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Mathematical Analysis of Inverter Fed Induction Motor Modeled by Caputo Fractional Derivative

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Keywords: Caputo fractional derivative, controllability, induction motor model, matlab/ simulink, observability, stability.

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Abstract Fractional order derivatives are preferred for accurate modeling of complex and memory-dependent systems and it is a better tool for analyzing their dynamics. In this paper, the Caputo derivative is used to explore the fractional order differential equations of the inverter fed induction motor drive. It is the most appropriate fractional operator for modeling real world systems such as power electronic converters, electric motors etc. The mathematical state space model of the induction motor is presented in which the stator currents $(d - q)$ and rotor fluxes are considered as the state variables. The rotor speed is derived from the obtained state variables and load torque. The important properties of the fractional order induction motor drive system such as the controllability, observability and stability are analyzed in detail and validated in simulation using fractional toolbox (FOMCON). The performance of the drive system is tested on simulink environment in closed loop with PI speed controller for different running conditions. The simulation results are presented for integer order and different fractional order. The results are compared and the most effective fractional order is identified with respect to the response of speed, torque, stator currents and rotor fluxes.

1 Introduction

Since 17th century, fractional calculus has been used in various fields like biologists, chemists, economists, engineers, and physicists began to examine how to define the fractional derivative and have been familiar in Fractional Differential Equation(FDE). FDE's have recently received a lot of attention due to their extensive use in the mathematical modeling of physics, engineering, biological phenomena, and viscoelasticity. It is a type of differential equation that has a fractional derivative. The subject of applied mathematics known as differential equations has grown in popularity. A generalization of an classical differential operator is treated as the fractional-order differential operator [27, 12]. It contains three widely used definitions: Grunwald-Letnikov, Riemann-Liouville (R-L) and Caputo. Furthermore, the Caputo derivative and Riemann-Liouville are frequently employed in fractional calculus scenarios in engineering and sciences. Since, these fractional derivatives show many benefits, they are not suitable in all scenarios. When using fractional differential equations to simulate real-world activities, the R-L derivative has some difficulties. A constant's R-L derivative is not zero. Furthermore, the fractional derivation of instant exponential and Mittag-Leffler functions have singularity at the origin if an arbitrary function is constant at the origin. The R-L fractional derivative's applicability spectrum is constrained by these difficulties. Higher regularity requirements are necessary for differentiability in order to compute a function's fractional derivative in the Caputo sense. To accomplish this, the function's derivative is estimated. Caputo derivatives are only derived for differentiable functions, while R-L fractional derivatives of all orders may exist for functions without a first-order derivative [28, 25, 29, 30].

FDE's are a powerful mathematical tool for modeling systems that exhibit non-local or memory-dependent behavior. In the case of induction motors, FDE's can be used to model the dynamic behavior of the rotor, which is inherently non-local due to the presence of magnetic fields. One application of FDE's in induction motors is in modeling the dynamic behavior of the rotor during startup and steady-state operation [14, 20, 21, 23]. Specifically, FDE's can be used to model the transient behavior of the rotor flux and current during the startup period, as well as the steady-state behavior of the rotor during normal operation. Another application of FDE's in induction motors is in the design of control systems for regulating the motor's speed and torque [22, 24, 26, 28]. FDE-based models can be used to develop advanced control algorithms that take into account the non-local and memory-dependent behavior of the rotor, which can lead to more accurate and efficient control of the motor's performance.

Simulation tools are used for the mathematical model of electrical machines to study their dynamic properties. Newer, user-friendly simulation software packages are now accessible in large part to the significant advancements in computer hardware and software technologies. Matlab is an excellent performance simulation tool for numerical computations. It combines computation, visuals and programming in a user-friendly environment where problem statements are described using well-known mathematical notations. Systems of all categories can be simulated using Matlab Simulink block sets. It is used for the simulation of vector control of an induction motor and other electric motors [9, 7, 11, 6, 10].

Inverter-fed induction motors are widely used in variable speed industry applications, renewable energy systems, and electric vehicles. FDE's can be used to model the behavior of the inverter-fed induction motor in a more accurate way. One application of FDE's in inverter-fed induction motors is in the control of the motor's speed and torque. By using FDE's, it is possible to design controllers that can provide better performance compared to traditional control techniques. FDE-based controllers can improve the transient response and reduce the steady-state error of the motor. Another application of FDE's in inverter-fed induction motors is in fault detection and diagnosis. It's models can provide better accuracy in detecting and diagnosing faults in the motor compared to traditional models. This is because FDE's can capture the non-integer order dynamics of the motor, which can affect the motor's behavior in the presence of faults. Overall, the use of FDE's in inverter-fed induction motors can improve the performance and reliability of the motor in various applications [2, 8, 7, 16, 17].

In [1], R. Gunabalan et al., investigated the simulation of inverter fed induction motor derive with LabVIEW using state space modeling, and software was chosen because of its powerful graphical user interface, flexible programming language, and integrated tools made especially for testing, measuring, and controlling.

Inspired by the above work the Caputo sense integer and non-integer order of the inverter fed induction motor model is discussed. Additionally, a comparison between several non-integer and integer order models have been made.

The following is a summary of the paper's layout: The inverter-fed induction motor modeling is presented in section (2) of this study. The mathematical results of observability and controllability are discussed in section (4). Additionally, the principles of the fractional order induction motor model of stability concepts are studied. The distinct fractional orders of induction motors are investigated in section (5) and simulation results are discussed.

2 Modeling of Induction Motor

For driving AC motors, induction heating, UPS, and other operations, voltage source inverters are frequently employed. Devices from the transistor family are typically preferred in voltage source inverters due to self commutation and high switching frequency. A voltage source inverter possesses DC voltage at the input side and converts it to AC voltage on output side. Depending on the speed control range, the AC voltage and frequency may be either variable or constant. Before creating a prototype, it is frequently convenient to simulate the system performance when creating a new converter circuit or a control strategy for a drive system. The technique enables for performance optimization in addition to validating the system's functionality. In addition to control the circuit parameters, the influence of plant parameter configuration can be investigated. Both linear and nonlinear system equations can be solved using state space equations. Any first order differential equations characterizing a control system could be converted into fractional order equations in state space models. In order to make it simple for software tools to identify the result, these equations are set up as matrix equations. The system structure and the performance objectives of the study influence the choice of state variables. Stator currents and rotor fluxes have been selected as state variables for induction motors. The state variables and motor parameters are used to estimate the rotor speed and torque.

The induction motor's continuous state space model appears as:

$$D^{\alpha}Y = AY + BU$$
$$X = CY$$

The state equation of the stator currents and rotor fluxes are as follows:

$$\begin{aligned}
{}^{C}D^{\alpha}y_1 &= -\beta\gamma y_1 + \omega_e y_2 + \frac{\mu}{\tau}y_3 + \mu\omega_r y_4 + \epsilon v_{ds}\\
{}^{C}D^{\alpha}y_2 &= -\omega_e y_1 - \beta\gamma y_2 - \mu\omega_r y_3 + \frac{\mu}{\tau}y_4 + \epsilon v_{qs}\\
{}^{C}D^{\alpha}y_3 &= \tau_1 y_1 - \frac{y_3}{\tau} + (\omega_e - \omega_r)y_4\\
{}^{C}D^{\alpha}y_4 &= \tau_1 y_2 - (\omega_e - \omega_r)y_3 - \frac{1}{\tau_r}y_4
\end{aligned} \tag{2.1}$$

The resulting stator currents and rotor fluxes are used to estimate the speed. The rotor speed equation is

$$\dot{wr} = \frac{3}{2}\sigma\left(y_3y_2 - y_4y_1\right) - \frac{1}{J}T_L \tag{2.2}$$

3 Basics Concepts

Definition 3.1. [24] The RL fractional order α integral for the function $f : \mathbb{J} \to \mathbb{R}$ is defined as

$$I^{\alpha}f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}f(s)ds, \quad t > 0,\ \mathbb{J} \in [0, T], \tag{3.1}$$

where $\Gamma(.)$ is the Euler function.

Definition 3.2. [12] The Caputo derivative of order α is defined by

$${}^{C}D^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^t \frac{f^n(s)}{(t-s)^{\alpha+1-n}}ds, \quad t > 0,\ n-1 < \alpha < n. \tag{3.2}$$

where ${}^{C}D^{\alpha}$ is an Caputo fractional derivative with order $\alpha \in (n-1, n)$ with limits t_0 to t.

4 Mathematical Analysis of Fed Induction Motor

In this section, the observability, controllability and stability results of induction motor are discussed. Control theory is widely used in induction motor control to regulate and improve the motor's performance. Some of the applications of control theory in induction motor are: Speed control, torque control, vector control, sensor-less control, etc. It plays a critical role in improving the performance and reliability of induction motors. It enables the motor to operate efficiently, accurately, and safely under a wide range of operating conditions.
Since the initial value of a FDE with a Caputo derivative is the same as that of an classical differential equation, the Caputo derivative is chosen.

Table 1: Parameters and Notations

(a) Parameter values

R_s=14.775 Ω	R_r=4.767 Ω
p=4	L_m=0.7485 H
fb=50 Hz	J=0.00296 H
L_s=0.8075 H	L_r=0.8075 H

(b) Notations

Parameter	Notation	Parameter	Notation
β	$L_r R_s + \frac{L_m^2}{\tau_r}$	σ	$\frac{n_p L_m}{J L_r}$
γ	$\frac{1}{\sigma L_s L_r}$	μ	$\frac{L_m}{\sigma L_s L_r}$
τ_1	$\frac{L_m}{\tau_r}$	ϵ	$\frac{1}{\sigma L_s}$

4.1 Observability

Observability is the ability to infer the internal state of a system based on its external outputs. In the context of induction motors, it is important because it allows the motor's condition to be monitored without direct internal parameters measurements. This is often achieved by measuring the motor's voltage, current, and speed; using this information the internal states are estimated. By monitoring the motor's condition, it is possible to detect faults and prevent damage to the motor.
Here, $Y(t, t_0)y_0$ satisfies the homogeneous equation

$$D^{\alpha}y(t) = A(t)y(t), \tag{4.1}$$

where $y(t) \in \mathbb{R}^n$ and the $n \times n$ matrix $A(t)$.

Definition 4.1. [13] The system (4.1), (4.2) is observable (that is pair $(H(t), A(t))$ is observable) on an interval $[0, T]$ if

$$x(t) = H(t)y(t) = 0, \quad t \in [0, T] \tag{4.2}$$

implies that $y(t) = 0 \quad t \in [0, T]$
(which is the same as saying $y(0) = y_0 = 0$)

Theorem 4.1. The observed linear system (4.1), (4.2) is observable on $[0, T]$ if there are distinct points $a_1, a_2, \cdots, a_k \in [0, T]$ such that

$$\text{rank } V(a_1, a_2, \cdots, a_k, 0) = n$$

where,

$$V(a_1, a_2, \cdots, a_k, 0) = \begin{bmatrix} H(a_1)Y(a_1, 0) \\ H(a_2)Y(a_2, 0) \\ \vdots \\ H(a_k)Y(a_k, 0) \end{bmatrix}$$

Example 4.1. Consider the Caputo fractional induction motor system form

$${}^{C}D^{\alpha}y(t) = A(t)y(t)$$

with observation $X = CY$. The corresponding fractional order system after substituting all motor parameters in Table 1 in the expression (4.1).

$$\begin{bmatrix} D^{\alpha}y_1 \\ D^{\alpha}y_2 \\ D^{\alpha}y_3 \\ D^{\alpha}y_4 \end{bmatrix} = \begin{bmatrix} -166 & 314.2 & 48.13 & 3859 \\ -314.2 & -166 & -3859 & 48.13 \\ 4.419 & 0 & -5.903 & -159.1 \\ 0 & 4.419 & 159.1 & -5.903 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

and

$$x = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

that is

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}.$$

It can be readily verified that

$$X(t,1) = \begin{bmatrix} 2.53885e-19t & -1.82053e-19t & -1.18613e-17t & -3.1822e-17t \\ 1.82053e-19t & 2.53885e-19t & 3.1822e-17t & -1.18613e-17t \\ -3.66045e-20t & 1.31263e-20t & 3.33644e-19t & 4.21402e-18t \\ -1.31263e-20t & -3.66045e-20t & -4.21402e-18t & 3.33644e-19t \end{bmatrix},$$

$$H(t)X(t,1) = \begin{bmatrix} 3.86207e-19t & 4.83542e-20t & 1.60804e-17t & -3.91356e-17t \end{bmatrix}.$$

Then, taking $a_1 = 1, a_2 = 2, a_3 = 3, a_4 = 4$,

$$\text{rank} \begin{bmatrix} H(1)X(1,1) \\ H(2)X(2,1) \\ H(3)X(3,1) \\ H(4)X(4,1) \end{bmatrix} = \text{rank} \begin{bmatrix} 3.86207e-19 & 4.83542e-20 & 1.60804e-17 & -3.91356e-17 \\ 7.72414e-19 & 9.67083e-20 & 3.21607e-17 & -7.82712e-17 \\ 1.15862e-18 & 1.45063e-19 & 4.82411e-17 & -1.17407e-16 \\ 1.54483e-18 & 1.93417e-19 & 6.43214e-17 & -1.56542e-16 \end{bmatrix} = 4$$

and the system (4.1), (4.2) is observable for the time interval $[0, T]$.

4.2 Controllability

In the context of induction motors, controllability refers to the ability to control the motor's behavior using an external input signal. Induction motors are typically controlled by adjusting the frequency and amplitude of the applied voltage, which affects the motor's speed and torque. By using a controller, it is possible to control the speed of the motor. Controllability is important because it allows the motor to be operated at specific speeds and torques, which is necessary for many industrial applications.

Definition 4.2. [12] The linear control system (4.5) is controllable on [0, T] if for every pair of vectors $y_0, \ y_1 \in \mathbb{R}^n$, there is a control $u \in \mathbb{L}^2_m[0, \ T]$ such that the solution $y(t)$ of (4.5) (with this control) satisfies

$$y(0) = y_0 \tag{4.3}$$

and

$$y(T) = y_1 \tag{4.4}$$

when u is the corresponding solution x of (4.5) satisfies (4.3) and (4.4).

Theorem 4.2. The constant coefficient of control system

$$^{C}D^{\alpha}y(t) = Ay(t) + Bu(t) \tag{4.5}$$

is controllable if and only if

$$\text{rank}\left[B, AB, \cdots, A^{n-1}B\right] = n,$$

where $u \in \mathbb{R}^m$ and A and B are $n \times n$ and $n \times m$ continuous matrices on $[0, T]$ respectively.

Example 4.2. Consider the fractional order inverter fed induction motor model system

$$\begin{aligned} ^{C}D^{\alpha}y_1 =& -\beta\gamma y_1 + \omega_e y_2 + \frac{\mu}{\tau}y_3 + \mu\omega_r y_4 + \epsilon u_1(t) \\ ^{C}D^{\alpha}y_2 =& -\omega_e y_1 - \beta\gamma y_2 - \mu\omega_r y_3 + \frac{\mu}{\tau}y_4 + \epsilon u_2(t) \\ ^{C}D^{\alpha}y_3 =& \tau_1 y_1 - \frac{y_3}{\tau} + (\omega_e - \omega_r)y_4 \\ ^{C}D^{\alpha}y_4 =& \tau_1 y_2 - (\omega_e - \omega_r)y_3 - \frac{1}{\tau_r}y_4. \end{aligned} \tag{4.6}$$

Rank Matrix Results

By comparing the system in (4.6) with the standard form (4.5), the state matrix A and input matrix B are represented as

$$A = \begin{bmatrix} -\beta\gamma & \omega_e & \frac{\mu}{\tau} & \mu\omega_r \\ -\omega_e & -\beta\gamma & -\mu\omega_r & \frac{\mu}{\tau} \\ \tau_1 & 0 & -\frac{1}{\tau} & (\omega_e - \omega_r) \\ 0 & \tau_1 & (\omega_r - \omega_e) & -\frac{1}{\tau_r} \end{bmatrix}, B = \begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The numerical values of matrix A and B are

$$A = \begin{bmatrix} -166 & 314.2 & 48.13 & 3859 \\ -314.2 & -166 & -3859 & 48.13 \\ 4.419 & 0 & -5.903 & -159.1 \\ 0 & 4.419 & 159.1 & -5.903 \end{bmatrix}, B = \begin{bmatrix} 8.796 & 0 \\ 0 & 8.796 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{4.7}$$

The controllability rank matrix is

$$[B\ AB\ A^2B] = \begin{bmatrix} 8.7959 & 0 & -1460.0 & 2763.3 & -623910.0 & -767370.0 \\ 0 & 8.7959 & -2763.3 & -1460.0 & 767370.0 & -623910.0 \\ 0 & 0 & 38.866 & 0 & -6680.8 & 6026.4 \\ 0 & 0 & 0 & 38.866 & -6026.4 & -6680.8 \end{bmatrix}$$

and

$$\text{rank}[B\ AB\ A^2B] = 4.$$

From the theorem (4.2), the inverter fed induction motor system is controllable.

4.3 Stability

In a stable system, the stator return to its steady-state condition after a disturbance. In the context of induction motors, stability is important because it ensures that the motor can operate reliably under varying load conditions. Induction motors are nonlinear systems with complex dynamics. However, by using a controller to regulate the motor's behavior, it is possible to achieve stability and ensure that the motor operates reliably over a wide range of load conditions.

Definition 4.3. [13] The solution $\phi(t)$ is called stable if for every $\epsilon > 0$ there exists $\delta(\epsilon) > 0$ such that every solution $y(t)$ of (4.8) with $\|y(0) - \phi(0)\| < \delta(\epsilon)$ exists and satisfies $\|y(t) - \phi(t)\| < \epsilon$ on $J = [0, \infty)$. The solution $\phi(t)$ is called asymptotically stable if it is stable and there exists constant $\sigma > 0$ such that $y(t) - \phi(t) \to 0$ as $t \to \infty$ whenever $\|y(0) - \phi(0)\| \le \sigma$.

Consider the induction motor system of the form

$$^{C}D^{\alpha}y(t) = A(t)y(t), \tag{4.8}$$

where $A(t)$ is continuous $n \times n$ matrix.

Theorem 4.3. (i) Let $Y(t)$ be a fundamental matrix of the system (4.8). Then it is stable if and only if there exist a constant $K > 0$ with

$$\|Y(t)\| \le K, \quad t \in J. \tag{4.9}$$

(ii) The system (4.8) is asymptotically stable if and only if

$$\|Y(t)\| \to 0, \quad t \to \infty. \tag{4.10}$$

Example 4.3. Consider the fractional induction motor system

$$^{C}D^{\alpha}y(t) = A(t)y(t), \tag{4.11}$$

where A is given in (4.7). The eigenvalues of the A matrix are $-40.0418 \pm 149.37i$, $-131.848 \pm 304.425i$.

The eigenvalues have negative real parts and it is concluded that the above system (4.11) is asymptotically stable.

5 Numerical Simulation and Result Discussions

The effectiveness of the inverter fed induction motor drive under closed loop operation is presented in this section. The rating of the induction motor is 1 HP, 1415 rpm. The fractional order is added in the state space modeling of the induction motor to study the motor characteristics. The performance parameters such as stator qd-currents, rotor dq –fluxes, speed and torque of the motor are measured. The entire system is constructed in MATLAB/Simulink.

The simulation study is conducted using the code ode45 (Dormand-Prince) solver for a total of 2 s including a relative tolerance of 1e-3. The essential data are entered in MATLAB m-file. The visual user interface is used for entering the data. Along with the m-file program, the state space equation is constructed in MATLAB. The speed of the motor is derived from the resultant state variables and load torque. The closed loop operation of the drive system is performed using a PI speed controller. The rotor flux is maintained constant. The set speed is 1000 rad/s and motor runs at no load. At t = 1 s, a load of 2.5 Nm is applied to the motor.
For the fractional-order induction motor model given in (2.1), a distinct fractional order $\alpha \in (0, 1]$ is considered to study its effect on the dynamic behavior of the drive system. Figure 1 illustrates the speed response for various fractional orders. The corresponding performance indices—settling time and peak overshoot—are listed in Table 2. It is observed that the fractional order $\alpha = 0.9$ provides superior performance, exhibiting the lowest peak overshoot and fastest settling time. This demonstrates the effectiveness of fractional-order modeling in enhancing the transient behavior of induction motors compared to traditional integer-order models.

5.1 Load Torque Response Analysis

One of the main advantages of using fractional-order models is evident from the load torque responses. As shown in Figure 2, the settling time of the load torque is significantly improved when fractional orders are employed. Figures 2a - 2d present the torque responses for $\alpha = 0.7, 0.8,$ and 0.9, respectively, while Figure 2d corresponds to the integer-order case ($\alpha = 1$). It is clearly inferred that the settling time is considerably lower for fractional orders ($\alpha = 0.7, 0.8, 0.9$), whereas the integer-order model exhibits longer settling under loaded conditions. Among these, the torque response corresponding to $\alpha = 0.9$ is the smoothest, showing minimal oscillations and fast damping. Therefore, $\alpha = 0.9$ may be considered the most efficient fractional order for load torque dynamics.

5.2 Stator Current Response

Figures 3 and 4 show the dq-axis stator current waveforms under different fractional orders. It can be noted that fractional orders improve current smoothness, with $\alpha = 0.9$ yielding the best overall performance. The state portrait of the stator current is provided in Figure 5, which confirms reduced oscillatory behavior and stable trajectories for the fractional order $\alpha = 0.9$.

5.3 Rotor Flux Response

Similarly, Figures 6 and 7 depict the dq-axis rotor flux responses for different fractional orders. The corresponding state portraits of the rotor flux are presented in Figure 8. A key observation is that, when the flux is held constant, the fractional-order model with $\alpha = 0.9$ results in smoother rotor flux trajectories. Furthermore, the overall harmonic distortion in the stator current is significantly reduced. This highlights an important benefit of fractional-order modeling: the ability to enhance power quality and reduce harmonics, while simultaneously improving transient stability.

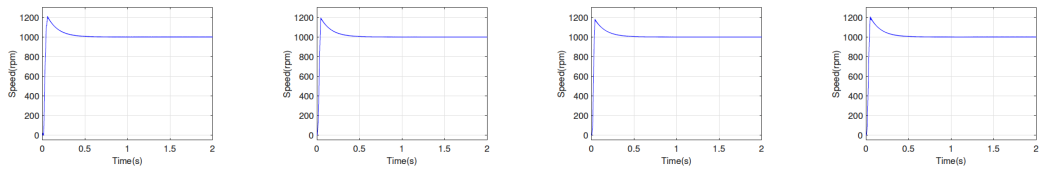
5.4 Summary of Observations

From the analysis of motor speed, torque, stator currents, and rotor fluxes, it is concluded that the fractional order $\alpha = 0.9$ consistently provides the most desirable performance characteristics:

- Reduced peak overshoot and faster settling time in speed response,

- Smooth torque transients with low oscillations under loaded conditions,
- Improved stator current and rotor flux responses with reduced harmonic distortion,
- Stable state portraits indicating better dynamic stability.

Overall, the results strongly suggest that fractional-order modeling of induction motors, particularly with $\alpha = 0.9$, offers a more efficient and robust representation than conventional integer-order approaches.

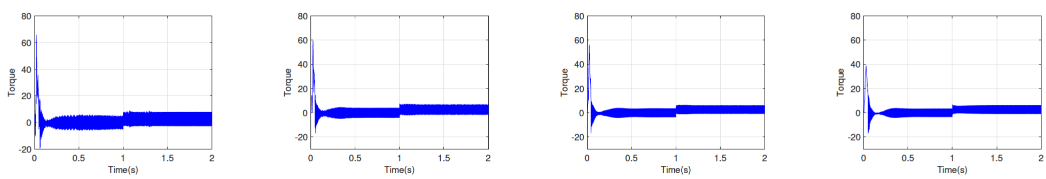


(a) order 0.7 (b) order 0.8 (c) order 0.9 (d) order 1

Figure 1: Motor speed response for various fractional order

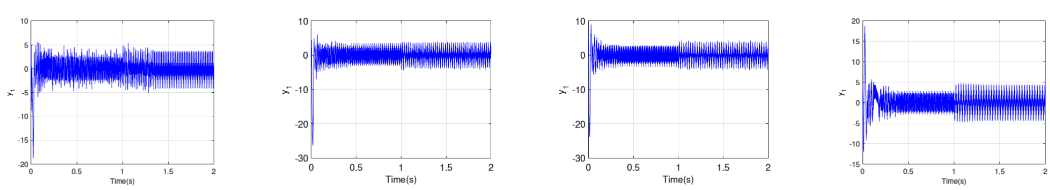
Table 2: Time domain response

Fractional Order	Rise Time (s)	Settling Time (s)	Peak Overshoot (%)
0.7	0.03	0.742	21.07
0.8	0.02	0.736	19.42
0.9	0.02	0.715	18.14
1	0.03	0.720	20.67



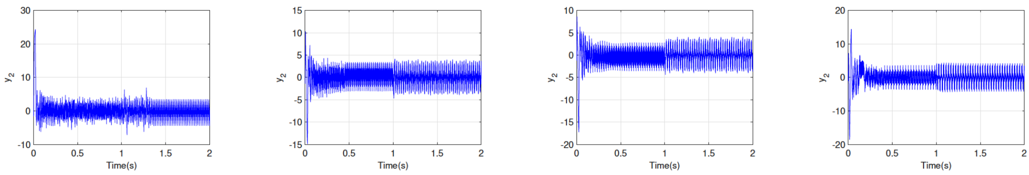
(a) fractional order 0.7 (b) fractional order 0.8 (c) fractional order 0.9 (d) integer order

Figure 2: Torque of integer and non-integer order induction motor model



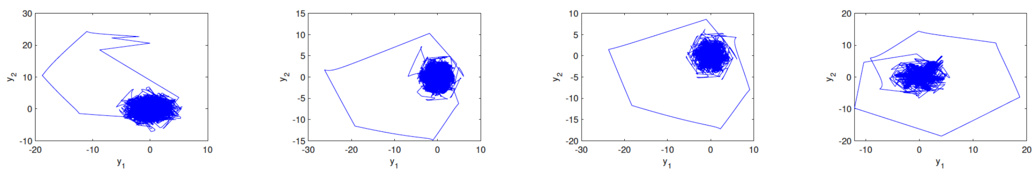
(a) order 0.7 (b) order 0.8 (c) order 0.9 (d) order 1

Figure 3: Stator current y_1 for various order



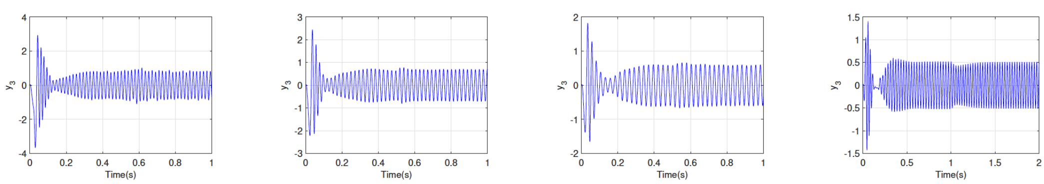
(a) order 0.7 (b) order 0.8 (c) order 0.9 (d) order 1

Figure 4: Stator current y_2 for various order



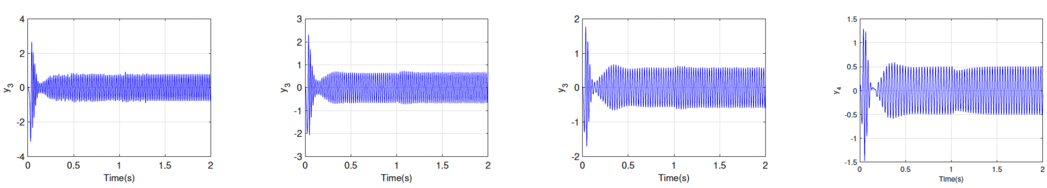
(a) order 0.7 (b) order 0.8 (c) order 0.9 (d) integer order

Figure 5: state space portraits of stator current with distinct fractional order



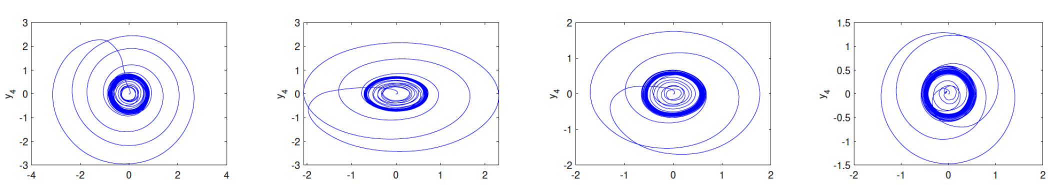
(a) order 0.7 (b) order 0.8 (c) order 0.9 (d) order 1

Figure 6: Rotor flux y_3 for distinct order



(a) order 0.7 (b) order 0.8 (c) order 0.9 (d) integer order

Figure 7: Rotor flux y_4 for various fractional order



(a) order 0.7 (b) order 0.8 (c) order 0.9 (d) order 1

Figure 8: State Space portraits of rotor fluxes with various order

Conclusion

In this paper, a simulation approach is carried out to evaluate the Caputo fractional order inverter fed induction motor's steady state and transient behaviors. The fractional order state space is used to model the induction motor. Induction motor's mechanical and electrical components are accomplished using intricate, described mathematical algorithms. Additionally, the controllability, observability and stability analysis of the fed induction motor are presented and validated with necessary theorems. The inverter fed induction motor's torque and rotor angular velocity are analyzed under different orders. The dynamic model described in this paper has produced highly accurate simulation results. Overall, FDE's offer a powerful and flexible framework for modeling and analyzing the dynamic behavior of induction motors, and they are an important tool for researchers and engineers working in the field of motor design and control. In future, the fractional inverter fed induction motor performance will be tested with fractional order PID controller.

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