# Study of fractional mathematical model for corruption in Nigeria using Caputo-Fabrizio derivative

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**Abstract** Corruption is a pervasive issue in Nigeria, gradually eroding societal structures. It is a global challenge to which individuals within any community may be exposed. This paper develops a dynamic model of corruption, dividing the population into six compartments: Susceptible, Exposed, Corrupt, Honest, Punished, and Recovered. The main objective is to provide an in-depth analysis of the transmission and control of corruption under the Caputo-Fabrizio derivative, with both qualitative and semi-analytical investigations. The Picard successive approximation method and Banach's fixed point theorem are employed to ensure the model's existence and stability. Numerical computations were carried out using the three-step Adams-Bashforth method, with the solutions graphically simulated in MATLAB. The paper concludes with observations and recommendations aimed at fostering a corruption-free society.

## 1 Introduction

The definition of corruption as stated by [30] and [31] Corruption is an illegal activity carried out for personal gain in a number of fields, such as law enforcement, security, public service, oil, and elections. According to Olujobi (2023) in [32], a number of reasons, including insufficient funding, a lack of political will on the part of the government to combat corruption, and a lackluster use of information and communication technology (ICT), can be blamed for the failure of anti-corruption organizations in Nigeria. The Nigerian Corruption Index as of September 2020 ranked 149 out of 180 countries with a score of 25 percent out of 100 percent , indicating high levels of perceived corruption in the public sector, according to Olujobi (2023) in [32]. It has been discovered that the effects of corruption in Nigeria not only undermine democracy, promote inequality, and exacerbate poverty and insecurity, but also have an impact on the social fabric of society and the government [32]. The delivery of essential services to Nigerian citizens has been severely impacted by corruption, particularly in the fields of health and education. Corruption frequently results in the misallocation and underestimation of public monies meant to enhance these vital sectors. Citizens in suffer from inadequate service delivery and restricted access to basic supplies as a result. Even now, though, the supply has not much decreased; villages are beginning to get it thanks to payments made by the Nigerian government. Consequently, in [33], they conducted a comparative research of legislative frameworks and real-world results in order to investigate and assess the efficacy of anti-corruption measures in Nigeria. The results imply that operational results may have been more successful for a number of reasons, even in the face of robust legislative frameworks to combat corruption.

Mathematical modeling facilitates precise definition and analysis of real-world events by employing various mathematical approaches, including fractional calculus. Its significance has grown notably within mathematical science, allowing us to transform the real-world issues into particularly equations, mathematical language, and then use appropriate approaches to predict results. Modeling has a wide range of applications, providing as a tool for anticipating or estimating future ramifications, making it an essential approach in many different fields [1, 2]. Over the past decades, several fractional operators, including Riemann-Liouville (RL), Hadamard [3, 4], Caputo [5, 6, 28], Caputo-Fabrizio (CF) [7, 27, 29], and Atangana-Baleanu, have been sug-

gested to enhance the understanding of model dynamics. Each operator has its own advantages and disadvantages. For example, the Caputo fractional operator utilizes initial conditions with integer-order derivatives that have clear physical meaning, but it exhibits singularities at certain points. To address this limitation, Caputo and Fabrizio ([8], [9]) recently proposed a unique fractional derivative operator.

Gambo et. al. [34] investigated the following model of Corruption as

$$\begin{cases} \dfrac{dS}{dt} = \Theta - \sigma\beta SC + (1-\kappa)\rho R - (\epsilon + k)S, \\[2mm] \dfrac{dL}{dt} = \sigma\beta SC - (\epsilon + \gamma)L, \\[2mm] \dfrac{dC}{dt} = \alpha\gamma L - (\epsilon + \pi + \varsigma)C, \\[2mm] \dfrac{dP}{dt} = \varsigma C - (\epsilon + \tau)P, \\[2mm] \dfrac{dH}{dt} = kS + \kappa\rho R - \epsilon H, \\[2mm] \dfrac{dR}{dt} = (1-\alpha)\gamma L + \pi C + \tau P - (\epsilon + \kappa\rho)R. \end{cases} \qquad (1.1)$$

with initial conditions

$$S(0) = S_0, \qquad (1.2)$$

$$L(0) = L_0, \qquad (1.3)$$

$$C(0) = C_0, \qquad (1.4)$$

$$P(0) = P_0. \qquad (1.5)$$

$$H(0) = H_0. \qquad (1.6)$$

and

$$R(0) = R_0. \qquad (1.7)$$

In this paper, we develop a dynamic model of corruption using the Caputo-Fabrizio derivative, dividing the population into six compartments: Susceptible, Exposed, Corrupt, Honest, Punished, and Recovered. The main objective is to provide an in-depth analysis of the transmission and control of corruption under the Caputo-Fabrizio derivative, with both qualitative and semi-analytical investigations. The Picard successive approximation method and Banach's fixed point theorem are employed to ensure the model's existence and stability. Numerical computations were carried out using the ILTM, with the solutions graphically simulated in MATLAB21a.

## 2  Fractional calculus

Fractional calculus stands as a prized asset within mathematics and offers exceptional significance in the realm of mathematical modeling. Its capacity to characterize natural phenomena exceeds that of classical calculus, providing more precise descriptions. Moreover, it proves highly advantageous for delineating nonlinear phenomena across various scientific and technological domains. Currently, numerous scholars harness this potent tool, enhancing the quality and depth of their research endeavors. In medical science, fractional calculus finds extensive application in the study of fractional-order disease models, like coronavirus dynamics [10, 11, 12, 13, 14], HIV [15], HBV [16, 17, 18, 19, 20, 21, 26] and serval others [22, 23, 24, 25]. Here, we present some useful definitions of fractional calculus that aid in our current research.

**Definition 2.1.** The Caputo derivative for $0 < \varpi < 1$, is defined as:

$$D_\xi^\varpi \varkappa(\xi) = \frac{1}{\Gamma(p-\varpi)} \int_0^\xi (\xi - \Psi)^{p-\varpi-1} \varkappa(\Psi)\, d\Psi, \ \xi > 0, \ p - 1 < \varpi \le p, \ p \in Z^+. \quad (2.1)$$

| Symbol | Description |
|--------|-------------|
| $S$ | Susceptible individuals |
| $L$ | Exposed individuals |
| $C$ | Corruption |
| $P$ | Punished individuals |
| $H$ | Honest individuals |
| $R$ | Recovered individuals |
| $\Theta$ | Recruitment rate of population |
| $\beta$ | Contact rate of individuals per contact |
| $\tau$ | Rate of punished individuals |
| $\sigma$ | Transmission rate per contact |
| $\gamma$ | Rate at which exposed individuals get corrupted |
| $\kappa$ | Proportion of individuals leaving the recovered group to enter the honest group |
| $\epsilon$ | Natural mortality rate for all individuals |
| $\pi$ | Rate at which corrupted individuals become punished |
| $k$ | Proportion of individuals joining the honest population |
| $\varsigma$ | Rate at which corrupted individuals move to the punished group |
| $\rho$ | Rate at which recovered individuals become honest |
| $\alpha$ | Rate of individuals leaving the exposed group to enter the corrupted group |

**Table 1.** Description of variables and parameters

**Definition 2.2.** Let $\varkappa \in G^1(a_1, b_1) \ b_1 > a_1$ and $0 < \varpi < 1$, then CF derivative ([8], [9]) is defined as:

$$^{CF}D_\xi^\varpi \varkappa(\xi) = \frac{1}{(1-\varpi)} \int_{a_1}^{\xi} \varkappa'(\Psi) \exp\left[-\varpi \frac{\xi-\Psi}{1-\varpi}\right] d\Psi, \tag{2.2}$$

here, $G^1(a_1, b_1) = \{\varkappa \mid \varkappa \in L^2(a_1, b_1) \ and \ \varkappa' \in L^2(a_1, b_1)\}$ and $L^2(a_1, b_1)$ is the space of square integrable functions on the interval $(a_1, b_1)$.

**Definition 2.3.** The CF fractional integral operator of order $0 < \varpi < 1$, is defined as:

$$^{CF}J_\xi^\varpi \varkappa(\xi) = \frac{2(1-\varpi)}{(2-\varpi)M(\varpi)} \varkappa(\xi) + \frac{2\varpi}{(2-\varpi)M(\varpi)} \int_0^\xi \varkappa(\Psi) \, d\Psi, \ \xi \geq 0, \tag{2.3}$$

where $^{CF}D_\xi^\varpi \varkappa(\xi) = 0$, if $\varkappa$ is a constant function.

**Definition 2.4.** The Laplace transform (LT) of the CF derivative of order $0 < \varpi \leq 1$ for $m \in \mathbb{N}$ is given by:

$$L\left(^{CF}D_\xi^{m+\varpi}\varkappa(\xi)\right)(q) = \frac{1}{1-\varpi}L\left(\varkappa^{(m+1)}(\xi)\right) L\left(exp\left(-\frac{\varpi}{1-\varpi}\xi\right)\right)$$

$$= \frac{q^{m+1}L\left(\varkappa(\xi)\right) - q^m \varkappa(0) - q^{m-1}\varkappa'(0) - .... - \varkappa^{(m)}(0)}{q + \varpi(1-q)} \tag{2.4}$$

In particular, we have

$$L\left(^{CF}D_\xi^\varpi \varkappa(\xi)\right)(q) = \frac{qL\left(\varkappa(\xi)\right) - \varkappa(0)}{q + \varpi(1-q)}, \ m = 0,$$

$$L\left(^{CF}D_\xi^{\varpi+1}\varkappa(\xi)\right)(q) = \frac{q^2 L\left(\varkappa(\xi)\right) - q\varkappa(0) - \varkappa'(0)}{q + \varpi(1-q)}, \ m = 1.$$

## 3 The fractionalized Model of Corruption

The Corruption model is rearranged using the CF fractional derivative operator, then we obtain following system

$$
\begin{cases}
{}^{CF}D_\xi^\varpi S\left(\xi\right) = \Theta - \sigma\beta SC + (1-\kappa)\rho R - (\epsilon + k)S, \\
{}^{CF}D_\xi^\varpi L\left(\xi\right) = \sigma\beta SC - (\epsilon + \gamma)L, \\
{}^{CF}D_\xi^\varpi C\left(\xi\right) = \alpha\gamma L - (\epsilon + \pi + \varsigma)C, \\
{}^{CF}D_\xi^\varpi P\left(\xi\right) = \varsigma C - (\epsilon + \tau)P, \\
{}^{CF}D_\xi^\varpi H\left(\xi\right) = kS + \kappa\rho R - \epsilon H, \\
{}^{CF}D_\xi^\varpi R\left(\xi\right) = (1-\alpha)\gamma L + \pi C + \tau P - (\epsilon + \kappa\rho)R.
\end{cases}
\tag{3.1}
$$

with initial conditions

$$S\left(0\right) = S_0, \tag{3.2}$$

$$L\left(0\right) = L_0, \tag{3.3}$$

$$C\left(0\right) = C_0, \tag{3.4}$$

$$P\left(0\right) = P_0. \tag{3.5}$$

$$H\left(0\right) = H_0. \tag{3.6}$$

and

$$R\left(0\right) = R_0. \tag{3.7}$$

## 4 Iterative scheme and stability analysis of the Caputo-Fabrizio fractional Corruption Model

### 4.1 Iterative scheme:

By applying the LT to both sides of the system given by the corruption model (3.1) along with the initial conditions (3.2)-(3.7), we can observe that the terms $SC$ in this model exhibit non-linearity.

$$
\frac{qL\left(S(\xi)\right) - S(0)}{q + \varpi(1-q)} = L\Big(\Theta - \sigma\beta S\,C + (1-\kappa)\rho R - (\epsilon + k)\,S\Big),
$$

$$
\frac{qL\left(L(\xi)\right) - L(0)}{q + \varpi(1-q)} = L\Big(\sigma\beta SC - (\epsilon + \gamma)L\Big),
$$

$$
\frac{qL\left(C(\xi)\right) - C(0)}{q + \varpi(1-q)} = L\Big(\alpha\gamma L - (\epsilon + \pi + \varsigma)C\Big),
$$

$$
\frac{qL\left(P(\xi)\right) - P(0)}{q + \varpi(1-q)} = L\Big(\varsigma C - (\epsilon + \tau)P\Big)
$$

$$
\frac{qL\left(H(\xi)\right) - H(0)}{q + \varpi(1-q)} = L\Big(kS + \kappa\rho R - \epsilon H\Big),
$$

$$
\frac{qL\left(R(\xi)\right) - R(0)}{q + \varpi(1-q)} = L\Big((1-\alpha)\gamma L + \pi C + \tau P - (\epsilon + \kappa\rho)R\Big).
$$

Rearranging, we get

$$L\left(S(\xi)\right) = \frac{S(0)}{q} + \left(\frac{q + \varpi(1-q)}{q}\right) L\Big(\Theta - \sigma\beta SC + (1-\kappa)\rho R - (\epsilon+k)S\Big),$$

$$L\left(L(\xi)\right) = \frac{L(0)}{q} + \left(\frac{q + \varpi(1-q)}{q}\right) L\Big(\sigma\beta SC - (\epsilon+\gamma)L\Big),$$

$$L\left(C(\xi)\right) = \frac{C(0)}{q} + \left(\frac{q + \varpi(1-q)}{q}\right) L\Big(\alpha\gamma L - (\epsilon+\pi+\varsigma)C\Big),$$

$$L\left(P(\xi)\right) = \frac{P(0)}{q} + \left(\frac{q + \varpi(1-q)}{q}\right) L\Big(\varsigma C - (\epsilon+\tau)P\Big),$$

$$L\left(H(\xi)\right) = \frac{H(0)}{q} + \left(\frac{q + \varpi(1-q)}{q}\right) L\Big(kS + \kappa\rho R - \epsilon H\Big),$$

$$L\left(R(\xi)\right) = \frac{R(0)}{q} + \left(\frac{q + \varpi(1-q)}{q}\right) L\Big((1-\alpha)\gamma L + \pi C + \tau P - (\epsilon+\kappa\rho)R\Big). \quad (4.1)$$

Further, the inverse LT of equations (4.1) yields

$$S(\xi) = S(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right) L\Big(\Theta - \sigma\beta SC + (1-\kappa)\rho R - (\epsilon+k)S\Big)\right],$$

$$L(\xi) = L(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right) L\Big(\sigma\beta SC - (\epsilon+\gamma)L\Big)\right],$$

$$C(\xi) = C(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right) L\Big(\alpha\gamma L - (\epsilon+\pi+\varsigma)C\Big)\right],$$

$$P(\xi) = P(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right) L\Big(\varsigma C - (\epsilon+\tau)P\Big)\right],$$

$$H(\xi) = H(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right) L\Big(kS + \kappa\rho R - \epsilon H\Big)\right],$$

$$R(\xi) = R(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right) L\Big((1-\alpha)\gamma L + \pi C + \tau P - (\epsilon+\kappa\rho)R\Big)\right]. \quad (4.2)$$

The series solutions achieved by the method are given by

$$S = \sum_{\tau=0}^{\infty} S_\tau \qquad L = \sum_{\tau=0}^{\infty} L_\tau \qquad C = \sum_{\tau=0}^{\infty} C_\tau,$$

$$P = \sum_{\tau=0}^{\infty} P_\tau \qquad H = \sum_{\tau=0}^{\infty} H_\tau \qquad R = \sum_{\tau=0}^{\infty} R_\tau. \quad (4.3)$$

The nonlinear term *SI* can be written as

$$SC = \sum_{\tau=0}^{\infty} G_\tau,$$

where $G_\tau$ is further decomposed as follows:

$$G_\tau = \sum_{r=0}^{\tau} S_r \sum_{r=0}^{\tau} C_r - \sum_{r=0}^{\tau-1} S_r \sum_{r=0}^{\tau-1} C_r.$$

We get the following recursive relation by using initial conditions

$$S_{n+1}(\xi) = S_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\left(\Theta - \sigma\beta S_n C_n + (1-\kappa)\rho R_n - (\epsilon+k)S_n\right)\right],$$

$$L_{n+1}(\xi) = L_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\left(\sigma\beta S_n C_n - (\epsilon+\gamma)L_n\right)\right],$$

$$C_{n+1}(\xi) = C_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\left(\alpha\gamma L_n - (\epsilon+\pi+\varsigma)C_n\right)\right],$$

$$P_{n+1}(\xi) = P_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\left(\varsigma C_n - (\epsilon+\tau)P_n\right)\right],$$

$$H_{n+1}(\xi) = H_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\left(kS_n + \kappa\rho R_n - \epsilon H_n\right)\right],$$

$$R_{n+1}(\xi) = R_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\left((1-\alpha)\gamma L_n + \pi C_n + \tau P_n - (\epsilon+\kappa\rho)R_n\right)\right].$$

$$(4.4)$$

## 4.2 Stability Analysis

Let Banach space $(B, ||.||)$ be an endomorphism $\Lambda$ on $B$. The recurrence formula $\zeta_{n+1} = p(\Lambda, \zeta_n)$ denotes an exact recurrence. The set of fixed points of $\Lambda$ is represented by $U(\Lambda)$. Moreover, $\Lambda$ has minimum one element $\zeta_n$ that converges to a point $x \in U(\Lambda)$. Let $\{\nu_n\} \in B$ and consider $\sigma_n = ||\nu_{n+1} - p(\Lambda, \nu_n)||$. If $\lim_{n\to\infty} \sigma_n = 0$ implies $\lim_{n\to\infty} \beta_n = x$, then the given iteration method $\zeta_{n+1} = p(\Lambda, \zeta_n)$ is known as $\Lambda$-stable. Thus, the sequence $\nu_n$ is bounded from above, and the process is recognized as Picard's iteration. Additionally, it achieves $\Lambda$-stability when the specified conditions hold true for $\zeta_{n+1} = \Lambda, \zeta_n$.

**Theorem 4.1.** *Let $\Lambda$ be a self-map on Banach space $(B, ||.||)$ that satisfies*

$$\|\Lambda a - \Lambda b\| \leq \Gamma\|a - \Lambda a\| + \epsilon\|a - b\|$$

$\forall\, a, b \in B$, *where $0 \leq \Gamma$ and $0 \leq \epsilon < 1$. Suppose $\Lambda$ is Picard $\Lambda$-stable. Consider equations* (4.4) *related to* (1.1).

**Theorem 4.2.** *Consider a self-map $\Lambda$ defined as*

$$\Lambda\left(S_n(\xi)\right) = S_{n+1}(\xi)$$
$$= S_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\left(\Theta - \sigma\beta S_n C_n + (1-\kappa)\rho R_n - (\epsilon+k)S_n\right)\right],$$

$$\Lambda\left(L_n(\xi)\right) = L_{n+1}(\xi)$$
$$= L_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\left(\sigma\beta S_n C_n - (\epsilon+\gamma)L_n\right)\right],$$

$$\Lambda\left(C_n(\xi)\right) = C_{n+1}(\xi)$$
$$= C_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\left(\alpha\gamma L_n - (\epsilon+\pi+\varsigma)C_n\right)\right],$$

$$\Lambda\left(P_n(\xi)\right) = P_{n+1}(\xi)$$
$$= P_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\left(\varsigma C_n - (\epsilon+\tau)P_n\right)\right],$$

$$\Lambda\left(H_n(\xi)\right) = H_{n+1}(\xi)$$
$$= H_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\left(kS_n + \kappa\rho R_n - \epsilon H_n\right)\right],$$

$$\Lambda\left(R_n(\xi)\right) = R_{n+1}(\xi)$$
$$= R_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\Big((1-\alpha)\gamma L_n + \pi C_n + \tau P_n - (\epsilon + \kappa\rho)R_n\Big)\right].$$
$$(4.5)$$

where Lagrange's multiplier in fractional form is defined as $\frac{q+\tau(1-q)}{q}$. It is $\Lambda$-stable in $L^1(a_1, b_1)$ if

$$\left(1 + \beta F(\varkappa) - \alpha K_1 G(\varkappa) - \alpha K_2 H(\varkappa)\right) < 1,$$

$$\left(1 - (\lambda + \zeta)F_1(\varkappa) + \alpha K_1 G_1(\varkappa) + \alpha K_2 H_1(\varkappa)\right) < 1,$$

$$\left(1 + \zeta F_2(\varkappa) - \beta G_2(\varkappa)\right) < 1,$$

$$\left(1 + \zeta F_3(\varkappa)\right) < 1.$$

*Proof.* The proof starts by showing that $\Lambda$ possesses a fixed point. Hence, for all $((m,n) \in \mathbb{N} \times \mathbb{N})$, we compute the following variances:

$$\Lambda\left(S_m(\xi)\right) - \Lambda\left(S_n(\xi)\right) = S_m(0) - S_n(0)$$
$$+ L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\Big(\Theta - \sigma\beta S_m C_m + (1-\kappa)\rho R_m - (\epsilon + k)S_m\Big)\right]$$
$$- L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\Big(\Theta - \sigma\beta S_n C_n + (1-\kappa)\rho R_n - (\epsilon + k)S_n\Big)\right],$$
$$\Lambda\left(L_m(\xi)\right) - \Lambda\left(L_n(\xi)\right) = L_m(0) - L_n(0)$$
$$+ L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\Big(\sigma\beta S_m C_m - (\epsilon + \gamma)L_m\Big)\right]$$
$$- L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\Big(\sigma\beta S_n C_n - (\epsilon + \gamma)L_n\Big)\right],$$
$$\Lambda\left(C_m(\xi)\right) - \Lambda\left(C_n(\xi)\right) = C_m(0) - C_n(0)$$
$$+ L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\Big(\alpha\gamma L_m - (\epsilon + \pi + \varsigma)C_m\Big)\right]$$
$$- L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\Big(\alpha\gamma L_n - (\epsilon + \pi + \varsigma)C_n\Big)\right],$$
$$\Lambda\left(P_m(\xi)\right) - \Lambda\left(P_n(\xi)\right) = P_m(0) - P_n(0)$$
$$+ L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\Big(\varsigma C_m - (\epsilon + \tau)P_m\Big)\right]$$
$$- L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\Big(\varsigma C_n - (\epsilon + \tau)P_n\Big)\right],$$
$$\Lambda\left(H_m(\xi)\right) - \Lambda\left(H_n(\xi)\right) = H_m(0) - H_n(0)$$
$$+ L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\Big(kS_m + \kappa\rho R_m - \epsilon H_m\Big)\right]$$
$$- L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)L\Big(kS_n + \kappa\rho R_n - \epsilon H_n\Big)\right],$$

$$\Lambda\left(R_m(\xi)\right) - \Lambda\left(R_n(\xi)\right) = R_m(0) - R_n(0)$$

$$+ L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right) L\left((1-\alpha)\gamma L_m + \pi C_m + \tau P_m - (\epsilon + \kappa\rho)R_m\right)\right]$$

$$- L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right) L\left((1-\alpha)\gamma L_n + \pi C_n + \tau P_n - (\epsilon + \kappa\rho)R_n\right)\right]. \quad (4.6)$$

Considering the first equation of (4.6) and applying the norm, we obtain the following expression, without loss of generality:

$$||\Lambda\left(S_m(\xi)\right) - \Lambda\left(S_n(\xi)\right)||$$

$$= \left|\left|S_m(0) - S_n(0) + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right) L\left(\Theta - \sigma\beta S_m C_m + (1-\kappa)\rho R_m - (\epsilon + k)S_m\right)\right]\right.$$

$$\left. - L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right) L\left(\Theta - \sigma\beta S_n C_n + (1-\kappa)\rho R_n - (\epsilon + k)S_n\right)\right]\right|\right|, \quad (4.7)$$

next, utilizing triangle inequality and simplifying (4.7), we get

$$||\Lambda\left(S_m(\xi)\right) - \Lambda\left(S_n(\xi)\right)|| \leq ||S_m(0) - S_n(0)|| + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)\right.$$

$$L\left[||-\sigma\beta S_n(C_m - C_n)|| + ||-\sigma\beta C_m(S_m - S_n)||\right.$$

$$\left.\left. + ||(1-\kappa)(R_m - R_n)|| + ||-(\epsilon + k)(S_m - S_n)||\right]\right]. \quad (4.8)$$

Since both solutions have similar impacts, it is presumed that

$$||S_m(\xi) - S_n(\xi)|| = ||L_m(\xi) - L_n(\xi)||$$
$$||S_m(\xi) - S_n(\xi)|| = ||C_m(\xi) - C_n(\xi)||$$
$$||S_m(\xi) - S_n(\xi)|| = ||P_m(\xi) - P_n(\xi)||$$
$$||S_m(\xi) - S_n(\xi)|| = ||H_m(\xi) - H_n(\xi)||$$
$$||S_m(\xi) - S_n(\xi)|| = ||R_m(\xi) - R_n(\xi)||$$

Substituting this into (4.8), we obtain the following equation:

$$||\Lambda\left(S_m(\xi)\right) - \Lambda\left(S_n(\xi)\right)|| \leq ||S_m(0) - S_n(0)|| + L^{-1}\left[\left(\frac{q + \varpi(1-q)}{q}\right)\right.$$

$$L\left[||-\sigma\beta S_n(S_m - S_n)|| + ||-\sigma\beta C_m(S_m - S_n)||\right.$$

$$\left.\left. + ||(1-\kappa)(S_m - S_n)|| + ||-(\epsilon + k)(S_m - S_n)||\right]\right]. \quad (4.9)$$

Since $S_n$ and $C_m$ are bounded sequences, we have three distinct positive constants, $K_1$, and $K_2$ for every $\xi$, such that

$$||S_n|| \leq K_1, \qquad ||C_m|| \leq K_2, \quad (m, n) \in \mathbb{N} \times \mathbb{N}. \quad (4.10)$$

Furthermore, taking into account equations (4.9) and (4.10), we obtain

$$||\Lambda\left(S_m(\xi)\right) - \Lambda\left(S_n(\xi)\right)||$$

$$\leq \left(1 + (1-\kappa-\epsilon-k)F(\varkappa) - \sigma\beta K_1 G(\varkappa) - \sigma\beta K_2 H(\varkappa)\right)||S_m - S_n||. \quad (4.11)$$

where $F$, $G$, and $H$ are functions of $L^{-1}\left\{L\left(\frac{q+\alpha(1-q)}{q}\right)\right\}$. Similarly, we can obtain

$$||\Lambda(L_m(\xi)) - \Lambda(L_n(\xi))|| \leq \left(1 - (\epsilon + \gamma)F_1(\varkappa) + \sigma\beta\,K_1G_1(\varkappa) + \sigma\beta\,K_2H_1(\varkappa)\right)||L_m - L_n||$$

$$||\Lambda(C_m(\xi)) - \Lambda(C_n(\xi))|| \leq \left(1 + \alpha\gamma\,F_2(\varkappa) - (\epsilon + \pi + \varsigma)\,G_2(\varkappa)\right)||C_m - C_n||$$

$$||\Lambda(P_m(\xi)) - \Lambda(P_n(\xi))|| \leq \left(1 + \varsigma\,F_3(\varkappa) - (\epsilon + \tau)\,G_3\right)||P_m - P_n||$$

$$||\Lambda(H_m(\xi)) - \Lambda(H_n(\xi))|| \leq \left(1 + k\,F_4(\varkappa) + \kappa\rho\,G_4 - \epsilon\,H_4\right)||H_m - H_n||$$

$$||\Lambda(R_m(\xi)) - \Lambda(R_n(\xi))|| \leq \left(1 + (1 - \alpha)\,F_5(\varkappa) + \pi\,G_5 + \tau\,H_5 - (\epsilon + k\,\rho)J_5\right)||R_m - R_n||$$

$$(4.12)$$

where

$$\left(1 + (1 - \kappa - \epsilon - k)\,F(\varkappa) - \sigma\beta\,K_1G(\varkappa) - \sigma\beta\,K_2H(\varkappa)\right) < 1,$$

$$\left(1 - (\epsilon + \gamma)F_1(\varkappa) + \sigma\beta\,K_1G_1(\varkappa) + \sigma\beta\,K_2H_1(\varkappa)\right) < 1,$$

$$\left(1 + \alpha\gamma\,F_2(\varkappa) - (\epsilon + \pi + \varsigma)\,G_2(\varkappa)\right) < 1,$$

$$\left(1 + \varsigma\,F_3(\varkappa) - (\epsilon + \tau)\,G_3\right) < 1,$$

$$\left(1 + k\,F_4(\varkappa) + \kappa\rho\,G_4 - \epsilon\,H_4\right) < 1,$$

$$\left(1 + (1 - \alpha)\,F_5(\varkappa) + \pi\,G_5 + \tau\,H_5 - (\epsilon + k\,\rho)J_5\right) < 1.$$

Consequently, the non-linear self-mapping ($\Lambda$) possesses a fixed point. Subsequently, we will demonstrate that ($\Lambda$) fulfills all the conditions outlined in Theorem 4.1. Given the validity of equations (4.11) and (4.12), we will proceed to show, we use $\varepsilon = (0, 0, 0, 0)$ and

$$\Gamma = \begin{cases} 1 + (1 - \kappa - \epsilon - k)\,F(\varkappa) - \sigma\beta\,K_1G(\varkappa) - \sigma\beta\,K_2H(\varkappa), \\ 1 - (\epsilon + \gamma)F_1(\varkappa) + \sigma\beta\,K_1G_1(\varkappa) + \sigma\beta\,K_2H_1(\varkappa), \\ 1 + \alpha\gamma\,F_2(\varkappa) - (\epsilon + \pi + \varsigma)\,G_2(\varkappa), \\ 1 + \varsigma\,F_3(\varkappa) - (\epsilon + \tau)\,G_3, \\ 1 + k\,F_4(\varkappa) + \kappa\rho\,G_4 - \epsilon\,H_4, \\ 1 + (1 - \alpha)\,F_5(\varkappa) + \pi\,G_5 + \tau\,H_5 - (\epsilon + k\,\rho)J_5. \end{cases}$$

$\square$

## 5   Data Fitting and Numerical discussions

In this section, we analyze the outcomes of numerical simulations conducted for various values of the fractional order and parameter sets listed in Table 2. Figure 1 illustrates the population of susceptible individuals, $S(\xi)$, for different fractional orders. Figures 2 and 3 depict the dynamics of exposed individuals transitioning into corruption for varying values of $\varpi$ over time. The model indicates that exposure to media and societal awareness reduces the prevalence of corruption within the population. Figure 4 demonstrates the rate of punished individuals at different fractional orders, while Figure 5 presents the trend of susceptible individuals choosing honesty.
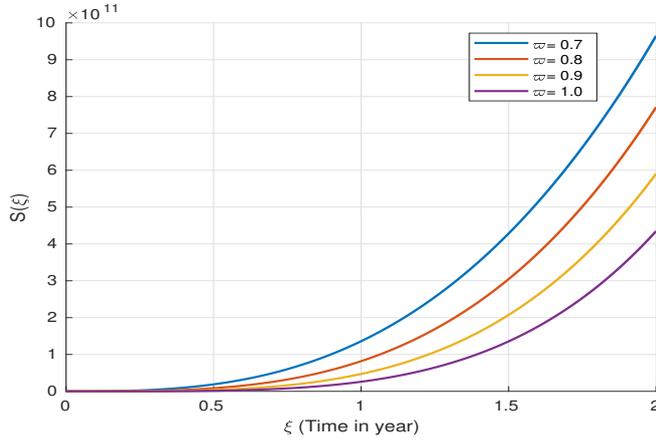
**Figure 1.** Graphical nature of susceptible individuals $S(\xi)$ with respect to $\xi$ at different order of $\varpi$.
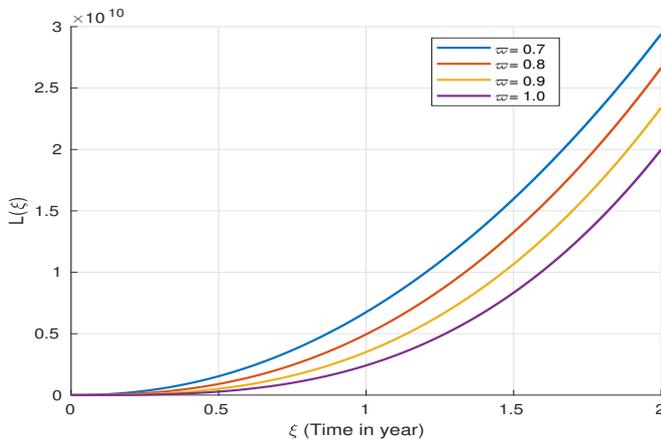


**Figure 2.** Graphical nature of Exposed individuals $L(\xi)$ with respect to $\xi$ at different order of $\varpi$.
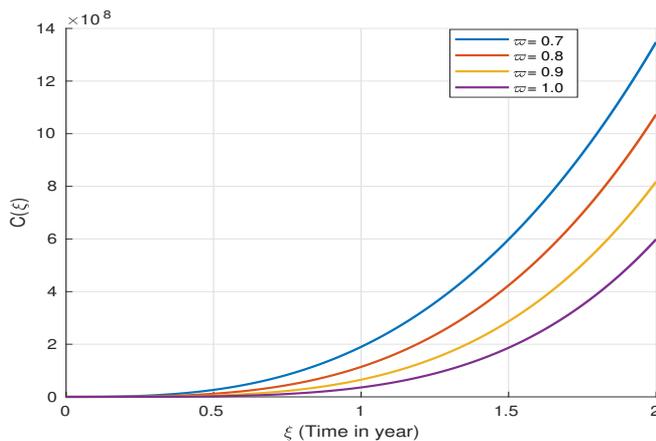


**Figure 3.** Graphical nature of Corrupt individuals $C(\xi)$ with respect to $\xi$ at different order of $\varpi$.

The increase in honest individuals, as shown in Figure 5, highlights the efficacy of information and communication technology in monitoring public officials, thereby reducing corruption. Additionally, Figure 6 shows the trajectory of punished individuals transitioning to recovery classes
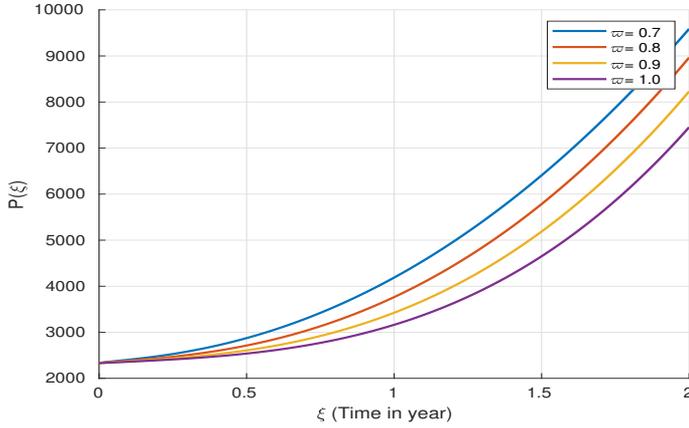
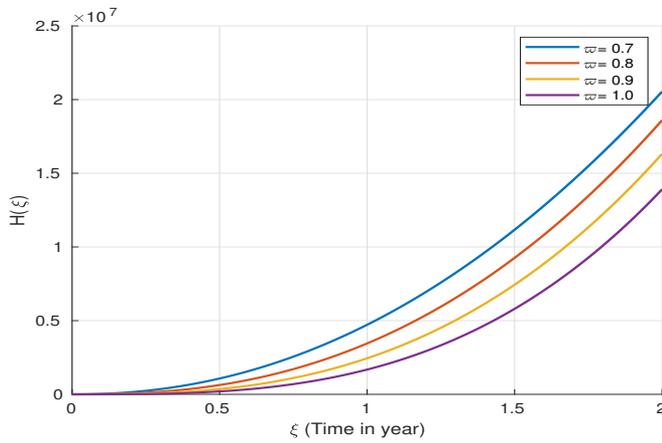**Figure 4.** Graphical nature of Honest Population $H(\xi)$ with respect to $\xi$ at different order of $\varpi$.



**Figure 5.** Graphical nature of Punished Population $P(\xi)$ with respect to $\xi$ at different order of $\varpi$.
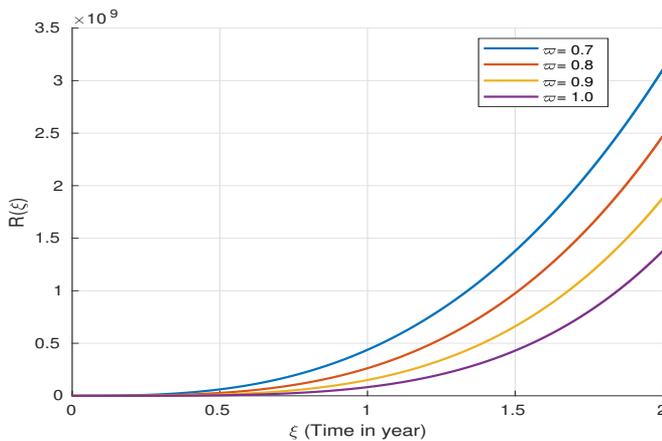


**Figure 6.** Graphical nature of Recovered Population $R(\xi)$ with respect to $\xi$ at different order of $\varpi$.

due to implemented control measures. The graph reveals that an increase in penalized individuals due to corruption-related offenses leads to a corresponding rise in recovered individuals and a simultaneous reduction in corruption levels. Overall, the findings suggest that exposing and

penalizing individuals involved in corruption significantly diminishes such practices. Measures like information and communication technology, media campaigns, public awareness initiatives, and legal penalties contribute to curbing corruption in society. Future research will incorporate optimal control strategies to further enhance these outcomes.

| Variable | Value | Variable | Value |
|---|---|---|---|
| $S_0$ | 200,000 | $\sigma$ | 0.2 |
| $L_0$ | 60,000 | $\gamma$ | 0.007 |
| $C_0$ | 50,000 | $\kappa$ | 0.35 |
| $P_0$ | 2330 | $\epsilon$ | 0.4 |
| $H_0$ | 500 | $\pi$ | 0.006 |
| $R_0$ | 250 | $k$ | 0.03 |
| $\Theta$ | 85 | $\varsigma$ | 0.0160 |
| $\beta$ | 0.036 | $\rho$ | 0.3 |
| $\tau$ | 0.0234 | $\alpha$ | 0.1 |

**Table 2.** Description of variables and parameters

## 6  Conclusions

In this paper, we analyzed fractional corruption model incorporating the Caputo–Fabrizio fractional derivative. Through the application of fixed-point theory, we established the existence and uniqueness of solutions and examined the system's stability. To obtain numerical results for the fractional corruption model, we employed an iterative Laplace transform scheme. Furthermore, numerical simulations were conducted for various fractional-order values. In conclusion, the findings confirm the effectiveness of the proposed method in addressing non-linear terms for semi-analytical results and provide simulations that facilitate the interpretation of these results. .

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