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ON ODD CHROMATIC NUMBER OF CORONA PRODUCT OF GRAPHS

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Abstract A proper vertex coloring of a graph G is said to be odd if, for every vertex $v \in V(G)$ with $deg(v) > 0$, there exists a color c such that the number of neighbors of v assigned color c is odd. Specifically, we determine $\chi_{od}(G \circ H)$, where $G \in \{P_q, K_q\}$ and $H \in \{P_r, K_r, C_r\}$.

1 Introduction

The conception of *odd coloring* [1, 2, 7, 8] was first instigated by Mirko Petruševski, Riste Škrekovski [2] in 2021. A proper vertex coloring of a graph G is said to be odd if, for every vertex $v \in V(G)$ with $deg(v) > 0$, there exists a color c such that the number of neighbors of v assigned color c is odd. The *odd chromatic number* denoted as $\chi_{od}(G)$ of a graph G is the minimum number of colors required to achieve any odd coloring of G. Mirko Petruševski, Riste Škrekovski [2] also proved that every simple planar graph admits an odd 9-coloring, and conjecture that 5 colors always suffice. The obvious inequality $\chi(G) \leq \chi_{od}(G)$ becomes an equality whenever the graph G is odd, that is, if it has only odd vertex degrees. The above said inequality may also be strict. The problem of determining $\chi_{od}(G)$ for a graph G is NP-hard [8]. They also came out with some upper bounds such that, if G is a connected graph with maximum degree Δ=3, then $\chi_{od}(G) \leq 4$. If G is a connected graph with maximum degree $\Delta \geq 3$, then $\chi_{od}(G) \leq \Delta+1$[7]. For every connected graph G with maximum degree Δ, then $\chi_{od}(G) \leq 2\Delta$[7]. The concept of odd coloring has got more interest in recent times which made many authors to work on it and come out with certain bounds and results. For instance, Rémy Belmonte et al.[4] determined that, any graph G of even order with maximum degree Δ, then $\chi_{od}(g) \leq 2\Delta - 1$. Similarly, Tianjiao Dai et al.[6] proved that for every Δ, $\chi_{od}(G) \leq \lfloor 3\Delta/2 \rfloor + 2$.

2 Preliminaries

The *Corona Product* of two graphs $G \circ H$ is obtained by taking one copy of G and $|V(G)|$ copies of H and by joining each vertex of the i^{th} copy of H to the i^{th} vertex of G, where $1 \leq i \leq |V(G)|$[5]. The corona product creats predictable and structured neighborhoods which makes it ideal for studying coloring constraints that depends on neighborhood behavior. It blends two graph families one governing global connectivity (via G) and the other local complexity (via H) which allows exploration of how local graph properties influence global coloring behavior. The corona product serves as a versatile graph operation with applications in hierarchical network modeling, cryptographic constructions, biological systems, and the analysis of graph invariants such as chromatic and domination parameters. A central tool that helped us to derive exact and tight bounds for the odd chromatic number across various corona product throughout this work is Corollary 3.1[3], which establishes that, for any two graphs G and H with chromatic numbers $\chi(G)$ and $\chi(H)$ respectively, then $\chi(H) + 1 \leq \chi_{od}(G \circ H) \leq \chi(H) + 2$, if $\chi(G) \leq \chi(H) + 1$ and $\chi_{od}(G \circ H) = \chi(G)$, otherwise.

3 Results

Theorem 3.1. *For any $q > 1$, $r > 2$, the odd chromatic number of corona product of path with complete graph is*

$$\chi_{od}(P_q \circ K_r) = \chi(K_r) + 1$$

Proof. Let $\{b_i : 1 \leq i \leq q\}$ be the q vertices of P_q and $\{d_j : 1 \leq j \leq r\}$ be the r vertices of K_r. Let the vertices of graph K_r in the i^{th} copy be $\{d_{ij} : 1 \leq i \leq q, 1 \leq j \leq r\}$. Define a mapping $c : V(P_q \circ K_r) \to \{1, 2, ...r + 1\}$ and the coloring as follows:

$$c(b_i) = (1, 2, 1, 2, ..., 1, 2)$$

$$c(d_{ij}) = \begin{cases} (2, 3, 4, 5, ..., r + 1)\ ; & i \text{ is } odd \\ (1, 3, 4, 5, ..., r + 1)\ ; & i \text{ is } even \end{cases}$$

This construction shows that $r + 1$ colors are sufficient, i.e., $\chi_{od}(P_q \circ K_r) \leq r + 1$. Since any attempt with less than $r + 1$ colors fails due to adjacency conflicts, we also have $\chi_{od}(P_q \circ K_r) \geq r + 1$. By Theorem 3.2[3], since $\chi(P_q) < \chi(K_r) + 1$, we have $\chi_{od}(P_q \circ K_r) = r + 1 = \chi(K_r) + 1$. This matches the construction, confirming that $r + 1$ colors are both sufficient and necessary.

□

Theorem 3.2. *For any $q > 1$, $r > 2$, the odd chromatic number of corona product of path with cycle graph is*

$$\chi_{od}(P_q \circ C_r) = \chi(C_r) + 1$$

Proof. Let $\{b_i : 1 \leq i \leq q\}$ be the q vertices of P_q and $\{a_j : 1 \leq j \leq r\}$ be the r vertices of C_r. Let the vertices of graph C_r in the i^{th} copy be $\{a_{ij} : 1 \leq i \leq q, 1 \leq j \leq r\}$.
Case 1: For odd r

Define a mapping $c : V(P_q \circ C_r) \to \{1, 2, 3, 4\}$ and the coloring as follows:

$$c(b_i) = (1, 2, 1, 2, ..., 1, 2)$$

$$c(a_{ij}) = \begin{cases} (2, 3, 2, 3, ..., 4)\ ; & i \text{ is } odd \\ (1, 3, 1, 3, ..., 4)\ ; & i \text{ is } even \end{cases}$$

This construction shows that 4 colors are sufficient, i.e., $\chi_{od}(P_q \circ C_r) \leq 4$. Since any attempt with less than 4 colors fails due to adjacency conflicts, we also have $\chi_{od}(P_q \circ C_r) \geq 4$. Moreover, for odd r, we have $\chi(P_q) \leq \chi(C_r) + 1$. Thus, corollary 3.1[3] implies:

$$\chi_{od}(P_q \circ C_r) \in [\chi(C_r) + 1,\ \chi(C_r) + 2] = [4,\ 5].$$

Our construction confirms that the lower bound is tight, establishing the result.

$$\chi_{od}(P_q \circ C_r) = 4 = \chi(C_r) + 1$$

Case 2: For even r

Define a mapping $c : V(P_q \circ C_r) \to \{1, 2, 3\}$ and the coloring as follows:
Subcase 1: if $r \equiv 0(mod\, 4)$

$$c(b_i) = (1, 2, 3, ..., 1, 2, 3)$$

$$c(a_{ij}) = \begin{cases} (2, 3, 2, 3, ..., 2, 3)\ ; & i + 2 \equiv 0(mod\, 3) \\ (1, 3, 1, 3, ..., 1, 3)\ ; & i + 1 \equiv 0(mod\, 3) \\ (1, 2, 1, 2, ..., 1, 2)\ ; & i \equiv 0(mod\, 3) \end{cases}$$

Subcase 2: if $r \not\equiv 0(mod\, 4)$

$$c(b_i) = (1, 2, ..., 1, 2)$$

$$c(a_{ij}) = \begin{cases} (2, 3, ..., 2, 3)\ ; & i \text{ is } odd \\ (1, 3, ..., 1, 3)\ ; & i \text{ is } even \end{cases}$$

The construction of subcases 1, 2 shows that 3 colors are sufficient, i.e., $\chi_{od}(P_q \circ C_r) \leq 3$. Since any attempt with less than 3 colors fails due to adjacency conflicts, we also have $\chi_{od}(P_q \circ C_r) \geq 3$. Moreover, for even r, we have $\chi(P_q) \leq \chi(C_r) + 1$, since $\chi(P_q) = 2$ and $\chi(C_r) = 2$. Thus, corollary 3.1[3] implies:

$$\chi_{od}(P_q \circ C_r) \in [\chi(C_r) + 1,\ \chi(C_r) + 2] = [3,\ 4].$$

Our construction confirms that the lower bound is tight, establishing the result.

$$\chi_{od}(P_q \circ C_r) = 3 = \chi(C_r) + 1$$

□

Theorem 3.3. *For any $q, r > 1$, the odd chromatic number of corona product of path with path graph is*

$$\chi_{od}(P_q \circ P_r) = \chi(P_r) + 1$$

Proof. Let $\{b_i : 1 \leq i \leq q\}$ be the q vertices of P_q and $\{p_j : 1 \leq j \leq r\}$ be the r vertices of P_r. Let the vertices of graph P_r in the i^{th} copy be $\{p_{ij} : 1 \leq i \leq q, 1 \leq j \leq r\}$.
Define a mapping $c : V(P_q \circ P_r) \to \{1, 2, 3\}$ and the coloring as follows:
Case 1: if $r \not\equiv 0, 1(mod\,4)$

$$c(b_i) = (1, 2, 1, 2, ..., 1, 2)$$

$$c(p_{ij}) = \begin{cases} (2, 3, 2, 3, ..., 2, 3)\ ; & i \text{ is } odd \\ (2, 3, 2, 3, ..., 2, 3)\ ; & i \text{ is } even \end{cases}$$

Case 2: if $r \equiv 0(mod\,4)$

$$c(b_i) = (1, 2, 3, ..., 1, 2, 3)$$

$$c(p_{ij}) = \begin{cases} (2, 3, 2, 3, ..., 2, 3)\ ; & i + 2 \equiv 0(mod\,3) \\ (1, 3, 1, 3, ..., 1, 3)\ ; & i + 1 \equiv 0(mod\,3) \\ (1, 2, 1, 2, ..., 1, 2)\ ; & i \equiv 0(mod\,3) \end{cases}$$

Case 3: if $r \equiv 1(mod\,4)$

$$c(b_i) = (1, 2, , ..., 1, 2)$$

$$c(p_{ij}) = \begin{cases} (3, 2, 3, 2, ..., 3, 2)\ ; & i = 1 \\ (3, 1, 3, 1, ..., 3, 1)\ ; & i = q \\ (2, 3, 2, 3, ..., 2, 3)\ ; & i \text{ is odd and } i \neq 1 \\ (1, 3, 1, 3, ..., 1, 3)\ ; & i \text{ is even and } i \neq q \end{cases}$$

The construction of above cases shows that 3 colors are sufficient $\chi_{od}(P_q \circ P_r) \leq 3$. Since any attempt with less than 3 colors fails due to adjacency conflicts, we also have $\chi_{od}(P_q \circ P_r) \geq 3$. Moreover, we have $\chi(P_q) \leq \chi(P_r) + 1$. Thus, corollary 3.1[3] implies:

$$\chi_{od}(P_q \circ P_r) \in [\chi(P_r) + 1,\ \chi(P_r) + 2] = [3,\ 4].$$

Our construction confirms that the lower bound is tight, establishing the result.

$$\chi_{od}(P_q \circ P_r) = 3 = \chi(P_r) + 1$$

□

Theorem 3.4. *For any $q, r > 2$, the odd chromatic number of corona product of complete with complete graph is*

$$\chi_{od}(K_q \circ K_r) = \begin{cases} \chi(K_r) + 1\ ; & if\ \ \chi(K_q) < \chi(K_r) + 1, \\ \chi(K_r) + 2\ ; & if\ \ \chi(K_q) = \chi(K_r) + 1, \\ \chi(K_q)\ ; & if\ \ \chi(K_q) \geq \chi(K_r) + 2 \end{cases}$$

Proof. Let $\{b_i : 1 \leq i \leq q\}$ be the q vertices of K_q and $\{B_j : 1 \leq j \leq r\}$ be the r vertices of K_r. Let the vertices of graph K_r in the i^{th} copy be $\{B_{ij} : 1 \leq i \leq q, 1 \leq j \leq r\}$.

Case 1: For $\chi(K_q) < \chi(K_r) + 1$

Define a mapping $c : V(K_q \circ K_r) \to \{1, 2, ...r + 1\}$ and the coloring as follows:

$$C(b_i) = i$$
$$c(B_{ij}) = (1, 2, \ldots, r + 1) \setminus \{i\}$$

This construction shows that $r + 1$ colors are sufficient, i.e., $\chi_{od}(K_q \circ K_r) \leq r + 1$. Since any attempt with less than $r + 1$ colors fails due to adjacency conflicts, we also have $\chi_{od}(K_q \circ K_r) \geq r+1$. By Theorem 3.2[3], since $\chi(K_q) < \chi(K_r)+1$, we have $\chi_{od}(K_q \circ K_r) = r+1 = \chi(K_r)+1$. This matches the construction, confirming that $r + 1$ colors are both sufficient and necessary.

Case 2: For $\chi(K_q) = \chi(K_r) + 1$

Define a mapping $c : V(K_q \circ K_r) \to \{1, 2, ...r + 2\}$ and the coloring as follows:

$$C(b_i) = i$$
$$c(B_{ij}) = (1, 2, \ldots, r + 2) \setminus \{i, r + 1\}$$

This construction shows that $r + 2$ colors are sufficient, i.e., $\chi_{od}(K_q \circ K_r) \leq r + 2$. Since any attempt with less than $r + 2$ colors fails due to adjacency conflicts, we also have $\chi_{od}(K_q \circ K_r) \geq r+2$. By Theorem 3.2[3], since $\chi(K_q) = \chi(K_r)+1$, we have $\chi_{od}(K_q \circ K_r) = r+2 = \chi(K_r)+2$. This matches the construction, confirming that $r + 2$ colors are both sufficient and necessary.

Case 3: For $\chi(K_q) \geq \chi(K_r) + 2$

Define a mapping $c : V(K_q \circ K_r) \to \{1, 2, ...q\}$ and the coloring as follows:

$$C(b_i) = i$$
$$c(B_{ij}) = (1, 2, \ldots, r + 1) \setminus \{i\}$$

This construction shows that q colors are sufficient, i.e., $\chi_{od}(K_q \circ K_r) \leq q$. Since any attempt with less than q colors fails due to adjacency conflicts, we also have $\chi_{od}(K_q \circ K_r) \geq q$. By Theorem 3.2[3], since $\chi(K_q) \geq \chi(K_r) + 2$, we have $\chi_{od}(K_q \circ K_r) = q = \chi(K_q)$. This matches the construction, confirming that q colors are both sufficient and necessary. □

Theorem 3.5. *For any $q > 2, r > 1$, the odd chromatic number of corona product of complete with path graph is*

$$\chi_{od}(K_q \circ P_r) = \begin{cases} \begin{cases} \chi(P_r) + 2\,; & \text{if } r \equiv 2 (mod\ 4) \\ \chi(P_r) + 1\,; & \text{if } r \equiv 0, 1, 3 (mod\ 4) \end{cases} \;;\; For\ \ q = 3 \\ \chi(K_q)\,;\ \ For\ \ q > 3 \end{cases}$$

Proof. Let $\{b_i : 1 \leq i \leq q\}$ be the q vertices of K_q and $\{p_j : 1 \leq j \leq r\}$ be the r vertices of P_r. Let the vertices of graph C_r in the i^{th} copy be $\{p_{ij} : 1 \leq i \leq q, 1 \leq j \leq r\}$.

Case 1: For $q = 3$

Subcase 1: if $r \equiv 2(mod\ 4)$

Define a mapping $c : V(K_q \circ P_r) \to \{1, 2, 3, 4\}$ and the coloring as follows:

$$c(b_1, b_2, b_3) = (1, 2, 3)$$

$$c(p_{ij}) = \begin{cases} (2, 3, 2, 3, ..., 4)\,; & i = 1 \\ (1, 3, 1, 3, ..., 4)\,; & i = 2 \\ (1, 2, 1, 2, ..., 4)\,; & i = 3 \end{cases}$$

This construction shows that 4 colors are sufficient, i.e., $\chi_{od}(K_q \circ P_r) \leq 4$. Since any attempt with less than 4 colors fails due to adjacency conflicts, we also have $\chi_{od}(K_q \circ P_r) \geq 4$. Moreover, for $q = 3$, we have $\chi(K_q) \leq \chi(P_r) + 1$. Thus, corollary 3.1[3] implies:

$$\chi_{od}(K_q \circ P_r) \in [\chi(P_r) + 1,\ \chi(P_r) + 2] = [3,\ 4].$$

Our construction confirms that the upper bound is tight, establishing the result.

$$\chi_{od}(K_q \circ P_r) = 4 = \chi(P_r) + 2$$

Subcase 2: if $r \equiv 1, 3(mod\, 4)$
Define a mapping $c : V(K_q \circ P_r) \rightarrow \{1, 2, 3\}$ and the coloring as follows:

$$c(b_1, b_2, b_3) = (1, 2, 3)$$

$$c(p_{ij}) = \begin{cases} (2,3,2,3,...,2)\ ; & i = 1 \\ (1,3,1,3,...,1)\ ; & i = 2 \\ (1,2,1,2,...,1)\ ; & i = 3 \end{cases}$$

Subcase 3: if $r \equiv 0(mod\, 4)$
Define a mapping $c : V(K_q \circ P_r) \rightarrow \{1, 2, 3\}$ and the coloring as follows:

$$c(b_1, b_2, b_3) = (1, 2, 3)$$

$$c(p_{ij}) = \begin{cases} (2,3,2,3,...,2,3)\ ; & i = 1 \\ (1,3,1,3,...,1,3)\ ; & i = 2 \\ (1,2,1,2,...,1,2)\ ; & i = 3 \end{cases}$$

The construction of subcases 2,3 shows that 3 colors are sufficient, i.e., $\chi_{od}(K_q \circ P_r) \leq 3$. Since any attempt with less than 3 colors fails due to adjacency conflicts, we also have $\chi_{od}(K_q \circ P_r) \geq 3$. Moreover, for $q = 3$, we have $\chi(K_q) \leq \chi(P_r) + 1$. Thus, corollary 3.1[3] implies:

$$\chi_{od}(K_q \circ P_r) \in [\chi(P_r) + 1,\ \chi(P_r) + 2] = [3,\ 4].$$

Our construction confirms that the lower bound is tight, establishing the result.

$$\chi_{od}(K_q \circ P_r) = 3 = \chi(P_r) + 1$$

Case 3: For $q > 3$

$$c(b_i) = i$$

The coloring pattern for $c(p_{ij})$ for this case cannot be specifically determined. However, the coloring pattern is same as the above case but sequel of colors may differ among q colors for the copies of path graph P_r, along which odd coloring must get satisfied. This construction shows that q colors are sufficient, i.e., $\chi_{od}(K_q \circ P_r) \leq q$. Since any attempt less than q colors fails due to adjacency conflicts, we also have $\chi_{od}(K_q \circ P_r) \geq q$. Moreover, for $q > 3$, we have $\chi(K_q) > \chi(P_r) + 1$. Hence, by corollary 3.1[3], it follows that:

$$\chi_{od}(K_q \circ P_r) = q = \chi(K_q)$$

□

Theorem 3.6. *For any $q > 2, r > 3$, the odd chromatic number of corona product of complete with cycle graph is*

$$\chi_{od}(K_q \circ C_r) = \begin{cases} \chi(C_r) + 1\ ; & For\ \ q = 3 \\ \chi(K_q) & ;\ \ For\ \ q > 3 \end{cases}$$

Proof. Let $\{b_i : 1 \leq i \leq q\}$ be the q vertices of K_q and $\{a_j : 1 \leq j \leq r\}$ be the r vertices of C_r. Let the vertices of graph C_r in the i^{th} copy be $\{a_{ij} : 1 \leq i \leq q, 1 \leq j \leq r\}$.
Case 1: For $q = 3$
Subcase 1: if $r \equiv 0(mod\, 4)$
Define a mapping $c : V(K_q \circ C_r) \rightarrow \{1, 2, 3\}$ and the coloring as follows:

$$c(b_1, b_2, b_3) = (1, 2, 3)$$

$$c(a_{ij}) = \begin{cases} (2,3,2,3,...,2,3)\ ; & i = 1 \\ (1,3,1,3,...,1,3)\ ; & i = 2 \\ (1,2,1,2,...,1,2)\ ; & i = 3 \end{cases}$$

The construction shows that 3 colors are sufficient, i.e., $\chi_{od}(K_q \circ C_r) \leq 3$. Since any attempt with less than 3 colors fails due to adjacency conflicts, we also have $\chi_{od}(K_q \circ C_r) \geq 3$. Moreover, for $q = 3$, we have $\chi(K_q) \leq \chi(C_r) + 1$. Thus, corollary 3.1[3] implies:

$$\chi_{od}(K_q \circ C_r) \in [\chi(C_r) + 1,\ \chi(C_r) + 2] = [3,\ 4].$$

Our construction confirms that the lower bound is tight, establishing the result.

$$\chi_{od}(K_q \circ C_r) = 3 = \chi(C_r) + 1$$

Subcase 2: if $r \equiv 1, 3 (mod\, 4)$

Define a mapping $c : V(K_q \circ C_r) \to \{1, 2, 3, 4\}$ and the coloring as follows:

$$c(b_1, b_2, b_3) = (1, 2, 3)$$

$$c(a_{ij}) = \begin{cases} (2, 3, 2, 3, ..., 4)\ ; & i = 1 \\ (1, 3, 1, 3, ..., 4)\ ; & i = 2 \\ (1, 2, 1, 2, ..., 4)\ ; & i = 3 \end{cases}$$

Subcase 3: if $r \equiv 2 (mod\, 4)$

Define a mapping $c : V(K_q \circ C_r) \to \{1, 2, 3, 4\}$ and the coloring as follows:

$$c(b_1, b_2, b_3) = (1, 2, 4)$$

$$c(a_{ij}) = \begin{cases} (2, 3, 2, 3, ..., 2, 3)\ ; & i = 1 \\ (1, 3, 1, 3, ..., 1, 3)\ ; & i = 2 \\ (1, 2, 1, 2, ..., 1, 2)\ ; & i = 3 \end{cases}$$

The construction of subcases 2,3 shows that 4 colors are sufficient, i.e., $\chi_{od}(K_q \circ C_r) \leq 4$. Since any attempt with less than 4 colors fails due to adjacency conflicts, we also have $\chi_{od}(K_q \circ C_r) \geq 4$. Moreover, for $q = 3$, we have $\chi(K_q) \leq \chi(C_r) + 1$. Thus, corollary 3.1[3] implies:

$$\chi_{od}(K_q \circ C_r) \in [\chi(C_r) + 1,\ \chi(C_r) + 2] = [4,\ 5].$$

Our construction confirms that the lower bound is tight, establishing the result.

$$\chi_{od}(K_q \circ C_r) = 4 = \chi(C_r) + 1$$

Case 2: For $q > 3$

$$c(b_i) = i$$

This construction shows that q colors are sufficient, i.e., $\chi_{od}(K_q \circ C_r) \leq q$. Since any attempt less than q colors fails due to adjacency conflicts, we also have $\chi_{od}(K_q \circ C_r) \geq q$. Moreover, for $q > 3$, we have $\chi(K_q) > \chi(C_r) + 1$. Hence, by corollary 3.1[3], it follows that:

$$\chi_{od}(K_q \circ C_r) = q = \chi(K_q)$$

□

4 Conclusion remarks

This work investigates the odd chromatic number $\chi_{od}(G)$ for corona products involving path graphs P_q, complete graphs K_q, and secondary graphs P_r, K_r, and C_r. By examining their structural and neighborhood properties, we derive exact values or tight bounds for $\chi_{od}(G)$, thereby advancing the study of parity-based coloring in composite graph constructions.

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