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Unified Analysis of Comprehensive Finite Integrals Using Srivastava Polynomial and ℵ-Function

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Abstract Special functions are pivotal in mathematical analysis, functional analysis, geometry, physics, and a wide array of practical applications due to their distinct properties and profound significance. These functions have become indispensable tools across various disciplines, effectively addressing complex mathematical challenges and offering valuable insights into science and engineering. This paper seeks to define comprehensive finite integrals by incorporating the Fresnel integral and the product of the incomplete ℵ-function with the Srivastava polynomial within a unified framework. The study derives a diverse set of novel results through a general and systematic approach, particularly in specialized cases. To emphasize the importance of these contributions, we present a thorough exposition of our findings, supported by specific corollaries derived from the fundamental results of the research. Including the Srivastava polynomial further enhances the scope of this work, demonstrating its significance and potential applications in advanced mathematics.

1 Introduction

Over the past four decades, the fields of fractional calculus and special functions have captivated the attention of mathematicians and scientists due to their profound applications in diverse domains, including medical science, ecology, biological systems, computer science, signal processing, fluid dynamics, probability theory, diffusive transport, control theory, viscoelasticity, and environmental science [1, 2, 3]. Special functions such as hypergeometric functions, Bessel functions, Mittag-Leffler functions, and Fox H-functions have established themselves as essential tools in addressing complex challenges in science and engineering.

The incomplete ℵ-function, introduced by Sudland et al. [4, 5], represents a significant extension of classical special functions. Researchers, including Bansal et al. [6], have explored its integral transformations and potential applications across applied mathematics, physics, and engineering. Furthermore, Srivastava et al. [7] analyzed the incomplete H-functions and incomplete $\overline{H}$-functions, introducing integral representations and fractional calculus formulations. Similarly, Jangid et al. [8] developed integral transformations for the incomplete I-function, showcasing its applications in diverse fields.

Inspired by the potential of special functions in integral representations, this study introduces a Fresnel-type integral involving the incomplete ℵ-function in the kernel. The innovative use of the Srivastava polynomial alongside the incomplete ℵ-function further enriches the scope of integral calculus, extending its applications to advanced mathematical modeling. By employing a systematic and unified approach, we aim to derive novel integral results and explore their specialized cases.

This paper's contributions build on the foundational work of numerous researchers [9, 10, 11], emphasizing the importance of integral equations with special functions in modern science and engineering. Through this framework, we present generalized finite integrals that highlight the interplay between fractional calculus and special functions, contributing to the growing body of knowledge in mathematical and applied sciences.

2 Mathematical Preliminaries

This section lays the groundwork for understanding special functions by presenting essential definitions and fundamental concepts. Key functions discussed include the incomplete Gamma function, the Aleph function, the Srivastava polynomial, and the Fresnel integral.

Incomplete Gamma Function

The standard incomplete gamma functions, $\gamma(\mathfrak{a}, \mathfrak{S})$ and $\Gamma(\mathfrak{a}, \mathfrak{S})$ as presented in [12], is given below:

$$\gamma(\mathfrak{a}, \mathfrak{S}) := \int_0^{\mathfrak{S}} \zeta^{\mathfrak{a}-1}\, e^{-\zeta}\, d\zeta, \qquad \big(\Re(\mathfrak{a}) > 0;\ \mathfrak{S} \geq 0\big), \tag{2.1}$$

and

$$\Gamma(\mathfrak{a}, \mathfrak{S}) := \int_{\mathfrak{S}}^{\infty} \zeta^{\mathfrak{a}-1}\, e^{-\zeta}\, d\zeta, \qquad \big(\Re(\mathfrak{a}) > 0;\ \mathfrak{S} \geq 0\big), \tag{2.2}$$

comply with the subsequent decomposition rule.

$$\gamma(\mathfrak{a}, \mathfrak{S}) + \Gamma(\mathfrak{a}, \mathfrak{S}) := \Gamma(\mathfrak{a}), \qquad \big(\Re(\mathfrak{a}) > 0\big), \tag{2.3}$$

where $\Re(\mathfrak{a})$ denote the real part of the parameter $\mathfrak{a}$. And, Setting $\mathfrak{S} = 0$ yields then we have $\Gamma(\mathfrak{a}, \mathfrak{S}) = \Gamma(\mathfrak{a})$.

Aleph Function

The $\aleph$-function, introduced by Sudland et al. in their research [4, 5], is a generalized higher transcendental function formally defined as follows:

$$\aleph(\varrho) = \aleph^{u,v}_{r_l, s_l, f_l; \wp}\left[\varrho \left| \begin{matrix} (\Theta_m, \mathfrak{E}_m)_{1,v},\ [f_m(\Theta_{ml}, \mathfrak{E}_{ml})]_{v+1,r_l} \\ (\mathcal{Q}_m, \mathcal{B}_m)_{1,u},\ [f_m(\mathcal{Q}_{ml}, \mathcal{B}_m)]_{u+1,s_l} \end{matrix} \right.\right] = \frac{1}{2\pi\iota} \int_{\$} \Omega(\mathbf{w})\, \varrho^{-\mathbf{w}}\, d\mathbf{w}, \tag{2.4}$$

where $\varrho \in \mathbb{C}/0$, $\iota = \sqrt{-1}$, and

$$\Omega(\mathbf{w}) = \frac{\prod\limits_{m=1}^{u} \Gamma(\mathcal{Q}_m + \mathcal{B}_m \mathbf{w}) \prod\limits_{m=1}^{v} \Gamma(1 - \Theta_m - \mathfrak{E}_m \mathbf{w})}{\sum\limits_{l=1}^{\wp} f_l \Big[\prod\limits_{m=u+1}^{s_l} \Gamma(1 - \mathcal{Q}_{ml} - \mathcal{B}_m \mathbf{w}) \prod\limits_{m=v+1}^{r_l} \Gamma(\Theta_{ml} + \mathfrak{E}_{ml} \mathbf{w}) \Big]}. \tag{2.5}$$

The integral path $\$ = \$_{\iota\gamma\infty}$ $(\gamma \in \mathbb{R})$ extends from $\gamma - \iota\infty$ and $\gamma + \iota\infty$; the poles of gamma function $\Gamma(1 - \Theta_m - \mathfrak{E}_m \mathbf{w})$ $(m = 1, \cdots, v)$ do not exactly match with the poles of gamma function $\Gamma(\mathcal{Q}_m + \mathcal{B}_m \mathbf{w})$ $(m = 1, \cdots, u)$; the parameters r_l, s_l are non-negative integers satisfying $0 \leq v \leq r_l$ and $1 \leq u \leq s_l$ for $l = 1, \cdots, r$; $\mathfrak{E}_m$, $\mathcal{B}_m$, $\mathfrak{E}_{ml}$, $\mathcal{B}_{ml}$ are positive real numbers, and Θ_m, $\mathcal{Q}_m$, Θ_{ml}, $\mathcal{Q}_{ml}$ are complex and the empty product is interpreted as unity. For the existence conditions and further details of $\aleph$-function one can refer to [4, 13].

Incomplete $\aleph$-Function

The incomplete $\aleph$-functions ${}^{\gamma}\aleph^{u,v}_{r_l, s_l, f_l; \wp}(\varrho)$ and ${}^{\Gamma}\aleph^{u,v}_{r_l, s_l, f_l; \wp}(\varrho)$ containig the incomplete gamma functions $\gamma(\mathfrak{a}, \mathfrak{S})$ and $\Gamma(\mathfrak{a}, \mathfrak{S})$ introduced by Bansal et al. [6] as define below:

$$\begin{aligned} {}^{\gamma}\aleph^{u,v}_{r_l, s_l, f_l; \wp}(\varrho) &= {}^{\gamma}\aleph^{u,v}_{r_l, s_l, f_l; \wp}\left[\varrho \left| \begin{matrix} (\Theta_1, \mathfrak{E}_1 : z), (\Theta_m, \mathfrak{E}_m)_{2,v},\ [f_m(\Theta_{ml}, \mathfrak{E}_{ml})]_{v+1,r_l} \\ (\mathcal{Q}_m, \mathcal{B}_m)_{1,u},\ [f_m(\mathcal{Q}_{ml}, \mathcal{B}_m)]_{u+1,s_l} \end{matrix} \right.\right] \\ &= \frac{1}{2\pi\iota} \int_{\$} \mathfrak{w}(\mathbf{w}, z)\, \varrho^{-\mathbf{w}}\, d\mathbf{w}, \end{aligned} \tag{2.6}$$

and

$$
{}^{\Gamma}\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}(\varrho) = {}^{\Gamma}\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}\left[\varrho \left| \begin{matrix} (\Theta_1, \mathfrak{E}_1 : z), (\Theta_m, \mathfrak{E}_m)_{2,v}, \, [f_m(\Theta_{ml}, \mathfrak{E}_{ml})]_{v+1,r_l} \\ (\mathcal{Q}_m, \mathcal{B}_m)_{1,u}, \, [f_m(\mathcal{Q}_{ml}, \mathcal{B}_m)]_{u+1,s_l} \end{matrix} \right. \right]
$$

$$
= \frac{1}{2\pi\iota}\int_{\$} \Psi(\mathbf{w}, z)\, \varrho^{-\mathbf{w}}\, d\mathbf{w}, \tag{2.7}
$$

where

$$
\mathfrak{w}(\mathbf{w}, z) = \frac{\gamma(1-\Theta_1-\mathfrak{E}_1\mathbf{w}; z)\prod_{m=1}^{u}\Gamma(\mathcal{Q}_m+\mathcal{B}_m\mathbf{w})\prod_{m=2}^{v}\Gamma(1-\Theta_m-\mathfrak{E}_m\mathbf{w})}{\sum_{l=1}^{\wp} f_l\left[\prod_{m=u+1}^{s_l}\Gamma(1-\mathcal{Q}_{ml}-\mathcal{B}_m\mathbf{w})\prod_{m=v+1}^{r_l}\Gamma(\Theta_{ml}+\mathfrak{E}_{ml}\mathbf{w})\right]}, \tag{2.8}
$$

$$
\Psi(\mathbf{w}, z) = \frac{\Gamma(1-\Theta_1-\mathfrak{E}_1\mathbf{w}; z)\prod_{m=1}^{u}\Gamma(\mathcal{Q}_m+\mathcal{B}_m\mathbf{w})\prod_{m=2}^{v}\Gamma(1-\Theta_m-\mathfrak{E}_m\mathbf{w})}{\sum_{l=1}^{\wp} f_l\left[\prod_{m=u+1}^{s_l}\Gamma(1-\mathcal{Q}_{ml}-\mathcal{B}_m\mathbf{w})\prod_{m=v+1}^{r_l}\Gamma(\Theta_{ml}+\mathfrak{E}_{ml}\mathbf{w})\right]}, \tag{2.9}
$$

for $\varrho \neq 0,\ z \geq 0$, the incomplete $\aleph$-functions ${}^{\gamma}\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}(\varrho)$ and ${}^{\Gamma}\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}(\varrho)$ in (2.6) and (2.7) exist under conditions [6]. Such as

$$
\delta_l > 0,\ |arg(\varrho)| < \tfrac{\pi}{2}\delta_l, \quad l = 1, \cdots, \wp,
$$
$$
\delta_l > 0,\ |arg(\varrho)| < \tfrac{\pi}{2}\delta_l, \ \text{and}\ \Re(\Delta)_l + 1 < 0,
$$

where

$$
\delta_l = \sum_{m=1}^{v}\mathfrak{E}_m + \sum_{m=1}^{u}\mathcal{B}_m - f_l\left(\sum_{m=v+1}^{r_l}\mathfrak{E}_{ml} + \sum_{m=u+1}^{s_l}\mathcal{B}_{ml}\right), \tag{2.10}
$$

$$
\Delta_l = \sum_{m=1}^{u}\mathcal{Q}_m - \sum_{m=1}^{v}\Theta_m + f_l\left(\sum_{m=u+1}^{s_l}\mathfrak{E}_{ml} - \sum_{m=v+1}^{r_l}\mathcal{B}_{ml}\right) + \frac{1}{2}(r_l - s_l), \quad l = 1, \cdots, \wp. \tag{2.11}
$$

The incomplete $\aleph$-functions ${}^{\gamma}\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}(\varrho)$ and ${}^{\Gamma}\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}(\varrho)$ defined in (2.6) and (2.7), simplified to numereous special functions such as $\aleph$-function, incomplete I-functions, I-functions, incomplete H-functions, Fox's H-function etc.

Srivastava Polynomial

The Srivastava [14] investigated a broader class of polynomials, which is summarised as follows:

$$
S_V^U[t] = \sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\, t^R, \tag{2.12}
$$

where $U \in \mathbb{Z}^+$ and $A_{V,\,R}$ are real or complex numbers arbitrary constant. The notations $[k]$ indicates the Floor function and $(\kappa)_\mu$ indicate the Pochhammer symbol described by:

$$
(\kappa)_0 = 1 \qquad and \qquad (\kappa)_\mu = \frac{\Gamma(\kappa+\mu)}{\Gamma(\kappa)}, \qquad\qquad (\mu \in \mathbb{C})
$$

in the form of the gamma function.

Fresnel Integral

The Fresnel integrals [15] find applications in diverse fields such as wave optics, diffraction phenomena, and various domains of signal processing. They are notable examples of oscillatory integrals, which pose significant analytical challenges. The Fresnel cosine integral, $\mathcal{C}(\mathrm{x})$, is defined as follows:

$$
\mathcal{C}(\mathrm{x}) := \int_0^{\mathrm{x}}\cos(t^2)dt. \tag{2.13}
$$

3 Solution of Fresnel Integral with Incomplete ℵ-Function and S_V^U Polynomial in the Kernel

This segment presents a general finite integral as given in (Section 4.5.4, Page 191, [16]). The integral is expressed as follows:

Lemma 3.1.

$$\int_0^{a} x^{s+\frac{1}{2}}(a-x)^{s}\mathcal{C}\left(b\sqrt[4]{x(a-x)}\right)dx = 2^{-2s-\frac{3}{4}}\, a^{2s+\frac{7}{4}}\, b^{\frac{1}{2}}\,\frac{\Gamma(2s+\frac{9}{4})}{\Gamma(2s+\frac{11}{4})} \times {}_2F_3\left[-\frac{ab^2}{8}\middle|\begin{array}{c}\frac{1}{4},\, 2s+\frac{9}{4}\\ \frac{1}{2},\, \frac{5}{4},\, 2s+\frac{11}{4}\end{array}\right], \tag{3.1}$$

where $\Re(s) > -\frac{9}{8}, a > 0$. *This integral is crucial for deriving solutions in integral calculus and the incomplete special functions.*

In this section, we investigate a generalization of the finite integral that incorporates the Srivastava polynomial and the incomplete ℵ-function, with $z > 0$ as an additional parameter. Including the parameter z enhances the integral's flexibility, formulating a broader range of solutions. Specifically, the incomplete ℵ-function introduces a dependence on z that influences the integral's behavior, facilitating more generalization in applied mathematics. This generalization allows for greater adaptability in addressing complex problems where classical functions may be insufficient or a higher level of precision is needed.

Theorem 3.2. *Let the following conditions hold:*

- $\Re(s) > -\frac{9}{8}$,
- $\Re(s) + (A+B)\min_{1\le m\le u}\left(\frac{\mathcal{Q}_m}{\mathcal{B}_m}\right) > -\frac{9}{8}$,
- $\delta_l > 0$, $|\arg(\varrho)| < \frac{\pi}{2}\delta_l$, *or* $\delta_l \ge 0$, $|\arg(\varrho)| < \frac{\pi}{2}\delta_l$, *for* $l = 1, \cdots, \wp$,
- $\Re(\Delta_l) + 1 > 0$,
- $A, B > 0$, *and* $a > 0$,
- *where* δ_l *and* Δ_l *are defined by Equations* (2.10) *and* (2.11) *respectively.*

Then the following result true

$$\int_0^{a} x^{s+\frac{1}{2}}(a-x)^{s}\mathcal{C}\left(b\sqrt[4]{x(a-x)}\right) S_V^U\left[\varrho x^{B}(a-x)^{B}\right]\, {}^{\Gamma}\aleph^{u,v}_{r_l,\, s_l,\, f_l;\,\wp}\left[\varrho x^{A}(a-x)^{A}\right] dx$$
$$= 2^{-2s-\frac{3}{4}}\, a^{2s+\frac{7}{4}}\, b^{\frac{1}{2}} \sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\, \left[\varrho(2a)^{2B}\right]^{R}\sum_{n=0}^{\infty}\frac{\left(\frac{1}{4}\right)_n}{\left(\frac{1}{2}\right)_n\left(\frac{5}{4}\right)_n}\left(-\frac{ab^2}{8}\right)^{n}$$
$$\times\, {}^{\Gamma}\aleph^{u,v+1}_{r_l+1,s_l+1,f_l;\wp}\left[\varrho\left(\frac{a}{2}\right)^{2A}\middle|\begin{array}{c}(\Theta_1,\mathfrak{C}_1:z),\mathfrak{T}_1,(\Theta_m,\mathfrak{C}_m)_{2,v},\,[f_m(\Theta_{ml},\mathfrak{C}_{ml})]_{v+1,r_l}\\ (\mathcal{Q}_m,\mathcal{B}_m)_{1,u},\,[f_m(\mathcal{Q}_{ml},\mathcal{B}_m)]_{u+1,s_l},\mathfrak{T}_2\end{array}\right], \tag{3.2}$$

where $\mathfrak{T}_1 = \left(-2s-2AR-\frac{5}{4}-n; 2A\right)$, $\quad\mathfrak{T}_2 = \left(-2s-2AR-\frac{7}{4}-n; 2A\right)$.

Proof. To prove the theorem, we express the incomplete ℵ-function as a Mellin-Barnes contour integral using Equation (2.6) and the Srivastava polynomial (2.12). We then interchange the order of integration, which is justified by the absolute convergence of the integrals involved in the process. We obtain the desired result by reinterpreting the resulting Mellin-Barnes contour integral in the context of the incomplete ℵ-function.

$$\mathcal{I}=\int_0^{\mathrm{a}} \mathrm{x}^{\mathrm{s}+\frac{1}{2}}(\mathrm{a}-\mathrm{x})^{\mathrm{s}}\mathcal{C}\left(\mathrm{b}\sqrt[4]{\mathrm{x}(\mathrm{a}-\mathrm{x})}\right) S_V^U\left[\varrho \mathrm{x}^{\mathrm{B}}(\mathrm{a}-\mathrm{x})^{\mathrm{B}}\right] {}^{\Gamma}\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}\left[\varrho \mathrm{x}^{\mathrm{A}}(\mathrm{a}-\mathrm{x})^{\mathrm{A}}\right]dx$$

$$=\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\,(\varrho)^R\,\frac{1}{2\pi\iota}$$

$$\times\int_{\$}\frac{\Gamma(1-\Theta_1-\mathfrak{E}_1\mathbf{w};z)\prod_{m=1}^{u}\Gamma(\mathcal{Q}_m+\mathcal{B}_m\mathbf{w})\prod_{m=2}^{v}\Gamma(1-\Theta_m-\mathfrak{E}_m\mathbf{w})}{\sum_{l=1}^{\wp} f_l\left[\prod_{m=u+1}^{s_l}\Gamma(1-\mathcal{Q}_{ml}-\mathcal{B}_m\mathbf{w})\prod_{m=v+1}^{r_l}\Gamma(\Theta_{ml}+\mathfrak{E}_{ml}\mathbf{w})\right]}\varrho^{-\mathbf{w}}$$

$$\times\int_0^{\mathrm{a}} \mathrm{x}^{\mathrm{s}-\mathrm{A}\mathbf{w}+\mathrm{B}R+\frac{1}{2}}(\mathrm{a}-\mathrm{x})^{\mathrm{s}-\mathrm{A}\mathbf{w}+\mathrm{B}R}\,\mathcal{C}\left(\mathrm{b}\sqrt[4]{\mathrm{x}(\mathrm{a}-\mathrm{x})}\right)dx\,d\mathbf{w}, \tag{3.3}$$

using Lemma 3.1 in Equation (3.3) we get:

$$\mathcal{I}=2^{-2\mathrm{s}-\frac{3}{4}}\,\mathrm{a}^{2\mathrm{s}+\frac{7}{4}}\,\mathrm{b}^{\frac{1}{2}}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\left[\varrho(2\mathrm{a})^{2\mathrm{B}}\right]^R$$

$$\times\frac{1}{2\pi\iota}\int_{\$}\frac{\Gamma(1-\Theta_1-\mathfrak{E}_1\mathbf{w};z)\prod_{m=1}^{u}\Gamma(\mathcal{Q}_m+\mathcal{B}_m\mathbf{w})\prod_{m=2}^{v}\Gamma(1-\Theta_m-\mathfrak{E}_m\mathbf{w})}{\sum_{l=1}^{\wp} f_l\left[\prod_{m=u+1}^{s_l}\Gamma(1-\mathcal{Q}_{ml}-\mathcal{B}_m\mathbf{w})\prod_{m=v+1}^{r_l}\Gamma(\Theta_{ml}+\mathfrak{E}_{ml}\mathbf{w})\right]}\varrho^{-\mathbf{w}}$$

$$\times\frac{\Gamma(2\mathrm{s}-2\mathrm{A}\mathbf{w}+2\mathrm{B}R+\frac{9}{4})}{\Gamma(2\mathrm{s}-2\mathrm{A}\mathbf{w}+2\mathrm{B}R+\frac{11}{4})}\,2^{2\mathrm{A}\mathbf{w}}\,\mathrm{a}^{-2\mathrm{A}\mathbf{w}}\times{}_2\mathrm{F}_3\left[-\frac{\mathrm{ab}^2}{8}\,\middle|\,\begin{matrix}\frac{1}{4},\,2\mathrm{s}-2\mathrm{A}\mathbf{w}+2\mathrm{B}R+\frac{9}{4}\\ \frac{1}{2},\,\frac{5}{4},\,2\mathrm{s}-2\mathrm{A}\mathbf{w}+2\mathrm{B}R+\frac{11}{4}\end{matrix}\right]d\mathbf{w}. \tag{3.4}$$

We replace the Gauss hypergeometric function by the series $\sum_{n=0}^{\infty}$, [17] under the hypothesis, we can interchanged this series, we have:

$$\mathcal{I}=2^{-2\mathrm{s}-\frac{3}{4}}\,\mathrm{a}^{2\mathrm{s}+\frac{7}{4}}\,\mathrm{b}^{\frac{1}{2}}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\left[\varrho(2\mathrm{a})^{2\mathrm{B}}\right]^R\sum_{n=0}^{\infty}\frac{\left(\frac{1}{4}\right)_n}{\left(\frac{1}{2}\right)_n\left(\frac{5}{4}\right)_n}\left(-\frac{\mathrm{ab}^2}{8}\right)^n$$

$$\times\frac{1}{2\pi\iota}\int_{\$}\frac{\Gamma(1-\Theta_1-\mathfrak{E}_1\mathbf{w};z)\prod_{m=1}^{u}\Gamma(\mathcal{Q}_m+\mathcal{B}_m\mathbf{w})\prod_{m=2}^{v}\Gamma(1-\Theta_m-\mathfrak{E}_m\mathbf{w})}{\sum_{l=1}^{\wp} f_l\left[\prod_{m=u+1}^{s_l}\Gamma(1-\mathcal{Q}_{ml}-\mathcal{B}_m\mathbf{w})\prod_{m=v+1}^{r_l}\Gamma(\Theta_{ml}+\mathfrak{E}_{ml}\mathbf{w})\right]}\varrho^{-\mathbf{w}}$$

$$\times\frac{\Gamma(2\mathrm{s}-2\mathrm{A}\mathbf{w}+2\mathrm{B}R+\frac{9}{4})}{\Gamma(2\mathrm{s}-2\mathrm{A}\mathbf{w}+2\mathrm{B}R+\frac{11}{4})}\,2^{2\mathrm{A}\mathbf{w}}\,\mathrm{a}^{-2\mathrm{A}\mathbf{w}}\,\frac{(2\mathrm{s}-2\mathrm{A}\mathbf{w}+2\mathrm{B}R+\frac{9}{4})_n}{(2\mathrm{s}-2\mathrm{A}\mathbf{w}+2\mathrm{B}R+\frac{11}{4})_n}\,d\mathbf{w}. \tag{3.5}$$

Now, apply the relation $\Gamma(a)_n=\frac{\Gamma(a+n)}{\Gamma(a)}, a\neq 0,-1,-2,-3,\cdots$, then we have:

$$\mathcal{I}=2^{-2\mathrm{s}-\frac{3}{4}}\,\mathrm{a}^{2\mathrm{s}+\frac{7}{4}}\,\mathrm{b}^{\frac{1}{2}}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\left[\varrho(2\mathrm{a})^{2\mathrm{B}}\right]^R\sum_{n=0}^{\infty}\frac{\left(\frac{1}{4}\right)_n}{\left(\frac{1}{2}\right)_n\left(\frac{5}{4}\right)_n}\left(-\frac{\mathrm{ab}^2}{8}\right)^n$$

$$\times\frac{1}{2\pi\iota}\int_{\$}\frac{\Gamma(1-\Theta_1-\mathfrak{E}_1\mathbf{w};z)\prod_{m=1}^{u}\Gamma(\mathcal{Q}_m+\mathcal{B}_m\mathbf{w})\prod_{m=2}^{v}\Gamma(1-\Theta_m-\mathfrak{E}_m\mathbf{w})}{\sum_{l=1}^{\wp} f_l\left[\prod_{m=u+1}^{s_l}\Gamma(1-\mathcal{Q}_{ml}-\mathcal{B}_m\mathbf{w})\prod_{m=v+1}^{r_l}\Gamma(\Theta_{ml}+\mathfrak{E}_{ml}\mathbf{w})\right]}\varrho^{-\mathbf{w}}$$

$$\times\frac{\Gamma(2\mathrm{s}-2\mathrm{A}\mathbf{w}+2\mathrm{B}R+\frac{9}{4}+n)}{\Gamma(2\mathrm{s}-2\mathrm{A}\mathbf{w}+2\mathrm{B}R+\frac{11}{4}+n)}\,2^{2\mathrm{A}\mathbf{w}}\,\mathrm{a}^{-2\mathrm{A}\mathbf{w}}\,d\mathbf{w}. \tag{3.6}$$

After some adjustment of terms, we obtain the intended outcomes. □

Theorem 3.3. *Let the following conditions hold:*

- $\Re(\mathrm{s}) > -\frac{9}{8}$,
- $\Re(\mathrm{s}) + (\mathrm{A}+\mathrm{B})\min_{1\le m\le u}\left(\frac{\mathcal{Q}_m}{\mathcal{B}_m}\right) > -\frac{9}{8}$,
- $\delta_l > 0$, $|\arg(\varrho)| < \frac{\pi}{2}\delta_l$, *or* $\delta_l \ge 0$, $|\arg(\varrho)| < \frac{\pi}{2}\delta_l$, *for* $l = 1,\cdots,\wp$,
- $\Re(\Delta_l) + 1 > 0$,
- $\mathrm{A},\mathrm{B} > 0$, *and* $\mathrm{a} > 0$,
- *where* δ_l *and* Δ_l *are defined by Equations* (2.10) *and* (2.11) *respectively.*

Then the following result true

$$\int_0^{\mathrm{a}} \mathrm{x}^{\mathrm{s}+\frac{1}{2}}(\mathrm{a}-\mathrm{x})^{\mathrm{s}}\mathcal{C}\left(\mathrm{b}\sqrt[4]{\mathrm{x}(\mathrm{a}-\mathrm{x})}\right) S_V^U\left[\varrho \mathrm{x}^{\mathrm{B}}(\mathrm{a}-\mathrm{x})^{\mathrm{B}}\right] {}^{\gamma}\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}\left[\varrho \mathrm{x}^{\mathrm{A}}(\mathrm{a}-\mathrm{x})^{\mathrm{A}}\right] d\mathrm{x}$$
$$= 2^{-2\mathrm{s}-\frac{3}{4}}\,\mathrm{a}^{2\mathrm{s}+\frac{7}{4}}\,\mathrm{b}^{\frac{1}{2}} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R}\,\left[\varrho(2\mathrm{a})^{2\mathrm{B}}\right]^R \sum_{n=0}^{\infty} \frac{\left(\frac{1}{4}\right)_n}{\left(\frac{1}{2}\right)_n\left(\frac{5}{4}\right)_n}\left(-\frac{\mathrm{ab}^2}{8}\right)^n$$
$$\times\, {}^{\gamma}\aleph^{u,v+1}_{r_l+1,s_l+1,f_l;\wp}\left[\varrho\left(\frac{\mathrm{a}}{2}\right)^{2\mathrm{A}} \middle| \begin{array}{c} (\Theta_1,\mathfrak{E}_1:z),\mathfrak{T}_1,(\Theta_m,\mathfrak{E}_m)_{2,v},\,[f_m(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}_m)_{1,u},\,[f_m(\mathcal{Q}_{ml},\mathcal{B}_m)]_{u+1,s_l},\mathfrak{T}_2 \end{array}\right], \tag{3.7}$$

where $\mathfrak{T}_1 = \left(-2\mathrm{s}-2\mathrm{AR}-\frac{5}{4}-n;2\mathrm{A}\right)$, $\quad \mathfrak{T}_2 = \left(-2\mathrm{s}-2\mathrm{AR}-\frac{7}{4}-n;2\mathrm{A}\right)$.

Proof. As the proof parallels Theorem 3.2, we omit the particulars of the proof for brevity and clarity. □

4 Special Cases

This section examines exceptional cases of generalized functions derived from incomplete I-functions, I-function, incomplete H-functions, H-function, and incomplete Meijer G-functions derived from incomplete $\aleph$-functions through appropriate substitutions for restrictive conditions. These functions extend their classical counterparts by incorporating Fresnel integration and reducing to their standard forms, offering applications in applied mathematics and theoretical physics.

(i) $\aleph$-Function: On setting $z = 0$, then Eq. (2.7) reduce to the $\aleph$-function proposed by Sudland [4, 5] :

$${}^{\Gamma}\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}\left[\varrho \middle| \begin{array}{c} (\Theta_1,\mathfrak{E}_1:0),(\Theta_m,\mathfrak{E}_m)_{2,v},\,[f_m(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\,[f_m(\mathcal{Q}_{ml},\mathcal{B}ml)]_{u+1,s_l} \end{array}\right]$$
$$= \aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}\left[\varrho \middle| \begin{array}{c} (\Theta_m,\mathfrak{E}_m)_{1,v},\,[f_m(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\,[f_m(\mathcal{Q}_{ml},\mathcal{B}ml)]_{u+1,s_l} \end{array}\right].$$

Corollary 4.1. *The integral*

$$\mathcal{I} = \int_0^{\mathrm{a}} \mathrm{x}^{\mathrm{s}+\frac{1}{2}}(\mathrm{a}-\mathrm{x})^{\mathrm{s}}\mathcal{C}\left(\mathrm{b}\sqrt[4]{\mathrm{x}(\mathrm{a}-\mathrm{x})}\right) S_V^U\left[\varrho \mathrm{x}^{\mathrm{B}}(\mathrm{a}-\mathrm{x})^{\mathrm{B}}\right] \aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}\left[\varrho \mathrm{x}^{\mathrm{A}}(\mathrm{a}-\mathrm{x})^{\mathrm{A}}\right] d\mathrm{x}$$
$$= 2^{-2\mathrm{s}-\frac{3}{4}}\,\mathrm{a}^{2\mathrm{s}+\frac{7}{4}}\,\mathrm{b}^{\frac{1}{2}} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R}\,\left[\varrho(2\mathrm{a})^{2\mathrm{B}}\right]^R \sum_{n=0}^{\infty} \frac{\left(\frac{1}{4}\right)_n}{\left(\frac{1}{2}\right)_n\left(\frac{5}{4}\right)_n}\left(-\frac{\varrho\mathrm{b}^2}{8}\right)^n$$
$$\times \aleph^{u,\,v+1}_{r_l+1,\,s_l+1,\,f_l;\,\wp}\left[\mathrm{x}\left(\frac{\mathrm{a}}{2}\right)^{2\mathrm{A}} \middle| \begin{array}{c} (\Theta_\iota,\mathfrak{E}_\iota)_{1,v},\,\mathfrak{T}_1,[f_m(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}m)_{2,u},\,\mathfrak{T}_2,\,[f_m(\mathcal{Q}_{ml},\mathcal{B}ml)]_{u+1,s_l},\,\mathfrak{T}_2 \end{array}\right], \tag{4.1}$$

where, $\mathfrak{T}_1 = \left(-2\mathrm{s}-2\mathrm{AR}-\frac{5}{4}-n;2\mathrm{A}\right)$ *and* $\mathfrak{T}_2 = \left(-2\mathrm{s}-2\mathrm{AR}-\frac{7}{4}-n;2\mathrm{A}\right)$.

(ii) Incomplete I-Function: Again, setting $f_l = 1$ in (2.6) and (2.7), then it becomes to the incomplete I-function suggested by Bansal and Kumar [18]:

$$ {}^{\gamma}\aleph^{u,v}_{r_l,\, s_l,\, 1;\, \wp}\left[\varrho \left| \begin{matrix} (\Theta_1,\mathfrak{E}_1:z),(\Theta_m,\mathfrak{E}_m)_{2,v},\ [1(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ [1(\mathcal{Q}_{ml},\mathcal{B}ml)]_{u+1,s_l} \end{matrix} \right.\right] $$

$$ = {}^{\gamma}I^{u,v}_{r_l,\, s_l;\, \wp}\left[\varrho \left| \begin{matrix} (\Theta_1,\mathfrak{E}_1:z),(\Theta_m,\mathfrak{E}_m)_{2,v},\ (\Theta_{ml},\mathfrak{E}_{ml})_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ (\mathcal{Q}_{ml},\mathcal{B}ml)_{u+1,s_l} \end{matrix} \right.\right], $$

and

$$ {}^{\Gamma}\aleph^{u,v}_{r_l,\, s_l,\, 1;\, \wp}\left[\varrho \left| \begin{matrix} (\Theta_1,\mathfrak{E}_1:z),(\Theta_m,\mathfrak{E}_m)_{2,v},\ [1(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ [1(\mathcal{Q}_{ml},\mathcal{B}ml)]_{u+1,s_l} \end{matrix} \right.\right] $$

$$ = {}^{\Gamma}I^{u,v}_{r_l,\, s_l;\, \wp}\left[\varrho \left| \begin{matrix} (\Theta_1,\mathfrak{E}_1:z),(\Theta_m,\mathfrak{E}_m)_{2,v},\ (\Theta_{ml},\mathfrak{E}_{ml})_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ (\mathcal{Q}_{ml},\mathcal{B}ml)_{u+1,s_l} \end{matrix} \right.\right]. $$

Corollary 4.2. *The integral*

$$ \mathcal{I} = \int_0^{\mathrm{a}} \mathrm{x}^{\mathrm{s}+\frac{1}{2}}(\mathrm{a}-\mathrm{x})^{\mathrm{s}}\mathcal{C}\left(\mathrm{b}\sqrt[4]{\mathrm{x}(\mathrm{a}-\mathrm{x})}\right) S^U_V\left[\varrho \mathrm{x}^{\mathrm{B}}(\mathrm{a}-\mathrm{x})^{\mathrm{B}}\right] \aleph^{u,v}_{r_l,\, s_l,\, f_l;\, \wp}\left[\varrho \mathrm{x}^{\mathrm{A}}(\mathrm{a}-\mathrm{x})^{\mathrm{A}}\right]dx $$

$$ = 2^{-2\mathrm{s}-\frac{3}{4}}\,\mathrm{a}^{2\mathrm{s}+\frac{7}{4}}\,\mathrm{b}^{\frac{1}{2}} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R}\left[\varrho(2\mathrm{a})^{2\mathrm{B}}\right]^R \sum_{n=0}^{\infty} \frac{\left(\frac{1}{4}\right)_n}{\left(\frac{1}{2}\right)_n\left(\frac{5}{4}\right)_n}\left(-\frac{\mathrm{ab}^2}{8}\right)^n $$

$$ \times\, {}^{\Gamma}I^{u,\, v+1}_{r_l+1,\, s_l+1;\, \wp}\left[\varrho\left(\frac{\mathrm{a}}{2}\right)^{2\mathrm{A}} \left| \begin{matrix} (\Theta_1,\mathfrak{E}_1:z),\ \mathfrak{T}_1,(\Theta_m,\mathfrak{E}_m)_{2,v},(\Theta_{ml},\mathfrak{E}_{ml})_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ (\mathcal{Q}_{ml},\mathcal{B}ml)_{u+1,s_l},\mathfrak{T}_2 \end{matrix} \right.\right], \tag{4.2} $$

where, $\mathfrak{T}_1 = \left(-2\mathrm{s}-2\mathrm{AR}-\frac{5}{4}-n;2\mathrm{A}\right)$ *and* $\mathfrak{T}_2 = \left(-2\mathrm{s}-2\mathrm{AR}-\frac{7}{4}-n;2\mathrm{A}\right)$.

(iii) I-Function: Next, setting $z = 0$ and $f_l = 1$ in (2.7), then it becomes to the I-function suggested by Saxena [19] :

$$ {}^{\Gamma}\aleph^{u,v}_{r_l,\, s_l,\, 1;\, \wp}\left[\varrho \left| \begin{matrix} (\Theta_1,\mathfrak{E}_1:0),(\Theta_m,\mathfrak{E}_m)_{2,v},\ [1(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ [1(\mathcal{Q}_{ml},\mathcal{B}ml)]_{u+1,s_l} \end{matrix} \right.\right] $$

$$ = I^{u,v}_{r_l,\, s_l;\, \wp}\left[\varrho \left| \begin{matrix} (\Theta_m,\mathfrak{E}_m)_{1,v},\ (\Theta_{ml},\mathfrak{E}_{ml})_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ (\mathcal{Q}_{ml},\mathcal{B}ml)_{u+1,s_l} \end{matrix} \right.\right]. $$

Corollary 4.3. *The integral*

$$ \mathcal{I} = \int_0^{\mathrm{a}} \mathrm{x}^{\mathrm{s}+\frac{1}{2}}(\mathrm{a}-\mathrm{x})^{\mathrm{s}}\mathcal{C}\left(\mathrm{b}\sqrt[4]{\mathrm{x}(\mathrm{a}-\mathrm{x})}\right) S^U_V\left[\varrho \mathrm{x}^{\mathrm{B}}(\mathrm{a}-\mathrm{x})^{\mathrm{B}}\right] \aleph^{u,v}_{r_l,\, s_l,\, f_l;\, \wp}\left[\varrho \mathrm{x}^{\mathrm{A}}(\mathrm{a}-\mathrm{x})^{\mathrm{A}}\right]dx $$

$$ = 2^{-2\mathrm{s}-\frac{3}{4}}\,\mathrm{a}^{2\mathrm{s}+\frac{7}{4}}\,\mathrm{b}^{\frac{1}{2}} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R}\left[\varrho(2\mathrm{a})^{2\mathrm{B}}\right]^R \sum_{n=0}^{\infty} \frac{\left(\frac{1}{4}\right)_n}{\left(\frac{1}{2}\right)_n\left(\frac{5}{4}\right)_n}\left(-\frac{\mathrm{ab}^2}{8}\right)^n $$

$$ \times\, I^{u,\, v+1}_{r_l+1,\, s_l+1;\, \wp}\left[\varrho\left(\frac{\mathrm{a}}{2}\right)^{2\mathrm{A}} \left| \begin{matrix} \mathfrak{T}_1,\ (\Theta_m,\mathfrak{E}_m)_{1,v},(\Theta_{ml},\mathfrak{E}_{ml})_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ (\mathcal{Q}_{ml},\mathcal{B}ml)_{u+1,s_l},\ \mathfrak{T}_2 \end{matrix} \right.\right], \tag{4.3} $$

where, $\mathfrak{T}_1 = \left(-2\mathrm{s}-2\mathrm{AR}-\frac{5}{4}-n;2\mathrm{A}\right)$ *and* $\mathfrak{T}_2 = \left(-2\mathrm{s}-2\mathrm{AR}-\frac{7}{4}-n;2\mathrm{A}\right)$.

(iv) Incomplete H-Function: Further setting $f_l = 1$ and $\wp = 1$ in (2.6) and (2.7), then it becomes to the incomplete H-function suggested by Srivastava [7]:

$$ {}^{\gamma}\aleph^{u,v}_{r_l,\, s_l,\, 1;\, 1}\left[\varrho \left| \begin{matrix} (\Theta_1,\mathfrak{E}_1:z),(\Theta_m,\mathfrak{E}_m)_{2,v},\ [1(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l} \\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ [1(\mathcal{Q}_{ml},\mathcal{B}ml)]_{u+1,s_l} \end{matrix} \right.\right] $$

$$ = {}^{\gamma}H^{u,v}_{r,\, s}\left[\varrho \left| \begin{matrix} (\Theta_1,\mathfrak{E}_1:z),(\Theta_m,\mathfrak{E}_m)_{2,v} \\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u} \end{matrix} \right.\right], $$

and

$$ {}^{\Gamma}\aleph^{u,v}_{r_l,\,s_l,\,1;\,1}\left[\varrho\left|\begin{matrix}(\Theta_1,\mathfrak{E}_1:z),(\Theta_m,\mathfrak{E}_m)_{2,v},\ [1(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l}\\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ [1(\mathcal{Q}_{ml},\mathcal{B}ml)]_{u+1,s_l}\end{matrix}\right.\right] = {}^{\Gamma}H^{u,v}_{r,\,s}\left[\varrho\left|\begin{matrix}(\Theta_1,\mathfrak{E}_1:z),(\Theta_m,\mathfrak{E}_m)_{2,v}\\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u}\end{matrix}\right.\right]. $$

Corollary 4.4. *The integral*

$$ \mathcal{I}=\int_0^{\mathrm{a}}\mathrm{x}^{\mathrm{s}+\frac12}(\mathrm{a}-\mathrm{x})^{\mathrm{s}}\mathcal{C}\left(\mathrm{b}\sqrt[4]{\mathrm{x}(\mathrm{a}-\mathrm{x})}\right)S_V^U\left[\varrho\mathrm{x}^{\mathrm{B}}(\mathrm{a}-\mathrm{x})^{\mathrm{B}}\right]\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}\left[\varrho\mathrm{x}^{\mathrm{A}}(\mathrm{a}-\mathrm{x})^{\mathrm{A}}\right]dx $$
$$ =2^{-2\mathrm{s}-\frac34}\,\mathrm{a}^{2\mathrm{s}+\frac74}\,\mathrm{b}^{\frac12}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\left[\varrho(2\mathrm{a})^{2\mathrm{B}}\right]^R\sum_{n=0}^{\infty}\frac{\left(\frac14\right)_n}{\left(\frac12\right)_n\left(\frac54\right)_n}\left(-\frac{\mathrm{ab}^2}{8}\right)^n $$
$$ \times\,{}^{\Gamma}H^{u,\,v+1}_{r+1,\,s+1}\left[\varrho\left(\frac{\mathrm{a}}{2}\right)^{2\mathrm{A}}\left|\begin{matrix}(\Theta_1,\mathfrak{E}_1:z),\ \mathfrak{T}_1(\Theta_m,\mathfrak{E}_m)_{2,v}\\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ \mathfrak{T}_2\end{matrix}\right.\right], \tag{4.4} $$

where, $\mathfrak{T}_1=\big(-2\mathrm{s}-2\mathrm{AR}-\frac54-n;2\mathrm{A}\big)$ *and* $\mathfrak{T}_2=\big(-2\mathrm{s}-2\mathrm{AR}-\frac74-n;2\mathrm{A}\big)$.

(v) H-Function: Next, we taking $z=0$, $f_l=1$, and $\wp=1$ in (2.7), then it becomes to the H-function suggested by Srivastava [20]:

$$ {}^{\Gamma}\aleph^{u,v}_{r_l,\,s_l,\,1;\,1}\left[\varrho\left|\begin{matrix}(\Theta_1,\mathfrak{E}_1:0),(\Theta_m,\mathfrak{E}_m)_{2,v},\ [1(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l}\\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ [1(\mathcal{Q}_{ml},\mathcal{B}ml)]_{u+1,s_l}\end{matrix}\right.\right]=H^{u,v}_{r,\,s}\left[\varrho\left|\begin{matrix}(\Theta_m,\mathfrak{E}_m)_{1,v}\\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u}\end{matrix}\right.\right]. $$

Corollary 4.5. *The integral*

$$ \mathcal{I}=\int_0^{\mathrm{a}}\mathrm{x}^{\mathrm{s}+\frac12}(\mathrm{a}-\mathrm{x})^{\mathrm{s}}\mathcal{C}\left(\mathrm{b}\sqrt[4]{\mathrm{x}(\mathrm{a}-\mathrm{x})}\right)S_V^U\left[\varrho\mathrm{x}^{\mathrm{B}}(\mathrm{a}-\mathrm{x})^{\mathrm{B}}\right]\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}\left[\varrho\mathrm{x}^{\mathrm{A}}(\mathrm{a}-\mathrm{x})^{\mathrm{A}}\right]dx $$
$$ =2^{-2\mathrm{s}-\frac34}\,\mathrm{a}^{2\mathrm{s}+\frac74}\,\mathrm{b}^{\frac12}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\left[\varrho(2\mathrm{a})^{2\mathrm{B}}\right]^R\sum_{n=0}^{\infty}\frac{\left(\frac14\right)_n}{\left(\frac12\right)_n\left(\frac54\right)_n}\left(-\frac{\mathrm{ab}^2}{8}\right)^n $$
$$ \times\,H^{u,\,v+1}_{r+1,\,s+1}\left[\varrho\left(\frac{\mathrm{a}}{2}\right)^{2\mathrm{A}}\left|\begin{matrix}\mathfrak{T}_1(\Theta_\iota,\mathfrak{E}_\iota)_{1,v}\\ (\mathcal{Q}_m,\mathcal{B}m)_{1,u},\ \mathfrak{T}_2\end{matrix}\right.\right], \tag{4.5} $$

where, $\mathfrak{T}_1=\big(-2\mathrm{s}-2\mathrm{AR}-\frac54-n;2\mathrm{A}\big)$ *and* $\mathfrak{T}_2=\big(-2\mathrm{s}-2\mathrm{AR}-\frac74-n;2\mathrm{A}\big)$.

(vi) Incomplete Meijer G-Function: Next, we taking $f_l=1,\mathfrak{E}_m=1,\mathcal{B}m=1$, and $\wp=1$ in (2.7), then it becomes to the incomplete G-function suggested by Meijer [21]:

$$ {}^{\Gamma}\aleph^{u,v}_{r_l,\,s_l,\,1;\,1}\left[\varrho\left|\begin{matrix}(\Theta_1,1:z),(\Theta_m,1)_{2,v},\ [1(\Theta_{ml},1)]_{v+1,r_l}\\ (\mathcal{Q}_m,1)_{1,u},\ [(\mathcal{Q}_{ml},1)]_{u+1,s_l}\end{matrix}\right.\right]={}^{\Gamma}G^{u,v}_{r,\,s}\left[\varrho\left|\begin{matrix}(\Theta_1:z),\ (\Theta_m)_{2,v}\\ (\mathcal{Q}_m)_{1,u}\end{matrix}\right.\right]. $$

Corollary 4.6. *The integral*

$$ \mathcal{I}=\int_0^{\mathrm{a}}\mathrm{x}^{\mathrm{s}+\frac12}(\mathrm{a}-\mathrm{x})^{\mathrm{s}}\mathcal{C}\left(\mathrm{b}\sqrt[4]{\mathrm{x}(\mathrm{a}-\mathrm{x})}\right)S_V^U\left[\varrho\mathrm{x}^{\mathrm{B}}(\mathrm{a}-\mathrm{x})^{\mathrm{B}}\right]\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}\left[\varrho\mathrm{x}^{\mathrm{A}}(\mathrm{a}-\mathrm{x})^{\mathrm{A}}\right]dx $$
$$ =2^{-2\mathrm{s}-\frac34}\,\mathrm{a}^{2\mathrm{s}+\frac74}\,\mathrm{b}^{\frac12}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\left[\varrho(2\mathrm{a})^{2\mathrm{B}}\right]^R\sum_{n=0}^{\infty}\frac{\left(\frac14\right)_n}{\left(\frac12\right)_n\left(\frac54\right)_n}\left(-\frac{\mathrm{ab}^2}{8}\right)^n $$
$$ \times\,{}^{\Gamma}G^{u,\,v+1}_{r+1,\,s+1}\left[\varrho\left(\frac{\mathrm{a}}{2}\right)^{2\mathrm{A}}\left|\begin{matrix}(\Theta_1:z),\ \mathfrak{T}_1,(\Theta_\iota)_{2,v}\\ (\mathcal{Q}_m)_{1,u},\ \mathfrak{T}_2\end{matrix}\right.\right], \tag{4.6} $$

where, $\mathfrak{T}_1=\big(-2\mathrm{s}-2\mathrm{AR}-\frac54-n;2\mathrm{A}\big)$ *and* $\mathfrak{T}_2=\big(-2\mathrm{s}-2\mathrm{AR}-\frac74-n;2\mathrm{A}\big)$.

5 Applications

This section addresses a few implications and applications of the aforementioned findings. By appropriately specializing the coefficient $A_{V,R}$, a wide range of existing polynomials can be derived, leading to certain unique cases of the resulting discoveries. The following instances are examined to illustrate this:

Example 5.1.

$$I = \int_0^{\mathrm{a}} \mathrm{x}^{\mathrm{s}+\frac{1}{2}}(\mathrm{a}-\mathrm{x})^{\mathrm{s}}\mathcal{C}\left(\mathrm{b}\sqrt[4]{\mathrm{x}(\mathrm{a}-\mathrm{x})}\right)\,{}^{\Gamma}\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}\left[\mathrm{z}\mathrm{x}^{\mathrm{A}}(\mathrm{a}-\mathrm{x})^{\mathrm{A}}\right]dx. \tag{5.1}$$

Solution: If we set $U=1, \mathrm{B}=0, \varrho=1, A_{V,0}=1$, and $A_{V,R}=0\ \forall\ R\neq 0$ (i.e $S_V^U\left[\varrho \mathrm{x}^{\mathrm{B}}(\mathrm{a}-\mathrm{x})^{\mathrm{B}}\right]=1$) in Equation (3.2), then solution of the Eq. (5.1) is

$$I = 2^{-2\mathrm{s}-\frac{3}{4}}\,\mathrm{a}^{2\mathrm{s}+\frac{7}{4}}\,\mathrm{b}^{\frac{1}{2}}\sum_{n=0}^{\infty}\frac{\left(\frac{1}{4}\right)_n}{\left(\frac{1}{2}\right)_n\left(\frac{5}{4}\right)_n}\left(-\frac{\mathrm{ab}^2}{8}\right)^n$$
$$\times\,{}^{\Gamma}\aleph^{u,\,v+1}_{r_l+1,\,s_l+1,\,f_l;\,\wp}\left[\mathrm{z}\left(\frac{\mathrm{a}}{2}\right)^{2\mathrm{A}}\,\middle|\,\begin{matrix}(\Theta_1,\mathfrak{E}_1:z),\mathfrak{T}_1,(\Theta_m,\mathfrak{E}_m)_{2,v},\,[f_m(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l}\\(\mathcal{Q}_m,\mathcal{B}_m)_{1,u},\,[f_m(\mathcal{Q}_{ml},\mathcal{B}_m)]_{u+1,s_l},\mathfrak{T}_2\end{matrix}\right]. \tag{5.2}$$

Where, $\mathfrak{T}_1=\left(-2\mathrm{s}-\frac{5}{4}-n;2\mathrm{A}\right)$ and $\mathfrak{T}_2=\left(-2\mathrm{s}-\frac{7}{4}-n;2\mathrm{A}\right)$ provided $\Re(\mathrm{s})>-\frac{9}{8}, \Re(\mathrm{s})+\mathrm{A}\min_{1\le m\le u}\left(\frac{\mathcal{Q}_m}{\mathcal{B}_m}\right)>-\frac{9}{8}$, $\delta_l>0$, $|arg(\varrho)|<\frac{\pi}{2}\delta_l, l=1,\cdots,\wp$, or $\delta_l\ge 0$, $|arg(\varrho)|<\frac{\pi}{2}\delta_l$, and $\Re(\Delta_l)+1>0$, δ_l and Δ_l is defined by (2.10) and (2.11) respectively, $\mathrm{A},\mathrm{B}>0$ and $\mathrm{a}>0$.

Example 5.2.

$$I = \int_0^{\mathrm{a}} \mathrm{x}^{\mathrm{s}+\frac{1}{2}}(\mathrm{a}-\mathrm{x})^{\mathrm{s}}\mathcal{C}\left(\mathrm{b}\sqrt[4]{\mathrm{x}(\mathrm{a}-\mathrm{x})}\right)H_V\left[\mathrm{x}(\mathrm{a}-\mathrm{x})\right]\,{}^{\Gamma}\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}\left[\mathrm{z}\mathrm{x}^{\mathrm{A}}(\mathrm{a}-\mathrm{x})^{\mathrm{A}}\right]dx. \tag{5.3}$$

Solution: If we set $\mathrm{B}=1, \varrho=1, A_{V,R}=(-1)^R$, and $U=2$ (i.e $S_V^2[t]\to t^{V/2}H_V\left(\frac{1}{2\sqrt{t}}\right)$, where $H_V(t)$ is Hermite polynomial) and making use of the connection, that is (see [14]):

$$H_V(\mathrm{x}) = \sum_{R=0}^{[V/2]}(-1)^R\frac{V!}{R!(V-2R)!}(2\mathrm{x})^{V-2R}, \tag{5.4}$$

in Equation (3.2), then solution of the Eq. (5.3) is

$$I = 2^{-2\mathrm{s}-\frac{3}{4}}\,\mathrm{a}^{2\mathrm{s}+\frac{7}{4}}\,\mathrm{b}^{\frac{1}{2}}\sum_{R=0}^{[V/2]}(-1)^R\frac{V!}{R!(V-2R)!}\left[8\mathrm{a}^2\right]^{V-2R}\sum_{n=0}^{\infty}\frac{\left(\frac{1}{4}\right)_n}{\left(\frac{1}{2}\right)_n\left(\frac{5}{4}\right)_n}\left(-\frac{\mathrm{ab}^2}{8}\right)^n$$
$$\times{}^{\Gamma}\aleph^{u,\,v+1}_{r_l+1,\,s_l+1,\,f_l;\,\wp}\left[\mathrm{z}\left(\frac{\mathrm{a}}{2}\right)^{2\mathrm{A}}\,\middle|\,\begin{matrix}(\Theta_1,\mathfrak{E}_1:z),\mathfrak{T}_1,(\Theta_m,\mathfrak{E}_m)_{2,v},\,[f_m(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l}\\(\mathcal{Q}_m,\mathcal{B}_m)_{1,u},\,[f_m(\mathcal{Q}_{ml},\mathcal{B}_m)]_{u+1,s_l},\mathfrak{T}_2\end{matrix}\right]. \tag{5.5}$$

Where, $\mathfrak{T}_1=\left(-2\mathrm{s}-2\mathrm{A}V+4\mathrm{A}R-\frac{5}{4}-n;2\mathrm{A}\right)$ and $\mathfrak{T}_2=\left(-2\mathrm{s}-2\mathrm{A}V+4\mathrm{A}R-\frac{7}{4}-n;2\mathrm{A}\right)$ provided $\Re(\mathrm{s})>-\frac{9}{8}, \Re(\mathrm{s})+(\mathrm{A}+1)\min_{1\le m\le u}\left(\frac{\mathcal{Q}_m}{\mathcal{B}_m}\right)>-\frac{9}{8}$, $\delta_l>0$, $|arg(\varrho)|<\frac{\pi}{2}\delta_l, l=1,\cdots,\wp$, or $\delta_l\ge 0$, $|arg(\varrho)|<\frac{\pi}{2}\delta_l$, and $\Re(\Delta_l)+1>0$, δ_l and Δ_l is defined by (2.10) and (2.11) respectively, $\mathrm{A},\mathrm{B}>0$ and $\mathrm{a}>0$.

Example 5.3.

$$I = \int_0^{\mathrm{a}} \mathrm{x}^{\mathrm{s}+\frac{1}{2}}(\mathrm{a}-\mathrm{x})^{\mathrm{s}}\mathcal{C}\left(\mathrm{b}\sqrt[4]{\mathrm{x}(\mathrm{a}-\mathrm{x})}\right)L_V^{\alpha}\left[\mathrm{x}(\mathrm{a}-\mathrm{x})\right]\,{}^{\Gamma}\aleph^{u,v}_{r_l,\,s_l,\,f_l;\,\wp}\left[\mathrm{z}\mathrm{x}^{\mathrm{A}}(\mathrm{a}-\mathrm{x})^{\mathrm{A}}\right]dx. \tag{5.6}$$

Solution: If we set $A_{V,R} = \binom{V+\alpha}{V}\frac{1}{(\alpha+1)_R}$ and $U = 1$ (i.e $S_V^1[t] \to L_V^{(\alpha)}(t)$, where $L_V^{(\alpha)}(t)$ is Laguerre polynomial) and making use of the connection, that is (see [14]).

$$L_V^{\alpha}(\mathrm{x}) = \sum_{R=0}^{V}\binom{V+\alpha}{V-R}\frac{(-\mathrm{x})^R}{R!}, \tag{5.7}$$

in Equation (3.2), then solution of the Eq. (5.6) is

$$I = 2^{-2\mathrm{s}-\frac{3}{4}}\,\mathrm{a}^{2\mathrm{s}+\frac{7}{4}}\,\mathrm{b}^{\frac{1}{2}}\sum_{R=0}^{V}\binom{V+\alpha}{V-R}\frac{(-1)^R}{R!}\left[4\mathrm{a}^2\right]^R\sum_{n=0}^{\infty}\frac{\left(\frac{1}{4}\right)_n}{\left(\frac{1}{2}\right)_n\left(\frac{5}{4}\right)_n}\left(-\frac{\mathrm{ab}^2}{8}\right)^n$$
$$\times{}^{\Gamma}\aleph^{u,\,v+1}_{r_l+1,\,s_l+1,\,f_l;\,\wp}\left[\mathrm{z}\left(\frac{\mathrm{a}}{2}\right)^{2\mathrm{A}}\,\middle|\,\begin{matrix}(\Theta_1,\mathfrak{E}_1:z),\mathfrak{T}_1,(\Theta_m,\mathfrak{E}_m)_{2,v},\,[f_m(\Theta_{ml},\mathfrak{E}_{ml})]_{v+1,r_l}\\(\mathcal{Q}_m,\mathcal{B}_m)_{1,u},\,[f_m(\mathcal{Q}_{ml},\mathcal{B}_m)]_{u+1,s_l},\mathfrak{T}_2\end{matrix}\right]. \tag{5.8}$$

Where, $\mathfrak{T}_1 = \left(-2\mathrm{s}-2\mathrm{A}R-\frac{5}{4}-n;2\mathrm{A}\right)$ and $\mathfrak{T}_2 = \left(-2\mathrm{s}-2\mathrm{A}R-\frac{7}{4}-n;2\mathrm{A}\right)$ provided $\Re(\mathrm{s}) > -\frac{9}{8}, \Re(\mathrm{s}) + (\mathrm{A}+1)\min_{1\le m\le u}\left(\frac{\mathcal{Q}_m}{\mathcal{B}_m}\right) > -\frac{9}{8}$, $\delta_l > 0$, $|arg(\varrho)| < \frac{\pi}{2}\delta_l, l = 1,\cdots,\wp$, or $\delta_l \ge 0$, $|arg(\varrho)| < \frac{\pi}{2}\delta_l$, and $\Re(\Delta_l) + 1 > 0$, δ_l and Δ_l is defined by (2.10) and (2.11) respectively, $\mathrm{A},\mathrm{B} > 0$ and $\mathrm{a} > 0$.

Remark: Similar to the above example, we can obtain assertions following Theorem 3.3, Corollary 4.1-4.6, respectively.

6 Conclusions

The conclusions of our study underscore the extensive significance and applicability of the explored functions and integrals. By integrating the Fresnel integral with the product of the incomplete ℵ-function and the Srivastava polynomial, we have established a unified framework capable of addressing complex mathematical problems. This approach facilitates the derivation of numerous novel results, encompassing a broad spectrum of special functions such as the I-function, H-function, Meijer's G-function, E-function, and hypergeometric functions. Furthermore, by specializing the parameters of the Fresnel integrals, we generate a diverse range of finite integrals involving the incomplete ℵ- functions. These findings reveal the versatility and profound utility of the derived functions, highlighting their potential applications across advanced mathematics, science, and engineering.

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References

[1] D. Baleanu, S. Rezapour, and Z. Saberpour, *On fractional integro-differential inclusions via the extended fractional Caputo–Fabrizio derivation*, Bound. Value Probl., **79(1)**, 2019.

[2] S.D. Purohit, D. Baleanu, and K. Jangid, *On the solutions for generalised multiorder fractional partial differential equations arising in physics*, Math Meth Appl Sci., **46**, 8139–8147, 2023.

[3] H.M. Srivastava and R.K. Saxena, *Some volterra-type fractional integro-differential equations with a multivariable confluent hypergeometric function as their kernel*, J. Integral Equ. Appl., 199–217, 2005.

[4] N. Südland, B. Baumann, and T.F. Nonnenmacher, *Who knows about the aleph ℵ-function*, Fract. Calc. Appl. Anal, **1(4)**, 401–402, 1998.

[5] N. Südland, B. Baumann, and T.F. Nonnenmacher, *Fractional driftless fokker-planck equation with power law diffusion coefficients*, In Computer Algebra in Scientific Computing CASC, 513–528, Springer, 2001.

[6] M.K. Bansal, D. Kumar, K.S. Nisar, and J. Singh, *Certain fractional calculus and integral transform results of incomplete ℵ-functions with applications*, Math. Meth. Appl. Sci., **43(8)**, 5602–5614, 2020.

[7] H.M. Srivastava, R.K. Saxena, and R.K. Parmar, *Some families of the incomplete H-functions and the incomplete $\bar{H}$-functions and associated integral transforms and operators of fractional calculus with applications*, Russ. J. Math. Phys., **25(1)**, 116–138, 2018.

[8] K. Jangid, S. Bhatter, S. Meena, and S.D. Purohit, *Certain classes of the incomplete I-functions and their properties*, Discontin. Nonlinearity Complex., **12(2)**, 437–454, 2023.

[9] H.M. Srivastava, M.A. Chaudhry, and R.P. Agarwal, *The incomplete pochhammer symbols and their applications to hypergeometric and related functions*, Integral Transforms and Special Functions, **23(9)**, 659–683, 2012.

[10] M.K. Bansal, D. Kumar, J. Singh, and K.S. Nisar, *On the solutions of a class of integral equations pertaining to incomplete H-function and incomplete $\bar{H}$-function*, Mathematics, **8(5)**, 819, 2020.

[11] D.L. Suthar, S.D. Purohit, and S. Araci, *Solution of fractional kinetic equations associated with the-Mathieu-type series*, Discrete Dyn. Nat. Soc., **2020**, 864516, 2020.

[12] M.A. Chaudhry and S.M. Zubair, On a class of incomplete gamma functions with applications Chapman and Hall/CRC, 2001.

[13] D.L. Suthar, H. Habenom, and H. Tadesse, *Certain integrals involving aleph function and wright's generalized hypergeometric function*, Acta Universitatis Apulensis, **52**, 1–10, 2017.

[14] H.M. Srivastava and N. Singh, *The integration of certain products of the multivariable H-function with a general class of polynomials*, Rend. Circ. Mat. Palermo, **32(2)**, 157–187, 1983.

[15] M. Abramowitz and I.A. Stegun, Handbook of mathematical functions with formulas, graphs, and mathematical tables, Volume 55. US Government printing office, New York. 1948.

[16] Y.A. Brychkov, Handbook of special functions: derivatives, integrals, series and other formulas, Chapman and Hall/CRC, 2008.

[17] L.J. Slater, Generalized hypergeometric functions, Cambridge University Press, 1966.

[18] M.K. Bansal and D. Kumar, *On the integral operators pertaining to a family of incomplete I-functions*, AIMS Math., **5(2)**, 1247–1259, 2020.

[19] V.P. Saxena, *Formal solution of certain new pair of dual integral equations involving H-functions*, Proc. Nat. Acad. Sci. India Sect. A, **52**, 366–375, 1982.

[20] H.M. Srivastava, K.C. Gupta, and S.P. Goyal, The H-functions of one and two variables, with applications, South Asian Publishers, 1982.

[21] C.S. Meijer, *On the G-function*, In Nederl. Akad. Wetensch. Proc., Volume 49, Page 234, 1946.

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