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OMEGA INVARIANT OF GENERALIZED MYCIELSKIAN GRAPHS

G. D. D'Souza, I. N. Cangul and M. I. Huilgol

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Abstract Omega invariant is a recently introduced graph invariant that gives substantial information about the graph such as its connectedness, cyclicness and so on. The most interesting thing about Omega invariant is that it is defined on the degree sequence of a graph, which in itself is a very interesting area in graph theory. In this paper, we determine the omega invariant of generalized Mycielskian graphs. Further, we obtain relations between various topological indices and omega invariant for some standard classes of graphs. The relations thus obtained are used to obtain relations between various topological indices and omega invariant of generalized Mycielskian graphs for some standard graphs. Also, we obtain new and interesting links between vertex and edge connectivity, omega invariant and some known topological indices.

1 Introduction

Mycielski graphs were first introduced by Mycielski in 1955, [1]. The Mycielski construction is one of the earliest and arguably simplest constructions of triangle-free graphs of arbitrary chromatic number. Mycielskian graphs have many interesting properties concerning various kinds of graph parameters such as chromatic number [1, 2], clique number [3], domination [3, 4, 5], packing [3, 5], hamiltonicity [5], connectivity [6], closeness [7], matchings [8] and so on.
The generalized Mycielskian of a graph G, also known as cones over graphs (as called by Tardif in [9]) are the natural generalization of the Mycielski graphs, which preserves some desirable properties of a good interconnection network.

Delen and Cangul, [10], defined a new graph invariant in 2018 in relation to the Euler characteristic and the cyclomatic number, which provides several combinatorial and topological properties about the realizations of a degree sequence D and it is denoted by $\Omega(D)$. Similarly, for a graph realization G, omega invariant of G is denoted by $\Omega(G)$.
Omega invariant is related to various graph parameters such as vertex degrees [10], vertex and edge deletion [11], average degree [12], cyclicness [13], connectedness [14], matching number [15], independence number [16], nullity [17], topological indices [18, 19, 20, 21]. Omega invariant of various families of graphs such as line graphs [22, 23], total graphs [22], Lucas graphs [24], Fibonacci graphs [25], Tribonacci graphs [26], complement graphs [27], graph products [28, 29, 30], Cayley graphs [31], Sierpiński graphs [32] are already studied.
Some applications of omega invariant were studied by Yurttas et al., [33]. Inverse problem have applications in the design of combinatorial libraries for drug discovery, [34]. The Inverse problem for Bell index, Sigma index and Zagreb index [19, 20, 21] were studied using the omega invariant.

A. Jerline et al., [35], studied the first and second Zagreb indices of the generalized Mycielskian graph, it's complement and some graph operations. Forgotten index, Hyper-Zagreb index, reformulated first Zagreb index and redefined Zagreb indices of generalized Mycielskians was studied by N. De [36]. A. Jerline et al., [37] studied the forgotten index of generalized Mycielskian graphs and product graphs. The F-coindex of generalized Mycielskians was studied by Jerline et al., [38].

In this paper, we determine the omega invariant of generalized Mycielskian graphs. We also give relations between many topological indices and the omega invariant for some standard graphs. Also we give relations between many topological indices and the omega invariant of generalized Mycielskian graphs for some standard graphs.

The following theorem will be useful to get our results.

Theorem 1.1. *[36]*

$$\begin{aligned} F(\mu_k(G)) &= (8k+1)F(G) + 3M_1(G) + 6m + n(n^2+1) \\ EM_1(\mu_k(G)) &= (8k-2)EM_1(G) + F(G) + 2HM(G) + (16k-5)M_1(G) \\ &\quad -6m(4k-1) + 4m(n-1) + n(n-1)^2 \\ HM(\mu_k(G)) &= (8k-4)HM(G) + 5F(G) + 7M_1(G) + 8M_2(G) + n(n+1)^2 \\ &\quad +4mn + 6m \end{aligned}$$

In the next section, we obtain the omega invariant of generalized Mycielskian graphs.

2 Generalized Mycielskian graphs

The generalized Mycielskian of a graph G, was introduced by Stiebitz in 1985, [39] and independently by Van Ngoc in 1987, [40]. They exhibit interesting properties related to various graph parameters such as circular clique number, total domination, fractional open packing, vertex cover number, determinant, spectrum and so on, see [3, 41]. They have applications in various fields, including chemical graph theory [35, 36, 37, 38] and coding theory [4].
The Generalized Mycielskian of a graph is defined as follows.

Definition 2.1. [39, 40] Let G be a graph with vertex set $V^0(G) = \{v_1^0, v_2^0, ..., v_n^0\}$ and edge set $E^0(G)$. Given an integer $k \geq 1$, the k-Mycielskian of G, denoted by $\mu_k(G)$, is the graph $\mu_k(G)$ with vertex set $V(\mu_k(G)) = V^0 \cup V^1 \cup V^2 \cup \cdots V^k \cup \{z\}$ where $V^i = \{v_j^i | v_j^0 \in V^0\}$ is the i^{th} distinct copy of V^0 for $i = 1, 2, ..., k$, and edge set $E^0 \bigcup \left(\bigcup_{i=0}^{k-1} \{v_j^i v_{j'}^{i+1} | v_j^0 v_{j'}^0 \in E^0\} \right) \bigcup \{v_j^k z | v_j^k \in V^k\}$.

The degree sequences of some standard graphs such as paths, cycles, complete graphs, star graphs and complete bipartite graphs are given in Table 1.

The following result gives the general form of the degree sequence for any generalized Mycielskian graphs.

Theorem 2.2. *Suppose that G is a connected graph having the degree sequence $DS(G) = \{d_1^{a_1}, d_2^{a_2}, ..., d_n^{a_n}\}$ then*
$DS(\mu_k(G)) = \{(d_1+1)^{a_1}, (d_2+1)^{a_2}, ..., (d_n+1)^{a_n}, (2d_1)^{a_1 k}, (2d_2)^{a_2 k}, ..., (2d_n)^{a_n k}, (a_1 + a_2 + ... + a_n)^1\}$.

Proof. Proof follows from the definition of generalized Mycielskian graphs. □

We illustrate Theorem 2.2 with an example.

Example 2.3. Consider $\mu_2(P_3)$ (see Figure 1). Here $k = 2$ and $DS(P_3) = \{1^2, 2^1\}$. Thus by Theorem 2.2, $DS(\mu_2(P_3)) = \{(1+1)^2, (2+1)^1, (2.1)^{(2.2)}, (2.2)^{1.2}, (2+1)^1\} = \{2^2, 3^1, 2^4, 4^2, 3^1\} = \{2^6, 3^2, 4^2\}$.

The Omega invariant of some standard generalized Mycielskian graphs is given in Table 1.

The next result gives a general formula for the omega invariant of generalized Mycielskian graphs.

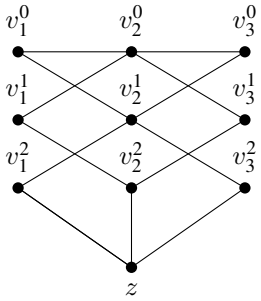


Figure 1. $\mu_2(P_3)$

Table 1. Some degree sequences and omega indices of some generalized Mycielskian graphs

Graphs G	$DS(\mu_k(G))$	$\Omega(\mu_k(G))$
$\mu_k(P_n)$	$\{2^{2k+2}, 3^{n-2}, 4^{(n-2)k}, n^1\}$	$(2n-4)k+(2n-4)$
$\mu_k(C_n)$	$\{3^n, 4^{nk}, n^1\}$	$2nk+2n-2$
$\mu_k(K_n)$	$\{n^{n+1}, (2n-2)^{nk}\}$	$(2n^2-4n)k+(n^2-n-2)$
$\mu_k(S_{n+1})$	$\{2^{nk+n}, (n+1)^2, (2n)^k\}$	$(2n-2)k+2n-2$
$\mu_k(K_{m,n})$	$\{(m+1)^n, (2m)^{nk}, (n+1)^m, (2n)^{mk}, (m+n)^1\}$	$\big((4m-2)n-2m\big)k+2mn-2$

Theorem 2.4. *Let G be a connected graph with n vertices and m edges. Then the Omega invariant of the generalized Mycielskian graph is* $\Omega(\mu_k(G)) = 4km+2m-2kn-2$.

Proof. By Theorem 2.2, we have
$DS(\mu_k(G)) = \{(d_1+1)^{a_1}, (d_2+1)^{a_2}, \cdots, (d_n+1)^{a_n}(2d_1)^{a_1k}, (2d_2)^{a_2k}, \cdots, (2d_n)^{a_nk}, (a_1+a_2+...+a_n)^1\}$.
Also, by the definition of Omega invariant, we have, $\Omega(G) = \sum_{i=1}^{\Delta}(i-2)a_i$.

$$\begin{aligned}\Omega(\mu_k(G)) &= (d_1+1-2)a_1+(d_2+1-2)a_2+....+(d_n+1-2)a_n+(2d_1-2)a_1k+\\ &\quad (2d_2-2)a_2k+....+(2d_n-2)a_nk+(a_1+a_2+...+a_n-2).\\ \Omega(\mu_k(G)) &= (d_1a_1+d_2a_2+...+d_na_n)-(a_1+a_2+...+a_n)\\ &\quad +2k(d_1a_1+d_2a_2+....+d_na_n)-2k(a_1+a_2+...+a_n)\\ &\quad +(a_1+a_2+...+a_n)-2.\end{aligned}$$

We know that $d_1a_1+d_2a_2+...+d_na_n = 2m$ and $a_1+a_2+...+a_n = n$.

Thus we get, $\Omega(\mu_k(G)) = 2m+4km-2kn-2$.

□

We illustrate Theorem 2.4 with an example.

Example 2.5. Consider $\mu_1(C_3)$ (see Figure 2). Here $k=1$, $m=n=3$. Thus by Theorem 2.4, $\Omega(\mu_1(C_3)) = 4\times1\times3+2\times3-2\times1\times3-2 = 12+6-6-2 = 10$. On the other hand, $DS(\mu_1(C_3)) = \{3^4, 4^3\}$. So it has 7 vertices and 12 edges. Hence $\Omega(\mu_1(C_3)) = 2(12-7) = 10$, approving the truth of it.

The following corollary gives the relation between the omega invariant of generalized Mycielskian graphs and omega invariant of any graph.

Corollary 2.6.

$$\Omega(\mu_k(G)) = 2kn+2k\Omega(G)+2m-2.$$

Proof. By Theorem 2.4, we have

$$\Omega(\mu_k(G)) = 4km+2m-2kn-2. \tag{2.1}$$

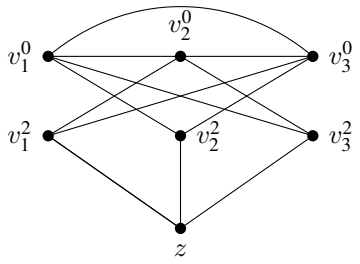


Figure 2. $\mu_1(C_3)$

Also we know that $\Omega(G) = 2(m-n) = 2m - 2n$. Thus, $2m = \Omega(G) + 2n$.

Substituting in 2.1 we get

$$\begin{aligned}\Omega(\mu_k(G)) &= 2k(\Omega(G) + 2n) + 2m - 2kn - 2\\ \Omega(\mu_k(G)) &= 2k\Omega(G) + 2nk + 2m - 2\end{aligned}$$

□

In the next section, we determine various relations between topological indices and the Omega invariant.

2.1 Topological indices of some standard graphs in terms of the Omega invariant

In this section, we obtain the exact expressions for the first and second Zagreb indices, the forgotten index, the first reformulated Zagreb index and the Hyper-Zagreb index in terms of their vertices and the omega invariant for some standard graphs such as paths, cycles, complete graphs, star graphs and complete bipartite graphs.

The following result gives a relation between the first Zagreb index and Omega invariant for some standard graphs.

Theorem 2.7.

$$\begin{aligned}M_1(P_n) &= n(\Omega(P_n))^2 + 3\Omega(P_n)\\ M_1(C_n) &= 4n + \Omega(C_n)\\ M_1(K_n) &= n\Omega(K_n) + n^2 + n\\ M_1(S_{n+1}) &= n\Omega(S_{n+1}) + n^2 + 3n\\ M_1(K_{m,n}) &= m^2n^2 - \frac{mn\Omega(K_{m,n})}{2}\end{aligned}$$

Proof. (i) From [42], we have $M_1(P_n) = 4n - 6$

$$\begin{aligned}&= -2(3 - 2n)\\ &= 3\Omega(P_n) - 2n\Omega(P_n)\ (\because \Omega(P_n) = -2)\\ &= n(\Omega(P_n))^2 + 3\Omega(P_n)\end{aligned}$$

(ii) From [42], we have $M_1(C_n) = 4n + \Omega(C_n)$ (since $\Omega(C_n) = 0$)

(iii) From [43], we have $M_1(K_n) = n(n-1)^2$

$$\begin{aligned}&= n(n^2 - 3n + n + 1)\\ &= n(\Omega(K_n) + n + 1)\ (\because \Omega(K_n) = n^2 - 3n)\\ &= n\Omega(K_n) + n^2 + n\end{aligned}$$

(iv) From [43], we have $M_1(S_{n+1}) = n^2 + n$

$$\begin{aligned}&= n^2 + 3n - 2n\\ &= n^2 + 3n + n\Omega(S_{n+1})\ (\because \Omega(S_{n+1}) = -2)\end{aligned}$$

(v) From [44], we have $M_1(K_{m,n}) = mn(m+n)$

$$m + n = \frac{M_1(K_{m,n})}{mn}$$

Also $\Omega(K_{m,n}) = 2mn - 2(m+n)$.
Substituting for $m+n$ in the above equation we get,

$$\begin{aligned}\Omega(K_{m,n}) &= 2mn - \frac{2M_1(K_{m,n})}{mn}\\ \frac{2M_1(K_{m,n})}{mn} &= 2mn - \Omega(K_{m,n})\\ 2M_1(K_{m,n}) &= 2m^2n^2 - mn\Omega(K_{m,n})\end{aligned}$$

Dividing by 2 throughout, we get the desired result. □

The next result gives a relation between the second Zagreb index and Omega invariant for some standard graphs.

Theorem 2.8.

$$\begin{aligned}M_2(P_n) &= n(\Omega(P_n))^2 + (\Omega(P_n))^3\\ M_2(C_n) &= 4n + \Omega(C_n)\\ M_2(K_n) &= \frac{n^2}{2}\Omega(K_n) + \frac{3}{2}n^2 - \frac{n}{2}\\ M_2(S_{n+1}) &= \Omega(S_{n+1}) + n^2 + 2\\ M_2(K_{m,n}) &= \frac{(\Omega(K_{m,n}))^2}{4} + (m+n)\Omega(K_{m,n}) + (m+n)^2\end{aligned}$$

Proof. Proof is similar to the proof of Theorem 2.7 □

The following result gives a relation between the Forgotten index and Omega invariant for some standard graphs.

Theorem 2.9.

$$\begin{aligned}F(P_n) &= 7\Omega(P_n) - n(\Omega(P_n))^3\\ F(C_n) &= 8n + \Omega(C_n)\\ F(K_n) &= n^2\Omega(K_n) + 3n^2 - n\\ F(S_{n+1}) &= n\Omega(S_{n+1}) + n^3 + 3n\\ F(K_{m,n}) &= \frac{m^2+n^2}{2}\Omega(K_{m,n}) + (m+n)(m^2+n^2)\end{aligned}$$

Proof. Proof is similar to the proof of Theorem 2.7 □

The next result gives a relation between the first reformulated Zagreb index and Omega invariant for some standard graphs.

Theorem 2.10.

$$\begin{aligned}EM_1(P_n) &= n(\Omega(P_n))^2 + 5\Omega(P_n)\\ EM_1(C_n) &= 4n + \Omega(C_n)\\ EM_1(K_n) &= 2n^2\Omega(K_n) - 2n\Omega(K_n) - 2n^3 + 10n^2 - 8n\\ EM_1(S_{n+1}) &= n^3 + n^2\Omega(S_{n+1}) + n\\ EM_1(K_{m,n}) &= \frac{(m+n-2)^2}{2}\Omega(K_{m,n}) + (m+n)(m+n-2)^2\end{aligned}$$

Proof. Proof is similar to the proof of Theorem 2.7 □

The following result gives a relation between the Hyper-Zagreb index and Omega invariant of some standard graphs.

Theorem 2.11.

$$\begin{aligned}HM(P_n) &= n(\Omega(P_n))^4 + 15\Omega(P_n)\\ HM(C_n) &= 16n + \Omega(C_n)\\ HM(K_n) &= 2n^2\Omega(K_n) + 6n^2 - 2n\\ HM(S_{n+1}) &= n^3 + n - n^2\Omega(S_{n+1})\\ HM(K_{m,n}) &= \frac{(m+n)^2}{2}\Omega(K_{m,n}) + (m+n)^3\end{aligned}$$

Proof. By applying the same technique as used in proof of Theorem 2.7, the desired result is obtained. □

In the next section, we obtain relations between various topological indices and omega invariant for generalized Mycielskian graphs.

2.2 Topological indices of Generalized Mycielskian graphs in terms of the Omega invariant

In this section, we obtain relations between the forgotten index, the first reformulated Zagreb index, the Hyper-Zagreb index and the omega invariant for some standard graphs such as paths, cycles, complete graphs, star graphs and complete bipartite graphs.
N. De [36] gave the exact expressions for the forgotten, the first reformulated and the Hyper-Zagreb index in terms of various other topological indices for any graph G. Here we give a simplified expression for forgotten, first reformulated, Hyper-Zagreb index in terms of the Omega invariant, k and their number of vertices n for some standard graphs.

The following corollary gives a relation between the forgotten index and the omega invariant of generalized Mycielskian graphs.

Corollary 2.12.

$$\begin{aligned}
F(\mu_k(P_n)) &= n^3+7n-6+56k\Omega(P_n)-n(\Omega(P_n))^3(8k+1)+3n(\Omega(P_n))^2+16\Omega(P_n).\\
F(\mu_k(C_n)) &= n^3+64kn+27n+8k\Omega(C_n)+4\Omega(C_n).\\
F(\mu_k(K_n)) &= n^3+(24k+9)n^2+(8k+1)n^2\Omega(K_n)+3n\Omega(K_n)-8kn.\\
F(\mu_k(S_{n+1})) &= (8k+2)n^3+6n^2+22n+(8k+4)n\Omega(S_{n+1})+24kn+2.\\
F(\mu_k(K_{m,n})) &= 4k(m^2+n^2)\Omega(K_{m,n})+\frac{(m^2+n^2-3mn)}{2}\Omega(K_{m,n})+8k(m+n)\\
&\quad\times(m^2+n^2)+2(m^3+n^3)+4(mn^2+nm^2)+(m+n)+(6mn+3m^2n^2).
\end{aligned}$$

Proof. By Theorem 1.1 [36] we have

$$F(\mu_k(G)) = (8k+1)F(G)+3M_1(G)+6m+n(n^2+1) \tag{2.2}$$

Applying Theorem 2.9 and Theorem 2.7 in Equation 2.2 and simplifying, the desired result is obtained. □

The next corollary gives a relation between the first reformulated Zagreb index and omega invariant for generalized mycielskian graphs of particular kind.

Corollary 2.13.

$$\begin{aligned}
EM_1(\mu_k(P_n)) &= n^3+2n^2-24kn-n+24k-2+2n(\Omega(P_n))^4-n(\Omega(P_n))^3\\
&\quad+(24k-7)n(\Omega(P_n))^2+(88k+12)\Omega(P_n).\\
EM_1(\mu_k(C_n)) &= n^3+2n^2+72kn+15n+24k\Omega(C_n)-4\Omega(C_n).\\
EM_1(\mu_k(K_n)) &= (7-16k)n^3+(84k-13)n^2+(6-36k)n+(16k+1)n^2\Omega(K_n)\\
&\quad-n\Omega(K_n).\\
EM_1(\mu_k(S_{n+1})) &= (8k+2)n^3+(32k-6)n+16kn^2+(8k-4)n^2\Omega(S_{n+1})\\
&\quad+(16k-4)\Omega(S_{n+1}).\\
2EM_1(\mu_k(K_{m,n})) &= (8k+1)m^2\Omega(K_{m,n})+(8k+1)n^2\Omega(K_{m,n})+5mn\Omega(K_{m,n})\\
&\quad-8m(4k-1)\Omega(K_{m,n})-8n(4k-1)\Omega(K_{m,n})+8(4k-1)\Omega(K_{m,n})\\
&\quad+(16k+4)m^3+(48k+16)mn^2+(48k+16)m^2n+(12-64k)m^2\\
&\quad+(28-176k)mn+(64k-14)m+(64k-14)n+(16k+4)n^3+\\
&\quad(12-64k)n^2+(32k-10)m^2n^2.
\end{aligned}$$

Proof. From Theorem 1.1 [36], we have

$$EM_1(\mu_k(G)) = (8k-2)EM_1(G) + F(G) + 2HM(G) + (16k-5)M_1(G) - 6m(4k-1) + 4m(n-1) + n(n-1)^2 \quad (2.3)$$

Using Theorem 2.10, Theorem 2.9, Theorem 2.11 and Theorem 2.7 in the above equation and simplifying, the required result is obtained. □

The following corollary gives a relation between the Hyper-Zagreb and omega invariant of generalized Mycielskian graphs of a particular kind.

Corollary 2.14.

$$\begin{aligned}
HM(\mu_k(P_n)) &= n^3 + 6n^2 + 3n - 6 + (8k-4)n(\Omega(P_n))^4 + (8-5n)(\Omega(P_n))^3 \\
&\quad +15n(\Omega(P_n))^2 + (120k-4)\Omega(P_n) \\
HM(\mu_k(C_n)) &= n^3 + 6n^2 + 128kn + 43n + 8k\Omega(C_n) + 16\Omega(C_n) \\
HM(\mu_k(K_n)) &= 3n^3 + (48k+13)n^2 + (4-16k)n + (16k+1)n^2\Omega(K_n) + 7n\Omega(K_n). \\
HM(\mu_k(S_{n+1})) &= (8k+2)n^3 + 24n^2 + (50+8k)n + 20 + 12n\Omega(S_{n+1}) + \\
&\quad 8\Omega(S_{n+1}) - (8k-4)n^2\Omega(S_{n+1}). \\
4HM(\mu_k(K_{m,n})) &= (16k+2)m^2\Omega(K_{m,n}) + (16k+2)n^2\Omega(K_{m,n}) + (32k-30)mn \\
&\quad \Omega(K_{m,n}) + (32k+8)m^3 + 96k(mn^2+m^2n) + (32k+8)n^3 + \\
&\quad 8(\Omega(K_{m,n}))^2 + (m+n) \times \big(32\Omega(K_{m,n}) + 4\big) + 40(m^2+n^2) \\
&\quad +mn(28mn+104).
\end{aligned}$$

Proof. From Theorem 1.1 [36], we have

$$HM(\mu_k(G)) = (8k-4)HM(G) + 5F(G) + 7M_1(G) + 8M_2(G) + n(n+1)^2 + 4mn + 6m \quad (2.4)$$

Applying Theorem 2.11, Theorem 2.9, Theorem 2.7 and Theorem 2.8, in Equation 2.4 and simplifying, the desired result is obtained. □

2.3 Relation between Omega invariant and connectivity

In this section, we obtain new results that gives an interesting connection between the omega invariant, vertex connectivity and edge connectivity for some standard graphs such as paths, cycles, complete graphs, star graphs, complete bipartite graphs and generalized mycielskian graphs. Further new relations between topological indices and connectivity is established.

Theorem 2.15. *The vertex and edge connectivity of the generalized Mycielskian graphs for some standard graphs is given by*

$$\begin{aligned}
\kappa(\mu_k(P_n)) = \kappa'(\mu_k(P_n)) &= 2 \\
\kappa(\mu_k(C_n)) = \kappa'(\mu_k(C_n)) &= 3 \\
\kappa(\mu_k(K_n)) = \kappa'(\mu_k(K_n)) &= n \\
\kappa(\mu_k(S_{n+1})) = \kappa'(\mu_k(S_{n+1})) &= 2 \\
\kappa(\mu_k(K_{m,n}) = \kappa'(\mu_k(K_{m,n})) &= m+1
\end{aligned}$$

Proof. i. We know that $DS(\mu_k(P_n)) = \{2^{2k+2}, 3^{n-2}, 4^{(n-2)k}\}$. As minimum degree is 2, the number of components increases by 1 only when 2 edges are removed. Thus, $\kappa'(\mu_k(P_n)) = 2$. Next we show that $\kappa(\mu_k(P_n)) = 2$. Consider a vertex of degree 2. On deleting all it's vertex neighbours, a disconnected graph is obtained. Thus, $\kappa(\mu_k(P_n)) = 2$.
Similar proof follows for the remaining four cases. □

The next result gives an interesting result between the Omega invariant and connectivity for some standard classes of graphs.

Corollary 2.16.

$$\begin{aligned}
\Omega(P_n) &= -2\kappa(P_n) = -2\kappa'(P_n)\\
\Omega(C_n) &= \kappa(C_n) - \kappa'(C_n)\\
\Omega(K_n) &= n\kappa(K_n) - 3n + \kappa(K_n) + 1\\
\Omega(S_{n+1}) &= -2\kappa(S_{n+1}) = -2\kappa'(S_{n+1})\\
\Omega(K_{m,n}) &= (2n-2)\kappa(K_{m,n}) - 2n \text{ if } m \leq n\\
\Omega(K_{m,n}) &= (2m-2)\kappa(K_{m,n}) - 2m \text{ if } m > n
\end{aligned}$$

Proof. We know that

$$\begin{aligned}
\kappa(C_n) = \kappa'(C_n) &= 2\\
\kappa(P_n) = \kappa'(P_n) &= 1\\
\kappa(K_n) = \kappa'(K_n) &= n-1\\
\kappa(S_{n+1}) = \kappa'(S_{n+1}) &= 1\\
\kappa(K_{m,n}) = \kappa'(K_{m,n}) &= \min(m,n)
\end{aligned} \tag{2.5}$$

Substituting these in omega invariant values, the desired result is obtained. □

The next result gives a relation between vertex connectivity and the first Zagreb index for some standard graphs.

Corollary 2.17.

$$\begin{aligned}
M_1(P_n) &= (4n-6)\kappa(P_n)\\
M_1(C_n) &= 4n + \kappa(C_n) - \kappa'(C_n)\\
M_1(K_n) &= n^2(\kappa(K_n) - 2) + n(\kappa(K_n) + 2)\\
M_1(S_{n+1}) &= n^2 + (3 - 2\kappa(S_{n+1}))n\\
M_1(K_{m,n}) &= m^2n^2 + (1 - \kappa(K_{m,n}))mn^2 + \kappa mn \text{ if } m \leq n\\
M_1(K_{m,n}) &= m^2n^2 + (1 - \kappa(K_{m,n}))m^2n + \kappa mn \text{ if } m > n
\end{aligned}$$

Proof. Proof follows from Theorem 2.7 and Equation 2.5.

□

The following result gives a relation between vertex connectivity and the second Zagreb index for some standard graphs.

Corollary 2.18.

$$\begin{aligned}
M_2(P_n) &= (4n-8)\kappa(P_n)\\
M_2(C_n) &= 4n + \kappa(C_n) - \kappa'(C_n)\\
M_2(K_n) &= \frac{n^3}{2}(\kappa(K_n) - 3) + \frac{n^2}{2}(\kappa(K_n) + 4) - \frac{n}{2}\\
M_2(S_{n+1}) &= -2\kappa(S_{n+1}) + n^2 + 2\\
M_2(K_{m,n}) &= (n-1)^2\kappa^2(K_{m,n}) + 2\kappa(K_{m,n})m(n-1) + m^2 + n^2 \text{ if } m \leq n\\
M_2(K_{m,n}) &= (m-1)^2\kappa^2(K_{m,n}) + 2\kappa(K_{m,n})n(m-1) + n^2 \text{ if } m > n
\end{aligned}$$

Proof. Proof follows from Theorem 2.8 and Equation 2.5.

□

The next result gives a relation between vertex connectivity and the forgotten index for some standard graphs.

Corollary 2.19.

$$\begin{aligned}
F(P_n) &= (8n-14)\kappa(P_n)\\
F(C_n) &= 8n + \kappa(C_n) - \kappa'(C_n)\\
F(K_n) &= n^3(\kappa(K_n) - 3) + (\kappa(K_n) + 4)n^2 - n\\
F(S_{n+1}) &= n^3 + 3n - 2n\kappa(S_{n+1})\\
F(K_{m,n}) &= (m^2 + n^2)(\kappa(K_{m,n})(n-1) + m) \text{ if } m \leq n\\
F(K_{m,n}) &= (m^2 + n^2)(\kappa(K_{m,n})(m-1) + n) \text{ if } m > n
\end{aligned}$$

Proof. Proof follows from Theorem 2.9 and Equation 2.5.

□

The following result gives a relation between vertex connectivity and the first reformulated zagreb index for some standard graphs.

Corollary 2.20.

$$\begin{aligned} EM_1(P_n) &= (4n-10)\kappa(P_n) \\ EM_1(C_n) &= 4n+\kappa(C_n)-\kappa'(C_n) \\ EM_1(K_n) &= 2n^3(\kappa(K_n)-4)+18n^2-2n\kappa(K_n)-10n \\ EM_1(S_{n+1}) &= n^3-2n^2\kappa(S_{n+1})+n \\ EM_1(K_{m,n}) &= (m+n-2)^2(\kappa(K_{m,n})(n-1)+m) \text{ if } m \leq n \\ EM_1(K_{m,n}) &= (m+n-2)^2(\kappa(K_{m,n})(m-1)+n) \text{ if } m > n \end{aligned}$$

Proof. Proof follows from Theorem 2.10 and Equation 2.5.

□

The next result gives a relation between vertex connectivity and the Hyper-Zagreb index for some standard graphs.

Corollary 2.21.

$$\begin{aligned} HM(P_n) &= (16n-30)\kappa(P_n) \\ HM(C_n) &= 16n+\kappa(C_n)-\kappa'(C_n) \\ HM(K_n) &= 2n^3(\kappa(K_n)-3)+2n^2(\kappa(K_n)+4)-2n \\ HM(S_{n+1}) &= n^3+n+2n^2\kappa(S_{n+1}) \\ HM(K_{m,n}) &= (m+n)^2(\kappa(K_{m,n})(n-1)-n) \text{ if } m \leq n \\ HM(K_{m,n}) &= (m+n)^2(\kappa(K_{m,n})(m-1)-m) \text{ if } m > n \end{aligned}$$

Proof. Proof follows from Theorem 2.11 and Equation 2.5. □

3 Conclusion remarks

In this paper, we have determined the general form of the degree sequence of any generalized Mycielskian graph. General formula for the Omega invariant was determined. We have given the exact expressions for first and second Zagreb index, Forgotten index, first Reformulated Zagreb index and Hyper-Zagreb index in terms of the Omega invariant for some standard classes of graphs such as paths, cycles, complete graphs, star graphs and complete bipartite graphs. Further, we give a simplified expression for Forgotten index, first Reformulated Zagreb index and Hyper-Zagreb index of generalized Mycielskian graphs such as paths, cycles, complete graphs, star graphs and complete bipartite graphs in terms of the Omega invariant. Also, we have determined the vertex and edge connectivity of generalized Mycielskian graphs of some standard graphs. Further, we have obtained new relationships between Omega invariant, connectivity and some known topological indices.

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Author information

G. D. D'Souza, Department of Mathematics, St Joseph's University, India.
E-mail: grace.divya@sju.edu.in

I. N. Cangul, Department of Mathematics, Bursa Uludag University, Turkey.
E-mail: cangul@uludag.edu.tr

M. I. Huilgol, Department of Mathematics, Bengaluru City University, India.
E-mail: medha@bub.ernet.in

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