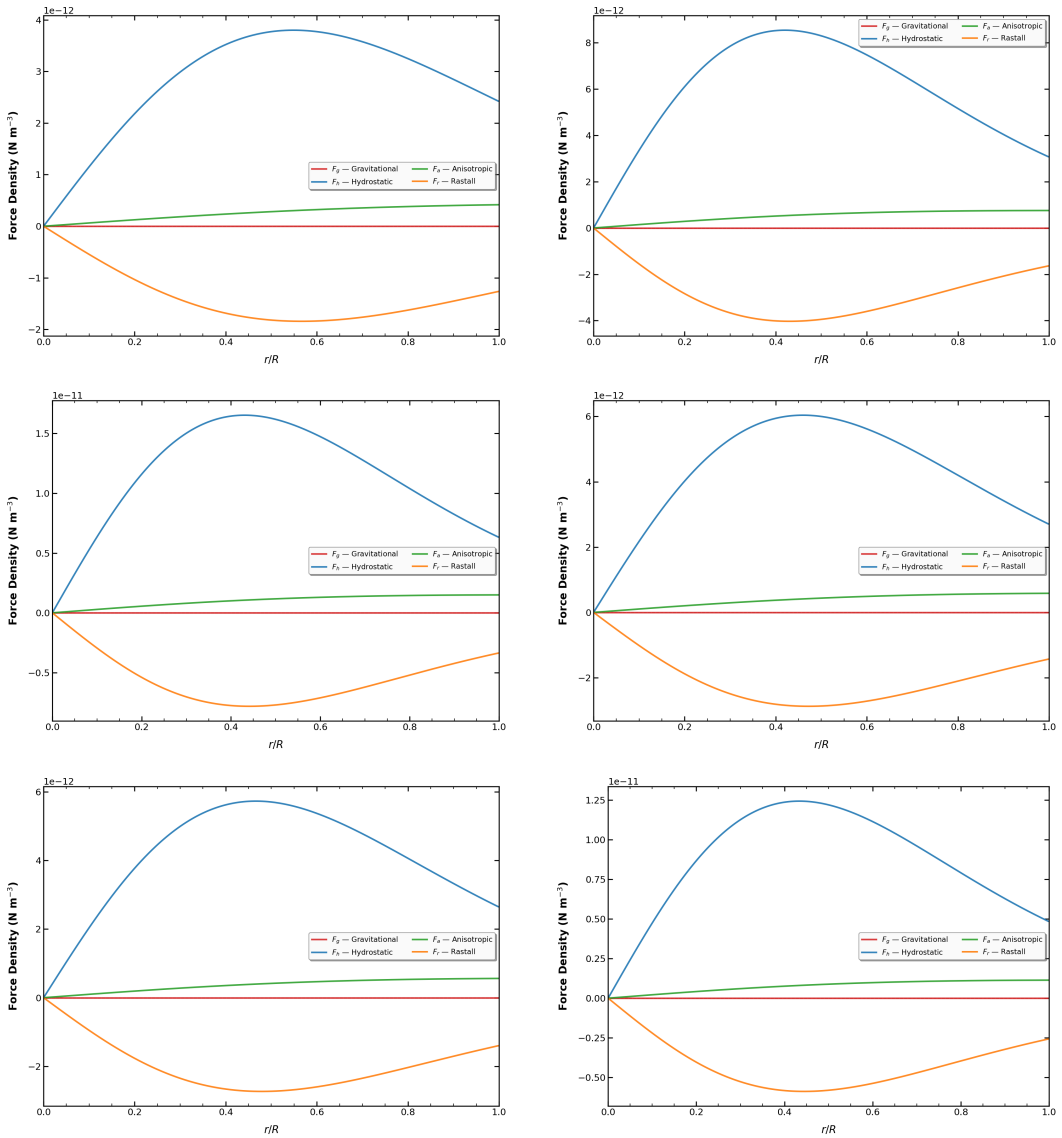


Figure 5. Force balance (F_h, F_g, F_a, F_r) for the six compact star candidates: (a) EXO 1785-248, (b) PSR J1614-2230, (c) Cen X-3, (d) Vela X-1, (e) 4U 1608-52, (f) KS 1731-260.

body ofnradius r is defined by

$$M(r) = 4\pi \int_0^r r'^2 \rho(r')\, dr' = \frac{r}{2}\left(1 - e^{-\lambda(r)}\right). \tag{8.9}$$

The graphical demeanor of the mass function has been illustrated in Fig. 6(a)

The compactness of the celestial object is defined as the mass (M) to radius (r) ratio as: $u(r) = M(r)/r$ must satisfy the Buchdahl bound $u \leq 4/9$ [55]. Corresponding to the compactness ,the surface redshift is defined as:

$$z_s(r) = (1 - 2u(r))^{-1/2} - 1. \tag{8.10}$$

According to Buchdahl [55], $z_s \leq 4.77$; for anisotropic stars Ivanov [56] extended this to $z_s \leq 5.211$.Fig. 6(b, c) shows the compactification and redshift of regular and acceptable profile

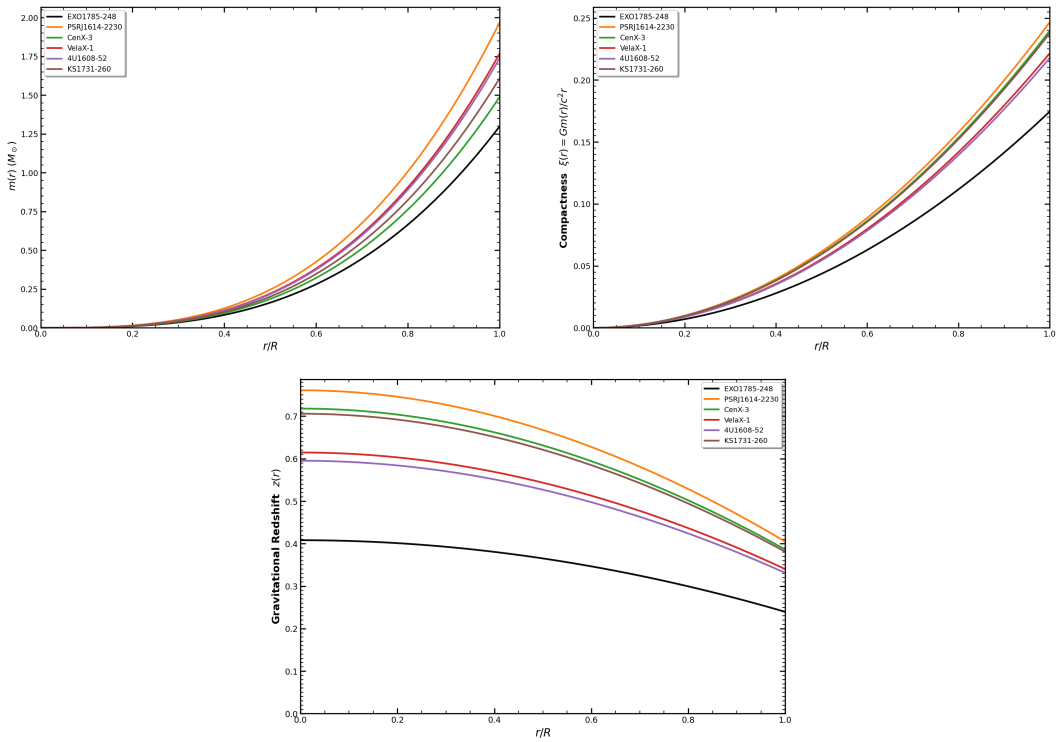


Figure 6. Variation of (a) gravitational mass function $m(r)$, (b) compactness parameter $\mu(r)$, and (c) redshift $Z(r)$.

9 Adiabatic Index

The adiabatic index Γ_r is defined as:

$$\Gamma_r = \left(\frac{P_r + \rho}{P_r}\right)\frac{dP_r}{d\rho}. \tag{9.1}$$

A relativistic stellar model is dynamically stable provided $\Gamma_r > 4/3$ [42]. This critical value was generalised to anisotropic matter by Herrera et al. [57]. For the present anisotropic configuration in Rastall gravity, the radial variation of Γ_r, displayed in Fig. 7 is smooth and monotonically increasing from the center toward the surface, and remains everywhere above the critical stability bound i.e., > 1.33. This behavior substantiates the dynamical stability of the stellar configuration throughout the entire interior.

10 Conclusion

In this work, we have presented a comprehensive study of anisotropic compact stars within the framework of Rastall gravity, employing the metric ansatz $e^{-\lambda(r)} = 1 + ar^2$ and $e^{\nu(r)} = C(1 + ar^2)^n$. Exact analytic solutions for $\rho(r)$, $P_r(r)$, $P_t(r)$, and $\Delta(r) = P_t(r) - P_r(r)$ were obtained directly from the Rastall field equations, without assuming any specific equation of state. The model parameters (a, n, B) were uniquely fixed by the Darmois–Israel junction conditions and vanishing radial pressure at the surface , with $\alpha = 1.80$ and $\beta = 0.47$. Predictions were compared against observational data for six neutron star candidates: **EXO 1785-248**, **PSR J1614–2230**, **Cen X-3**, **Vela X-1**, **4U 1608-52**, and **KS 1731-260**.

The principal findings are as follows.

(i) **Regularity and positivity.** The metric potentials are finite, positive, and singularity-free, satisfying $e^{\nu(0)} > 0$ and $e^{-\lambda(0)} = 1$. Energy density and pressures attain maxima at the stellar centre and decrease monotonically outward.

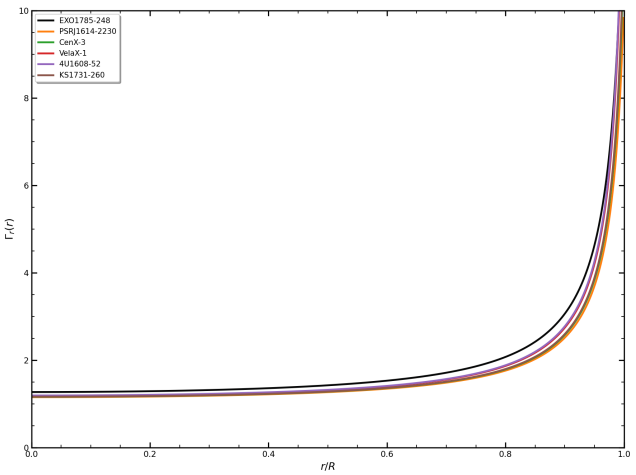


Figure 7. Profile of the adiabatic index versus the radial coordinate.

(ii) **Pressure anisotropy.** The anisotropy $\Delta(r) = P_t(r) - P_r(r)$ is positive throughout the interior and vanishes at the centre. This positive anisotropy provides an additional outward force, enabling the model to accommodate observed high masses without unrealistically stiff equations of state .

(iii) **Energy conditions.** All four standard energy conditions (NEC, WEC, SEC, DEC) are satisfied everywhere for all six compact stars, confirming physically reasonable matter content.

(iv) **Causality and Herrera cracking stability.** Sound speeds satisfy $0 < v_r^2 < 1$ and $0 < v_t^2 < 1$, and the cracking criterion $0 < v_r^2 - v_t^2 < 1$ holds without sign reversal, confirming stability against local perturbations .

(v) **Hydrostatic equilibrium.** The modified TOV equation decomposes into four forces satisfying $F_g + F_h + F_a + F_r = 0$ throughout, demonstrating the non-trivial role of the Rastall correction in strong-field regimes .

(vi) **Mass, compactness, and redshift.** The mass function $M(r)$ increases monotonically; compactness $u(r) < 4/9$; and surface redshift Z_s lies within the bounds $Z_s \leq 4.77$ and $Z_s \leq 5.211$ for all candidates.

(vii) **Adiabatic stability.** The adiabatic index $\Gamma_r > 4/3$ everywhere inside each stellar candidate , confirming dynamical stability in agreement with the generalised criterion of Herrera et al.

(viii) **Model parameters.** Parameters ξ, a, n, B are tabulated in Table 1, and ρ_c, ρ_s, p_c, μ_s, Z_s in Table 2, confirming that the model correctly reproduces observed stellar masses and radii for $\alpha = 1.80$ and $\beta = 0.47$.

Taken together, these results establish that Rastall gravity, combined with pressure anisotropy, provides a physically consistent and observationally viable framework for modelling compact stars. The non-minimal geometry–matter coupling supports higher stellar masses and broader compactness ranges compared to standard General Relativity , without violating any classical energy or stability bound .

Future extensions may include a systematic study of (α, β) dependence, incorporation of charge or rotation, exploration of quark-star configurations, and comparison with gravitational wave constraints.

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