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UNIDERS: A COMMON FRAMEWORK FOR IDEALS AND FILTERS

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Abstract. In this study, we introduce the concept of unider by reconsidering the notions of ideal and filter within a common framework. Uniders, defined with the help of uninorms, are explained through various examples, their relations with ideals and filters are revealed, the subclasses of t-ideals, t-filters, prime uniders, ultrauniders, and principal uniders are introduced, the unider closure is described, and the ideal–filter duality is generalized in the context of uniders.

1 Introduction

Algebraic structures play a fundamental role in both the theoretical and applied areas of mathematics. Among these, lattices hold a particularly important place in order theory, set theory, subobjects in a wide range of abstract structures, and in the field of logic, standing at the very center of algebraic logic.

In the period following the emergence of fuzzy logic, t-norms and t-conorms appeared as the building blocks of many-valued logic. These operations make it possible to draw inferences under different criteria in systems involving uncertainty or partial truth. However, within the existing frameworks, the focus has been mainly on the clusterings around truth or falsity. In this context, attention has sometimes been given to filters and ideals. Yet, in certain cases, it may also be important to explain the acceptable neighborhoods of a particular equilibrium point.

The transition between Boolean algebras and Boolean rings establishes a bridge between the two branches of the concept of ideals, one reflecting order theory and the other ring theory. Ideals have found their place in an astonishing variety of structures [3, 5, 7, 15, 18]. In the context of order theory, the duals of ideals are filters, and they hold a very important position especially in topology and algebraic logic [2, 11]. Like sequences, filters can be discussed in terms of their convergence within a topological space, and they are quite powerful tools that can be used for fully characterizing topologies. In convergence theories, not only filters but also ideals have been used in different ways. For instance, the concept of ideal convergence has been proposed as a generalization of convergence for sequences and has been extensively studied [1, 9, 13, 14, 16, 19].

On the other hand, in response to needs arising in theory, the concept of a uninorm, bringing together the notions of t-norm and t-conorm under one framework, has emerged and has been investigated in many directions [17]. Uninorms are binary operations that generally share many common properties of t-norms and t-conorms, while also behaving in many respects like an interesting hybrid of the two, and have been studied in detail. In addition to being convenient tools for generalized logic theories, they have also been discussed in applied contexts such as decision-making. Native to the lattice $[0, 1]$, this concept has also been defined on other lattices, and has given rise to variations such as the unima and the unidefiners [4, 10].

Inspired by the fact that the concepts of t-norm and t-conorm have been reconciled under the framework of uninorms, and that uninorms have subsequently become the subject of extensive research and applications, in this study we attempt to reconcile the concepts of ideals and filters within a single notion called unider[1]. The new concept we propose here is itself closely related to uninorms; moreover, uniders actually depend on uninorms in the sense that each unider is given modulo a uninorm, just as each module is given over a particular ring.

Although the above discussion illustrates that uniders form a natural algebraic structure combining ideals and filters through uninorms, their relevance is not merely algebraic. In most practical applications, particularly in decision-making and reasoning under uncertainty, acceptance regions are not necessarily strictly downward closed, as in the case of ideals, or strictly upward closed, as in the case of filters. Rather, there exists a tolerance zone around an equilibrium point where alternatives are deemed acceptable. The identity element of a uninorm provides a natural model of this equilibrium point, and uniders model the acceptance regions in a mathematically stable manner.

In multi-criteria decision-making problems, alternatives are often compared in partially ordered structures rather than totally ordered structures. In such problems, the acceptable alternatives form a set that is neither an ideal nor a filter: some slightly worse alternatives may still be acceptable, while some strictly better alternatives may be ruled out because of violated constraints. Uniders can offer a flexible mathematical framework for modeling these acceptable regions as convex and stable sets with respect to a suitable aggregation operator modeled by a uninorm.

A similar phenomenon occurs in fuzzy rule-based systems and approximate reasoning, where the aggregation of truth values does not follow purely conjunction or disjunction but rather a context-dependent aggregation. Since uninorms generalize both t-norms and t-conorms, uniders can be seen as stable sets of acceptable truth degrees with respect to these hybrid aggregation processes.

From the point of view of uncertainty modeling, the convexity condition in the definition of uniders guarantees that if two states are acceptable, then all states in between are acceptable as well. This is a robustness property that naturally emerges in tolerance-based systems and reliability theory.

After a brief preliminary section introducing the basic concepts and notation, in Section 3 we present the definition of unider, discuss several concrete examples, and clarify its connections with ideals and filters. We then provide a decomposition for uniders via two subclasses, t-ideals and t-filters, which are very closely related to ideals and filters. Furthermore, we transfer the ideal–filter duality to the case of uniders when an appropriate negation is given, construct the unider closure for arbitrary subsets of a lattice with a uninorm, and discuss some special subclasses such as principal, maximal (ultra), and prime uniders.

Hence the main contributions of this paper can be summarized as follows: We define the concept of unider in bounded lattices with a uninorm; we demonstrate that classical ideals and filters are special cases; we develop the theory of prime, and maximal uniders; we build unider closures and principal uniders; we establish a duality theory under certain assumptions.

2 Preliminaries

For the sake of notational consistency, we revisit some of the concepts appearing in this study.

Throughout the study, it is assumed that the set $\mathbb{N}$ contains the element 0.

For two given sets A and B, $A \triangle B$ stands for the symmetric difference of these sets; that is $A \triangle B = (A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$.

We denote a general lattice as $(L, \lesssim)$ by reference to its sequence structure or as $(L, \wedge, \vee)$ by reference to its algebraic structure, and for simplicity in notation, we assume that when we refer to a non-special lattice L, the symbols $\lesssim$, $\wedge$, and $\vee$ are used in this sense.

Given a bounded lattice L, we denote the greatest and the least elements of L respectively by $\top$ and $\bot$. For any given $a, b \in L$, the closed interval $[a, b]$ is defined as

$$[a, b] = \{x \in L : a \lesssim x \lesssim b\}.$$

[1] A neologism inspired by the uni-, id-, and -er of the words uninorm, ideal, and filter, which can be pronounced as /ˈjuːnɪ.də/ or /juː.ˈnaɪ.də/.

This notation makes sense only if $a \lesssim b$; otherwise $[a, b] = \varnothing$. Then, the convex closure of a subset S of L is defined by

$$\mathrm{conv}(S) = \bigcup\{[a, b] : a, b \in S\}.$$

For a given element $a \in L$, $\uparrow a$ and $\downarrow a$ respectively denote upper and lower sets, that is

$$\uparrow a = \{x \in L : a \lesssim x\},$$

and

$$\downarrow a = \{x \in L : x \lesssim a\}.$$

An ideal of a bounded lattice L is a nonempty subset $I \subseteq L$ such that
(I1) $x, y \in I$ implies $x \vee y \in I$,
(I2) $x \lesssim y$ and $y \in I$ implies $x \in I$.
for all $x, y \in L$. Dually, a filter is a nonempty subset $F \subseteq L$ such that
(F1) $x, y \in F$ implies $x \wedge y \in F$,
(F2) $x \lesssim y$ and $x \in F$ implies $y \in F$.
for all $x, y \in L$
A unary operation $\sim$ on L is called a negation, if
(N0) $\widetilde{\top} = \bot$ and $\widetilde{\bot} = \top$
(N1) $x \lesssim y$ implies $\widetilde{y} \lesssim \widetilde{x}$ for all $x, y \in L$
and it is said to be involutive, if also the condition
(IN) $\widetilde{\widetilde{x}} = x$ for all $x \in L$
is satisfied [20].

A uninorm on a bounded lattice is a monotone commutative monoid operation [12]. More precisely, $\diamond$ is a uninorm on L, if
(C) $x \diamond y = y \diamond x$ for all $x, y \in L$.
(A) $(x \diamond y) \diamond z = x \diamond (y \diamond z)$ for all $x, y, z \in L$.
(M) $x_1 \lesssim y_1$ and $x_2 \lesssim y_2$ implies $x_1 \diamond x_2 \lesssim y_1 \diamond y_2$, for all $x_1, x_2, y_1, y_2 \in L$.
(I) There is an $e \in L$, such that $x \diamond e = x$ for all $x \in L$.
A uninorm with the identity $\top$ is called a t-norm, and a uninorm with the identity $\bot$ is called a t-conorm. If $x \diamond x = x$ for all $x \in L$, then the uninorm $\diamond$ is called idempotent.

3 Main Results

In this section, we introduce the concept of unider, which generalizes the notions of ideal and filter in a common framework, and we investigate some of its examples and properties.

The concept of a unider, defined below, can be seen as a stability area around the identity element of the uninorm. In the extreme cases where the uninorm coincides with the join or the meet operation, this stability region will collapse to the classical notions of ideal and filter.

Definition 3.1. A unider of a bounded lattice L is a pair $(U, \diamond)$, where U is a nonempty subset of L and $\diamond$ is a uninorm on L with an identity $e \in L$, such that
(U0) $e \in U$,
(U1) $x, y \in U$ implies $x \diamond y \in U$,
(U2) $x \in [a, b]$ and $a, b \in U$ implies $x \in U$
for all $a, b, x, y \in L$. If $U \neq L$, then $(U, \diamond)$ is called a proper unider.

Condition (U1) states closure with respect to the aggregation process; for any two admissible elements aggregated by the uninorm, the resulting element is also admissible. This states that the set is self-consistent with respect to the operation modeling the interaction of the elements.

Condition (U2) has a geometric order-theoretic interpretation. It states that for any two elements in the set, all elements in between them in the lattice order are also in the set. This states that a unider is a convex substructure of the lattice. This captures a natural robustness requirement; once two states are judged acceptable, all states in between are also acceptable.

From this point of view, a unider is neither downward closed like an ideal nor upward closed like a filter. Rather, it defines a tolerance area around the identity element of the uninorm, which

plays the role of an equilibrium level separating the conjunctive and disjunctive parts of the behavior of the operation.

Before discussing structures and properties of uniders, we first present a series of illustrative examples.

Example 3.2. Consider the bounded lattice $L = \{0, a, b, c, e, m, n, 1\}$ given with order $0 < a, b, c < e < m, n < 1$, where a, b, c are pairwise incomparable and m, n are incomparable. A Hasse diagram for L is given in Figure 1.

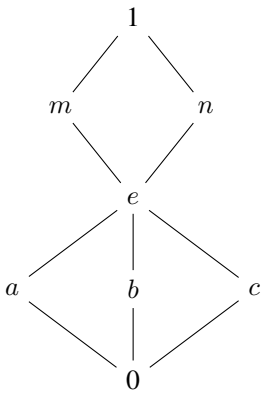


Figure 1. Hasse diagram of the lattice L in Example 3.2

Define a binary operation $\diamond$ on L by

$$x \diamond y = \begin{cases} x \vee y, & x, y \geq e, \\ x \wedge y, & \text{otherwise.} \end{cases}$$

Since every element of L is comparable with e, this operation is commutative, associative and monotone on L with identity element e. Hence it is a uninorm.

According to this uninorm, many uniders of L can be given. Some of these are $\{b, e, m\}$, $\{0, a, b, c, e\}$, $\{e\}$, $\{b, e, m, n, 1\}$ and $\{c, e, n\}$. These are visualized in Figure 2. It can be noted that each of them captures elements with a certain proximity to e (the unit), where e acts as an equilibrium point.

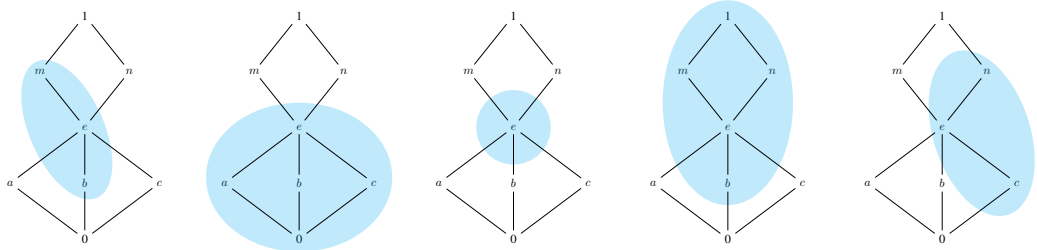


Figure 2. Some uniders in the context of Examaple 3.2

Example 3.3. Define the operation

$$A \Diamond B := \begin{cases} A \cup B, & \text{if } 0 \in A \cap B \\ (A \cup B) \setminus \{0\}, & \text{otherwise.} \end{cases}$$

on the lattice $(\mathcal{P}(\mathbb{N}), \subseteq)$. This is clearly monotone, commutative, and has $\{0\}$ as the identity. Associativity also holds, as the sets $A \Diamond (B \Diamond C)$ and $(A \Diamond B) \Diamond C$ are almost equal to $A \cup B \cup C$, except the possible distinction on including the element 0, or not, and both $A \Diamond (B \Diamond C)$ and $(A \Diamond B) \Diamond C$ include 0 when $0 \in A \cap B \cap C$, and do not include 0 when $0 \notin A \cap B \cap C$. Hence $\Diamond$ is a uninorm on $\mathcal{P}(\mathbb{N})$.

Then for $\mathcal{U} = \{A \subseteq \mathbb{N} : 0 \in A,\ 1 \notin A\}$, $(\mathcal{U}, \Diamond)$ is a unider of $\mathcal{P}(\mathbb{N})$. In fact; $\{0\} \in \mathcal{U}$; if $A, B \in \mathcal{U}$, then $A \Diamond B = A \cup B$, thus $A \Diamond B \in \mathcal{U}$; and if $A, B \in \mathcal{U}$, $C \in [A, B]$, that is $A \subseteq C \subseteq B$, and $0 \in A$, $1 \notin A$, $0 \in B$, $1 \notin B$, then $0 \in C$, $1 \notin C$, hence $C \in \mathcal{U}$.

Example 3.4. Let X be a set and E be an arbitrary subset of X. Consider the lattice $(\mathcal{P}(X), \subseteq)$ and the operation $\square$ on $\mathcal{P}(X)$ given by

$$A \square B = (A \cap B \cap E) \cup \big((A \cup B) \cap E^c\big).$$

We show that $\square$ is a uninorm on $\mathcal{P}(X)$.

Commutativity and monotonicity are immediately seen.

E serves as the identity, since $A \square E = (A \cap E \cap E) \cup \big((A \cup E) \cap E^c\big) = (A \cap E) \cup (A \cap E^c) \cup (E \cap E^c) = (A \cap \big(E \cup E^c)\big) \cup \varnothing = A$.

Finally,

$$\begin{aligned}(A \square B) \square C &= \Big(\big((A \cap B \cap E) \cup \big((A \cup B) \cap E^c\big)\big) \cap C \cap E\Big) \\ &\quad \cup \Big(\big((A \cap B \cap E) \cup \big((A \cup B) \cap E^c\big) \cup C\big) \cap E^c\Big) \\ &= (A \cap B \cap C \cap E) \cup \big((A \cup B \cup C) \cap E^c\big),\end{aligned}$$

and similarly $A \square (B \square C) = (B \square C) \square A = (B \cap C \cap A \cap E) \cup \big((B \cup C \cup A) \cap E^c\big)$. Hence the equality follows.

Now, suppose that $\mathcal{I}$ is an ideal of $\mathcal{P}(X)$, and define the class

$$\mathcal{U} = \{A \subseteq X : A \triangle E \in \mathcal{I}\}.$$

Then $(\mathcal{U}, \square)$ is a unider.

U0) $E \triangle E = \varnothing \in \mathcal{I}$.

U1) If $A, B \in \mathcal{U}$, then $A \triangle E, B \triangle E \in \mathcal{I}$, and

$$\begin{aligned}(A \square B) \triangle E &= \Big(\big((A \cap B \cap E) \cup \big((A \cup B) \cap E^c\big)\big) \cup E\Big) \\ &\quad \setminus \Big(\big((A \cap B \cap E) \cup \big((A \cup B) \cap E^c\big)\big) \cap E\Big) \\ &= (A \cup B \cup E) \setminus (A \cap B \cap E) \\ &= (A \cup B \cup E) \cap (A^c \cup B^c \cup E^c) \\ &= (A \triangle B) \cup (A \triangle E) \cup (B \triangle E)\end{aligned}$$

For $x \in A \setminus B$, either $x \in A \setminus E \subseteq A \triangle E$ or $x \in A \cap E$, and since $x \notin B$, $x \in E \setminus B \subseteq B \triangle E$. That is, $A \setminus B \subseteq (A \triangle E) \cup (B \triangle E)$. Similarly $B \setminus A \subseteq (A \triangle E) \cup (B \triangle E)$. These give $A \triangle B \subseteq (A \triangle E) \cup (B \triangle E)$, so that $(A \square B) \triangle E = (A \triangle B) \cup (A \triangle E) \cup (B \triangle E) = (A \triangle E) \cup (B \triangle E) \in \mathcal{I}$. Hence $A \square B \in \mathcal{U}$.

U2) Suppose that $C \in [A, B]$ and $A, B \in \mathcal{U}$. Then $A \subseteq C \subseteq B$, $A \triangle E, B \triangle E \in \mathcal{I}$, and $(A \triangle E) \cup (B \triangle E) \in \mathcal{I}$. On the other hand, $(A \triangle E) \cup (B \triangle E) = (A \setminus E) \cup (E \setminus A) \cup (B \setminus E) \cup (E \setminus B) \supseteq (B \setminus E) \cup (E \setminus A) \supseteq (C \setminus E) \cup (E \setminus C) \in \mathcal{I}$ by *(I2)*.

Not that, if $E = \varnothing$, then $A \square B = A \cup B, \mathcal{U} = \{A \subseteq X : A \in \mathcal{I}\} = \mathcal{I}$ by $A \triangle \varnothing = A$, and thus $\mathcal{U}$ is the given ideal of X. In contrast, if $E = X$, then $A \square B = A \cap B, \mathcal{U} = \{A \subseteq X : A^c \in \mathcal{I}\}$ by $A \triangle X = A^c$, and thus $\mathcal{U}$ is a filter of X, namely the dual filter corresponding to the ideal $\mathcal{I}$.

Example 3.5. The construction in Example 3.4 can be used to obtain a large class of uniders on powerset lattices.

For instance, consider the lattice $\mathcal{P}(\mathbb{N})$, and let $E = 2\mathbb{N}$ be the set of all even natural numbers. Then $A \square B$ is the set of even numbers in $A \cap B$ and odd numbers in $A \cup B$, that is the set of all numbers in $A \cap B$, and only odd numbers in $(A \cup B) \setminus (A \cap B) = A \triangle B$. For example $\{1, 3, 4, 6, 7, 9, 10, 12\} \square \{2, 3, 5, 6, 8, 9, 11, 12\} = \{3, 6, 9, 12, 1, 5, 7, 11\}$.

In this case, $(\mathcal{U}, \square)$ is a unider, where $\mathcal{U}$ is the class of the subsets of $\mathbb{N}$ containing at most finitely many odd numbers and all but finite number of even numbers, like $\{0, 1, 3, 4, 6, 8, 10, 12, 14, \ldots\}$.

Example 3.6. Let V and W be two vector spaces over the same field, and consider the sublattice

$$\mathcal{L} = \{R \oplus S : R \text{ is a subspace of } V \text{ and } S \text{ is a subspace of } W\}$$

of the lattice of subspaces of direct sum $V \oplus W$.

Define the operation $\circledast$ on $\mathcal{L}$ by

$$(R \oplus S) \circledast (T \oplus U) = (R \cap T) \oplus (S + U).$$

This is clearly commutative, associative, monotone, and has $V \oplus \{0_W\}$ as the identity. Hence it is a uninorm on $\mathcal{L}$.

In this framework, $(\mathcal{U}, \circledast)$ is a unider, where

$$\mathcal{U} = \{R \oplus S : \dim(S) + \operatorname{codim}(R) < \infty\},$$

and where $\operatorname{codim}(R)$ is the codimension of R in V.

$V \oplus \{0_W\} \in \mathcal{U}$ by $\dim(0_W) + \operatorname{codim}(V) = 0 < \infty$.

If $R \oplus S, T \oplus U \in \mathcal{U}$, then $\dim(S) + \operatorname{codim}(R) < \infty$, $\dim(U) + \operatorname{codim}(T) < \infty$. By $\dim(S + U) \leq \dim(S) + \dim(U)$ and $\operatorname{codim}(R \cap T) \leq \operatorname{codim}(R) + \operatorname{codim}(T)$, we also have $\dim(S + U) + \operatorname{codim}(R \cap T) < \infty$. Hence $(R \oplus S) \circledast (T \oplus U) = (R \cap T) \oplus (S + U) \in \mathcal{U}$.

Let $P \oplus Q \in [R \oplus S, T \oplus U]$ and $R \oplus S, T \oplus U \in \mathcal{U}$. Then $\dim(S) + \operatorname{codim}(R) < \infty$, $\dim(U) + \operatorname{codim}(T) < \infty$, $R \oplus S \subseteq P \oplus Q \subseteq T \oplus U$, and necessarily $R \subseteq P \subseteq T$, $S \subseteq Q \subseteq U$. Therefore $\dim(Q) + \operatorname{codim}(P) \leq \dim(U) + \operatorname{codim}(R) < \infty$.

Example 3.7. Let $\mathcal{F}(\mathbb{R})$ be the lattice of all fuzzy subsets of $\mathbb{R}$, and define the operation $*$ on $\mathcal{F}(\mathbb{R})$ by

$$(\mu * \nu)(x) = \begin{cases} \min\big(\mu(x), \nu(x)\big), & \text{if } x \in \mathbb{Q} \\ \max\big(\mu(x), \nu(x)\big), & \text{otherwise.} \end{cases}$$

Thanks to the pointwise structure, $*$ is clearly associative, commutative, monotone, and it has $\chi_{\mathbb{Q}}$ as the identity, where

$$\chi_{\mathbb{Q}}(x) = \begin{cases} 1, & \text{if } x \in \mathbb{Q} \\ 0, & \text{otherwise,} \end{cases}$$

since $(\mu * \chi_{\mathbb{Q}})(x) = \min\big(\mu(x), 1\big) = \mu(x)$ if $x \in \mathbb{Q}$, and $(\mu * \chi_{\mathbb{Q}})(x) = \max\big(\mu(x), 0\big) = \mu(x)$ if $x \notin \mathbb{Q}$.

Let $\mathcal{U}$ be the class of all fuzzy subsets μ of $\mathbb{R}$, such that $\mu(x) \neq \chi_{\mathbb{Q}}(x)$ for at most a finite number of $x \in \mathbb{R}$. Then clearly $\chi_{\mathbb{Q}} \in \mathcal{U}$. If $\mu, \nu \in \mathcal{U}$, then there are only finite number of real numbers such that $\mu(x) \neq \chi_{\mathbb{Q}}(x)$ or $\nu(x) \neq \chi_{\mathbb{Q}}(x)$, and by the definition of $*$, this is also the case for the fuzzy subset $\mu * \nu$. If $\mu, \nu \in \mathcal{U}$, $\xi \in [\mu, \nu]$, then $\mu(x) = \nu(x) = \chi_{\mathbb{Q}}(x)$ for all but finite number of $x \in \mathbb{R}$, and by $\mu(x) \leq \xi(x) \leq \nu(x)$, also there are at most finite number of $x \in \mathbb{R}$ such that $\xi(x) \neq \chi_{\mathbb{Q}}(x)$, so that $\xi \in \mathcal{U}$. Hence $(\mathcal{U}, *)$ is a unider.

In the sequel, a bounded lattice L with a uninorm $\diamond$ having an identity e is supposed to be given.

Let us now explain how uniders bring together the notions of ideals and filters.

Theorem 3.8. *Ideals of a bounded lattice are exactly uniders with respect to the t-conorm $\vee$, and filters are exactly uniders with respect to the t-norm $\wedge$.*

Proof. Let $(U, \vee)$ be a unider. Then the identity is $\bot$, and $\bot \in U$ by (U0), so that U is nonempty. If $x, y \in U$, then $x \vee y \in U$ by (U1). Finally, suppose that $x \lesssim y$ and $y \in U$. Since $\bot \lesssim x$, we have $x \in [\bot, y]$, hence $\bot, y \in U$ gives $x \in U$. Therefore U is an ideal. The dual case for filters is observed similarly. □

Since uniders are defined with respect to a uninorm, it will be appropriate to treat separately the special classes of uniders corresponding to t-norms and t-conorms.

Definition 3.9. We call a unider with respect to a t-norm a t-filter, and a unider with respect to a t-conorm a t-ideal.

The notions of t-ideal and t-filter are, in fact, strengthened variations of the concepts of ideal and filter.

Theorem 3.10. *Considered as a set, every t-ideal corresponds to an ideal, and every t-filter corresponds to a filter.*

Proof. Suppose that $(U, \star)$ is a t-ideal of L. Then $\star$ is a t-conorm on L. We show that U is an ideal.

U is nonempty, since it includes $\bot$, the identity of the t-conorm.

Let $x, y \in U$. Then $x \star y \in U$ by (U1). But since $x = x \star \bot \lesssim x \star y$ and $y = \bot \star y \lesssim x \star y$, we have $x \vee y \lesssim x \star y$. In this case, $\bot \lesssim x \vee y \lesssim x \star y$ and $\bot, x \star y \in U$, guarantee $x \vee y \in U$ by (U2). Hence (I1) holds.

To see (I2), note that, if $x \lesssim y$ and $y \in U$, then $x \in [\bot, y]$ gives $x \in U$ by (U0) and (U2).

The dual result for t-filters is observed similarly. □

Clearly, every ideal is a t-ideal with respect to the t-conorm $\vee$. The above result means that every t-ideal is also an ideal when considered merely as a set. Of course, this does not mean that the concepts of ideal and t-ideal are equivalent; it only means that given an arbitrary t-ideal $(U, \star)$, then automatically $(U, \vee)$ will also be a t-ideal, that is, U will be an ideal in the classical sense. In this respect, the concept of t-ideal extends the concept of ideal with a stronger version. For example, under this framework, interesting questions can be asked, such as for which family of t-conorms a given ideal is a t-ideal. Below, an example is given where an ideal may fail to be a t-ideal with respect to a particular t-conorm.

Example 3.11. Consider the t-conorm $x \oplus y = x + y - xy$ on the lattice $([0, 1], \leq)$. While $[0, \frac{1}{2})$ is an ideal of $[0, 1]$, it is not a t-ideal with respect to $\oplus$, since $\frac{1}{3} \in [0, \frac{1}{2})$, but $\frac{1}{3} \oplus \frac{1}{3} = \frac{5}{9} \notin [0, \frac{1}{2})$.

We state a result for uniders reminiscent of the well-known decomposition theorems for uninorms. Accordingly, uniders exhibit a behavior similar to ideals upward from the identity of the corresponding uninorm, and similar to filters downward from it. However, this decomposition is not sufficient to characterize uniders, since it gives no clue about the situation of those elements that are incomparable with the identity.

Theorem 3.12. *Let $(U, \diamond)$ be a unider and let $\diamond$ have e as the identity. Then $(U \cap \uparrow e, \diamond)$ is a t-ideal of the sublattice $\uparrow e$, and $(U \cap \downarrow e, \diamond)$ is a t-filter of the sublattice $\downarrow e$.*

Proof. If $x, y \in \uparrow e$, then $e \lesssim x$ and $e \lesssim y$ gives $e = e \diamond e \lesssim x \diamond y \in \uparrow e$, that is $\uparrow e$ is closed under the operation $\diamond$, which has the least element e of $\uparrow e$ as the identity. Hence $\diamond$ is a t-conorm on $\uparrow e$, and this completes the proof. The dual result is seen similarly. □

Theorem 3.10 and Theorem 3.12 together yield the following result.

Corollary 3.13. *Let $(U, \diamond)$ be a unider with the identity e. Then $U \cap \uparrow e$ is an ideal of the sublattice $\uparrow e$, and $U \cap \downarrow e$ is a filter of the sublattice $\downarrow e$.*

The duality between ideals and filters extends as well to the case of uniders.

Theorem 3.14. *Let $(U, \diamond)$ be a unider on L given with an involutive negator, and let $\widetilde{\diamond}$ be the dual of the uninorm $\diamond$, that is*

$$x \mathbin{\widetilde{\diamond}} y = \widetilde{(\widetilde{x} \diamond \widetilde{y})}$$

for all $x, y \in L$. Then also $(\widetilde{U}, \widetilde{\diamond})$ is a unider, where $\widetilde{U} = \{\widetilde{u} : u \in U\}$.

Proof. $\widetilde{e}$ is the identity for the uninorm $\widetilde{\diamond}$, since $x \mathbin{\widetilde{\diamond}} \widetilde{e} = \widetilde{(\widetilde{x} \diamond e)} = \widetilde{\widetilde{x}} = x$, and clearly $\widetilde{e} \in \widetilde{U}$. If $x, y \in \widetilde{U}$, then $\widetilde{x}, \widetilde{y} \in U$, and $\widetilde{x} \diamond \widetilde{y} \in U$, hence $x \mathbin{\widetilde{\diamond}} y = \widetilde{(\widetilde{x} \diamond \widetilde{y})} \in \widetilde{U}$. Finally, if $x \in [a, b]$ and $a, b \in \widetilde{U}$, then $\widetilde{a}, \widetilde{b} \in U$ and $a \lesssim x \lesssim b$, that is $\widetilde{b} \lesssim \widetilde{x} \lesssim \widetilde{a}$, or $\widetilde{x} \in [\widetilde{b}, \widetilde{a}]$. Hence $\widetilde{x} \in U$, or equivalently $x \in \widetilde{U}$. □

Definition 3.15. The unider $(\widetilde{U}, \widetilde{\diamond})$ introduced in Theorem 3.14 called the dual of the unider $(U, \diamond)$. Moreover, if $\widetilde{\diamond}$ distributes over $\diamond$, and $x \mathbin{\widetilde{\diamond}} u \in U$ for all $x \in L$, $u \in U$, then the negation is called $(U, \diamond)$-compatible.

Example 3.16. Let $(B, \vee, \wedge, ^{\sim}, 0, 1)$ be a Boolean algebra, and I be an ideal of B. Then $(I, \vee)$ is the corresponding unider, its dual is $(\widetilde{I}, \wedge)$, and $\widetilde{I}$ is a filter of B. $\wedge$ distributes over $\vee$ by the definition of a Boolean algebra. Moreover for $x \in L$, $i \in I$, $x \wedge i \leq i$ implies $x \wedge i \in I$. These show that for every ideal I of a Boolean algebra, the negation is $(I, \vee)$-compatible. Likewise, the negation of a Boolean algebra is $(F, \wedge)$-compatible for its every filter F.

For describing the lattice structure of uniders with respect to a fixed uninorm and for constructing new uniders, the following result is as useful as it is clear.

Theorem 3.17. *If $\{(U_i, \diamond)\}_{i \in I}$ is a family of uniders with respect to the same uninorm, then also $\left(\bigcap_{i \in I} U_i, \diamond\right)$ is a unider.*

Proof. Follows directly. □

Given a lattice with a uninorm, we proceed to construct the unider generated by the elements of an arbitrary subset of the lattice.

Definition 3.18. For a given subset $S \subseteq L$, let $M_\diamond(S)$ denote the monoidal closure of S, that is the intersection of all submonoids (with the same unit) of L containing S. Then its convex closure $\mathrm{conv}(M_\diamond(S))$ is denoted by $\mathrm{ucl}(S)$, and called the unider closure of S.

The unider closure of a set can be considered as the smallest stability region generated by the set. From an arbitrary set, one first requires closure with respect to the uninorm, which means combining elements in a way that no new elements are generated anymore. This step ensures internal consistency with respect to the aggregation process.

The second step is to take the convex closure with respect to the lattice order. This step ensures that if two generated elements are comparable, then all elements in between are also included. In this way, the set becomes order-theoretically closed.

Thus, the unider closure of a set is the smallest unider containing the set. It can be interpreted as the set of all elements that are unavoidably admissible if the initial elements are accepted and the stability conditions of Definition 3.1 are fulfilled.

From a constructive point of view, this process can be described as an iterative generation process, where one alternates between aggregation by the uninorm and interval completion in the lattice until a fixed point is reached. In particular, for finite lattices, this process terminates after a finite number of steps.

If the closure is generated by a single element, then the structure describes all elements that become admissible by repeated aggregation with the element and the convexity condition. This gives a natural interpretation of principal uniders, which will be described in Definition 3.22.

Before presenting the next result, we illustrate the construction of a unider closure by an explicit finite example.

Example 3.19. Consider the bounded lattice and the uninorm given in Example 3.2.

Let $S = \{a, b, m\}$. Since e is the identity of the monoid corresponding to given uninorm, we first consider the addition

$$S \cup \{e\} = \{a, b, m, e\}.$$

Using $\diamond$ we also obtain

$$a \diamond b = a \wedge b = 0,$$

and no further new element is generated. Hence the set generated under $\diamond$ is

$$M_\diamond(S) = \{0, a, b, e, m\}.$$

To obtain the unider closure we take the convex closure with respect to the lattice order. Since $0 \leq c \leq e$, and $0, e \in M_\diamond(S)$ the element c must also be included. Therefore

$$\mathrm{ucl}(S) = \{0, a, b, c, e, m\}.$$

The previous example suggests that the unider closure of finite lattices can be computed through a finite generation procedure. This procedure starts with an initial set of states and repeatedly applies the aggregation process and then closes the set with respect to order convexity.

This procedure generates the smallest stable region containing the initial data and can be considered as a fixed-point construction in a discrete state space.

In a finite lattice the construction of the unider closure admits an algorithmic interpretation. Starting with a subset S, one first considers $S \cup \{e\}$ and generates all elements obtained by successive applications of the uninorm. Repeating these step produces an increasing sequence of subsets. When the lattice is finite, this sequence stabilizes after finitely many iterations. The obtained fixed point coincides with $M_\diamond(S)$. Then this resulting set is completed with respect to order convexity to obtain $\mathrm{ucl}(S)$. This provides a constructive way to compute unider closures in finite lattices.

Theorem 3.20. *For each subset S of L, $(\mathrm{ucl}(S), \diamond)$ is a unider of L.*

Proof. Since $M_\diamond(S)$ is a submonoid of $(L, \diamond)$ with the same unit, $e \in M_\diamond(S) \subseteq \mathrm{conv}(M_\diamond(S)) = \mathrm{ucl}(S)$.

Let $x, y \in \mathrm{ucl}(S)$. Then $x \in [a, b]$ and $y \in [c, d]$ for some $a, b, c, d \in M_\diamond(S)$. Since $M_\diamond(S)$ is a monoid $a \diamond c, b \diamond d \in M_\diamond(S)$, and $a \lesssim x \lesssim b$, $c \lesssim y \lesssim d$ gives $a \diamond c \lesssim x \diamond y \lesssim b \diamond d$. Hence $x \diamond y \in [a \diamond c, b \diamond d]$, and $x \diamond y \in \mathrm{ucl}(S)$.

Suppose that $x \in [y, z]$ and $y, z \in \mathrm{ucl}(S)$. Then there are $a, b, c, d \in M_\diamond(S)$ such that $y \in [a, b]$, $z \in [c, d]$. Then $a \lesssim y \lesssim x \lesssim z \lesssim d$, so that $x \in [a, d]$, which gives $x \in \mathrm{ucl}(S)$. □

The following results follows from Theorems 3.17 and 3.20.

Corollary 3.21. *The family of all uniders of a lattice with respect to a fixed uninorm $\diamond$ has a lattice structure with $(U, \diamond) \wedge (V, \diamond) = (U \cap V, \diamond)$ and $(U, \diamond) \vee (V, \diamond) = (\mathrm{ucl}(U \cup V), \diamond)$.*

We propose a notion that generalizes the case of principal ideals and filters.

Definition 3.22. A unider in the form $(\mathrm{ucl}(\{a\}), \diamond)$, where $a \in L$, is called a principal unider.

Example 3.23. Consider the uninorm $\diamond$ defined by

$$x \diamond y = \begin{cases} \min(x, y), & \text{if } (x, y) \in [0, \frac{1}{2}]^2 \\ \max(x, y), & \text{elsewhere.} \end{cases}$$

on the lattice $\big([0, 1], \leq\big)$ [6]. Then $\mathrm{ucl}(\{\frac{1}{3}\}) = \mathrm{conv}(M_\diamond(\{\frac{1}{3}\})) = \mathrm{conv}(\{\frac{1}{3}, \frac{1}{2}\}) = [\frac{1}{3}, \frac{1}{2}]$, hence $\big([\frac{1}{3}, \frac{1}{2}], \diamond\big)$ is the principal unider at $\frac{1}{3}$ with respect to $\diamond$. On the other hand, if we consider the uninorm

$$x * y = \begin{cases} 2xy, & \text{if } (x, y) \in [0, \frac{1}{2}]^2 \\ \max(x, y), & \text{elsewhere,} \end{cases}$$

then we get values like $\frac{1}{3} * \frac{1}{3} = \frac{2}{9}$, $\frac{2}{9} * \frac{1}{3} = \frac{4}{27}$ in the monoidal closure, so that $\mathrm{ucl}(\{\frac{1}{3}\}) = \mathrm{conv}(M_*(\{\frac{1}{3}\})) = \mathrm{conv}(\{\ldots, \frac{4}{27}, \frac{2}{9}, \frac{1}{3}, \frac{1}{2}\}) = (0, \frac{1}{2}]$, and thus also $\big((0, \frac{1}{2}], *\big)$ is a principal unider.

In the preceding example, while the uninorm $\diamond$ is idempotent, $*$ is not. The fact that in the first case the connection between the given element and the result appears more immediate from an intuitive perspective is not accidental. Indeed, the characterization of principal uniders with respect to idempotent uninorms is rather straightforward.

Theorem 3.24. *Let $\diamond$ be an idempotent uninorm with an identity $e \in L \setminus \{\bot, \top\}$. Then the principal unider at an $a \in L$ is in the form $(U, \diamond)$, where*

$$U = \begin{cases} [a, e], & \text{if } a < e \\ [e, a], & \text{if } e < a \\ \{a, e\}, & \text{otherwise.} \end{cases}$$

Proof. In each case the monoidal closure $M_\diamond(\{a\}) = \{a, e\}$, since the operation is idempotent and e is the identity. If $a < e$, then $\mathrm{conv}(\{a, e\}) = [a, e]$, if $e < a$, then $\mathrm{conv}(\{a, e\}) = [e, a]$, and otherwise, if $a = e$ or if a and e are not comparable, then $\mathrm{conv}(\{a, e\}) = \{a, e\}$. □

Definition 3.25. A proper unider $(U, \diamond)$ on L with respect to a uninorm $\diamond$ is called an ultraunider, if there is no unider $(V, \diamond)$ on L such that $U \subsetneq V \subsetneq L$. If L has an involutive negation, then an unider $(U, \diamond)$ is called prime, if $x \mathbin{\widetilde{\diamond}} y \in U$ implies $x \in U$ or $y \in U$, for all $x, y \in L$.

When ideals and filters are viewed as uniders, it is clear that ultrauniders correspond to maximal ideals and ultrafilters, while prime uniders correspond to prime ideals and filters. In fact, when an ideal I is considered as the unider $(I, \vee)$, it is an ultraunider iff there is no unider $(J, \vee)$ such that $J \subsetneq I \subsetneq L$, which corresponds the maximality of I; and it is a prime unider iff $x \wedge y \in I$ implies $x \in I$ or $y \in I$; and this means that I is a prime ideal. Similar arguments are valid for the case of filters. Below, a partial extension of a known property of ideals and filters is presented in the context of uniders.

Theorem 3.26. *An ultraunider $(U, \diamond)$ is a prime unider, if the involutive negation on L is $(U, \diamond)$-compatible.*

Proof. Suppose that $x \mathbin{\widetilde{\diamond}} y \in U$, but $y \notin U$. We show that $x \in U$.

Since U is an ultraunider and $y \notin U$, we must have $\mathrm{ucl}(U \cup \{y\}) = L$. In particular $\widetilde{e} \in \mathrm{ucl}(U \cup \{y\})$. So there are $a, b \in M_\diamond(U \cup \{y\})$ such that $\widetilde{e} \in [a, b]$. In this case, there are possibilities like $a = u$, $a = u \diamond y$, $a = u \diamond y \diamond y$, $a = u \diamond y \diamond y \diamond y$, and so on, where $u \in U$. (Note that also $a = y$, $a = y \diamond y$, ... also fall into these forms as $y = e \diamond y$, $y \diamond y = e \diamond y \diamond y$, ... for $e \in U$.) These cases yield equalities like

$$x \mathbin{\widetilde{\diamond}} a = x \mathbin{\widetilde{\diamond}} u$$

for $a = u$,

$$x \mathbin{\widetilde{\diamond}} a = x \mathbin{\widetilde{\diamond}} (u \diamond y) = (x \mathbin{\widetilde{\diamond}} u) \diamond (x \mathbin{\widetilde{\diamond}} y)$$

for $a = u \diamond y$,

$$x \mathbin{\widetilde{\diamond}} a = x \mathbin{\widetilde{\diamond}} (u \diamond y \diamond y) = (x \mathbin{\widetilde{\diamond}} u) \diamond (x \mathbin{\widetilde{\diamond}} y) \diamond (x \mathbin{\widetilde{\diamond}} y)$$

for $a = u \diamond y \diamond y$, and so on, by $(U, \diamond)$-compatiblity. In each case $x \mathbin{\widetilde{\diamond}} u \in U$ by $u \in U$ and since the negation is $(U, \diamond)$-compatible, and $x \mathbin{\widetilde{\diamond}} y \in U$ by the assumption. Hence $x \mathbin{\widetilde{\diamond}} a \in U$ in every possible case. The similar arguments also go for b, and we also have $x \mathbin{\widetilde{\diamond}} b \in U$. $\widetilde{e} \in [a, b]$ implies $x = x \mathbin{\widetilde{\diamond}} \widetilde{e} \in [x \mathbin{\widetilde{\diamond}} a, x \mathbin{\widetilde{\diamond}} b]$. Therefore $x \in U$. □

We confirm with an example that the compatibility condition in the theorem cannot be omitted.

Example 3.27. Consider the lattice $([0, 1], \leq)$, with the negation $\widetilde{x} = 1 - x$. Consider the drastic t-conorm

$$x \sqcup y = \begin{cases} 1, & \text{if } x, y \neq 0 \\ \max(x, y), & \text{otherwise.} \end{cases}$$

Then $(\{0\}, \sqcup)$ is an ultraunider, since addition of any $a > 0$ give $1 = a \sqcup a$, and the convex closure gives the whole $[0, 1]$. $\widetilde{\sqcup}$ is the drastic t-norm $\sqcap$ given by

$$x \sqcap y = \begin{cases} 0, & \text{if } x, y \neq 1 \\ \min(x, y), & \text{otherwise.} \end{cases}$$

The ultraunider $(\{0\}, \sqcup)$ is not prime, since $\frac{1}{3} \sqcap \frac{2}{3} = 0$, but neither $\frac{1}{3} \in \{0\}$, nor $\frac{2}{3} \in \{0\}$. The problem here is that the negation is not $(\{0\}, \sqcup)$-compatible, as $\sqcap$ does not distribute over $\sqcup$. For instance,

$$\frac{1}{2} \sqcap \left(\frac{1}{3} \sqcup \frac{2}{3} \right) = \frac{1}{2} \sqcap 1 = \frac{1}{2},$$

but

$$\left(\frac{1}{2} \sqcap \frac{1}{3} \right) \sqcup \left(\frac{1}{2} \sqcap \frac{2}{3} \right) = 0 \sqcup 0 = 0.$$

The idea of a unider is conceptualized as a unified framework that encompasses both ideal-type and filter-type stability conditions in a single aggregation step, expressed by a uninorm. As an additional note, we place uniders within the context of classical ideals and filters, and we make a brief connection to the most commonly used fuzzy generalizations. From a general point of view, uniders are complementary to degree-based methods, such as fuzzy ideals and filters. While fuzzy theories usually generalize membership to a graded assessment, uniders retain crisp membership and generalize the stability process using a hybrid aggregation. This makes uniders a good candidate for a structural component that could potentially be merged with probabilistic or fuzzy elements in future work. A more detailed integration is left for possible future researches.

Finally, we conclude this section by offering some theoretical perspectives on how uniders can potentially be interpreted in various fields.

Partial ordering of a set of alternatives based on their performance criteria can be represented as a bounded lattice. The aggregation of criteria is assumed to be modeled by a uninorm with identity element e representing a neutral evaluation level. In this context, a unider represents a stable region of acceptable alternatives, which is closed under the aggregation process and robust with respect to intermediate evaluations. Thus, once two alternatives are found to be acceptable, all alternatives between them in the order are also acceptable.

In approximate reasoning, aggregation operators tend to work in a conjunctive manner for low values and in a disjunctive manner for high values. This hybrid behavior of aggregation operators can be easily represented by a uninorm. In this context, a unider represents a set of admissible states, which is stable under the hybrid aggregation process. The convexity condition ensures that if two states are found to be admissible, all states between them are also admissible, which represents a robustness condition in uncertainty modeling.

4 Conclusion remarks

In this study, the duality between ideals and filters is carried over to a common flexible framework provided by uninorms, thereby introducing the novel concept of unider. The results obtained show that uniders not only encompass classical ideals and filters but also offer a broad generalization. In addition, certain subclasses and the unider closure have been explained. It is believed that this concept may open new avenues of research in both theoretical and applied contexts.

The unider axioms merge two demands: stability under the selected aggregation ($\diamond$-closure) and stability under the lattice order (convexity). In particular, if the uninorm is the join (meet) operation on the entire lattice, the $\diamond$-closure is just $\vee$-closure ($\wedge$-closure) and convexity corresponds to an ideal-type (filter-type) acceptance region. In this way, the unider approach provides a unified treatment of ideal-like and filter-like stability through a single operation, the explicit use of the identity element e as an equilibrium level distinguishing conjunctive/disjunctive behavior of the aggregation, and a natural closure operator ucl with a constructive meaning in finite lattices.

On the other hand, there are some restrictions. Some aspects are dependent on the selected uninorm and the underlying lattice. For instance, properties concerning duality, primeness/maximality, or representation might need supplementary assumptions (like compatibility conditions involving negation or distributivity-type constraints). In addition, for big lattices, the computations could be expensive in the worst case, and thus approximation or incremental methods could be preferred.

Several possible directions for future research emerge naturally from the current work. First, there is the open problem of extending uniders to non-bounded lattices, where the interaction of identity elements and convex closure is a problem that still needs to be addressed. Another significant direction is the study of uniders induced by non-idempotent uninorms, where some of the structural properties proved in this paper might not hold. Moreover, the duality results proved under compatibility conditions on negation naturally lead to the question of which results are robust with respect to the logical setting. Finally, from a more general point of view, some of the most promising directions for future research seem to be the categorical framework, dynamic versions of uniders, and hybrid models that combine uniders with probabilistic or fuzzy models of uncertainty.

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