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Certain Generating Function Involving The Incomplete Yang Y-Function

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Abstract In this paper, we have developed a comprehensive set of generating functions for the incomplete Yang Y-functions. These generating functions are derived through various techniques and provide insight into the functional relationships and properties of the incomplete Yang Y-functions. Bilateral and linear generating relations have been established, enabling a deeper understanding of these functions and their applications. The results obtained in this study are generalized, offering a broad spectrum of potential utility in mathematical and applied contexts.

1 Introduction and definitions

Special functions are particular mathematical functions, and the field of special functions is vast. Special functions are widely used in engineering and other fields. The emergence of new challenges in engineering and applied sciences has driven the steady evolution of this extensive area of study. In line with the needs and interests of the academic and scientific communities, the present study explores various aspects of these functions and their potential applications.

In certain situations, working with a continuous function can be significantly simpler than dealing with a series. Therefore, generating functions that are linear, bilinear, and bilateral play a crucial role in analyzing the beneficial properties of the sequences they produce. Generating functions have numerous applications, including their use in recurrence equations, combinatorial problems, and physics. Examples include equations such as Hermite, Legendre, Laguerre, and other types of equations, which are applied in various fields.

Researchers from diverse disciplines have shown considerable interest in the extensive applications of generating functions, particularly in connection with special functions. The literature provides several techniques for generating functions involving special functions (see [1, 2, 3, 4]). Inspired by the work referenced in [5], this article introduces specific generating functions in the form of linear and bilateral functions involving incomplete Yang Y-functions. To establish the main results, we first review some of the fundamental concepts available in the literature.

One of the most studied functions is the Yang Y-function. The H-function was initially developed by Fox, and the Yang Y-function serves as an extension of the H-function. In this study, we derive certain generating functions involving the incomplete Yang Y-function of the Mellin-Barnes type contour [6].

The Yang Y-function [6], recognized as a generalized hypergeometric function, is defined using a contour integral.

$$Y_{r,s}^{p,q}\left[w;v;u\ \middle|\ \begin{matrix}(e_i,E_i)_1^r\\(f_j,F_j)_1^s\end{matrix}\right]=\frac{1}{2\pi i}\int_L\theta_{r,s}^{p,q}(\nu)w^{-\nu}e^{\nu v}\nu^u d\nu, \tag{1.1}$$

where

$$\theta_{r,s}^{p,q}(\nu)=\frac{[\prod_{j=1}^{p}\Gamma(f_j-F_j\nu)]\prod_{i=1}^{q}[\Gamma(1-e_i+E_i\nu)]}{[\prod_{j=p+1}^{s}\Gamma(1-f_j+F_j\nu)]\prod_{i=q+1}^{r}\Gamma(e_i-E_i\nu)}, \tag{1.2}$$

here u,v,w are complex number and a contour L in the complex plane the orders p,q,r,s are nonnegative integers so that $0\le p\le s$, $0\le q\le r$ the parameters $E_m>0, F_n>0$ are positive and $e_i, f_j, i = 1, \cdots, r; j = 1, \cdots, s$ are arbitrary complex which are satisfying the following conditions:

$$E_i(f_j + \varrho) \neq F_j(e_i - \grave{\varrho} - 1), \tag{1.3}$$

$$(\varrho, \grave{\varrho} \in \mathbb{N}_0(0, 1, 2\cdots); i = 1, 2, 3\cdots r; j = 1, 2, 3\cdots s).$$

Our new step here will be to remember about the that familiar incomplete function $\gamma(\rho, \delta)$ and $\Gamma(\rho, \delta)$ are defined as (see [7, 8])

$$\gamma(\rho, \delta) = \int_0^{\delta} k^{\rho-1} e^{-k} dk,\ (\Re(\rho) > 0, \delta \geq 0), \tag{1.4}$$

and

$$\Gamma(\rho, \delta) = \int_{\delta}^{\infty} k^{\rho-1} e^{-k} dk,\ (\Re(\rho) > 0, \delta \geq 0), \tag{1.5}$$

respectively.
Definitions of incomplete gamma function (1.4) and (1.5) easily yield to following decomposition formula:

$$\gamma(\rho, \delta) + \Gamma(\rho, \delta) = \Gamma(\rho),\ (\Re(\rho) > 0). \tag{1.6}$$

2 The incomplete *Y*-functions

Here, In this paper, we present the Yang Y-function corresponding incomplete Y-functions using the incomplete gamma functions in Eqs. (1.4) and (1.5). The incomplete Y-functions are defined as follows:

$$\begin{aligned} {}^{\gamma}Y_{r,s}^{p,q}(w; v; u) &= {}^{\gamma}Y_{r,s}^{p,q}\left[w; v; u \,\middle|\, \begin{matrix} (e_1, E_1, \delta), (e_i, E_i)_2^r \\ (f_j, F_j)_1^s \end{matrix} \right] \\ &= \frac{1}{2\pi i} \int_L {}^{\gamma}\vartheta_{r,s}^{p,q}(\nu, \delta) w^{-\nu} e^{\nu v} \nu^u d\nu, \end{aligned} \tag{2.1}$$

where

$${}^{\gamma}\vartheta_{r,s}^{p,q}(\nu, \delta) = \frac{\gamma(1 - e_1 + E_1\nu, \delta)[\prod_{j=1}^{p} \Gamma(f_j - F_j\nu)] \prod_{i=2}^{q} [\Gamma(1 - e_i + E_i\nu)]}{[\prod_{j=p+1}^{s} \Gamma(1 - f_j + F_j\nu)] \prod_{i=q+1}^{r} \Gamma(e_i - E_i\nu)}, \tag{2.2}$$

and

$$\begin{aligned} {}^{\Gamma}Y_{r,s}^{p,q}(w; v; u) &= {}^{\Gamma}Y_{p,q}^{m,n}\left[w; v; u \,\middle|\, \begin{matrix} (e_1, E_1, \delta), (e_j, E_j)_2^r \\ (f_j, F_j)_1^s \end{matrix} \right] \\ &= \frac{1}{2i\pi} \int_L {}^{\Gamma}\Theta_{r,s}^{p,q}(\nu) w^{-\nu} e^{\nu v} \nu^u d\nu, \end{aligned} \tag{2.3}$$

where

$${}^{\Gamma}\Theta_{r,s}^{p,q}(\nu, \delta) = \frac{\Gamma(1 - e_1 + E_1\nu, \delta)[\prod_{j=1}^{p} \Gamma(f_j - F_j\nu)] \prod_{j=2}^{q} [\Gamma(1 - e_j + E_j\nu)]}{[\prod_{j=p+1}^{s} \Gamma(1 - f_j + F_j\nu)] \prod_{j=q+1}^{r} \Gamma(e_j - E_j\nu)}. \tag{2.4}$$

The incomplete Y-functions ${}^{\gamma}Y_{r,s}^{p,q}(w; v; u)$ and ${}^{\Gamma}Y_{r,s}^{p,q}(w; v; u)$ in (2.1) and (2.3) exist for all $\delta \geq 0$ under the same contour and the same sets of conditions as stated above.
The decomposition formula that follows is easily obtained from these definitions:

$${}^{\gamma}Y_{r,s}^{p,q}(w; v; u) + {}^{\Gamma}Y_{r,s}^{p,q}(w; v; u) = Y_{r,s}^{p,q}(w; v; u). \tag{2.5}$$

The result from Equation (2.5) is equal to the Equation (1.1), hence Yang Y-function follows incomplete function.

We now go over some fundamental properties of the incomplete Yang Y-functions. The incomplete Yang Y-functions defined in (2.1) and (2.3) exist for all $\delta \geq 0$, under the set of conditions given by Yang [6] with

$$\Delta \geq 0, |arg(w; v; u)| < \Delta\frac{\pi}{2},$$

$$\Delta = \sum_{j=1}^{p} F_j - \sum_{j=p+1}^{s} F_j + \sum_{j=1}^{q} E_j - \sum_{j=q+1}^{r} E_j.$$

3 Main results

We developed several new generating functions for the incomplete Yang Y-functions in this section.

Theorem 3.1. *If $p, q, r, s \in \mathbb{N}_0$ with $0 \leq q \leq r$, $0 \leq p \leq s$, E_j (j = 2,$\cdots$, r-1), F_j (j=2,$\cdots$, s) $\in \mathbb{R}^+$ and $e_j, f_j, \kappa \in \mathbb{C}$ then the following linear generating relation holds:*

$$\sum_{\mu=0}^{\infty} \binom{\kappa+\mu-1}{\mu} {}^{\Gamma}Y_{r,s}^{p,q}\left[w; v; u \,\middle|\, \begin{matrix} (e_1, E_1; \delta), (e_j, E_j)_{2,r-1}, (\kappa+\mu, 0) \\ (\kappa+\mu, 1), (f_j, F_j)_{2,s} \end{matrix}\right] t^{\mu}$$

$$= (1-t)^{-\kappa}\, {}^{\Gamma}Y_{r,s}^{p,q}\left[w(1-t); v; u \,\middle|\, \begin{matrix} (e_1, E_1; \delta), (e_j, E_j)_{2,r-1}, (\kappa, 0) \\ (\kappa, 1), (f_j, F_j)_{2,s} \end{matrix}\right]. \tag{3.1}$$

Proof. To prove the result, let us take the left-hand side of (3.1)

$$LHS = \sum_{\mu=0}^{\infty} \binom{\kappa+\mu-1}{\mu} {}^{\Gamma}Y_{r,s}^{p,q}\left[w; v; u \,\middle|\, \begin{matrix} (e_1, E_1; \delta), (e_j, E_j)_{2,r-1}, (\kappa+\mu, 0) \\ (\kappa+\mu, 1), (f_j, F_j)_{2,s} \end{matrix}\right] t^{\mu},$$

write incomplete Yang Y-function in terms of Mellin-Barnes type contour integral, we obtain

$$LHS = \sum_{\mu=0}^{\infty} \binom{\kappa+\mu-1}{\mu} \left(\frac{1}{2\pi i}\int_L \Theta(\nu, \delta) w^{-\nu} e^{\nu v} \nu^u d\nu\right) t^{\mu},$$

now, change the order of the summation and integration, then after some calculation, we have

$$LHS = \frac{1}{2\pi i}\int_L \frac{[\Gamma(1-e_1+E_j\nu, \delta)[\prod_{j=1}^{p}\Gamma(f_j - F_j\nu]\prod_{j=2}^{q}[\Gamma(1-e_j+E_j\nu)]}{\prod_{j=p+1}^{s}[\Gamma(1-f_j+F_j\nu)]\prod_{j=q+1}^{r-1}[\Gamma(e_j - E_j\nu)]}$$

$$\times \sum_{\mu=0}^{\infty} \binom{\kappa+\mu-\nu-1}{\mu} \frac{\Gamma(\kappa-\nu)}{\Gamma(\kappa)} t^{\mu} w^{-\nu} e^{\nu v} \nu^u d\nu,$$

using the following generalized binomial expansion formula;

$$\sum_{l=0}^{\infty} \binom{\lambda+l-1}{l} x^l = (1-x)^{-\lambda},$$

we have

$$LHS = \frac{(1-t)^{-\kappa}}{2\pi i}\int_L \frac{[\Gamma(1-e_1+E_1, \delta)[\prod_{j=1}^{p}\Gamma(f_j - F_j\nu]\prod_{j=2}^{q}[\Gamma(1-e_j+E_j\nu)]}{\prod_{j=p+1}^{s}[\Gamma(1-f_j+F_j\nu)]\prod_{j=q+1}^{r-1}[\Gamma(e_j - E_j\nu)]}$$

$$\times\frac{\Gamma(\kappa-\nu)}{\Gamma(\kappa)}(w^{-1}(1-t))^{\nu}e^{\nu v}\nu^{u}d\nu,$$

lastly, we derive the right-hand side of Theorem 3.1 's claim (3.1) using (2.3) continuing to follow the related Theorem 3.1 process and then using the definition (2.1). The following Theorem 3.2 about the lower incomplete Yang Y-functions is obtained. □

Theorem 3.2. *If $p,q,r,s\in\mathbb{N}_0$ with $0\le q\le r$, $0\le p\le s$, E_j (j = 2,⋯, r-1), F_j (j=2,⋯, s) $\in\mathbb{R}^+$ and $e_j,f_j,\kappa\in\mathbb{C}$ then the following linear generating relation holds:*

$$\sum_{\mu=0}^{\infty}\binom{\kappa+\mu-1}{\mu}\,{}^{\gamma}Y_{r,s}^{p,q}\left[w;v;u\,\middle|\,\begin{matrix}(e_1,E_1;\delta),(e_j,E_j)_{2,r-1},(\kappa+\mu,0)\\(\kappa+\mu,1),(f_j,F_j)_{2,s}\end{matrix}\right]t^{\mu}$$

$$=(1-t)^{-\kappa}\,{}^{\gamma}Y_{r,s}^{p,q}\left[w(1-t);v;u\,\middle|\,\begin{matrix}(e_1,E_1;\delta),(e_j,E_j)_{2,r-1},(\kappa,0)\\(\kappa,1),(f_j,F_j)_{2,s}\end{matrix}\right].\tag{3.2}$$

Theorem 3.3. *If $p,q,r,s\in\mathbb{N}_0$ with $0\le q\le r$, $0\le p\le s$, E_j (j = 2,⋯, r), F_j (j=1,⋯, s) $\in\mathbb{R}^+$ and α, β, e_j, $f_j\in\mathbb{C}$ then the following linear generating relation holds:*

$$\sum_{n=0}^{\infty}P_{n,m}^{\alpha,\beta}(k_1,k_2)\frac{t^n}{n!}=\frac{(1+\eta)^{\alpha+1}}{1-\beta\eta}\varphi[k_1(-\eta)^m]$$

$$\times{}^{\Gamma}Y_{r,s}^{p,q}\left[k_2(1+\eta)^{\xi};v;u\,\middle|\,\begin{matrix}(e_1,E_1;\delta),(e_j,E_j)_{2,r}\\(f_j,F_j)_{1,s}\end{matrix}\right],\tag{3.3}$$

where,
$\eta=t(1+\eta)^{\beta+1},\varphi[s]=\sum_{n=0}^{\infty}\Omega_n s^n,(\Omega_n)_{n=0}^{\infty}$ *is an arbitrary complex sequence and*

$$P_{n,m}^{\alpha,\beta}(k_1,k_2)=\sum_{k=0}^{[n/m]}(-n)_{mk}\Omega_k k_1^k$$

$$\times{}^{\Gamma}Y_{r+1,s+1}^{p,q+1}\left[k_2;v;u\,\middle|\,\begin{matrix}(e_1,E_2;\delta),(-\alpha-(\beta+1)n,\xi),(e_j,E_j)_{2,r}\\(f_j,F_j)_{1,s},(-\alpha-\beta n-mk,\xi)\end{matrix}\right].$$

Proof. To prove the result, we initially use the Mellin-Barnes type contour integral form of the incomplete Yang Y-function defined in (2.3), we have

$$P_{n,m}^{\alpha,\beta}(k_1,k_2)=\frac{1}{2\pi i}\int_L\Theta(\nu,\delta)k_2^{-\nu}e^{\nu v}\nu^{u}\{\sum_{k=0}^{[n/m]}\frac{\Gamma(1+\alpha+(\beta+1)n+\xi\nu)}{\Gamma(1+\alpha+\beta n+mk+\xi\nu)}$$
$$\times(-n)_{mk}\Omega_k k_1^k\}d\nu,\tag{3.4}$$

where, $\Theta(\nu,\delta)$ is given in (2.4), substituting (3.4) into the LHS of the bilateral generating relation (3.3) and after changing the order of the summation and integration, we get

$$I=\sum_{n=0}^{\infty}P_{n,m}^{\alpha,\beta}(k_1,k_2)\frac{t^n}{n!}=\frac{1}{2\pi i}\int_L\Theta(\nu,\delta)k_2^{-\nu}e^{\nu v}\nu^{u}$$
$$\times\left\{\sum_{n=0}^{\infty}\binom{\alpha+\xi\nu+(\beta+1)n}{n}t^n\cdot\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}\Omega_k k_1^k}{(1+\alpha+\xi\nu+\beta n)_{mk}}\right\}d\nu,\tag{3.5}$$

on using Srivastava's result [9];

$$\sum_{n=0}^{\infty}\binom{\alpha+(\beta+1)n}{n}t^n\cdot\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}\Omega_k D_k}{(1+\alpha+\beta n)_{mk}}\frac{k_1^k}{k!}=\frac{(1+\eta)^{\alpha+1}}{1-\beta\eta}\cdot\sum_{k=0}^{\infty}D_k\frac{[k_1(-\eta)^m]^k}{k!},$$

with $\alpha \to \alpha + \xi\nu$ and $D_k \to \Gamma(k+1).\Omega_k, k \geq 0$ using (3.5) we obtain;

$$I = \frac{(1+\eta)^{\alpha+1}}{1-\beta\eta} \sum_{k=0}^{\infty} \Omega_k k_1^k (-\eta)^{mk} \times \frac{1}{2\pi i} \int_L \Theta(\nu,\delta) k_2^{-\nu} (1+\eta)^{\xi\nu} e^{\nu v} \nu^u d\nu, \tag{3.6}$$

where $\eta = t(1+\eta)^{\beta+1}$.

Finally, using the definition of (2.3) and $\varphi[s]$ we get the desired result (3.3) following Theorem 3.4 about the lower incomplete Yang Y-functions is obtained. □

Theorem 3.4. *If* $p, q, r, s \in \mathbb{N}_0$ *with* $0 \leq q \leq r$, $0 \leq p \leq s$, E_j *(j = 2, $\cdots$, r),* F_j *(j=1,$\cdots$, s)* $\in \mathbb{R}^+$ *and* α, β, e_j, $f_j \in \mathbb{C}$ *then the following linear generating relation holds:*

$$\sum_{n=0}^{\infty} P_{n,m}^{\alpha,\beta}(k_1,k_2)\frac{t^n}{n!} = \frac{(1+\eta)^{\alpha+1}}{1-\beta\eta}\varphi[k_1(-\eta)^m]$$

$$\times {}^{\gamma}Y_{r,s}^{p,q}\left[k_2(1+\eta)^{\xi}; v; u \,\middle|\, \begin{matrix} (e_1,E_1;\delta),(e_j,E_j)_{2,r} \\ (f_j,F_j)_{1,s} \end{matrix}\right], \tag{3.7}$$

where,
$\eta = t(1+\eta)^{\beta+1}, \varphi[s] = \sum_{n=0}^{\infty} \Omega_n s^n, (\Omega_n)_{n=0}^{\infty}$ *is an arbitrary complex sequence and*

$$P_{n,m}^{\alpha,\beta}(k_1,k_2) = \sum_{k=0}^{[n/m]} (-n)_{mk}\Omega_k k_1^k$$

$$\times {}^{\gamma}Y_{r+1,s+1}^{p,q+1}\left[k_2; v; u \,\middle|\, \begin{matrix} (e_1,E_2;\delta),(-\alpha-(\beta+1)n,\xi),(e_j,E_j)_{2,r} \\ (f_j,F_j)_{1,s},(-\alpha-\beta n-mk,\xi) \end{matrix}\right].$$

Theorem 3.5. *If* $p, q, r, s \in \mathbb{N}_0$ *with* $0 \leq q \leq r$, $0 \leq p \leq s$, E_j *(j = 2, $\cdots$, r),* F_j*(j=1,$\cdots$, s)* $\in \mathbb{R}^+$ *and* α, β, e_j, f_j, $\omega \in \mathbb{C}$ *then the following linear generating relation holds:*

$$\sum_{n=0}^{\infty} Q_{n,m}^{\alpha,\beta}(w;k_1,k_2)\frac{t^n}{n!} = \frac{(1+\eta)^{\alpha+1}}{1-\beta\eta}\varphi[k_1(-\eta)^m(1+\eta)^w]$$

$$\times {}^{\Gamma}Y_{r,s}^{p,q}\left[k_2(1+\eta)^{\xi}; v; u \,\middle|\, \begin{matrix} (e_1,E_1;\delta),(e_j,E_j)_{2,r} \\ (f_j,F_j)_{1,s} \end{matrix}\right] \tag{3.8}$$

where,
$\eta = t(1+\eta)^{\beta+1}, \varphi[s] = \sum_{n=0}^{\infty} \Omega_n s^n, (\Omega_n)_{n=0}^{\infty}$ *is an arbitrary complex sequence and*

$$Q_{n,m}^{\alpha,\beta}(w;k_1,k_2) = \sum_{k=0}^{[n/m]} (-n)_{mk}\Omega_k k_1^k$$

$$\times {}^{\Gamma}Y_{r+1,s+1}^{p,q+1}\left[k_2; v; u \,\middle|\, \begin{matrix} (e_1,E_1;\delta),(-\alpha-(\beta+1)n-wk,\xi),(e_j,E_j)_{2,r} \\ (f_j,F_j)_{1,s},(-\alpha-\beta n-(w+m)k,\xi) \end{matrix}\right]$$

Proof. To prove the result, we initially use the Mellin-Barnes type contour integral form of the incomplete Yang Y-function defined in (2.3) we obtain

$$Q_{n,m}^{\alpha,\beta}(w;k_1,k_2) = \frac{1}{2\pi i}\int_L \Theta(\nu,\delta)k_2^{-\nu}\{\sum_{k=0}^{[n/m]} \frac{\Gamma(1+\alpha+(\beta+1)n+\nu\xi+wk)}{\Gamma(1+\alpha+\beta n+(w+m)k+\xi\nu)}$$

$$\times(-n)_{mk}\Omega_k k_1^k\}d\nu \tag{3.9}$$

where, $\Theta(\nu,\delta)$ is given in (2.4), substituting (3.9) into the LHS of the bilateral generating relation (3.8) and after changing the order of the summation and integration, we get

$$I=\sum_{n=0}^{\infty}Q_{n,m}^{\alpha,\beta}(w;k_1,k_2)\frac{t^n}{n!}=\frac{1}{2\pi i}\int_L\Theta(\nu,\delta)k_2^{-\nu}e^{\nu v}\nu^u\times$$

$$\left\{\sum_{n=0}^{\infty}\binom{\alpha+\xi\nu+(\beta+1)n}{n}t^n\cdot\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}(1+\alpha+\nu\xi+(\beta+1)n)_{wk}\Omega_k k_1^k}{(1+\alpha+\xi\nu+\beta n)_{(w+m)k}}\right\}d\nu,\quad(3.10)$$

using Srivastava and Bushman's result [10];

$$\sum_{n=0}^{\infty}\binom{\alpha+(\beta+1)n}{n}t^n\cdot\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}(1+\alpha+(\beta+1)n)_{\omega k}\Omega_k}{(1+\alpha+\beta n)_{(m+w)k}}k_1^k$$

$$=\frac{(1+\eta)^{\alpha+1}}{1-\beta\eta}\varphi[k_1(-\eta)^m)(1+\eta)^w)],$$

with $\alpha\to\alpha+\xi\nu$ and we have from (3.10)

$$I=\frac{(1+\eta)^{\alpha+1}}{1-\beta\eta}\varphi[k_1(-\eta)^m)(1+\eta)^w)]\times\frac{1}{2\pi i}\int_L\Theta(\nu,\delta)k_2^{-\nu}e^{\nu v}\nu^u(1+\eta)^{\xi\nu}d\nu,\qquad(3.11)$$

where $\eta=t(1+\eta)^{\beta+1}$.

Finally, using the definition of (2.3) and $\varphi[s]$ we get the desired bilateral generating relation (3.8). □

Theorem 3.6. *If $p,q,r,s\in\mathbb{N}_0$ with $0\le q\le r$, $0\le p\le s$, E_j (j = 2, $\cdots$, r), F_j (j=1,$\cdots$, s) $\in\mathbb{R}^+$ and α, β, e_j, $f_j,\omega\in\mathbb{C}$ then the following linear generating relation holds:*

$$\sum_{n=0}^{\infty}Q_{n,m}^{\alpha,\beta}(w;k_1,k_2)\frac{t^n}{n!}=\frac{(1+\eta)^{\alpha+1}}{1-\beta\eta}\varphi[k_1(-\eta)^m(1+\eta)^w]$$

$$\times{}^{\gamma}Y_{r,s}^{p,q}\left[k_2(1+\eta)^{\xi};v;u\;\middle|\;\begin{matrix}(e_1,E_1;\delta),(e_j,E_j)_{2,r}\\(f_j,F_j)_{1,s}\end{matrix}\right],\qquad(3.12)$$

where,
$\eta=t(1+\eta)^{\beta+1},\varphi[s]=\sum_{n=0}^{\infty}\Omega_n s^n,(\Omega_n)_{n=0}^{\infty}$ *is an arbitrary complex sequence and*

$$Q_{n,m}^{\alpha,\beta}(w;k_1,k_2)=\sum_{k=0}^{[n/m]}(-n)_{mk}\Omega_k k_1^k$$

$$\times{}^{\gamma}Y_{r+1,s+1}^{p,q+1}\left[k_2;v;u\;\middle|\;\begin{matrix}(e_1,E_1;\delta),(-\alpha-(\beta+1)n,\xi),(e_j,E_j)_{2,r}\\(f_j,F_j)_{1,s},(-\alpha-\beta n-(w+m)k,\xi)\end{matrix}\right].$$

Remark 3.7. If we set $w=0$ then we get Theorem 3.3 and Theorem 3.4.

Remark 3.8. If we set $\alpha=\lambda-1,\beta=0$ and $\eta=t/(1-t)$ in the Theorem 3.3 and Theorem 3.4 we get the below Theorem 3.9 and 3.10.

Theorem 3.9. *If $p,q,r,s\in\mathbb{N}_0$ with $0\le q\le r$, $0\le p\le s$, E_j (j = 2, $\cdots$, r), F_j (j=1,$\cdots$, s) $\in\mathbb{R}^+$ and Λ, e_j, $f_j\in\mathbb{C}$ then the following linear generating relation holds:*

$$\sum_{n=0}^{\infty}\sigma_n^m(\tau)^{\Gamma}Y_{r+1,s}^{p,q+1}\left[k_2;v;u\;\middle|\;\begin{matrix}(e_1,E_1;\delta),(1-\Lambda-n,\xi),(e_j,E_j)_{2,r}\\(f_j,F_j)_{1,s}\end{matrix}\right]\frac{t^n}{n!}$$

$$= (1-t)^{-\Lambda} \sum_{k=0}^{\infty} \frac{\Omega_k}{(mk)!} \left[\tau \left(\tfrac{t}{1-t} \right)^m \right]^k$$

$$^{\Gamma}Y^{r+1,s}_{p,q+1} \left[\frac{k_2}{(1-t)^{\xi}}; v; u \,\middle|\, \begin{matrix} (e_1, E_1; \delta), (1-\Lambda-n, \xi), (e_j, E_j)_{2,r} \\ (f_j, F_j)_{1,s} \end{matrix} \right], \tag{3.13}$$

where, $\sigma_n^m(\tau) = \sum_{k=0}^{[n/m]} \binom{n}{mk} \Omega_k \tau^k$ *and* $\left(\Omega_k\right)_{n=0}^{\infty}$ *is an arbitrary complex sequence.*

Theorem 3.10. *If* $p, q, r, s \in \mathbb{N}_0$ *with* $0 \le q \le r$, $0 \le p \le s$, E_j *(j = 2, $\cdots$, r),* F_j*(j=1, $\cdots$, s)* $\in \mathbb{R}^+$ *and* Λ, e_j, $f_j \in \mathbb{C}$ *then the following linear generating relation holds:*

$$\sum_{n=0}^{\infty} \sigma_n^m(\tau)^{\gamma}Y^{p,q+1}_{r+1,s} \left[k_2; v; u \,\middle|\, \begin{matrix} (e_1, E_1; \delta), (1-\Lambda-n, \xi), (e_j, E_j)_{2,r} \\ (f_j, F_j)_{1,s} \end{matrix} \right] \frac{t^n}{n!}$$

$$= (1-t)^{-\Lambda} \sum_{k=0}^{\infty} \frac{\Omega_k}{(mk)!} \left[\tau \left(\tfrac{t}{1-t} \right)^m \right]^k$$

$$^{\gamma}Y^{r+1,s}_{p,q+1} \left[\frac{k_2}{(1-t)^{\xi}}; v; u \,\middle|\, \begin{matrix} (e_1, E_1; \delta), (1-\Lambda-n, \xi), (e_j, E_j)_{2,r} \\ (f_j, F_j)_{1,s} \end{matrix} \right], \tag{3.14}$$

where, $\sigma_n^m(\tau) = \sum_{k=0}^{[n/m]} \binom{n}{mk} \Omega_k \tau^k$ *and* $\left(\Omega_k\right)_{n=0}^{\infty}$ *is an arbitrary complex sequence.*

4 Special cases

In this section, as specific cases of the theorems involving the incomplete Y-function, the Y-function, the incomplete H-function, and the H-function, we derive the following results. By specifying certain parameters, we obtain special cases that illustrate the application of the fundamental results.

(i) Yang Y-function:- If we replace $\delta = 0$ in the (2.1) and (2.3) then it reduces Yang Y-function(1.1),as follow:

$$^{\gamma}Y^{p,q}_{r,s} \left[w; v; u \,\middle|\, \begin{matrix} (e_1, E_1, 0), (e_j, E_j)_2^r \\ (f_j, F_j)_1^s \end{matrix} \right] = Y^{p,q}_{r,s} \left[w; v; u \,\middle|\, \begin{matrix} (e_j, E_j)_1^r \\ (f_j, F_j)_1^s \end{matrix} \right]. \tag{4.1}$$

Applying the relationships outlined in Theorem (3.1) to Theorem (3.10), we obtain the following corollaries.

Corollary 4.1. *If* $p, q, r, s \in \mathbb{N}_0$ *with* $0 \le q \le r$, $0 \le p \le s$, E_j *(j = 1,$\cdots$, r-1),* F_j *(j=2,$\cdots$, s)* $\in \mathbb{R}^+$ *and* $e_j, f_j, \kappa \in \mathbb{C}$ *then the following linear generating relation holds:*

$$\sum_{\mu=0}^{\infty} \binom{\kappa+\mu-1}{\mu} Y^{p,q}_{r,s} \left[w; v; u \,\middle|\, \begin{matrix} (e_j, E_j)_{1,r-1}, (\kappa+\mu, 0) \\ (\kappa+\mu, 1), (f_j, F_j)_{2,s} \end{matrix} \right] t^{\mu}$$

$$= (1-t)^{-\kappa}\, Y^{p,q}_{r,s} \left[w(1-t); v; u \,\middle|\, \begin{matrix} (e_j, E_j)_{1,r-1}, (\kappa, 0) \\ (\kappa, 1), (f_j, F_j)_{2,s} \end{matrix} \right]. \tag{4.2}$$

Similarly we will get results for all the Theorem given above.

(ii) Incomplete H-Function:- If we replace $u = v = 0$ and $\frac{1}{w}$ in place of w in (2.1) and (2.3), it reduces to the incomplete H-function [11], as follows:

$$^{\gamma}\mathbb{Y}^{p,q}_{r,s} \left[\frac{1}{w}; 0; 0 \,\middle|\, \begin{matrix} (e_1, E_1; \delta), (e_j, E_j)_2^r \\ (f_j, F_j)_1^s \end{matrix} \right] = \gamma^{p,q}_{r,s} \left[w \,\middle|\, \begin{matrix} (e_1, E_1; \delta), (e_j, E_j)_2^r \\ (f_j, F_j)_1^s \end{matrix} \right]. \tag{4.3}$$

Applying the relationships outlined in Theorem (3.2), Theorem (3.4), Theorem (3.6) and Theorem (3.10), we obtain the following corollaries.

Similarly for upper Yang Y-function, applying the relationships outlined in Theorem (3.1), Theorem (3.3), Theorem (3.5) and Theorem (3.9), we obtain the following corollaries.

Corollary 4.2. *If $p, q, r, s \in \mathbb{N}_0$ with $0 \leq q \leq r$, $0 \leq p \leq s$, E_j (j = 2,$\cdots$, r-1), F_j (j=2,$\cdots$, s) $\in \mathbb{R}^+$ and $e_j, f_j, \kappa \in \mathbb{C}$ then the following linear generating relation holds:*

$$\sum_{\mu=0}^{\infty} \binom{\kappa+\mu-1}{\mu} \gamma_{r,s}^{p,q}\left[w \,\middle|\, \begin{matrix} (e_1, E_1; \delta), (e_j, E_j)_{2,r-1}, (\kappa+\mu, 0) \\ (\kappa+\mu, 1), (f_j, F_j)_{2,s} \end{matrix}\right] t^{\mu}$$

$$= (1-t)^{-\kappa}\, \gamma_{r,s}^{p,q}\left[w(1-t) \,\middle|\, \begin{matrix} (e_1, E_1; \delta), (e_j, E_j)_{2,r-1}, (\kappa, 0) \\ (\kappa, 1), (f_j, F_j)_{2,s} \end{matrix}\right]. \tag{4.4}$$

Similarly we will get results for all the Theorem given above.

(iii) H-Function:- If we replace $u = v = \delta = 0$ and $\frac{1}{w}$ in place of w in (2.1) and (2.3), it reduces to the H-function [12], as follows:

$${}^{\gamma}\mathbb{Y}_{r,s}^{p.q}\left[\frac{1}{w}; 0; 0 \,\middle|\, \begin{matrix} (e_1, E_1; 0), (e_j, E_i)_2^r \\ (f_j, F_j)_1^s \end{matrix}\right] = H_{r,s}^{p,q}\left[w \,\middle|\, \begin{matrix} (e_j, E_j)_1^r \\ (f_j, F_j)_1^s \end{matrix}\right]. \tag{4.5}$$

Applying the relationships outlined in Theorem (3.1) to Theorem (3.10), we obtain the following corollaries.

Corollary 4.3. *If $p, q, r, s \in \mathbb{N}_0$ with $0 \leq q \leq r$, $0 \leq p \leq s$, E_j (j = 1,$\cdots$, r-1), F_j (j=2,$\cdots$, s) $\in \mathbb{R}^+$ and $e_j, f_j, \kappa \in \mathbb{C}$ then the following linear generating relation holds:*

$$\sum_{\mu=0}^{\infty} \binom{\kappa+\mu-1}{\mu} H_{r,s}^{p,q}\left[w \,\middle|\, \begin{matrix} (e_j, E_j)_{1,r-1}, (\kappa+\mu, 0) \\ (\kappa+\mu, 1), (f_j, F_j)_{2,s} \end{matrix}\right] t^{\mu}$$

$$= (1-t)^{-\kappa}\, H_{r,s}^{p,q}\left[w(1-t) \,\middle|\, \begin{matrix} (e_j, E_j)_{1,r-1}, (\kappa, 0) \\ (\kappa, 1), (f_j, F_j)_{2,s} \end{matrix}\right]. \tag{4.6}$$

Similarly we will get results for all the Theorem given above.

5 Conclusion

In this work, we derived bilateral and linear generating functions involving incomplete Yang Y-functions. The results obtained are consistent with expectations and demonstrate the robustness of the approach. These outcomes are general in nature, offering the flexibility to produce specific cases that involve various well-known special functions, such as the H-function, G-function, Fox-Wright function, and hypergeometric functions, under appropriate parameter conditions. This highlights the versatility and broad applicability of the results within the realm of special functions and their extensions.

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