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Some Extended Summations and Transformations of Basic Hypergeometric Series

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Abstract In the present work, we have used q- Beta integral as an operator to extent some known results in the theory of basic hypergeometric function. Some interesting special cases of our results have also been mentioned.

1 Introduntction

The transformations and summations of basic (or q-) hypergeometric series not only provides a deep insight into the structure and properties of these functions but also facilitate the research and further developments in many areas of sciences. A systematic treatment to the theory basic hypergeometric series was first given by Bailey [17]. After Bailey, Slater [12], Sears [4, 5, 6, 7], Agarwal [14], Verma [1], Andrews [9] and many others contributed to further developments in the field. These developments helped the subject to flourish as it cached the attention of others working in divers areas of mathematics and many other branches of science and technology. The researches in the basic hypergeometric series have got further impetus due to its wide range of applications in combinatorics, orthogonal polynomials, statistical mechanics, quantum algebra and physics, modular forms and mock theta functions; see Andrews [8].
Bailey in his work [17] discussed various techniques from for finding the transformations and summations of basic hypergeometric series. In recent years it has been shown that operator approach is also one of the useful methods. Besides, Laplace integral [13], the Beta integrals [1] have also been frequently used in discovering and generalizing the known results in the theory of basic hypergeometric series. The Beta integral has been used with advantage by MacRobert [16] to augment the parameters to obtain the identities of generalized hypergeometric series. Krattenthalera and Rao [3] used the Beta integrals to generate the new identities of hypergeometric series from known identities. In the present paper, we have investigated many transformations and summations of basic hypergeometric series via q-Beta integral approach. In what follows, we have used the definitions and notations of [10].
We define a basic hypergeometric series as

$$ {}_r\phi_s(a_1, a_2, ..., a_r; b_1, b_2, ..., b_s; q, z) = \sum_{n=0}^{\infty} \frac{(a_1, a_2, ..., a_r; q)_n}{(q, b_1, b_2, ..., b_s; q)_n} [(-1)^n q^{\frac{n(n-1)}{2}}]^{1+s-r} z^n, \quad (1.1)$$

where

$$(a; q)_n = \frac{(a; q)_\infty}{(aq^n; q)_\infty}.$$

The series is absolutely convergent for all z, if $0 < |q| < 1$ where $r \leq s$ and for $|z| < 1$ where $r = s + 1$. The series is also absolutely convergent for $|z| < \frac{|b_1 b_2 ... b_s q|}{|a_1 a_2 ... a_r|}$, if $|q| > 1$.
The q-Beta function is defined [10] as

$$B_q(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1 - tq)_{\beta-1} d(q, t) = \frac{\Gamma_q(\alpha)\Gamma_q(\beta)}{\Gamma_q(\alpha + \beta)}, \quad (1.2)$$

provided Re $\alpha > 0$, Re $\beta > 0$. For $q \to 1$ in (1.2), we get

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1 - t)^{\beta-1} dt = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)}, \quad (1.3)$$

provided Re $\alpha > 0$, Re $\beta > 0$. Also $\Gamma_q(\alpha)$ is the q- extension of gamma function.
In the next section, we have used the following known results

$$ {}_2\phi_1(a^2, aq; a; q, z) = (1 + az)\frac{(a^2qz; q)_\infty}{(z; q)_\infty}. \tag{1.4} $$

(Page 21 [10])

$$ {}_3\phi_2(a, q\sqrt{a}, -q\sqrt{a}; \sqrt{a}, -\sqrt{a}; q, z) = (1 - aqz^2)\frac{(azq^2; q)_\infty}{(z; q)_\infty}, \tag{1.5} $$

(Page 50 [10])

$$ {}_3\phi_2(a, b, e; aq, de; q, z) = \frac{(bz/q, e, deq/b; q)_\infty}{(z, eq/b, de; q)_\infty} {}_3\phi_2(q, aq/b, q/b; aq, deq/b; q, bz/q). \tag{1.6} $$

(Equ. 2.2 [15])

$$ {}_4\phi_3(a, b, \sqrt{ab}, -\sqrt{ab}; ab, \sqrt{abq}, -\sqrt{abq}; q, z) = {}_2\phi_1(a, b; abq; q^2, z)\,{}_2\phi_1(a, b; abq; q^2, zq). \tag{1.7} $$

(Equ. 1.12 [11])

$$ {}_6\phi_5(a, q\sqrt{a}, -q\sqrt{a}, b, c, a/bc; \sqrt{a}, -\sqrt{a}, aq/b, aq/c, bcq; q, zq) = (1 - z)\,{}_4\phi_3(aq, bq, cq, aq/bc; aq/b, aq/c, bcq; q, z). \tag{1.8} $$

(Equ. 4.22 [2])

2 Some Extended Transformation and Summation Identities

In this section, we have established some extended transformations and summations of basic hypergeometric series. We have used the method of augmentation of parameters in the known results via q-beta integral. Here, our claim is that under suitable convergence conditions, the following identities are true

$$ \begin{aligned} {}_{p+2}\phi_{p+1}(a^2, aq, \alpha_1, ..., \alpha_p; a, \beta_1, ..., \beta_p; q, z) &= {}_{p+1}\phi_p(a^2q, \alpha_1, ..., \alpha_p; \beta_1, ..., \beta_p; q, z) \\ &+ az\,\frac{(1-\alpha_1)...(1-\alpha_p)}{(1-\beta_1)...(1-\beta_p)}\,{}_{p+1}\phi_p(a^2q, \alpha_1 q, ..., \alpha_p q; \beta_1 q, ..., \beta_p q; q, z). \end{aligned} \tag{2.1} $$

$$ \begin{aligned} {}_{p+3}\phi_{p+2}\left(a, q\sqrt{a}, -q\sqrt{a}, \alpha_1, ..., \alpha_p; \sqrt{a}, -\sqrt{a}, \beta_1, ..., \beta_p; q, z\right) &= {}_{p+1}\phi_p\left(aq^2, \alpha_1, ..., \alpha_p; \beta_1, ..., \beta_p; q, z\right) \\ &- aqz^2\frac{(1-\alpha_1)...(1-\alpha_p)}{(1-\beta_1)...(1-\beta_p)}\frac{(1-\alpha_1 q)...(1-\alpha_p q)}{(1-\beta_1 q)...(1-\beta_p q)}\times \\ &\quad {}_{p+1}\phi_p(aq^2, \alpha_1 q^2, ..., \alpha_p q^2; \beta_1 q^2, ..., \beta_p q^2; q, z). \end{aligned} \tag{2.2} $$

$$ \begin{aligned} {}_{p+3}\phi_{p+2}\left(a, b, e, \alpha_1, \alpha_2, ..., \alpha_p; aq, de, \beta_1, \beta_2, ..., \beta_p; q, z\right) &= \frac{(e, deq/b; q)_\infty}{(eq/b, de; q)_\infty}\sum_{n=0}^{\infty}\frac{(aq/b, q/b, \alpha_1, \alpha_2, ..., \alpha_p; q)_n}{(aq, deq/b, \beta_1, \beta_2, ..., \beta_p; q)_n}\left(\frac{bz}{q}\right)^n \times \\ &\quad {}_{p+1}\phi_p\left(b/q, \alpha_1 q^n, \alpha_2 q^n, ..., \alpha_p q^n; \beta_1 q^n, \beta_2 q^n, ..., \beta_p q^n; q, z\right). \end{aligned} \tag{2.3} $$

$$
\begin{aligned}
{}_{2p+4}\phi_{2p+3} &\Big(a, b, \sqrt{ab}, -\sqrt{ab}, \sqrt{\alpha_1}, -\sqrt{\alpha_1}, ..., \sqrt{\alpha_p}, -\sqrt{\alpha_p}; ab, \sqrt{abq}, \\
&\qquad -\sqrt{abq}, \sqrt{\beta_1}, -\sqrt{\beta_1}, ..., \sqrt{\beta_p}, -\sqrt{\beta_p}; q, z\Big) \\
&= \sum_{n=0}^{\infty} \frac{(a, b, \alpha_1, ..., \alpha_p; q^2)_n}{(q^2, abq, \beta_1, ..., \beta_p; q^2)_n} z^n \times \\
&\qquad {}_{p+2}\phi_{p+1}(a, b, \alpha_1 q^{2n}, ..., \alpha_p q^{2n}; abq, \beta_1 q^{2n}, ..., \beta_p q^{2n}; q^2, z). \qquad (2.4)
\end{aligned}
$$

$$
\begin{aligned}
{}_{p+6}\phi_{p+5} &\left(a, q\sqrt{a}, -q\sqrt{a}, b, c, a/bc, \alpha_1, ..., \alpha_p; \sqrt{a}, -\sqrt{a}, aq/b, aq/c,\right. \\
&\qquad \left. bcq, \beta_1, ..., \beta_p; q, zq\right) \\
&= {}_{p+4}\phi_{p+3}(aq, bq, cq, aq/bc, \alpha_1, ..., \alpha_p; aq/b, aq/c, bcq, \beta_1, ..., \beta_p; q, z) \\
&\qquad - z\frac{(1-\alpha_1)...(1-\alpha_p)}{(1-\beta_1)...(1-\beta_p)} \times \\
&\qquad {}_{p+4}\phi_{p+3}\left(aq, bq, cq, aq/bc, \alpha_1 q, ..., \alpha_p q; aq/b, aq/c, bcq, \beta_1 q, ..., \beta_p q; q, z\right). \qquad (2.5)
\end{aligned}
$$

We give below an outline of the proof of the identity (2.1). The other identities (2.2) - (2.5) can be proved by similar method.
In (1.4) on using q-binomial theorem, we get

$$
{}_2\phi_1\left(a^2, aq; a; q, z\right) = {}_1\phi_0\left(a^2 q; q, z\right) + az\ {}_1\phi_0\left(a^2 q; q, z\right). \qquad (2.6)
$$

Taking $z \to zt$ in (2.6), multiplying $t^{\alpha_1-1}(1-qt)_{\beta_1-\alpha_1-1}$ and integrating with respect to t and after simplifying, we have

$$
\begin{aligned}
\sum_{n=0}^{\infty} \frac{(a^2, aq; q)_n}{(a, q; q)_n}(z)^n \int_0^1 t^{\alpha_1+n-1}(1-qt)_{\beta_1-\alpha_1-1} d(q,t) \\
= \sum_{n=0}^{\infty} \frac{(a^2 q; q)_n}{(q; q)_n}(z)^n \int_0^1 t^{\alpha_1+n-1}(1-qt)_{\beta_1-\alpha_1-1} d(q,t) \\
+ a\sum_{n=0}^{\infty} \frac{(a^2 q; q)_n}{(q; q)_n}(z)^{n+1} \int_0^1 t^{\alpha_1+n}(1-qt)_{\beta_1-\alpha_1-1} d(q,t).
\end{aligned}
$$

By using (1.2) in above, we get

$$
\begin{aligned}
{}_3\phi_2\left(a^2, aq, \alpha_1; a, \beta_1; q, z\right) = {}_2\phi_1\left(a^2 q, \alpha_1; \beta_1; q, z\right) \\
+ az\frac{(1-\alpha_1)}{(1-\beta_1)} {}_2\phi_1\left(a^2 q, \alpha_1 q; \beta_1 q; q, z\right). \qquad (2.7)
\end{aligned}
$$

Next, in (2.7), we are taking $z \to zt$, after multiplying $t^{\alpha_2-1}(1-qt)_{\beta_2-\alpha_2-1}$ and integrate with respect to t both sides, we have

$$
\begin{aligned}
\sum_{n=0}^{\infty} \frac{(a^2, aq, \alpha_1; q)_n}{(a, \beta_1 q; q)_n}(z)^n \int_0^1 t^{\alpha_2+n-1}(1-qt)_{\beta_2-\alpha_2-1} d(q,t) \\
= \sum_{n=0}^{\infty} \frac{(a^2 q, \alpha_1; q)_n}{(\beta_1, q; q)_n}(z)^n \int_0^1 t^{\alpha_2+n-1}(1-qt)_{\beta_2-\alpha_2-1} d(q,t) \\
+ a\frac{(1-\alpha_1)}{(1-\beta_1)} \sum_{n=0}^{\infty} \frac{(a^2 q, \alpha_1 q; q)_n}{(\beta_1 q, q; q)_n}(z)^{n+1} \int_0^1 t^{\alpha_2+n}(1-qt)_{\beta_2-\alpha_2-1} d(q,t).
\end{aligned}
$$

Now, applying (1.2) in above, we get

$$
{}_4\phi_3\left(a^2, aq, \alpha_1, \alpha_2; a, \beta_1, \beta_2; q, z\right) = {}_3\phi_2\left(a^2q, \alpha_1, \alpha_2; \beta_1, \beta_2; q, z\right) + az\,\frac{(1-\alpha_1)(1-\alpha_2)}{(1-\beta_1)(1-\beta_2)}\,{}_3\phi_2\left(a^2q, \alpha_1q, \alpha_2q; \beta_1q, \beta_2q; q, z\right). \quad (2.8)
$$

Repeating the process p times, we obtain (2.1).

3 Some Special Cases

For $\alpha_1 = a, \beta_1 = a^2$, (2.7) becomes

$$
{}_2\phi_1\left(\begin{matrix} a^2q, & a; & q, & z \\ & a^2 & & \end{matrix}\right) = \left[1 - az\frac{(1-a)}{(1-a^2)}\right]\frac{(aqz;q)_\infty}{(z;q)_\infty}. \quad (3.1)
$$

Substituting $\beta_1 = a^2$, $z = \beta_2/\alpha_1\alpha_2 q$ in (2.8), we get

$$
\sum_{n=0}^{\infty}\frac{(1-aq^n)}{(1-a)}\frac{(\alpha_1,\alpha_2;q)_n}{(\beta_2,q;q)_n}\left(\frac{\beta_2}{\alpha_1\alpha_2 q}\right)^n = \sum_{n=0}^{\infty}\frac{(1-a^2q^n)}{(1-a^2)}\frac{(\alpha_1,\alpha_2;q)_n}{(\beta_2,q;q)_n}\left(\frac{\beta_2}{\alpha_1\alpha_2 q}\right)^n + \frac{a\beta_2}{\alpha_1\alpha_2 q}\frac{(1-\alpha_1)(1-\alpha_2)}{(1-a^2)(1-\beta_2)}\frac{(\beta_2/\alpha_1,\beta_2/\alpha_2;q)_\infty}{(\beta_2/\alpha_1\alpha_2 q,\beta_2 q;q)_\infty}. \quad (3.2)
$$

For $\alpha_1 = b_1, ..., \alpha_p = b_p$, $\beta_1 = b_1q^{m_1}, ..., \beta_p = b_pq^{m_p}$, $z = 1/a$, in (2.1)

$$
{}_{p+2}\phi_{p+1}\left(\begin{matrix} a^2, & aq, & b_1, ..., b_p; & q, & 1/a \\ & a, & b_1q^{m_1}, ..., b_pq^{m_p} & & \end{matrix}\right) = {}_{p+1}\phi_p\left(\begin{matrix} a^2q, & b_1, ..., b_p; & q, & 1/a \\ & b_1q^{m_1}, ..., b_pq^{m_p} & & \end{matrix}\right) + \frac{(b_1;q)_{m_1}}{(b_1q;q)_{m_1}}\frac{...}{...}\frac{(b_p;q)_{m_p}}{(b_pq;q)_{m_p}}\times {}_{p+1}\phi_p\left(\begin{matrix} a^2q, & b_1q, ..., b_pq; & q, & 1/a \\ & b_1q^{m_1+1}, ..., b_pq^{m_p+1} & & \end{matrix}\right). \quad (3.3)
$$

Substituting $\alpha_1 = b_1/q, ..., \alpha_p = b_p/q$, $\beta_1 = b_1q^{m_1}, ..., \beta_p = b_pq^{m_p}$ & $z = 1/\sqrt{aq}$, in (2.2), we get

$$
{}_{p+3}\phi_{p+2}\left(\begin{matrix} a, & q\sqrt{a}, & -q\sqrt{a}, & b_1/q, ..., b_p/q; & q, & 1/\sqrt{aq} \\ & \sqrt{a}, & -\sqrt{a}, & b_1q^{m_1}, ..., b_pq^{m_p} & & \end{matrix}\right) = {}_{p+1}\phi_p\left(\begin{matrix} aq^2, & b_1/q, ..., b_p/q; & q, & 1/\sqrt{aq} \\ & b_1q^{m_1}, ..., b_pq^{m_p} & & \end{matrix}\right) - \frac{(b_1/q;q)_{m_1}...(b_p/q;q)_{m_p}}{(b_1q;q)_{m_1}...(b_pq;q)_{m_p}}\times {}_{p+1}\phi_p\left(\begin{matrix} aq^2, & b_1q, ..., b_pq; & q, & 1/\sqrt{aq} \\ & b_1q^{m_1+2}, ..., b_pq^{m_p+2} & & \end{matrix}\right). \quad (3.4)
$$

In (2.2), substitute $\alpha_1 = b, \beta_1 = c, \alpha_2 = \beta_2, ..., \alpha_p = \beta_p$ and $z = c/abq^2$ then we have

$$
{}_4\phi_3\left(\begin{matrix} a, & q\sqrt{a}, & -q\sqrt{a}, & b; & q, & c/abq^2 \\ & \sqrt{a}, & -\sqrt{a}, & c & & \end{matrix}\right) = \frac{(c/aq^2, c/b;q)_\infty}{(c/abq^2, c;q)_\infty} - \left[\frac{c(1-b)(1-bq)}{ab^2q^3}\right]\frac{(c/a, c/b;q)_\infty}{(c/abq^2, c;q)_\infty}. \quad (3.5)
$$

Taking $\alpha_1 = q^{-n}$ and $\beta_1 = q/b$ in (2.3), we have

$$
{}_{p+3}\phi_{p+2}\left(\begin{matrix} a, & b, & e, & q^{-n}, & \alpha_2,...,\alpha_p; & q, & z \\ & aq, & de, & q/b, & \beta_2,...,\beta_p & & \end{matrix}\right)
= \frac{(e, deq/b; q)_\infty}{eq/b, de; q)_\infty} {}_{p+2}\phi_{p+1}\left(\begin{matrix} q, & aq/b, & q^{-n}, & \alpha_2,...,\alpha_p; & q, & bz/q \\ & aq, & deq/b, & \beta_2,...,\beta_p & & \end{matrix}\right). \tag{3.6}
$$

For $\alpha_1 = f, \beta_1 = g, \alpha_2 = \beta_2, ..., \alpha_p = \beta_p$ & $z = gq/fb$ in (2.3), we have

$$
{}_4\phi_3\left(\begin{matrix} a, & b, & e, & f; & q, & gq/fb \\ & aq, & de, & g & & \end{matrix}\right) = \frac{(e, deq/b, g/f, gq/b; q)_\infty}{(eq/b, de, g, gq/bf; q)_\infty} \times
{}_4\phi_3\left(\begin{matrix} q, & aq/b, & q/b, & f; & q, & g/f \\ & aq, & deq/b, & gq/b & & \end{matrix}\right). \tag{3.7}
$$

Putting $b = aq$ in (3.7), we get

$$
{}_3\phi_2\left(\begin{matrix} a, & e, & f; & q, & g/af \\ & de, & g & & \end{matrix}\right) = \frac{(e, de/a, g/f, g/a; q)_\infty}{(e/a, de, g, g/af; q)_\infty}. \tag{3.8}
$$

On substituting $e = 0$, (3.8) gives q-analogue of Gauss summation formula.

$$
{}_2\phi_1\left(\begin{matrix} a, & f; & q, & g/af \\ & g & & \end{matrix}\right) = \frac{(g/f, g/a; q)_\infty}{(g, g/af; q)_\infty}. \tag{3.9}
$$

Putting $\alpha_1 = q^{-2n}$ and $\beta_1 = ab$ in (2.4), we obtain

$$
{}_{2p+2}\phi_{2p+1}\left(\begin{matrix} a, & b, & q^{-n}, & -q^{-n}, & \sqrt{\alpha_2}, & -\sqrt{\alpha_2}, ..., \sqrt{\alpha_p}, & -\sqrt{\alpha_p}; & q, & z \\ & ab, & \sqrt{abq}, & -\sqrt{abq}, & \sqrt{\beta_2}, & -\sqrt{\beta_2}, ..., \sqrt{\beta_p}, & -\sqrt{\beta_p} & & \end{matrix}\right)
= {}_{p+2}\phi_{p+1}\left(\begin{matrix} a, & b, & q^{-2n}, & \alpha_2,...,\alpha_p; & q^2, & z \\ & abq, & ab, & \beta_2,...,\beta_p & & \end{matrix}\right). \tag{3.10}
$$

Putting $\alpha_1 = abq, \beta_1 = ab, \alpha_2 = \beta_2, ..., \alpha_p = \beta_p$ in (2.4), we get

$$
\sum_{n=0}^{\infty} \frac{(a, b; q^2)_n}{(q^2, ab; q^2)_n} z^n \sum_{m=0}^{n} \frac{(a, b; q^2)_m}{(abq, q^2; q^2)_m} \frac{(1 - abq^{2n+m})}{(1 - abq^{2n})} (zq)^m
= {}_2\phi_1\left(\begin{matrix} a, & b; & q, & z \\ & ab & & \end{matrix}\right). \tag{3.11}
$$

In (2.5), we substitute $\alpha_1 = b_1, ..., \alpha_p = b_p,\quad \beta_1 = b_1 q^{m_1}, ..., \beta_p = b_p q^{m_p}$ & $z = q^{m_1 + ... + m_p}$ then

$$
{}_{p+6}\phi_{p+5}\left(\begin{array}{c} a, \quad q\sqrt{a}, \quad -q\sqrt{a}, \quad b, \quad c, \quad a/bc, \quad b_1,...,b_p; \\ \sqrt{a}, \quad -\sqrt{a}, \quad aq/b, \quad aq/c, \quad bcq, \quad b_1q^{m_1},...,b_pq^{m_p} \end{array} q, \quad q^{m_1+...+m_p+1}\right)
$$

$$
= {}_{p+4}\phi_{p+3}\left(\begin{array}{c} aq, \quad bq, \quad cq, \quad aq/bc, \quad b_1,...,b_p; \\ aq/b, \quad aq/c, \quad bcq, \quad b_1q^{m_1},...,b_pq^{m_p} \end{array} q, \quad q^{m_1+...+m_p}\right)
$$

$$
- q^{m_1+...+m_p}\frac{(b_1;q)_{m_1}...(b_p;q)_{m_p}}{(b_1q;q)_{m_1}...(b_pq;q)_{m_p}}\times
$$

$$
{}_{p+4}\phi_{p+3}\left(\begin{array}{c} aq, \quad bq, \quad cq, \quad aq/bc, \quad b_1q,...,b_pq; \\ aq/b, \quad aq/c, \quad bcq, \quad b_1q^{m_1+1},...,b_pq^{m_p+1} \end{array} q, \quad q^{m_1+...+m_p}\right). \tag{3.12}
$$

For $\alpha_1 = aq/b, \alpha_2 = aq/c, \beta_1 = b, \beta_2 = c, \alpha_3 = \beta_3, ..., \alpha_p = \beta_p$, (2.5) becomes

$$
{}_4\phi_3\left(\begin{array}{c} a, \quad q\sqrt{a}, \quad -\sqrt{a}, \quad a/bc; \\ \sqrt{a}, \quad -\sqrt{a}, \quad bcq \end{array} q, \quad zq\right)
= {}_4\phi_3\left(\begin{array}{c} aq, \quad bq, \quad cq, \quad aq/bc; \\ b, \quad c, \quad bcq \end{array} q, \quad z\right)
$$

$$
- \frac{z}{bc}\frac{(b-aq)(c-aq)}{(1-b)(1-c)}{}_4\phi_3\left(\begin{array}{c} aq, \quad aq/bc, \quad aq^2/b, \quad aq^2/c; \\ aq/b, \quad aq/c, \quad bcq \end{array} q, \quad z\right). \tag{3.13}
$$

If in (3.13), we substitute $a = 1$ then

$$
\sum_{n=0}^{\infty}\frac{(bq, cq, q/bc;q)_n}{(b, c, bcq;q)_n}z^n = 1 + \frac{z}{bc}\frac{(b-q)(c-q)}{(1-b)(1-c)}\sum_{n=0}^{\infty}\frac{(q/bc, q^2/b, q^2/c;q)_n}{(q/b, q/c, bcq;q)_n}z^n. \tag{3.14}
$$

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