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EXISTENCE OF SIMILAR BOUNDARY LAYER FLOW AND MASS TRANSFER OVER A PLATE WITH STEFAN BLOWING/SUCTION

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Abstract : In this paper, the existence of 'boundary layer' flow over a permeable plate by imposing Stefan blowing/suction at the boundary wall has been examined by employing the continuous group of transformations on the governing equations. This study reveals that a thinner 'boundary layer' structure is developed over the plate for a Blasius 'boundary layer' equation under Stefan suction while blowing makes it thicker. The symmetries and scaling laws are obtained. Numerical solutions of the 'self-similar' equations are found out for various values of the parameters and presented graphically and analyzed in detail. The results are discussed with physical reasoning. It is noted that due to Stefan suction, the momentum 'boundary layer' and the solute 'boundary layer' thicknesses are diminished when the Lewis number of the problem is considered as fixed value. However, both thicknesses show increasing for Stefan blowing.

1 Introduction

In viscous fluid flow with little viscosity, a boundary layer is a narrow shear layer of liquid in the neighbourhood of a bounding surface when viscous fluid flows along it. Contact of fluid with the solid wall produces a no-slip boundary condition i.e., zero velocity at the wall. This induces the creation of flow vorticity and the velocity increases monotonically above the surface attaining the free stream velocity at the edge of the boundary layer. A velocity boundary layer is a tiny shear layer of viscous fluid with little viscosity. This boundary layer flow has several applications in environmental science, heat and mass transfer engineering, and natural phenomena including drag reduction, ship hull construction, flows through pipe and channel. The boundary layer flow structure is an important idea to comprehend fluid behavior close to surfaces. H. Blasius [1] studied first steady boundary layer flow over a semi-infinite flat plate subject to zero pressure gradient and uniform free stream in 1908, and obtained power series solution of the self-similar nonlinear equation. The 'boundary layer' parameters of Blasius flow agreed well with experimental data. Howarth [2] had been able to acquire the numerical solution to the Blasius flow problem. Weidman et al. [3] reported the influences of transpiration on self-similar 'boundary layer' flow over a moving surface. The results showed that when suction increases and blowing decreases, the range of dual solutions grows. Aziz [4] showed that a similarity solution may be achieved if the convective heat transfer from the hot fluid on the plate's bottom surface is proportionate to the streamwise distance as $x^{-\frac{1}{2}}$. Kusukawa et al. [5] recently investigated boundary layer flow past a flat plate using an iterative method. The results reveal that the solution increasingly approaches the well-known solution given by Howarth [2]. There are several works that

have been carried out by taking various kinds of fluids specifically non-Newtonian fluid models, 'heat and mass' transfer. The 'boundary layer' flow past a stretching porous surface with 'suction/blowing' and heat absorption/generation near a stagnation-point were investigated by Layek et al. [6], and they noticed the decrease in temperature at the sheet owing to 'suction' but the temperature increases due to injection.
Lie-group analysis, also known as symmetry analysis was developed by Sophius Lie in the nineteenth century through the identification of point transformations that transfer partial differential equations (pdes) to corresponding ordinary differential equations (odes). It delivers all possible symmetries of the governing system. Group analysis reduces the number of independent variables in the governing equation keeping the system and boundary conditions invariant. Knowledge of symmetries of a system often leads to a completely new way of looking at the phenomenon, which enriched understanding of the mechanism of the process. Sometimes new scaling laws are found which are priori unknown. The fundamental equations may be invariant under simple rotation and translation of the reference frame. This invariance is a direct consequence of homogeneity and isotropy of the system. Recently, Layek et al. [10, 11, 12], Mandal and Layek [13] employed symmetry analysis extensively to ascertain symmetries, and scaling rules for both turbulent and laminar boundary layer flows. The new scaling laws are obtained particularly for turbulent flows. It is noteworthy to mention that all possible symmetries may not be represented the fluid dynamical relevant solutions of the flow problem. The judgment of fluid dynamical relevant solutions is crucially important and needs experimental data for verification.

In fluid dynamics, the idea of blowing/suction is frequently employed through porous wall, especially for 'boundary layer' flow for obtaining the desire level of flow quantities. However, Stefan blowing/suction is completely different from ordinary suction/blowing and it frequently occurs in case of mass and heat transfer process. Bhattacharyya and Layek [14] obtained the full similarity of boundary layer-driven convection flow in presence of reactive solutes through a permeable plate using blowing and suction at the boundary. The result shows that for large Schmidt number the concentration attain negative value, in presence of chemically reaction. Recently, Palanichamy [15] explored the variations in fluid velocity on a plate with injection/suction. Some of the recent researches showed the impacts of suction/blowing and Stefan blowing in fluid flow under different aspects, see the works [16], [17] and reference therein.
In this paper, Lie group of transformations have been employed for boundary layer 'flow and mass transfer' past a permeable plate with Stefan blowing/suction. The system remains invariant due to some relationships among the transformation group parameters. It converts the non-linear coupled pdes leading to the flow and solute distribution into a system of nonlinear odes by minimizing the number of independent variables along with the boundary conditions. The equations are then solved using the shooting technique with Runge-Kutta (4th order) and determined the numerical solutions for velocity and concentration taking various effects. Both velocity and solute 'boundary layer' thicknesses are obtained and explained physically.

2 Flow description

Consider a steady two-dimensional 'boundary layer' motion of Newtonian fluid and solute distribution past a flat plate. The x and y axes are respectively representing the horizontal and vertical axes. The plate is placed along the x axis . The equations for fluid flow and the distribution of solute may be expressed in standard notation using the 'boundary layer approximations', see [1] [17] as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}, \tag{2.2}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2}. \tag{2.3}$$

2.1 Boundary conditions at the plate

The two types of boundary conditions are imposed viz., the permeable plate wall $y = 0$ and the boundary conditions far away from plate, that is, $y \to \infty$. The appropriate boundary conditions are expressed by

$$u = 0, \quad v = -\frac{D_B}{1 - C_w}\frac{\partial C}{\partial y}, \quad C = C_w \quad \text{at the wall} \quad y = 0, \tag{2.4}$$

$$u \to U, \quad C \to C_\infty, \quad \text{at far away from the wall} \quad y \to \infty. \tag{2.5}$$

Here u and v are velocity components, respectively in the x and y directions, μ is the coefficient of the fluid viscosity, ρ is the fluid density, $\nu = \mu/\rho$ is the kinematic coefficient of viscosity, C is the solute concentration. Stefan blowing/suction is a fluid dynamics process where mass is injected into a fluid or removed from the fluid through a permeable wall. So, it increases or decreases the fluid velocity and 'mass transfer' rates. This mechanism is also known as wall injection/suction. Stefan blowing is a natural phenomenon driven by mass diffusion at the boundary whereas ordinary blowing involves forced injection of substance into fluid through porous surface. Stefan suction arises due to molecular movement of different species during chemical reaction while ordinary suction is a movement of fluid caused by pressure difference. However, D_B is the diffusion coefficient. It gives how particles, such as uniform tiny colloidal particles travel in a fluid due to random motion. U stands for the uniform free stream velocity, C_w is the concentration at the plate wall and C_∞ is the concentration far away from the plate. The above specifications of 'boundary conditions' for both velocity and solute are standard for steady two-dimensional boundary layer flows.

2.2 Introduction to stream function in two-dimension flow

We define the stream function $\psi(x, y)$, for this two-dimensional steady flow satisfying $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$, and so the equation of continuity (2.1) automatically satisfied. We take $\phi = \frac{C - C_\infty}{C_w - C_\infty}$, as a dimensionless solute concentration. The momentum boundary layer Eq. (2.2) and solute concentration Eq. (2.3) now become

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \nu \frac{\partial^3 \psi}{\partial y^3}, \tag{2.6}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2}. \tag{2.7}$$

The boundary conditions (2.4) and (2.5) take the following forms:

$$\frac{\partial \psi}{\partial y} = 0, \quad \frac{\partial \psi}{\partial x} = \frac{D_B}{1 - C_w}(C_W - C_\infty)\frac{\partial \phi}{\partial y}, \quad \phi = 1 \quad \text{at} \quad y = 0, \tag{2.8}$$

$$\frac{\partial \psi}{\partial y} \to U, \quad \phi \to 0 \quad \text{as} \quad y \to \infty. \tag{2.9}$$

2.3 Scaling laws and self-similar equations

Lie continuous group theory is a well-recognized mathematical method for identifying all symmetries of a system and is described in the books [7], [8], and [9]. The method is used to obtain all symmetries of fluid dynamical equations, both laminar and turbulent flows [10] and [11]. The main idea in Lie group of transformations is to determine the continuous transformations which keep a given system of equations invariant. The basic concept is to find the infinitesimal generator of a symmetry group and the infinitesimal criterion. The generator is a vector field on the manifold in which the flow coincides with the one-parameter group generating over the manifold. The invariance conditions give a system of linear equations, known as the defining equations. It can split into an overdetermined system of pde's which are generally linear and are easy to solve in many cases by hand. However, several symbolic manipulation computer

programs are now available for this symmetry group calculations, e.g., IntroToSymmetry.m by B.J. Cantwell [8], and others. Lie's theory calculates the symmetry groups of a system in an algorithmic procedure and is given in the Appendix for easy understanding for finding symmetry groups, self-similar equations of a system. Its key benefit is that all possible invariants of the system may be obtained without any prior knowledge. However, all symmetries may not represent physically relevant and so experimental data are needed to select physically relevant solutions. We employ the Lie symmetry analysis to the system of nonlinear pdes represented by the Eqs. (2.6) and (2.7) along with the boundary conditions Eqs. (2.8) and (2.9). The Eqs. (2.6) and (2.7) have two independent variables x, y and two dependent variables (unknowns) ψ and ϕ. According to Lie group theory the infinitesimal generator of the equations has obtained as

$$\chi = \xi_1 \frac{\partial}{\partial x} + \xi_2 \frac{\partial}{\partial y} + \eta_1 \frac{\partial}{\partial \psi} + \eta_2 \frac{\partial}{\partial \phi}. \tag{2.10}$$

Since the governing equation is of third order, therefore the infinitesimal generator of Eq. (2.10) can be stretched or prolonged three times. The extended generator upto three times prolongation can be written as

$$\begin{aligned} \chi^{[3]} = \xi_1 \frac{\partial}{\partial x} + \xi_2 \frac{\partial}{\partial y} + \eta_1 \frac{\partial}{\partial \psi} + \eta_2 \frac{\partial}{\partial \phi} + \zeta_1 \frac{\partial}{\partial \psi_x} + \zeta_2 \frac{\partial}{\partial \psi_y} + \zeta_{12} \frac{\partial}{\partial \psi_{xy}} + \\ \zeta_{22} \frac{\partial}{\partial \psi_{yy}} + \zeta_{222} \frac{\partial}{\partial \psi_{yyy}} + \gamma_1 \frac{\partial}{\partial \phi_x} + \gamma_2 \frac{\partial}{\partial \phi_y} + \gamma_{22} \frac{\partial}{\partial \phi_{yy}}. \end{aligned} \tag{2.11}$$

The coefficients of $\zeta_l, \zeta_{lm}, \zeta_{lmn}, \gamma_l$ and $\gamma_{lm}, (l, m, n = 1, 2)$ can be obtained using the infinitesimal $\xi_i, \eta_j, (i, j = 1, 2)$ by the following recursive formulae, (see the algorithm for symmetry analysis of the present system in the Appendix)

$$\left.\begin{aligned} \zeta_1 &= D_x(\eta_1) - \psi_x D_x(\xi_1) - \psi_y D_x(\xi_2) \\ \zeta_2 &= D_y(\eta_1) - \psi_x D_y(\xi_1) - \psi_y D_y(\xi_2) \\ \zeta_{12} &= D_x(\zeta_2) - \psi_{xy} D_x(\xi_1) - \psi_{yy} D_x(\xi_2) \\ \zeta_{22} &= D_y(\zeta_2) - \psi_{xy} D_y(\xi_1) - \psi_{yy} D_y(\xi_2) \\ \zeta_{222} &= D_y(\zeta_{22}) - \psi_{xyy} D_y(\xi_1) - \psi_{yyy} D_y(\xi_2) \\ \gamma_1 &= D_x(\eta_2) - \phi_x D_x(\xi_1) - \phi_y D_x(\xi_2) \\ \gamma_2 &= D_y(\eta_2) - \phi_x D_y(\xi_1) - \phi_y D_y(\xi_2) \\ \gamma_{22} &= D_y(\gamma_2) - \phi_{xy} D_y(\xi_1) - \phi_{yy} D_y(\xi_2) \end{aligned}\right\} \tag{2.12}$$

The package IntroToSymmetry.m [8] has been used for finding the infinitesimals. Now solving the equation (invariant conditions) $\chi^{[3]} F|_{F=0} = 0$, the infinitesimals are obtained as

$$\left.\begin{aligned} \xi_1 &= a_0 + a_1 x, \\ \xi_2 &= g(x) + a_2 y, \\ \eta_1 &= a_3 + (a_1 - a_2)\psi, \\ \eta_2 &= a_4 + a_5 \phi. \end{aligned}\right\} \tag{2.13}$$

Here g is a function of x only and the constants $a_0, a_1, a_2, a_3, a_4, a_5 \in \mathbb{R}$ are the group parameters. The group of transformations keeps the equations along with the boundary conditions invariant. At the surface $y = 0$, we find $g(x) = 0$. For the simplification of calculation, we neglect now all the translation group of parameters, $a_0 = a_3 = a_4 = 0$. This will not change the qualitative property of the flow system. Finally, the infinitesimal can be written as

$$\left.\begin{aligned} \xi_1 &= a_1 x, \\ \xi_2 &= a_2 y, \\ \eta_1 &= (a_1 - a_2)\psi, \\ \eta_2 &= a_5 \phi. \end{aligned}\right\} \tag{2.14}$$

Eqs. (2.6) and (2.7) combined with the boundary conditions given by Eqs. (2.8) and (2.9), allow us three-parameter symmetry group of transformations. We write the generators of three groups as

$$\chi_1 = x\frac{\partial}{\partial x} + \psi\frac{\partial}{\partial \psi}, \chi_2 = y\frac{\partial}{\partial y} - \psi\frac{\partial}{\partial \psi}, \chi_3 = \phi\frac{\partial}{\partial \phi}.$$

The group of point transformations of the system are

$$\left.\begin{aligned} G_{a_1} &: x^* = e^{a_1}x, y^* = y, \psi^* = e^{a_1}\psi, \phi^* = \phi, \\ G_{a_2} &: x^* = x, y^* = e^{a_2}y, \psi^* = e^{-a_2}\psi, \phi^* = \phi, \\ G_{a_3} &: x^* = x, y^* = y, \psi^* = \psi, \phi^* = e^{a_5}\phi. \end{aligned}\right\} \tag{2.15}$$

One can easily calculate the commutator table of three group of generators, where each entries become zero, it represents the abelian Lie group and so it forms a solvable Lie algebra. Therefore, Eqs. (2.6) and (2.7) combined with the boundary conditions (2.8) and (2.9) may be solved. The required characteristic equations can be obtained as

$$\frac{dx}{a_1 x} = \frac{dy}{a_2 y} = \frac{d\psi}{(a_1 - a_2)\psi} = \frac{d\phi}{a_5\phi},$$

$$\Rightarrow \frac{dx}{x} = \frac{dy}{ay} = \frac{d\psi}{(1-a)\psi} = \frac{d\phi}{b\phi}.$$

Solving the above equations we found the following invariants as

$$\eta^* = \frac{y}{x^a}, \psi = x^{(1-a)}F(\eta^*), \phi = x^b\phi(\eta^*), \tag{2.16}$$

We call the variable η^* as the similarity variable, and the parameters $a = \frac{a_2}{a_1}, b = \frac{a_5}{a_1}, (a_1, a_2 \neq 0)$ are arbitrary real constants. It is found that the self-similar equations are obtained for the present system when the scaling group parameters satisfy the relations $a_1 = 2a_2$ and $a_5 = 0$. Hence, there is a symmetry-breaking phenomenon in the system. So, the following self-similar equations are obtained under the above symmetry group of transformations

$$\nu F''' + \frac{1}{2}FF'' = 0, \tag{2.17}$$

$$D_B\phi'' + \frac{1}{2}F\phi' = 0. \tag{2.18}$$

The invariant boundary conditions are

$$F'(\eta^*) = 0, \quad F(\eta^*) = \frac{D_B}{1 - C_w}(C_W - C_\infty)\phi'(\eta^*), \quad \phi(\eta^*) = 1 \quad \text{at} \quad \eta^* = 0, \tag{2.19}$$

and

$$F'(\eta^*) \to U, \quad \phi(\eta^*) \to 0 \quad \text{at} \quad \eta^* \to \infty. \tag{2.20}$$

Again, we apply the following transformations to the non-dimensional variables η, F and ψ

$$\eta^* = \nu^{\frac{1}{2}}U^{-\frac{1}{2}}\eta, \quad F(\eta^*) = \nu^{\frac{1}{2}}U^{\frac{1}{2}}f(\eta), \quad \phi(\eta^*) = \phi(\eta).$$

Eqs. (2.17) and (2.18), along with the boundary conditions in Eqs. (2.19) and (2.20) are transformed into the following forms :

$$2f''' + ff'' = 0, \tag{2.21}$$

$$2\phi'' + Lef\phi' = 0. \tag{2.22}$$

The boundary conditions become

$$f' = 0, \quad f = \frac{2S}{Le}\phi', \quad \phi = 1 \quad \text{at} \quad \eta = 0, \tag{2.23}$$

$$f' \to 1, \quad \phi \to 0, \quad \text{as} \quad \eta \to \infty. \tag{2.24}$$

Where, $Le = \frac{\nu}{D_B}$ is the Lewis number and the Stefan blowing/suction parameter is denoted by $S = \frac{C_w - C_\infty}{1 - C_w}$, see [17], when $S > 0$ indicates mass transfer rate from plate to fluid, often known as blowing, whereas $S < 0$ indicates mass transformation rate from fluid to plate, or mass suction from the fluid. The Lewis number is basically the ratio of fluid and mass diffusion coefficients. When $Le > 1$ implies that fluid diffusion coefficient is greater than solute diffusion. This relation holds for highly viscous liquids, oil and water. The practically relevant relation is $Le < 1$, for example, Le for air $= 0.6$ and for gases < 1. Fore some special class of fluids and solutes the two fluid and solute diffusion coefficients are approximately equal to one i.e. $Le = 1$. The air is treated as liquid having Lewis number approximately 1. If we take the value of Stefan blowing/suction parameter S is zero, i.e. no mass flux at the surface then the Eq. (2.21) along with the boundary condition (2.23) coincide with the Blasius boundary layer equation.

2.4 Numerical solution procedure

In this sub-section we find self-similar solutions numerically of the non-linear odes (2.21) and (2.22) with the 'boundary conditions' (2.23) and (2.24) . To achieve numerical solutions, the bvp ('boundary value problem') is converted to an ivp ('initial value problem') and then the shooting technique with the Runge-Kutta method is used. Newton's iterative scheme is used to acquire approximately missing slopes of the system. This approach uses a first-order system of equations to generate numerical solutions. The above system Eqs. (2.21) and (2.22) can be described using a set of first order equations as follows:

$$\left.\begin{aligned} f' = z, \quad z' = p, \quad p' = -\frac{fp}{2}, \\ \phi' = q, \quad q' = -\frac{Lefq}{2}, \end{aligned}\right\} \tag{2.25}$$

with the following initial conditions

$$f(0) - \frac{2S}{Le}\phi'(0) = 0, \quad f'(0) = 0, \quad \phi(0) = 1. \tag{2.26}$$

To solve the initial value problem (2.25) subject to (2.26), $f''(0), \phi'(0)$ are required. After selecting initial guesses, the 4th-order Runge-Kutta method is utilized to determine the numerical solutions. The computed values $f'(\eta), \phi(\eta)$ at $\eta_\infty (\eta \to \infty)$ are compared with the specified boundary conditions; $f'(\eta_\infty) = 1, \phi(\eta_\infty) = 0$. To improve approximate solution, the values of $f''(0), \phi'(0)$ are modified by Newton's method. This approach uses a fixed step size of $\Delta\eta = 0.01$. Repeated usage of this procedure yields accurate results up to the desired accuracy level of 10^{-5}.

3 Results and Discussions

3.1 Validation of numerical results

The proposed equations represent Blasius self-similar equation over a semi-infinite plate when the Stefan blowing/suction effects are considered to be nullified. The self-similar velocity profile is agreed with the Blasius similar velocity profile. So, the fluid dynamical relevant solutions are obtained through Lie symmetry analysis of the system.

3.2 Analysis of results

Numerical simulations of reduced self-similar equations with specified boundary conditions were performed for specific values of the parameters. The outcomes are shown in some figures Fig. 1(a) to Fig. 3(b). Details discussions along with physical explanations of the 'flow and mass transfer' are provided.

Figs. 1(a)-1(b) show how the velocity and concentration are affected by Stefan suction or blowing through the permeable flat plate taking the typical value of the Lewis number $Le = 1$.

The Lewis number is equal to one, when the fluid and solute have same diffusion coefficients and is possible for special kind of fluid and solute concentration, e.g., vapour of air and water mixture. Fluid velocity [Fig. 1(a)] decreases at a point as Stefan blowing at the plate increases. And the opposite effect is observed, when Stefan suction increases. The concentration [Fig. 1(b)] at a point increases for higher values of blowing at the permeable plate. Conversely, when suction is applied, the opposite result is observed. Because the thickness of solute 'boundary layer' increases with blowing and decreases with suction, the aforementioned effects are physically relevant, according to the presence of Stefan suction/blowing at the plate wall as explained earlier.

Figs. 2(a)-2(b) illustrate how the Lewis number affects velocity distributions for Stefan blowing

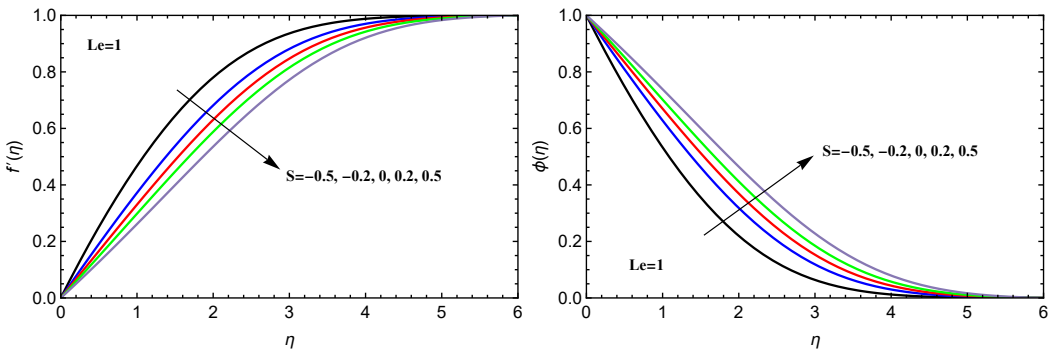


Figure 1. Velocity and concentration profiles for different values of S taking the Lewis number $Le = 1$.

[Fig. 2(a)] and Stefan suction [Fig. 2(b)]. The velocity increases for increasing Lewis number for the case blowing, on the other hand for the case of suction, velocity decreases when the Lewis number increases.

Figs. 3(a)-3(b) show how Lewis number affects the concentration distribution. In both the cases

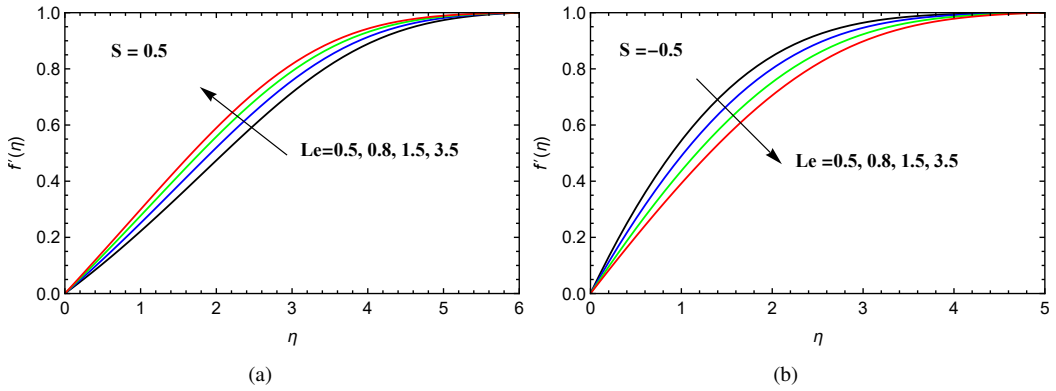


Figure 2. Velocity profile for different values of Lewis number Le for the case of (a) blowing $S = 0.5$ (thicker boundary layer thickness) and (b) suction $S = -0.5$ (thinner boundary layer thickness).

of Stefan blowing and Stefan suction, as Lewis number Le increases, the concentration $\phi(\eta)$ at a given η rapidly decreases. However, the profile is slowly decreasing for $Le < 1$, while it decreases sharply for $Le > 1$.

Table **1** presents the thickness of boundary layer for Stefan suction/blowing parameter S, taking the typical value of the Lewis number $Le = 1$. The 'momentum boundary layer' thickness η_m increases as a result of increasing blowing at the plate. However, increasing Stefan suction causes the decrease in 'boundary layer' thickness η_m. And the concentration 'boundary layer' η_s

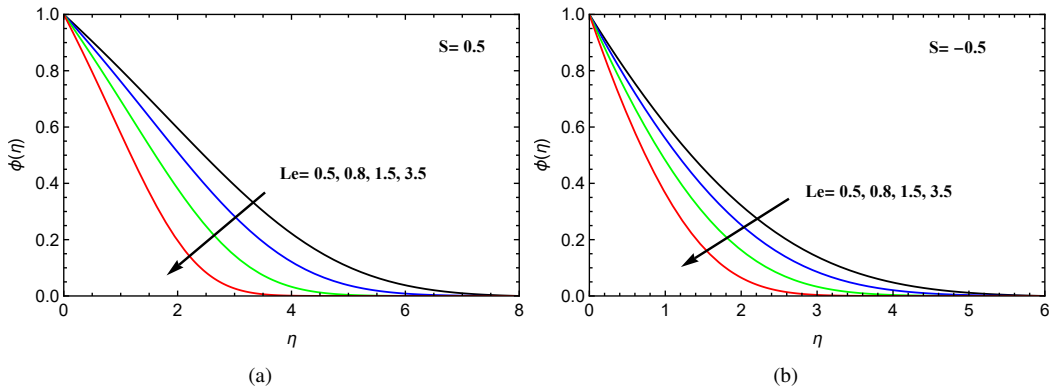


Figure 3. Self-similar concentration profile for different values of Lewis number Le for the particular case (a) $S = 0.5$ (thicker boundary layer thickness) and (b) $S = -0.50$ (thinner boundary layer thickness).

Table 1. Boundary layer thicknesses for various values of S in steady flow

Stefan suction/blowing parameter (S)	Velocity boundary layer thickness (η_m)	Solute boundary layer thickness (η_s)
-0.5	4.22	4.38
-0.2	4.76	4.88
0	4.98	5.11
0.2	5.18	5.30
0.5	5.44	5.47

increases for rise in Stefan blowing, and its value decreases with an increase of Stefan suction. When there is no mass flux at the permeable wall, it is noteworthy to mention, see Table **1** for $S = 0$, that both thicknesses are higher values than the values obtained in presence of the Stefan suction parameter. This also proves physically correctness of the results.

4 Conclusions

We examine the existence of self-similar solutions of Blasius type boundary layer flow over a flat plate in presence of Stefan blowing or suction at the permeable wall. The governing nonlinear equations are transformed into self-similar equations by employing the Lie-group of continuous transformations. The symmetry analysis shows that the self-similar equations exist for the given boundary layer flow with specified boundary conditions. The three groups G_{a_1}, G_{a_2} and G_{a_3} are found for the present model. The fluid dynamical relevant solution is indulged through the comparison with the classical Blasius boundary layer flow. Adopting the shooting methodology and the fourth-order Runge-Kutta method, the numerical solutions of the self-similar equations are obtained. Both 'boundary layer thicknesses' for velocity and solute are obtained. The study leads to the following important conclusions and novelties :

(a) The present model equations exhibit the self-similar solutions under the Lie group of continuous transformations when the scaling group parameters satisfy the relations $a_1 = 2a_2$ and $a_5 = 0$.

(b) The thickness of the momentum 'boundary layer' (velocity 'boundary layer') increases as a result of increased blowing at the plate. However, the increasing Stefan suction causes the thickness of the 'boundary layer' to decrease. And the concentration 'boundary layer' increases with the increasing Stefan blowing, and the 'boundary layer thickness' decreases with an increase in Stefan suction, when the value of Lewis number is fixed. We con-

clude that blowing makes both momentum and concentration boundary layer thicker, while opposite trend (thinner) is developed for the case of suction.

(c) The velocity 'boundary layer thickness' decreases for increasing Lewis number for the case of Stefan blowing. On the other hand, for the case of Stefan suction the 'boundary layer thickness' increases when the Lewis number increases. This indicates that the role of Lewis number is crucially important for overall boundary layer characteristics.

(d) The concentration 'boundary layer thickness' decreases with the increasing Lewis number in both Stefan blowing and Stefan suction cases. Because a bigger Lewis number indicates lower mass diffusivity, which inhibits species diffusion and results in a thinner concentration boundary layer regardless of the flow direction.

(e) The Lie group theory is not a self-reliant method for finding fluid dynamical and solute transfer relevant solutions and experimental verification of results are needed.
The selection of scaling group parameters is crucially important. Finally, the future scope of this work is to verify the results with experimental data.

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Appendix

Algorithm for performing symmetry analysis of the present coupled nonlinear system :

- Step 1: The present system contains two independent variables x, y and two dependent or unknowns ψ, ϕ, and expressed by

$$F(x, y, \psi, \phi) = 0 \tag{4.1}$$

- Step 2: Write the infinitesimal generator associated with the above differential Eq. (4.1).
- Step 3: Utilize the extended infinitesimal generator for the differential Eq. (4.1), and project it into the invariance condition $\chi^{[3]} F|_{F=0} = 0$ and find the infinitesimals.
- Step 4: Write the equations $F = 0$ in a solvable form, where the highest-order linear derivative is normally represented in terms of the others. Then, substitute the relevant term in the invariance conditions.
- Step 5: Separate the terms that have derivatives of all orders of the dependent variable with respect to the independent variable and the different product terms from those that don't have these terms, and set the coefficients of each term equal to zero.
- Step 6: To obtain the infinitesimal coefficients by solving the above way obtained overdetermined system. These equations are usually linear and can be easier to solved by hand or with symbolic software, e.g., IntroToSymmetry.m.
- Step 7: Finally, the group of transformations, continuous groups, self-similar equations and the invariant boundary conditions are found.

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