Palestine Journal of Mathematics

Vol 15(Special Issue II)(2026) , 153–159

An Extensive Investigation of the Corona Covering Number in the Tensor Product of Path Graphs

Niveditha P, G. Mahadevan and C. Sivagnanam

MSC 2010 Classifications: Primary 33C20; Secondary 33C65.

Keywords and phrases: Vertex covering, pendant vertex, support vertex, tensor product.

The authors would like to thank the reviewers and editor for their constructive comments and valuable suggestions that improved the quality of our paper.

Corresponding Author: G. Mahadevan

Abstract *Let G be a graph. A subset $S \subseteq V(G)$ is called a vertex cover if every edge in $E(G)$ has at least one endpoint in S. The smallest possible size of a vertex cover in G is known as the vertex covering number, denoted by $\alpha(G)$. Recently, G. Mahadevan et al. introduced the concept of the corona covering number. A Vertex cover set $S \subseteq V(G)$ is considered a corona cover if every vertex in the induced subgraph $\langle S \rangle$ is either a pendant or a support vertex. The minimum size of such a set is called the corona covering number, represented as τ_C. In this study, we analyze this parameter for the tensor product of path graphs.*

1 Introduction

The Simple, finite, undirected graphs are considered. The vertex set and edge set of G are denoted by V and E, respectively. A vertex set $S \subseteq V(G)$ is a vertex cover set if $\langle V - S \rangle$ is an empty graph. The minimum cardinality of a vertex cover set is called the vertex covering number and it is expressed as $\alpha(G)$. Based on corona domination [1] and vertex covering, G. Mahadevan et al. defined the notion of corona covering numbers in [2]. A vertex cover set $S \subseteq V(G)$ is a corona cover set if every vertex $v \in S$ such that $d_{\langle S \rangle}(v) = 1$ or there exist a vertex $u \in S$ with $d_{\langle S \rangle}(u) = 1$ and $uv \in E$. The least cardinality of a corona cover set is the corona covering number of a graph and it is expressed as $\tau_C(G)$. The corona covering number of certain standard graphs and specific types of graphs has been examined in [2] and [3]. The cartesain product of two path has been discussed in [4] . In this article we analyse the tensor product of two paths. Let $P_n = (u_1, u_2, ..., u_n)$ and $P_k = (v_1, v_2, ..., v_m)$ be the paths. The vertex set of tensor product of paths P_n and P_k, $P_t \bigotimes P_k$ is the cartesian product of vertices of P_n and P_k and $\mid V(P_n \times P_k) \mid= mn$ and any two vertices in $P_t \bigotimes P_k$ is adjacent say (u_p, v_q) is adjacent to (u_r, v_s) if $d(u_p, u_r) = 1$ in P_n and $d(v_q, v_s) = 1$ in P_k [5][6]. The corona covering number for tensor product of two paths will be discussed in the section 2. Throughout this paper the collection of white vertices denote the corona cover set of the given graph.

2 Corona Covering number of Tensor product of paths

Let $P_t \bigotimes P_k$ be the tensor product of paths P_n and P_k, $V(P_t \bigotimes P_k) = \{v_{(h,g)} : 1 \leq h \leq n, 1 \leq g \leq m\}$ and $E(P_t \bigotimes P_k) = \{v_{(h,g)}v_{(h+1,g+1)} : 1 \leq h \leq n-1, 1 \leq g \leq m-1\} \cup \{v_{(h,g)}v_{(h+1,g-1)} : 1 \leq h \leq n-1, 2 \leq g \leq m\}$.

Observation
Since $P_2 \bigotimes P_k \cong 2P_k$,
$$\tau_C(P_2 \bigotimes P_k) = \tau_C(2P_k) = \begin{cases} 4 & for\ k = 2, \\ 2k - 2\lceil \frac{k}{3} \rceil & otherwise. \end{cases}$$

Theorem 2.1. If P_k is a path of order k, $k \geq 2$, then
$$\tau_C(P_3 \bigotimes P_k) = \begin{cases} 6(\frac{k}{4}) & for\ k \equiv 0\ (mod\ 4), \\ 6\lceil \frac{k}{4} \rceil - 4 & for\ k \equiv 1\ (mod\ 4), \\ 6\lceil \frac{k}{4} \rceil - 2 & for\ k \equiv 2\ (mod\ 4), \\ 6\lceil \frac{k}{4} \rceil - 1 & for\ k \equiv 3\ (mod\ 4). \end{cases}$$

Proof. Let $S_1 = \{v_{(2,g)} : 1 \leq g \leq k\} \cup \{v_{(3,g)} : g \equiv 2\ or\ 3\ (mod\ 4)\}$, $S_2 = \{v_{(2,g)} : 1 \leq g \leq k-1\} \cup \{v_{(3,g)} : g \equiv 2\ or\ 3\ (mod\ 4)\} \cup \{v_{(3,k-1)}\} \cup \{v_{(1,k-1)}\}$
$$\text{Assume } S = \begin{cases} S_1 & for\ k \equiv 0\ or\ 3\ (mod\ 4), \\ S_1 \cup \{v_{(3,k-1)}\} & for\ k \equiv 1\ (mod\ 4), \\ S_2 & for\ k \equiv 2\ (mod\ 4). \end{cases}$$
Then S is a corona cover set of $P_3 \bigotimes P_k$ and hence
$$\tau_C(P_3 \bigotimes P_k) \leq |S| = \begin{cases} 6(\frac{k}{4}) & for\ k \equiv 0\ (mod\ 4), \\ 6\lceil \frac{k}{4} \rceil - 4 & for\ k \equiv 1\ (mod\ 4), \\ 6\lceil \frac{k}{4} \rceil - 2 & for\ k \equiv 2\ (mod\ 4), \\ 6\lceil \frac{k}{4} \rceil - 1 & for\ k \equiv 3\ (mod\ 4). \end{cases}$$
Let S' be a corona cover set of $P_3 \bigotimes P_k$. Suppose D be a vertex cover set of cardinality at most
$$N = \begin{cases} 6(\frac{k}{4}) - 1 & for\ k \equiv 0\ (mod\ 4), \\ 6\lceil \frac{k}{4} \rceil - 5 & for\ k \equiv 1\ (mod\ 4), \\ 6\lceil \frac{k}{4} \rceil - 3 & for\ k \equiv 2\ (mod\ 4), \\ 6\lceil \frac{k}{4} \rceil - 2 & for\ k \equiv 3\ (mod\ 4). \end{cases}$$
Then $< D >$ has an isolated vertex, thus we have
$$|S'| \geq N + 1 = \begin{cases} 6(\frac{k}{4}) & for\ k \equiv 0\ (mod\ 4), \\ 6\lceil \frac{k}{4} \rceil - 4 & for\ k \equiv 1\ (mod\ 4), \\ 6\lceil \frac{k}{4} \rceil - 2 & for\ k \equiv 2\ (mod\ 4), \\ 6\lceil \frac{k}{4} \rceil - 1 & for\ k \equiv 3\ (mod\ 4). \end{cases}$$
Hence the result follows. □

Example 2.2.

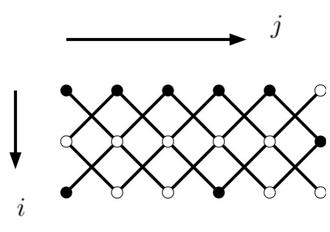

Figure 1. $P_3 \bigotimes P_6$

In the graph in figure 1, the collection of white vertices is corona cover set of minimum cardinality and hence $\tau_C(P_3 \bigotimes P_6) = 10$.

Theorem 2.3. If P_k is a path of order k, $k \geq 2$, then
$$\tau_C(P_4 \bigotimes P_k) = \begin{cases} 12(\frac{k}{5}) - 2 & for\ k \equiv 0\ (mod\ 5), \\ 12\lfloor \frac{k}{5} \rfloor & for\ k \equiv 1\ (mod\ 5), \\ 12\lfloor \frac{k}{5} \rfloor + 4 & for\ k \equiv 2\ (mod\ 5), \\ 12\lfloor \frac{k}{5} \rfloor + 6 & for\ k \equiv 3\ (mod\ 5), \\ 12\lceil \frac{k}{5} \rceil - 4 & for\ k \equiv 4\ (mod\ 5). \end{cases}$$

Proof. Let $S_1 = \{v_{(h,g)} : g \equiv 0 \text{ or } 2 \ (mod\ 5), 1 \leq h \leq 4, 1 \leq g \leq k-1\} \cup \{v_{(h,k-2)} : h = 2,3\} \cup \{v_{(h,k-1)} : 1 \leq h \leq 4\}$, $S_2 = \{v_{(h,g)} : g \equiv 0 \text{ or} 2 \ (mod\ 5), 1 \leq h \leq 4, 1 \leq g \leq k\} \cup \{v_{(h,g)} : g \equiv 3 \text{ or } 4 \ (mod\ 5), h = 2,3, 1 \leq g \leq k\}$, $S_3 = \{v_{(h,g)} : g \equiv 0 \text{ or } 2 \ (mod\ 5), 1 \leq g \leq k-5\} \cup \{v_{(h,g)}, v_{(h,k)} : g \equiv 3 \text{ or } 4 \ (mod\ 5), h = 2,3, 1 \leq g \leq k-4\} \cup \{v_{(h,g)} : 1 \leq h \leq 4, g = k-2, k-1\}$

Assume $S = \begin{cases} S_1 & for\ k \equiv 0 \ (mod\ 5), \\ S_3 & for\ k \equiv 2 \ (mod\ 4), \\ S_2 & otherwise. \end{cases}$ Then S is a corona cover set of $P_4 \bigotimes P_k$ and hence

$$\tau_C(P_4 \bigotimes P_k) \leq |S| = \begin{cases} 12(\frac{k}{5}) - 2 & for\ k \equiv 0 \ (mod\ 5), \\ 12\lfloor \frac{k}{5} \rfloor & for\ k \equiv 1 \ (mod\ 5), \\ 12\lfloor \frac{k}{5} \rfloor + 4 & for\ k \equiv 2 \ (mod\ 5), \\ 12\lfloor \frac{k}{5} \rfloor + 6 & for\ k \equiv 3 \ (mod\ 5), \\ 12\lceil \frac{k}{5} \rceil - 4 & for\ k \equiv 4 \ (mod\ 5). \end{cases}$$

Let S' be a corona cover set of $P_4 \bigotimes P_k$. Suppose D be a vertex cover set of cardinality at most

$N = \begin{cases} 12(\frac{k}{5}) - 3 & for\ k \equiv 0 \ (mod\ 5), \\ 12\lfloor \frac{k}{5} \rfloor - 1 & for\ k \equiv 1 \ (mod\ 5), \\ 12\lfloor \frac{k}{5} \rfloor + 3 & for\ k \equiv 2 \ (mod\ 5), \\ 12\lfloor \frac{k}{5} \rfloor + 5 & for\ k \equiv 3 \ (mod\ 5), \\ 12\lceil \frac{k}{5} \rceil - 5 & for\ k \equiv 4 \ (mod\ 5). \end{cases}$ Then either $< D >$ has an isolated vertex, thus we

have $|S'| \geq N + 1 = \begin{cases} 12(\frac{k}{5}) - 2 & for\ k \equiv 0 \ (mod\ 5), \\ 12\lfloor \frac{k}{5} \rfloor & for\ k \equiv 1 \ (mod\ 5), \\ 12\lfloor \frac{k}{5} \rfloor + 4 & for\ k \equiv 2 \ (mod\ 5), \\ 12\lfloor \frac{k}{5} \rfloor + 6 & for\ k \equiv 3 \ (mod\ 5), \\ 12\lceil \frac{k}{5} \rceil - 4 & for\ k \equiv 4 \ (mod\ 5). \end{cases}$

Hence the result follows. □

Example 2.4.

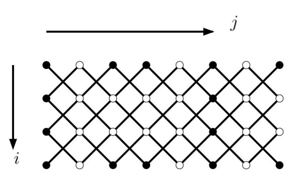

Figure 2. $P_4 \bigotimes P_8$

In the graph in figure 2, the collection of white vertices is corona cover set of minimum cardinality and hence $\tau_C(P_4 \bigotimes P_8) = 18$.

Theorem 2.5. Let P_n and P_k be the paths of order n and k respectively, $n \geq 5$, $k \geq 2$ and $n \equiv 0 \ (mod\ 4)$. Then $\tau_C(P_t \bigotimes P_k) = \begin{cases} \frac{5kn}{8} - 2 & for\ k \equiv 0 \ (mod\ 4), \\ \frac{5n}{2} \lfloor \frac{k}{4} \rfloor & for\ k \equiv 1 \ (mod\ 4), \\ \frac{5n}{2} \lfloor \frac{k}{4} \rfloor + (\frac{6n}{4} - 2) & for\ k \equiv 2 \ (mod\ 4), \\ \frac{5n}{2} \lfloor \frac{k}{4} \rfloor + \frac{3n}{2} & for\ k \equiv 3 \ (mod\ 4). \end{cases}$

Proof. Let $S_1 = \{v_{(h,g)} : 1 \leq h \leq n, g \equiv 0 \ (mod\ 2)\}$, $S_2 = \{v_{(h,g)} : 1 \leq h \leq n, g \equiv 1 \ (mod\ 2)\}$, $S_3 = \{v_{(h,g)} : h \equiv 2 \text{ or } 3 \ (mod\ 4), g \equiv 2 \ (mod\ 4)\}$, $S_4 = \{v_{(h,g)} : h \equiv 2 \text{ or } 3 \ (mod\ 4), g \equiv 3 \ (mod\ 4)\}$

Assume $S = \begin{cases} (S_1 \cup S_4) - \{v_{(1,k)}, v_{(n,k)}\} & for\ k \equiv 0 \ (mod\ 4), \\ S_1 \cup S_4 & for\ k \equiv 1 \text{ or } 3 \ (mod\ 4), \\ (S_2 \cup S_3) - \{v_{(1,1)}, v_{(1,k)}\} & for\ k \equiv 2 \ (mod\ 4). \end{cases}$

Then S is a corona cover set of $P_t \bigotimes P_k$ and hence

$$\tau_C(P_t \bigotimes P_k) \leq |S| = \begin{cases} \frac{5kn}{8} - 2 & for\ k\ \equiv 0\ (mod\ 4), \\ \frac{5n}{2}\lfloor \frac{k}{4} \rfloor & for\ k\ \equiv 1\ (mod\ 4), \\ \frac{5n}{2}\lfloor \frac{k}{4} \rfloor + (\frac{6n}{4} - 2) & for\ k\ \equiv 2\ (mod\ 4), \\ \frac{5n}{2}\lfloor \frac{k}{4} \rfloor + \frac{3n}{2} & for\ k\ \equiv 3\ (mod\ 4). \end{cases}$$

Let S' be a corona cover set of $P_t \bigotimes P_k$. Suppose D be a vertex cover set of cardinality at most $N = \begin{cases} \frac{5kn}{8} - 3 & for\ k\ \equiv 0\ (mod\ 4), \\ \frac{5n}{2}\lfloor \frac{k}{4} \rfloor - 1 & for\ k\ \equiv 1\ (mod\ 4), \\ \frac{5n}{2}\lfloor \frac{k}{4} \rfloor + (\frac{6n}{4} - 2) - 1 & for\ k\ \equiv 2\ (mod\ 4), \\ \frac{5n}{2}\lfloor \frac{k}{4} \rfloor + \frac{3n}{2} - 1 & for\ k\ \equiv 3\ (mod\ 4). \end{cases}$ Then $< D >$ has an isolated vertex,

thus we have $|S'| \geq N + 1 = \begin{cases} \frac{5kn}{8} - 2 & for\ k\ \equiv 0\ (mod\ 4), \\ \frac{5n}{2}\lfloor \frac{k}{4} \rfloor & for\ k\ \equiv 1\ (mod\ 4), \\ \frac{5n}{2}\lfloor \frac{k}{4} \rfloor + (\frac{6n}{4} - 2) & for\ k\ \equiv 2\ (mod\ 4), \\ \frac{5n}{2}\lfloor \frac{k}{4} \rfloor + \frac{3n}{2} & for\ k\ \equiv 3\ (mod\ 4). \end{cases}$

Hence the result follows. □

Example 2.6.

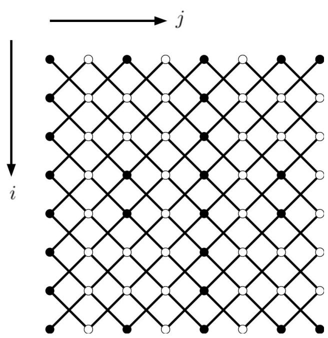

Figure 3. $P_8 \bigotimes P_8$

In the graph in figure 3, the collection of white vertices is corona cover set of minimum cardinality and hence $\tau_C(P_8 \bigotimes P_8) = 18$.

Theorem 2.7. Let P_t and P_k be the paths of order t and k respectively, $t \geq 5$, $k \geq 2$, if both t and k are not equal 5 and $t\ \equiv\ 1\ (mod\ 4)$.

Then $\tau_C(P_t \bigotimes P_k) = \begin{cases} 10\lfloor \frac{t}{4} \rfloor (\frac{k}{4}) & for\ k\ \equiv 0\ (mod\ 4), \\ 10\lfloor \frac{t}{4} \rfloor \lfloor \frac{k}{4} \rfloor + 6\lfloor \frac{t}{4} \rfloor & for\ k\ \equiv 2\ (mod\ 4), \\ 10\lfloor \frac{t}{4} \rfloor \lfloor \frac{k}{4} \rfloor + 8\lfloor \frac{t}{4} \rfloor & for\ k\ \equiv 3\ (mod\ 4), \\ does\ not\ exists & for\ k\ \equiv 1\ (mod\ 4). \end{cases}$

Proof. Let $S_1 = \{v_{(h,g)} : 1 \leq g \leq k, h \equiv 0\ (mod\ 2)\}$, $S_2 = \{v_{(h,g)} : g \equiv 2\ or\ 3\ (mod\ 4), h \equiv 3\ (mod\ 4)\}$, $S_3 = \{v_{(h,g)} : g \equiv 1\ or\ 2\ (mod\ 4), h \equiv 3\ (mod\ 4)\}$

Assume $S = \begin{cases} S_1 \cup S_2 & for\ k\ \equiv 0\ (mod\ 4), \\ S_1 \cup S_3 & for\ k\ \equiv 2\ or\ 3\ (mod\ 4), \\ Does\ not\ exists & for\ k\ \equiv 1\ (mod\ 4). \end{cases}$

Then S is a corona cover set of $P_t \bigotimes P_k$ and hence

$$\tau_C(P_t \bigotimes P_k) \leq |S| = \begin{cases} 10\lfloor \frac{t}{4} \rfloor (\frac{k}{4}) & for\ k\ \equiv 0\ (mod\ 4), \\ 10\lfloor \frac{t}{4} \rfloor \lfloor \frac{k}{4} \rfloor + 6\lfloor \frac{t}{4} \rfloor & for\ k\ \equiv 2\ (mod\ 4), \\ 10\lfloor \frac{t}{4} \rfloor \lfloor \frac{k}{4} \rfloor + 8\lfloor \frac{t}{4} \rfloor & for\ k\ \equiv 3\ (mod\ 4), \\ does\ not\ exists & for\ k\ \equiv 1\ (mod\ 4). \end{cases}$$

Let S' be a corona cover set of $P_t \bigotimes P_k$. Suppose D be a corona cover set of cardinality at most

$$N = \begin{cases} 10\lfloor\frac{t}{4}\rfloor(\frac{k}{4}) - 1 & for\ k\ \equiv 0\ (mod\ 4), \\ 10\lfloor\frac{t}{4}\rfloor\lfloor\frac{k}{4}\rfloor + 6\lfloor\frac{t}{4}\rfloor - 1 & for\ k\ \equiv 2\ (mod\ 4), \\ 10\lfloor\frac{t}{4}\rfloor\lfloor\frac{k}{4}\rfloor + 8\lfloor\frac{t}{4}\rfloor - 1 & for\ k\ \equiv 3\ (mod\ 4), \\ does\ not\ exists & for\ k\ \equiv 1\ (mod\ 4). \end{cases}$$

Then $< D >$ has an isolated vertex thus we have

$$|S'| \geq N + 1 = \begin{cases} 10\lfloor\frac{t}{4}\rfloor(\frac{k}{4}) & for\ k\ \equiv 0\ (mod\ 4), \\ 10\lfloor\frac{t}{4}\rfloor\lfloor\frac{k}{4}\rfloor + 6\lfloor\frac{t}{4}\rfloor & for\ k\ \equiv 2\ (mod\ 4), \\ 10\lfloor\frac{t}{4}\rfloor\lfloor\frac{k}{4}\rfloor + 8\lfloor\frac{t}{4}\rfloor & for\ k\ \equiv 3\ (mod\ 4), \\ does\ not\ exists & for\ k\ \equiv 1\ (mod\ 4). \end{cases}$$

Hence the result follows. □

Remark
If t and k are equal to 5, then $\tau_C(P_5 \bigotimes P_5) = 13$

Example 2.8.

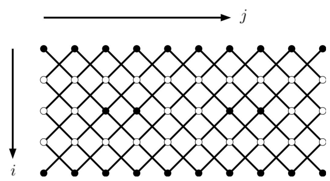

Figure 4. $P_5 \bigotimes P_{10}$

In the graph in figure 4, the collection of white vertices is corona cover set of minimum cardinality and hence $\tau_C(P_5 \bigotimes P_{10}) = 26$.

Theorem 2.9. Let P_t and P_k be the paths of order t and k respectively, $t \geq 5$, $k \geq 2$ and $t \equiv 2\ (mod\ 4)$. Then

$$\tau_C(P_t \bigotimes P_k) = \begin{cases} (10\lfloor\frac{t}{4}\rfloor + 6)(\frac{k}{4}) - 2 & for\ k\ \equiv 0\ (mod\ 4), \\ (10\lfloor\frac{t}{4}\rfloor + 6)\lfloor\frac{k}{4}\rfloor & for\ k\ \equiv 1\ (mod\ 4), \\ (10\lfloor\frac{t}{4}\rfloor + 6)\lfloor\frac{k}{4}\rfloor + (6\lfloor\frac{t}{4}\rfloor + 2) & for\ k\ \equiv 2\ (mod\ 3), \\ (10\lfloor\frac{t}{4}\rfloor + 6)\lfloor\frac{k}{4}\rfloor + (6\lfloor\frac{t}{4}\rfloor + 4) & for\ k\ \equiv 3\ (mod\ 4). \end{cases}$$

Proof. Let $S_1 = \{v_{(h,g)} : g \equiv 0\ (mod\ 2), 1 \leq h \leq t\}$, $S_2 = \{v_{(h,g)} : g \equiv 3\ (mod\ 4), h \equiv 3\ (mod\ 4), h \equiv 1\ or\ 2\ (mod\ 4)\}$, $S_3 = \{v_{(h,g)} : g \equiv 2\ (mod\ 4), h \equiv 1\ or\ 2\ (mod\ 4)\}$, $S_4 = \{\{v_{(h,1)} : 2 \leq h \leq t-1\} \cup \{v_{(h,g)} : g \equiv 1\ (mod\ 2), g \geq 3, 1 \leq h \leq t\}\}$

Assume $S = \begin{cases} S_1 \cup S_2 & for\ k\ \equiv 1\ or\ 3\ (mod\ 4), \\ S_3 \cup S_4 & for\ k\ \equiv 0\ or\ 2\ (mod\ 4). \end{cases}$

Then S is a corona cover set of $P_t \bigotimes P_k$ and hence

$$\tau_C(P_t \bigotimes P_k) \leq |S| = \begin{cases} (10\lfloor\frac{t}{4}\rfloor + 6)(\frac{k}{4}) - 2 & for\ k\ \equiv 0\ (mod\ 4), \\ (10\lfloor\frac{t}{4}\rfloor + 6)\lfloor\frac{k}{4}\rfloor & for\ k\ \equiv 1\ (mod\ 4), \\ (10\lfloor\frac{t}{4}\rfloor + 6)\lfloor\frac{k}{4}\rfloor + (6\lfloor\frac{t}{4}\rfloor + 2) & for\ k\ \equiv 2\ (mod\ 3), \\ (10\lfloor\frac{t}{4}\rfloor + 6)\lfloor\frac{k}{4}\rfloor + (6\lfloor\frac{t}{4}\rfloor + 4) & for\ k\ \equiv 3\ (mod\ 4). \end{cases}$$

Let S' be a corona cover set of $P_t \bigotimes P_k$. Suppose D be a vertex cover set of cardinality at most

$$N = \begin{cases} (10\lfloor\frac{t}{4}\rfloor + 6)(\frac{k}{4}) - 3 & for\ k\ \equiv 0\ (mod\ 4), \\ (10\lfloor\frac{t}{4}\rfloor + 6)\lfloor\frac{k}{4}\rfloor - 1 & for\ k\ \equiv 1\ (mod\ 4), \\ (10\lfloor\frac{t}{4}\rfloor + 6)\lfloor\frac{k}{4}\rfloor + (6\lfloor\frac{t}{4}\rfloor + 2) - 1 & for\ k\ \equiv 2\ (mod\ 3), \\ (10\lfloor\frac{t}{4}\rfloor + 6)\lfloor\frac{k}{4}\rfloor + (6\lfloor\frac{t}{4}\rfloor + 4) - 1 & for\ k\ \equiv 3\ (mod\ 4). \end{cases}$$

Then $< D >$ has an isolated vertex, thus we have

$$|S'| \geq N + 1 = \begin{cases} (10\lfloor\frac{t}{4}\rfloor + 6)(\frac{k}{4}) - 2 & for\ k\ \equiv 0\ (mod\ 4), \\ (10\lfloor\frac{t}{4}\rfloor + 6)\lfloor\frac{k}{4}\rfloor & for\ k\ \equiv 1\ (mod\ 4), \\ (10\lfloor\frac{t}{4}\rfloor + 6)\lfloor\frac{k}{4}\rfloor + (6\lfloor\frac{t}{4}\rfloor + 2) & for\ k\ \equiv 2\ (mod\ 3), \\ (10\lfloor\frac{t}{4}\rfloor + 6)\lfloor\frac{k}{4}\rfloor + (6\lfloor\frac{t}{4}\rfloor + 4) & for\ k\ \equiv 3\ (mod\ 4). \end{cases}$$

Hence the result follows. □

Example 2.10.

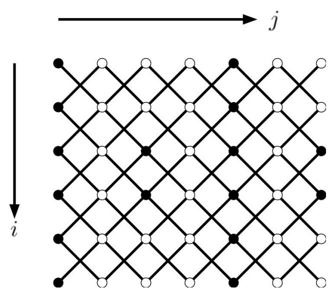

Figure 5. $P_6 \bigotimes P_7$

In the graph in figure 5, the collection of white vertices is corona cover set of minimum cardinality and hence $\tau_C(P_6 \bigotimes P_7) = 26$.

Theorem 2.11. Let P_t and P_k be the paths of order t and k respectively, $t \geq 5$, $k \geq 2$ and $t \equiv 3 \ (mod\, 4)$. Then

$$\tau_C(P_t \bigotimes P_k) = \begin{cases} (10\lfloor \frac{t}{4} \rfloor + 6)(\frac{k}{4}) & for\ k \equiv 0 \ (mod\, 4), \\ (10\lfloor \frac{t}{4} \rfloor + 8)\lfloor \frac{k}{4} \rfloor & for\ k \equiv 1 \ (mod\, 4), \\ (10\lfloor \frac{t}{4} \rfloor + 6)\lfloor \frac{k}{4} \rfloor + (6\lfloor \frac{t}{4} \rfloor + 4) & for\ k \equiv 2 \ (mod\, 3), \\ (10\lfloor \frac{t}{4} \rfloor + 8)\lfloor \frac{k}{4} \rfloor + (6\lfloor \frac{t}{4} \rfloor + 5) & for\ k \equiv 3 \ (mod\, 4). \end{cases}$$

Proof. Let $S_1 = \{v_{(h,g)} : h \equiv 0 \ (mod\, 2), 1 \leq g \leq k\}$, $S_2 = \{v_{(h,g)} : h \equiv 3 \ (mod\, 4), g \equiv 2 \ or\ 3 \ (mod\, 4)\}$, $S_3 = \{v_{(h,g)} : h \equiv 3 \ (mod\, 4), g \equiv 1 \ or\ 2 \ (mod\, 4)\}$, $S_4 = \{v_{(h,g)} : g \equiv 0 \ (mod\, 2), 1 \leq h \leq t\}$, $S_5 = \{v_{(h,g)} : g \equiv 3 \ (mod\, 4), h \equiv 2 \ or\ 3 \ (mod\, 4)\}$

Assume $S = \begin{cases} S_1 \cup S_2 & for\ k \equiv 0 \ (mod\, 4), \\ S_4 \cup S_5 & for\ k \equiv 1 \ or\ 3 \ (mod\, 4), \\ S_1 \cup S_3 & for\ k \equiv 2 \ (mod\, 4). \end{cases}$

Then S is a corona cover set of $P_t \bigotimes P_k$ and hence

$$\tau_C(P_t \bigotimes P_k) \leq |S| = \begin{cases} (10\lfloor \frac{t}{4} \rfloor + 6)(\frac{k}{4}) & for\ k \equiv 0 \ (mod\, 4), \\ (10\lfloor \frac{t}{4} \rfloor + 8)\lfloor \frac{k}{4} \rfloor & for\ k \equiv 1 \ (mod\, 4), \\ (10\lfloor \frac{t}{4} \rfloor + 6)\lfloor \frac{k}{4} \rfloor + (6\lfloor \frac{t}{4} \rfloor + 4) & for\ k \equiv 2 \ (mod\, 3), \\ (10\lfloor \frac{t}{4} \rfloor + 8)\lfloor \frac{k}{4} \rfloor + (6\lfloor \frac{t}{4} \rfloor + 5) & for\ k \equiv 3 \ (mod\, 4). \end{cases}$$

Let S' be a corona cover set of $P_t \bigotimes P_k$. Suppose D be a corona cover set of cardinality at most

$$N = \begin{cases} (10\lfloor \frac{t}{4} \rfloor + 6)(\frac{k}{4}) - 1 & for\ k \equiv 0 \ (mod\, 4), \\ (10\lfloor \frac{t}{4} \rfloor + 8)\lfloor \frac{k}{4} \rfloor - 1 & for\ k \equiv 1 \ (mod\, 4), \\ (10\lfloor \frac{t}{4} \rfloor + 6)\lfloor \frac{k}{4} \rfloor + (6\lfloor \frac{t}{4} \rfloor + 4) - 1 & for\ k \equiv 2 \ (mod\, 3), \\ (10\lfloor \frac{t}{4} \rfloor + 8)\lfloor \frac{k}{4} \rfloor + (6\lfloor \frac{t}{4} \rfloor + 5) - 1 & for\ k \equiv 3 \ (mod\, 4). \end{cases}$$

Then $< D >$ has an isolated vertex, thus we have

$$|S'| \geq N + 1 = \begin{cases} (10\lfloor \frac{t}{4} \rfloor + 6)(\frac{k}{4}) & for\ k \equiv 0 \ (mod\, 4), \\ (10\lfloor \frac{t}{4} \rfloor + 8)\lfloor \frac{k}{4} \rfloor & for\ k \equiv 1 \ (mod\, 4), \\ (10\lfloor \frac{t}{4} \rfloor + 6)\lfloor \frac{k}{4} \rfloor + (6\lfloor \frac{t}{4} \rfloor + 4) & for\ k \equiv 2 \ (mod\, 3), \\ (10\lfloor \frac{t}{4} \rfloor + 8)\lfloor \frac{k}{4} \rfloor + (6\lfloor \frac{t}{4} \rfloor + 5) & for\ k \equiv 3 \ (mod\, 4). \end{cases}$$

Hence the result follows. □

Example 2.12.
In the graph in figure 6, the collection of white vertices is corona cover set of minimum cardinality and hence $\tau_C(P_7 \bigotimes P_9) = 36$.

3 Conclusion

In this paper, the corona covering number of the tensor product of paths has been investigated and established. Future work will extend this study to other graph products, aiming to analyze

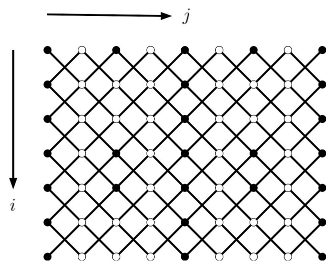

Figure 6. $P_7 \bigotimes P_9$

their corona covering numbers.

References

[1] MAHADEVAN, G., VIMALA SUGANTHI, M., AND SIVAGNANAM, C. (2021): *Corona Domination Number of Graphs.* In International Conference on mathematical Modelling and Computational Intelligence Techniques (pp. 255-265). Singapore: Springer Nature Singapore.

[2] G. MAHADEVAN, NIVEDITHA P AND C. SIVAGNANAM *Corona covering number of graphs* (Communicated)

[3] NIVEDITHA. P, G. MAHADEVAN AND C. SIVAGNANAM (2017): *An Inquisition of corona covering number for notable types of graphs*(Communicated)

[4] G. MAHADEVAN, NIVEDITHA. P AND C. SIVAGNANAM *An Excerpting of Corona covering number of cartesian product of path graphs* (Communicated)

[5] MILLER, D. J. (1968): *The categorical product of graphs.* Canadian Journal of Mathematics, 20, 1511-1521.

[6] PRAVEENKUMAR, L., MAHADEVAN, G., SIVAGNANAM, C., AND TYAGI, S. K. (2023): *Explicating the Exact γ_{CD} Value for the Tensor Product of Graphs.* In International Conference on Discrete Mathematics and Mathematical Modelling in Digital Era (pp. 201-209). Singapore: Springer Nature Singapore.

Author information

Niveditha P, Department of Mathematics, The Gandhigram Rural Institute-Deemed to be University, Dindigul, Tamil Nadu, India, India.
E-mail: `nivedithapandian06@gmail.com`

G. Mahadevan, Department of Mathematics, The Gandhigram Rural Institute-Deemed to be University, Dindigul, Tamil Nadu, India, India.
E-mail: `drgmaha2014@gmail.com`

C. Sivagnanam, Mathematics and Computing Skills Unit, University of Technology and Applied Sciences- Sur, Sultanate of Oman.
E-mail: `choshi71@gmail.com`