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VIBRATIONS OF ROTATING DOUBLE NANOROD SYSTEM USING THE TIMOSHENKO BEAM THEORY

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Abstract To analyze and simulate the behaviour of a rotating double nanorod system using Timoshenko beam theory, one can gain valuable insights into the stress distribution, natural frequencies, and stability of the system under various rotating conditions. Understanding the mechanical behaviour of these nanostructures can contribute to the development of advanced nanoscale devices and applications.

1 Introduction

The study of rotating double nanorod system with the effect of Timoshenko beam theory in rotating rods is an advanced topic in nanomechanics and nanotechnology. Zhang et al.[1]discussed about rotational symmetry of Chiral plasmonic nanorods and their properties. Junot et al. [2] explained about the propulsion regimes that can be implemented in microfluidic devices to perform precise operations based on the selective sorting of microscopic cargoes. Hohenau et al. [3]investigated about the angle-invariant resonances of nano-rod dimers that can promote their controlled application in surface enhanced spectroscopy in sensing. Cui et al. [4] discussed about the exploration of the rotational degrees of freedom which in this process has remained limited, primarily because of the predominant focus on spherical nanoparticles, for which individual particle orientation cannot be determined. Wang et al. [5] studied the elastic buckling phenomenon of micro nanorods and nanotubes based on nonlocal Timoshenko theory. Ponnusamy and Amuthalakshmi[6] examined that the the effect of thermal and magnetic fields on the transverse vibration of double-walled carbon nanotubes using nonlocal Timoshenko beam theory while increasing magnetic field strength raises natural frequencies, while the nonlocal parameter reduces them. Manghi et al.[7] studied the dynamics of rotating elastic filament using Stokesian simulations. Gu et al. [8] discussed the rotational modes of functionalized gold nanorods on live cell membranes. Abouelregal et al. [9] developed a nonlinear model for rotating viscoelastic nanorods under moving heat sources using Klein–Gordon-type nonlocal elasticity and fractional heat conduction and it showed that fractional and nonlocal parameters significantly impacted thermal dissipation, wave propagation, and size-dependent behaviors. Gu and Kornev [10] analyzed the nanorod alignment in thin films by external magnetic field and characterized these films using Magnetic rotational spectroscopy.

Schrittwieser et al. [11] developed empirical models based on numerical solutions of the Fokker-Planck equation for describing the dynamics of magnetic nanoparticles in rotating magnetic fields. Chen [12] discussed about the different valence states and coordination units of vanadium in low optical basicity glasses for conduction, photoluminescence and catalysis applications. Bhimraddi [13] developed a higher order theory for the free vibration analysis of circular cylindrical shell. Zhang [14]investigated the parametric analysis of frequency of rotating laminated composite cylindrical shell using wave propagation approach.

In this paper, the rotating effect on the propagation of flexural waves in double nanorod system is discussed. It also demonstrates that the application of a rotating field can affect the stability of structures without altering their geometrical or material properties. The study of these effects on nanorods is in high demand, the extensive analysis and research have been conducted on the static stability of nanostructures under rotating fields, no research has yet been done on the dynamic stability of Timoshenko beams with magnetic fields in the literature. Overall, this paper contributes to the understanding of flexural wave propagation in double nanorod system by considering the influence of rotating effect, providing valuable insights for researchers in the field.

2 Formulation Of the Problem

Consider a single nanorod system of length L based on Timoshenko beam model subjected to external rotation force Ω as shown in the Figure 1. This study used a cartesian coordinate system x, y to represent a graphene rod subjected to external force and additional deformation, as seen in Figure 2. For finite element analysis the Timoshenko beam model provides a stiffness matrix that includes both bending and shear stiffness, providing more accurate modeling of deflections and stresses.

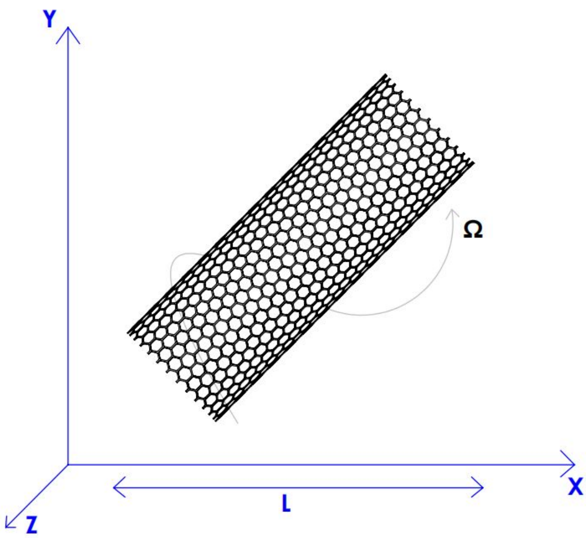

Figure 1. Rotating Single nanorod system

The relationship between the bending moment and curvature is as follows

$$\frac{\partial y}{\partial x} = \psi + \gamma_o \tag{2.1}$$

Since γ_o is the shear strain at the centroidal axis, then $G\gamma_o A$ is the shear force, where y is a displacement to the centroidal plane, x is the axial coordinate, t is the time, ψ is the effect of bending and γ_o is the shear effects.The expressions for bending moment is given by

$$\frac{M}{EI} = \frac{\partial \psi}{\partial x} \tag{2.2}$$

Where M is the bending moment,E is the Young's Modulus and I is the moment of inertia. The shear force$V$ at the cross section is given in terms of shear strain γ and shear modulus G is given as

$$V = G\int_A \gamma dA \tag{2.3}$$

Where A is the area of cross section and κ is the adjustment coefficient is introduced to balance the equation and is given by

$$V = (G\gamma_o A)\kappa$$

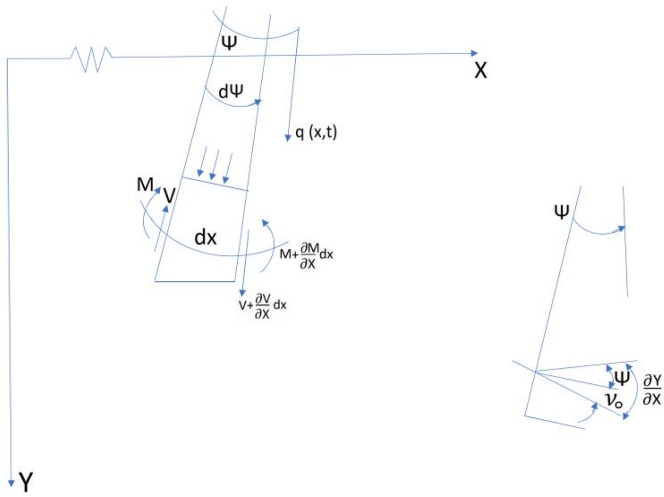

Figure 2. Cartesian co-ordinate of rod subjected to external force and the additional deformations.

$$\Rightarrow V = GA\kappa(\frac{\partial y}{\partial x} - \psi) \tag{2.4}$$

Equation of motion in the vertical direction for the element is of the form

$$-V + (V + \frac{\partial V}{\partial x}dx) + qdx = \rho A dx \frac{\partial^2 y}{\partial t^2}$$

$$\Rightarrow \frac{\partial V}{\partial x} + q = \rho A \frac{\partial^2 y}{\partial t^2} \tag{2.5}$$

Where ρ is the element of mass density and q is the external force Summing moments about an axis perpendicular to x,y-plane and passing through the center of the element, we get

$$M - (M + \frac{\partial M}{\partial x})dx + \frac{1}{2}Vdx + \frac{1}{2}(V + \frac{\partial V}{\partial x}dx)dx = \rho I \frac{\partial^2 \psi}{\partial t^2}dx \tag{2.6}$$

$$\Rightarrow V - \frac{\partial M}{\partial x} = \rho I \frac{\partial^2 \psi}{\partial t^2} \tag{2.7}$$

Substituting the bending moment equation (2.5) and shear force equation (2.4) into equations (2.5) and (2.7), the equations of the motion becomes

$$GA\kappa(\frac{\partial \psi}{\partial x} - \frac{\partial^2 y}{\partial x^2}) + \rho A \frac{\partial^2 y}{\partial t^2} = q(x,t) \tag{2.8}$$

$$GA\kappa(\frac{\partial y}{\partial x} - \psi) + EI \frac{\partial^2 \psi}{\partial x^2} = \rho I \frac{\partial^2 \psi}{\partial t^2} \tag{2.9}$$

Consider a constant axial force due to rotation effects q given by

$$q = -\frac{\rho A \Omega^2 L^2}{2} \frac{\partial^2 y}{\partial x^2} \tag{2.10}$$

Substituting equation (2.10) in the equations of the motion (2.8) and (2.9) gives

$$GA\kappa(\frac{\partial \psi}{\partial x} - \frac{\partial^2 y}{\partial x^2}) + \rho A \frac{\partial^2 y}{\partial t^2} = -\frac{\rho A \Omega^2 L^2}{2} \frac{\partial^2 y}{\partial x^2} \tag{2.11}$$

$$GA\kappa(\frac{\partial y}{\partial x} - \psi) + EI \frac{\partial^2 \psi}{\partial x^2} = \rho I \frac{\partial^2 \psi}{\partial t^2} \tag{2.12}$$

Distribution of waves in graphene nanorod under a rotating field is studied by considering the harmonic wave in the infinite beam. Assuming the solutions in the form

$$y = B_1 e^{i(\gamma x - \omega t)} \qquad \psi = B_2 e^{i(\gamma x - \omega t)} \tag{2.13}$$

where B_1 and B_2 are amplititutes, γ is the wavenumber and ω is the frequency. Substituting the harmonic solution equation(2.13) in the equation of motion equation (2.11)-(2.12), we get

$$(GA\kappa\gamma^2 - \rho A\omega^2 - \frac{\rho A\Omega^2 L^2}{2}\gamma^2)B_1 + iGA\kappa\gamma B_2 = 0 \tag{2.14}$$

$$iGA\kappa\gamma B_1 - (GA\kappa + EI\gamma^2 - \rho I\omega^2)B_2 = 0 \tag{2.15}$$

The Equations (2.14)-(2.15) can be written in the form of matrix,

$$\begin{bmatrix} (GA\kappa\gamma^2 - \rho A\omega^2 - \frac{\rho A\Omega^2 L^2}{2}\gamma^2) & iGA\kappa\gamma \\ iGA\kappa\gamma & -(GA\kappa + EI\gamma^2 - \rho I\omega^2) \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2.16}$$

A trivial solution is obtained by solving the matrix given in Equation (2.16), so as to achieve a significant solution compare the coefficient of the determinant arrangement to nonexistent as follows

$$\begin{vmatrix} GA\kappa\gamma^2 - \rho A\omega^2 - \frac{\rho A\Omega^2 L^2}{2}\gamma^2) & iGA\kappa\gamma \\ iGA\kappa\gamma & -(GA\kappa + EI\gamma^2 - \rho I\omega^2) \end{vmatrix} = 0 \tag{2.17}$$

By solving the determinant given in equation (2.17), a fourth order frequency equation is obtained in the form

$$\frac{\rho I\omega^4}{G\kappa A} + \frac{EI}{\rho A}(1 + \frac{EA\alpha\theta}{G\kappa})\gamma^4 - \frac{I}{A}(1 + \frac{E}{G\kappa} + \frac{E}{G\kappa}\alpha\theta)]\omega^2\gamma^2 - \omega^2 - \frac{\rho A_2\Omega^2 L^2}{2}\gamma = 0 \tag{2.18}$$

Using the identity $\omega = \gamma c$ in the above equation, the dispersion equation of motion becomes

$$\Rightarrow \frac{\rho I\gamma^4 c^4}{G\kappa A} + \frac{EI}{\rho A}(1 + \frac{\rho A\Omega^2 L^2}{2g\kappa})\gamma^4 - \frac{I}{A}[(1 + \frac{E}{G\kappa} + \frac{\rho A\Omega^2 L^2}{2})]c^2\gamma^4 - \gamma c^2\frac{\rho A_2\Omega^2 L^2}{2\rho} = 0 \tag{2.19}$$

Equation (2.19) represents the relationship between the frequency and wavenumber of the graphene nanorod based on Timoshenko beam theory. The phase velocity of the wave propagation $\bar{c} = \frac{c}{\bar{c}_o}$ the dispersion characteristics of the nanorod are analyzed and non-dimensional quantities such as $\epsilon = \frac{E}{G\kappa}$,$\bar{R} = \frac{\rho\Omega^2 L^2}{G\kappa}$ and $\bar{\gamma} = \frac{I\gamma^2}{A}$ are introduced in the equation(19),we get

$$\bar{\gamma}\bar{c}^4 - \frac{\bar{\gamma}((1 + \epsilon + \bar{R}) - 1)\bar{c}^2}{\epsilon} + \frac{\bar{\gamma}(1 + \bar{R})}{\epsilon} - \frac{\bar{R}}{\epsilon^2} = 0 \tag{2.20}$$

3 Double Nanorod Systems

Consider a double nanorod system subjected to the rotating field as shown in Figure (3). Assume $y_i (i = 1, 2)$ be the deflection in the nanorod. Thus the governing equation of motion for the vibration of double nanorod system can be written as

$$GA_1\kappa(\frac{\partial\psi_1}{\partial x} - \frac{\partial^2 y_1}{\partial x^2}) + \rho A_1\frac{\partial^2 y_1}{\partial t^2} = -\frac{\rho A_1\Omega^2 L^2}{2}\frac{\partial^2 y_1}{\partial x^2} + p_1 \tag{3.1}$$

$$GA_1\kappa(\frac{\partial y_1}{\partial x} - \psi) + EI_1\frac{\partial^2\psi_1}{\partial x^2} = \rho I_1\frac{\partial^2\psi_1}{\partial t^2} \tag{3.2}$$

$$GA_2\kappa(\frac{\partial\psi_2}{\partial x} - \frac{\partial^2 y_2}{\partial x^2}) + \rho A_2\frac{\partial^2 y_2}{\partial t^2} = -\frac{\rho A_2\Omega^2 L^2}{2}\frac{\partial^2 y_2}{\partial x^2} + p_2 \tag{3.3}$$

$$GA_2\kappa(\frac{\partial y_2}{\partial x} - \psi) + EI_2\frac{\partial^2\psi_2}{\partial x^2} = \rho I_2\frac{\partial^2\psi_2}{\partial t^2} \tag{3.4}$$

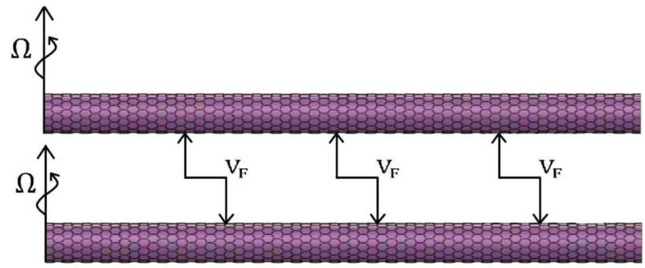

Figure 3. Rotating Double nanorod system

Where p_i is the vanderwalls interaction between layers is given by,

$$p_i = c_{ij}(y_i - y_j) \tag{3.5}$$

Where c is the interaction coefficient, subscripts $i, j = 1, 2$ represents the layers respectively, y_1 and y_2 are the deflections of the layers of double nanorod system. VanderWaals interactions play a crucial role in the behavior of nanorods, especially when they are in close proximity. These interactions arise from induced forces between the rods and can significantly influence the dynamic properties of nanorods. Substituting the vanderwalls interaction equation (3.5) in the governing equations of motions equations(3.1)-(3.4), can be extended into four equations, namely

$$GA_1\kappa(\frac{\partial\psi_1}{\partial x} - \frac{\partial^2 y_1}{\partial x^2}) + \rho A_1\frac{\partial^2 y_1}{\partial t^2} = -\frac{\rho A_1\Omega^2 L^2}{2}\frac{\partial^2 y_1}{\partial x^2} + c(y_1 - y_2) \tag{3.6}$$

$$GA_1\kappa(\frac{\partial y_1}{\partial x} - \psi) + EI_1\frac{\partial^2\psi_1}{\partial x^2} = \rho I_1\frac{\partial^2\psi_1}{\partial t^2} \tag{3.7}$$

$$GA_2\kappa(\frac{\partial\psi_2}{\partial x} - \frac{\partial^2 y_2}{\partial x^2}) + \rho A_2\frac{\partial^2 y_2}{\partial t^2} = -\frac{\rho A_2\Omega^2 L^2}{2}\frac{\partial^2 y_2}{\partial x^2} + c(y_2 - y_1) \tag{3.8}$$

$$GA_2\kappa(\frac{\partial y_2}{\partial x} - \psi) + EI_2\frac{\partial^2\psi_2}{\partial x^2} = \rho I_2\frac{\partial^2\psi_2}{\partial t^2} \tag{3.9}$$

The four equation in Equations(3.6)-(3.9) can be reduced to a two equation by eliminating the angle displacements of ψ_1 and ψ_2.The solution of the two equation are assumed that

$$y_n = Y_n e^{i(\gamma x - \omega t)} \tag{3.10}$$

Where Y_n is the amplitude of the wave mode, The Equations (3.6)-(3.9) can be written in the form of matrix

$$\begin{bmatrix} g_1 & g_2 \\ g3 & g_4 \end{bmatrix}\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3.11}$$

Where

$$g_1 = \frac{EI_1}{\rho A_1}\gamma^4 - \frac{I_1}{A_1}\left[1 + \frac{E}{G\kappa} + \frac{EA_1\Omega^2 L^2}{2G\kappa}\right]\omega^2\gamma^2 + \omega^2\left[1 - \frac{EA_1\Omega^2 L^2}{2G\kappa} - \frac{CI_1}{G\kappa A_1^2}\right] + \frac{\rho I_1}{G\kappa A_1}\left[1 + \frac{EA_1\Omega^2 L^2}{2G\kappa}\right]\omega^4 - \frac{C}{\rho A_1} + \frac{CEI_1}{G\kappa\rho A_1^2}\gamma^2$$

$$g_2 = \frac{C}{\rho A_1} + \frac{CI_1}{G\kappa A_1^2}\omega^2 - \frac{CEI_1}{G\kappa\rho A_1^2}\gamma^2, g_3 = \frac{C}{\rho A_2} + \frac{CI_2}{G\kappa A_2^2}\omega^2 - \frac{CEI_2}{G\kappa\rho A_2^2}\gamma^2$$

$$g_4 = \frac{EI_2}{\rho A_2}\gamma^4 - \frac{I_2}{A_2}\left[1 + \frac{E}{G\kappa} + \frac{EA_2\Omega^2L^2}{2G\kappa}\right]\omega^2\gamma^2 + \omega^2\left[1 - \frac{EA_2\Omega^2L^2}{2G\kappa} - \frac{CI_2}{G\kappa A_2^2}\right] + \frac{\rho I_2}{G\kappa A_2}\left[1 + \frac{EA_2\Omega^2L^2}{2G\kappa}\right]\omega^4 - \frac{C}{\rho A_2} + \frac{CEI_2}{G\kappa\rho A_2^2}\gamma^2$$

Introducing the additional non dimenional terms such as $\bar{I}_1 = \frac{I_1\gamma^2}{A_1}$, $\delta_1 = \frac{CI_1}{G\kappa A_1^2}$,$\bar{I}_2 = \frac{I_2\gamma^2}{A_2}$, $\delta_2 = \frac{CI_2}{G\kappa A_2^2}$ in equation (3.11), can be rewritten in form of

$$\begin{bmatrix} g_5\bar{c}^4 - \bar{c}^2g_6 + g_7 & g_8 + g_9\bar{c}^2 \\ g_{13} + g_{14}\bar{c}^2 & g_{10}\bar{c}^4 - \bar{c}^2g_{11} + g_{12} \end{bmatrix}\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3.12}$$

Where

$$g_5 = \bar{I}_1\left[1 + \bar{R}\right], g_6 = \left[\frac{\bar{I}_1}{\epsilon}(1 + \epsilon - \bar{R}) - \bar{I}_1(1 + \bar{R} - \epsilon)\right], g_7 = \frac{\bar{I}_1}{\epsilon} - \delta_1\bar{I}_1\epsilon^2 + \delta_1\epsilon, g_8 = \delta\bar{I}_1\epsilon^2 - \delta_1\epsilon$$

$$g_9 = \delta_1\bar{I}_1, g_{10} = \bar{I}_2\left[1 + \bar{R}\right], g_{11} = \left[\frac{\bar{I}_2}{\epsilon}(1 + \epsilon - \bar{R}) - \bar{I}_2(1 + \bar{R} - \epsilon)\right], g_{12} = \frac{\bar{I}_2}{\epsilon} - \delta_1\bar{I}_2\epsilon^2 + \delta_1\epsilon$$

$$g_{13} = \delta\bar{I}_2\epsilon^2 - \delta_1\epsilon, g_{14} = \delta_1\bar{I}_2$$

A trivial solution is obtained by solving the matrix given in Equation(3.12) so as to achieve a significant solution compare the coefficient of the determinant arrangement to non-existence as follows

$$\begin{vmatrix} g_5\bar{c}^4 - \bar{c}^2g_6 + g_7 & g_8 + g_9\bar{c}^2 \\ g_{13} + g_{14}\bar{c}^2 & g_{10}\bar{c}^4 - \bar{c}^2g_{11} + g_{12} \end{vmatrix} = \begin{bmatrix} 0 \end{bmatrix} \tag{3.13}$$

By solving the determinant the eigth order frequency equation is obtained in the form of

$$g_5g_{10}\bar{c}^8 - \bar{c}^6\left[g_5g_{11} + g_6g_{10}\right] + \bar{c}^4\left[g_{12}g_5 + g_6g_{11} + g_7g_{10} - g_9g_{14}\right] - \bar{c}^2\left[g_6g_{12} + g_7g_{11} + g_8g_{14} + g_{13}g_9\right] + g_7g_{12} - g_8g_{12} = 0 \tag{3.14}$$

Equation(3.14) represents the dispersed relation between dimensionless phase speed and dimensionless wave number of a double nanorod system in the presence of rotating field.

4 Numerical Results

The frequency equation is derived using beam theory and parameters of the beam are taken in the order from [15] as $E = 1 \times 10^{12}Tpa$, $\rho = 2.27 \times 10^3kgm^{-3}$, $G = 0.4 \times 10^{12}pa$, $A = 3 \times 10^{-19}m^2$and $I = 1.78 \times 10^{-38}m^4$.

Dispersion curves of a rotating double nanorod for different rotation is drawn and is shown in Figure (4). From the Figure (4), it is observed that all the three different rotating effects show significant peak indicates a strong resonance effect, possibly due to a dominant mode of vibration. The difference in peak heights suggests that the rotating force affects the resonance frequency. This trend is typical for flexural waves, where higher the wave numbers, shorter the wavelengths corresponding to higher modes of vibration. The relative positions of different rotations remain consistent, indicating that the rotating force continually affects the wave propagation characteristics. The rotating force might be increasing the effective rotational inertia of the nanorod. Higher rotational inertia would raise the resonance frequency. The rotating force increases the relationship between wave number and frequency by introducing additional vibrational modes and modifying the existing ones. The Timoshenko beam theory explains the characteristics by considering shear deformations and rotational inertia, which are significant in understanding the dynamic response of double nanorods under rotating forces.

Dispersion curves of a rotating double nanorod system for different length is drawn and is shown in Figure (5). From the Figure (5), it is observed that the length of the nanorod affects its stiffness and mass distribution, which in turn influences the natural frequencies of vibration. Longer rods generally have lower natural frequencies for a given mode compared to shorter

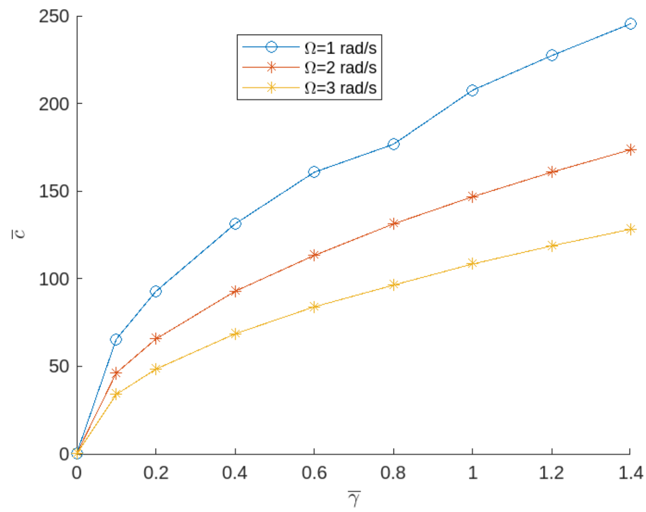

Figure 4. Dispersion curve for rotating double nanorod system at different rotating field

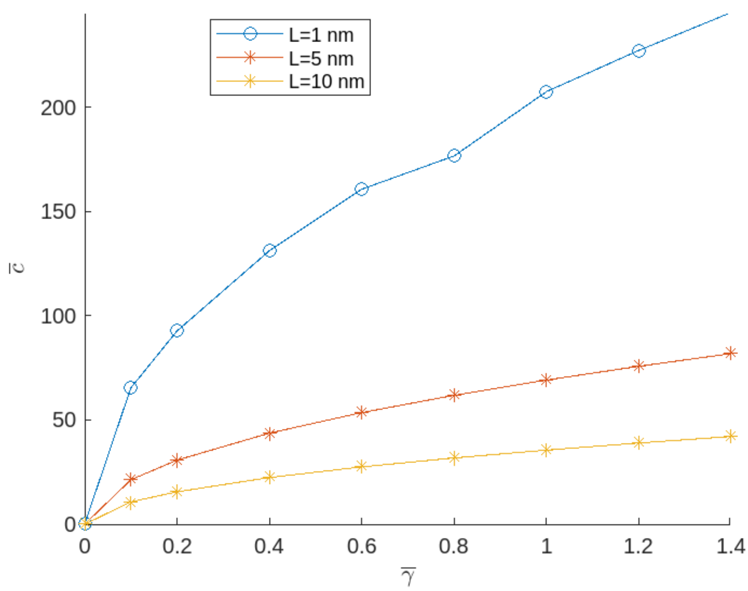

Figure 5. Dispersion curve for rotating double nanorod system at different length

rods due to increased flexibility. The peak frequency decreases as the length increases. The rotating force can introduce torsional effects and modify the shear deformation, impacting the frequency response. This is more pronounced in shorter rods L=1 nm where the torsional effects can significantly alter the natural frequencies. The fundamental mode is heavily influenced by the length. Longer rods (L=10 nm) are more flexible, leading to lower natural frequencies. The rotating force affects the shear deformation, causing higher frequencies in shorter rods. As the wave number increases, the higher modes of vibration become prominent. These modes are less affected by the length compared to the fundamental mode but still show a dependency. The rotational inertia and shear effects accounted for by Timoshenko beam theory become significant in these higher modes, particularly for shorter rods. This would provide a comprehensive understanding of the vibrational behaviour under different conditions.

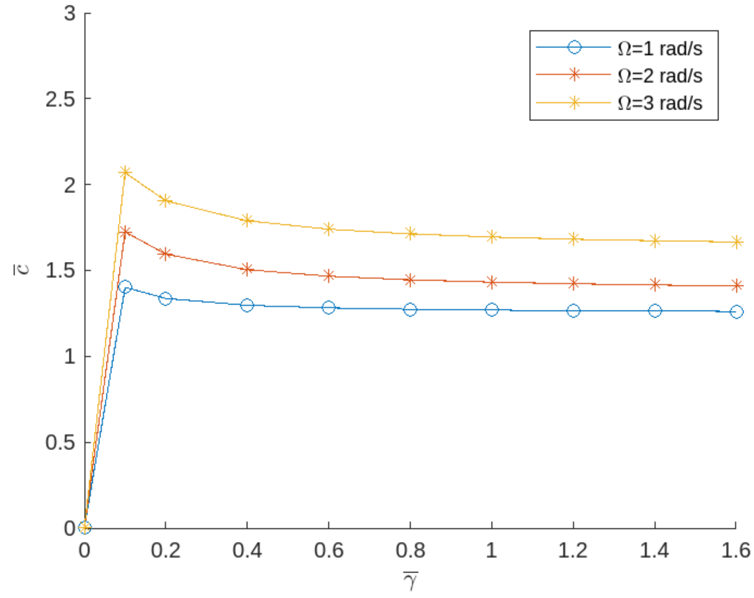

Figure 6. Dispersion curve for rotating single nanorod system at different rotating field

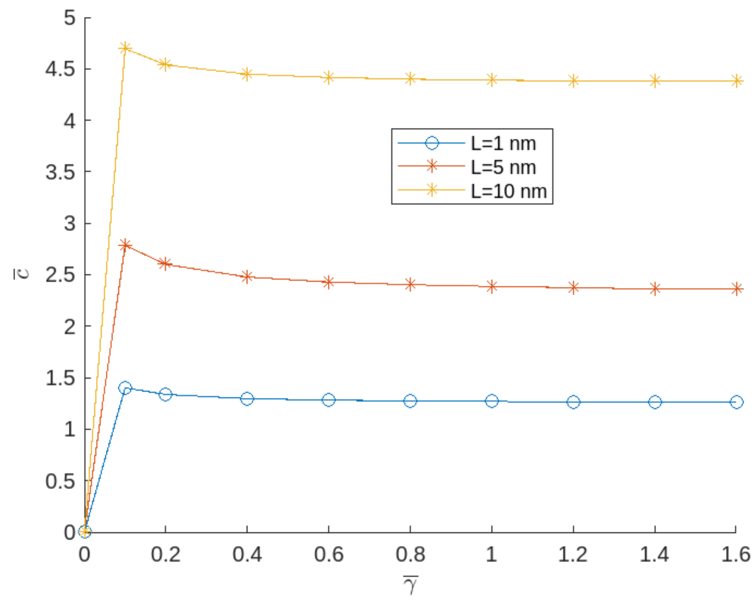

Figure 7. Dispersion curve for rotating single nanorod system at different length

Dispersion curves of a rotating single nanorod for simultaneous rotation and length are drawn and is shown in Figure (6)and (7). From the Figure (6)and (7), it is observed that the simultaneous change in length and rotating force significantly affects the frequency response of the single nanorod. Increasing L and Ω leads to higher peak frequencies, suggesting that longer nanorods under higher rotating forces exhibit more pronounced vibrational modes. This could be due to the combined effects of increased mass due to length and greater energy due to rotating force, which enhance the vibrational characteristics.The study of lower modes in a single nanorod, particularly using Timoshenko beam theory, has broad implications in sensing, wave propagation, robotics, nonlocal mechanics, and nanophotonics. The relationship between wavenumber and phase velocity is key to designing more efficient nanostructures and devices with improved mechanical and dynamic performance.

5 CONCLUDING REMARKS

The relationship between phase velocity and wavenumber is obtained by solving the governing equations in matrix form, and the frequency equations for flexural vibration modes are derived. Numerical results are presented as dispersion curves to study wave propagation characteristics. The results confirm that phase velocity increases with wavenumber for all vibration modes, demonstrating the dispersive nature of the system. These insights provide a deeper understanding of dynamic behavior in nanorod structures. Beyond theoretical importance, such findings are valuable for the design of nanoscale devices where vibration control and wave propagation play a critical role. Potential applications include nanosensors, actuators, and advanced composite materials. Thus, the study contributes both to fundamental nanomechanics and to practical advancements in materials science.

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