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SOME NEW TYPE OF MAPPINGS IN PYTHOGOREAN NEUTROSOPHIC TOPOLOGICAL SPACES

P. Basker

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Corresponding Author: P. Basker

Abstract We developed the ideas of continuous and irresolute mappings in pythogorean neutrosophic topological spaces, as well as open and closed functions. Using open sets, we generated a locally indiscrete space. The notion of Linked Space was shown and segmented in PNTS, which is an intersection of a non-empty set whose closure is empty in a space with separated and connected spaces. Furthermore, we built and derived different outcomes for various types of functions utilizing the present functions.

1 Introduction

After Zadeh's creation of fuzzy sets [25], Fuzzy topological space was originally conceptualised by Chang [13], and a number of researchers modified standard topological principles for use in fuzzy topology, most importantly. One generalisation of the fuzzy set is Atanassov's intuitionistic fuzzy sets ([2]). Afterwards, Coker [14] used the notion of the IFS to establish the important concept of intuitionistic fuzzy topological space. Intuitionistic fuzzy-continuity and pre-continuity were defined and studied by Jeon et al. [15]. After Smarandache [**?**], [23] presented the ideas of neutrosophy and the neutrosophic set. Neutronosophic crisp topological spaces and neutrosophic crisp sets were first presented by Salama and Alblowi [20]. A distinct class of mathematical ideas that generalise both their crisp and fuzzy counterparts are based on neurophysiology. The IFS is expanded upon by the neutrosophic set. They heralded the arrival of the notion of neutrosophic topological spaces. Neurosophic topological space is an extension of intuitionistic fuzzy topological space with a neutrosophic set that contains the degree of membership, non-membership, and indeterminacy of each element.

The degrees of independence and interdependence of the fuzzy and neutrosophic components, the intuitionistic neutrosophic soft set, and the neutrosophic logic and its scientific applications were originally proposed by Smarandache. References: [14], [12]. The total of the three membership functions, not to exceed three, is the fuzzy neutrosophic set. Arokiarani et al. were the ones who first conceptualised it [1]. In 2017, presented fundamental operations and a Fuzzy Neutrosophic topological space. The variables $N\psi_{\alpha}^{\#0}$ and $N\psi_{\alpha}^{\#1}$ spaces in neutrosophic topological spaces, Neutrosophic homeomorphisms through neutrosophic functions P. Basker and Broumi Said [2021, 2022, 2023] investigated and analysed $(\beta_{\rho n})$-OS in pythagorean neutrosophic topological spaces, applications of sets and functions by using an open sets in fuzzy neutrosophic topological spaces, intuitionistic and intuitionistic fuzzy topological spaces and in M-Structures derived and investigated. [4], [6], [8], [9], [10], [11]. Modak, Noiri, and Islam [18] have introduced and investigated a weaker version of connectivity. Noiri.T and Modak.S completed half of the b-connectedness [19].

Tyagi has discussed a few potent types of connectivity [24]. Hibrid Δ-statistical convergence for neutrosophic normed space, yielded slightly pre-continuous functions and faintly upper and lower precontinuous multifunctions [3], [7].n-Cylindrical Fuzzy Neutrosophic Sets, where T and F are dependent components and I is an independent component, were introduced by Sarannya et al. [17] examined and defined [16] along with several related concepts of fuzzy neutrosophic topological spaces that are n-cylindrical. Florentin Smarandache [23] presented a review of fuzzy soft topological spaces, intuitionistic fuzzy soft topological spaces, and neutrosophic fuzzy soft topological spaces. S. Ganesan, C. Alexander, A. Pandi, and F. Schmandache have examined and explored neutronsophic micro ideal topological structure [22].

2 $(\beta_{\rho n})$-WC functions

Definition 2.1. A function $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$ is said to be $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ at $e \in \Omega^{\delta_{[1]}}$ if for each open set $Q_{\theta(2)}$ of $\Omega^{\delta_{[2]}}$ containing $\tilde{\varpi}(e)$, $\exists$ a $(\beta_{\rho n})$-OS, $Q_{\theta(1)}$ containing e such that $\tilde{\varpi}(Q_{\theta(1)}) \subseteq Cl(Q_{\theta(2)})$. If for each $e \in \Omega^{\delta_{[1]}}$, $\tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ at $e \in \Omega^{\delta_{[1]}}$, $\tilde{\varpi}$ is said to be $(\beta_{\rho n})$-$W^{\theta}C_{\#}$.

Example 2.2. Let $\Omega^{\delta_{[1]}} = \{\alpha^1, \alpha^2\}$ and $\Omega^{\delta_{[2]}} = \{\beta^1, \beta^2\}$. Then, $\theta(1)_{PN} = \{0_N,\ 1_N,\ \xi_1,\ \xi_2\}$ and $\theta(2)_{PN} = \{0_N,\ 1_N,\ \zeta_1\}$ is a Pythagorean neutrosophic topological spaces on $\Omega^{\delta_{[1]}}$ and $\Omega^{\delta_{[2]}}$ respectively, where
$\xi_1 = \left\langle p, (\frac{9}{40}^{\delta_1}, \frac{17}{40}^{\delta_2}, \frac{29}{40}^{\delta_3}), (\frac{17}{40}^{\delta_1}, \frac{17}{40}^{\delta_2}, \frac{17}{40}^{\delta_3}) \right\rangle$,
$\xi_2 = \left\langle p, (\frac{13}{40}^{\delta_1}, \frac{21}{40}^{\delta_2}, \frac{5}{8}^{\delta_3}), (\frac{21}{40}^{\delta_1}, \frac{17}{40}^{\delta_2}, \frac{5}{8}^{\delta_3}) \right\rangle$ and
$\zeta_1 = \left\langle q, (\frac{13}{40}^{\delta_1}, \frac{21}{40}^{\delta_2}, \frac{5}{8}^{\delta_3}), (\frac{21}{40}^{\delta_1}, \frac{9}{40}^{\delta_2}, \frac{29}{40}^{\delta_3}) \right\rangle$. Then, the function $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$ is a $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ at $e \in \Omega^{\delta_{[1]}}$ as it is defined as $\tilde{\varpi}(\alpha^1) = \beta^1$ and $\tilde{\varpi}(\alpha^2) = \beta^2$, since $\forall OS\ \zeta_1$ of $\Omega^{\delta_{[2]}}$ containing $\tilde{\varpi}(p)$, $\exists$ a $(\beta_{\rho n})$-OS ξ_1 containing p such that $\tilde{\varpi}(\xi_1) \subseteq Cl(\xi_2)$. If for each $p \in \Omega^{\delta_{[1]}}$, $\tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ at $p \in \Omega^{\delta_{[1]}}$ then $\tilde{\varpi}$ is a $(\beta_{\rho n})$-$W^{\theta}C_{\#}$.

Example 2.3. Let $\Omega^{\delta_{[1]}} = \{\alpha^1, \alpha^2\}$ and $\Omega^{\delta_{[2]}} = \{\beta^1, \beta^2\}$. Then, $\theta(1)_{PN} = \{0_N,\ 1_N,\ \xi_1,\ \xi_2\}$ and $\theta(2)_{PN} = \{0_N,\ 1_N,\ \zeta_1\}$ is a Pythagorean neutrosophic topological spaces on $\Omega^{\delta_{[1]}}$ and $\Omega^{\delta_{[2]}}$ respectively, where
$\xi_1 = \left\langle p, (\frac{21}{40}^{\delta_1}, \frac{5}{8}^{\delta_2}, \frac{13}{40}^{\delta_3}), (\frac{21}{40}^{\delta_1}, \frac{9}{40}^{\delta_2}, \frac{9}{40}^{\delta_3}) \right\rangle$,
$\xi_2 = \left\langle p, (\frac{5}{8}^{\delta_1}, \frac{9}{40}^{\delta_2}, \frac{13}{40}^{\delta_3}), (\frac{13}{40}^{\delta_1}, \frac{5}{8}^{\delta_2}, \frac{9}{40}^{\delta_3}) \right\rangle$ and
$\zeta_1 = \left\langle q, (\frac{9}{40}^{\delta_1}, \frac{9}{40}^{\delta_2}, \frac{13}{40}^{\delta_3}), (\frac{13}{40}^{\delta_1}, \frac{5}{8}^{\delta_2}, \frac{13}{40}^{\delta_3}) \right\rangle$. Then, the function $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$ is a $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ at $e \in \Omega^{\delta_{[1]}}$ as it is defined as $\tilde{\varpi}(\alpha^1) = \beta^1$ and $\tilde{\varpi}(\alpha^2) = \beta^2$, since $\forall OS\ \zeta_1$ of $\Omega^{\delta_{[2]}}$ containing $\tilde{\varpi}(p)$, $\exists$ a $(\beta_{\rho n})$-OS ξ_1 containing p such that $\tilde{\varpi}(\xi_1) \subseteq Cl(\xi_2)$. If for each $p \in \Omega^{\delta_{[1]}}$, $\tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ at $p \in \Omega^{\delta_{[1]}}$ then $\tilde{\varpi}$ is a $(\beta_{\rho n})$-$W^{\theta}C_{\#}$.

Theorem 2.4. *For a function $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$, the following are equivalent:*
(a) $\tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ at $e \in \Omega^{\delta_{[1]}}$,
(b) $e \in \rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)})))$ for each neighborhood $Q_{\theta(1)}$ of $\tilde{\varpi}(e)$.

Proof. $(a) \Longrightarrow (b)$ Let $Q_{\theta(1)}$ be any neighborhood of $\tilde{\varpi}(e)$. Then $\exists$ a $(\beta_{\rho n})$-OS, $Q^{\#}$ consisting of e in which case $\tilde{\varpi}(Q^{\#}) \subset pnCl(Q_{\theta(1)})$. Since $Q^{\#} \subset \tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)}))$ and $Q^{\#}$ is $(\beta_{\rho n})$-OS, then $e \in Q^{\#} \subset \rho n^{\#I}_{\beta}(Q^{\#}) \subset \rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)})))$.
$(b) \Longrightarrow (a)$ Let $e \in \rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)})))$ for each neighborhood $Q_{\theta(1)}$ of $\tilde{\varpi}(e)$. Take $Q_{\theta(2)} = \rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)})))$. This implies that $\tilde{\varpi}(Q_{\theta(2)}) \subset pnCl(Q_{\theta(1)})$ and $Q_{\theta(2)}$ is $(\beta_{\rho n})$-OS. Hence, $\tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ at $e \in \Omega^{\delta_{[1]}}$. □

Theorem 2.5. *For a function $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$, the following are equivalent:*

(a) $L^{\#}$ *is* $(\beta_{\rho n})$-$W^{\theta}C_{\#}$,
(b) $\rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(pnInt(pnCl(Q_{\theta(2)})))) \subset \tilde{\varpi}^{-1}(pnCl(Q_{\theta(2)}))$ $\forall$ *subset* $Q_{\theta(2)} \subset \Omega^{\delta_{[}2]}$,
(c) $\rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(pnInt(L^{\#}))) \subset \tilde{\varpi}^{-1}(L^{\#})$ $\forall$ pn-RCS *subset* $L^{\#} \subset \Omega^{\delta_{[}2]}$,
(d) $\rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(Q_{\theta(1)}))$ *is a subset of* $\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)}))$ $\forall$ pn-OS $Q_{\theta(1)}$ *is a subset of* $\Omega^{\delta_{[}2]}$,
(e) $\tilde{\varpi}^{-1}(Q_{\theta(1)}) \subset \rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)})))$ $\forall$ pn-OS $Q_{\theta(1)} \subset \Omega^{\delta_{[}2]}$,
(f) $\rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(Q_{\theta(1)}))$ *is a subset of* $\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)}))$ $\forall$ pn-POS *subset* $Q_{\theta(1)} \subset \Omega^{\delta_{[}2]}$,
(g) $\tilde{\varpi}^{-1}(Q_{\theta(1)}) \subset \rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)})))$ $\forall$ pn-POS *subset* $Q_{\theta(1)} \subset \Omega^{\delta_{[}2]}$.

Proof. $(a) \Longrightarrow (b)$ Let $Q_{\theta(2)} \subset \Omega^{\delta_{[}2]}$ & $e \in \Omega^{\delta_{[}1]}/\tilde{\varpi}^{-1}(pnCl(Q_{\theta(2)}))$. Then $\tilde{\varpi}(e) \in \Omega^{\delta_{[}2]}/pnCl(Q_{\theta(2)})$ and $\exists$ an OS, $Q_{\theta(1)}$ containing $\tilde{\varpi}(e)$ such that $Q_{\theta(1)} \cap Q_{\theta(2)} = \phi$. We've $pnCl(Q_{\theta(1)})/pnInt(pnCl(Q_{\theta(2)})) = \phi$. Since $\tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$, then $\exists$ a $(\beta_{\rho n})$-OS, $Q_{\theta(3)}$ containing e such that $\tilde{\varpi}(Q_{\theta(3)}) \subset pnCl(Q_{\theta(1)})$. Then $Q_{\theta(3)} \cap \tilde{\varpi}^{-1}(pnInt(pnCl(Q_{\theta(2)}))) = \phi$ and $e \in \Omega^{\delta_{[}1]}/\rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(pnInt(pnCl(Q_{\theta(2)}))))$.
Hence, $\rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(pnInt(pnCl(Q_{\theta(2)})))) \subset \tilde{\varpi}^{-1}(pnCl(Q_{\theta(2)}))$.
$(b) \Longrightarrow (c)$ Let $L^{\#}$ be any pn-RCS in $\Omega^{\delta_{[}2]}$. Then $\rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(pnInt(L^{\#})))$
$= \rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(pnInt(pnCl(pnInt(L^{\#}))))) \subset \tilde{\varpi}^{-1}(pnCl(pnInt(L^{\#}))) = \tilde{\varpi}^{-1}(L^{\#})$.
$(c) \Longrightarrow (d)$ Let $Q_{\theta(1)}$ be an pn-OS subset of $\Omega^{\delta_{[}2]}$. Since $pnCl(Q_{\theta(1)})$ is pn-RCS in $\Omega^{\delta_{[}2]}$, then $\rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(Q_{\theta(1)})) \subset \rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(pnInt(pnCl(Q_{\theta(1)})))) \subset \tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)}))$.
$(d) \Longrightarrow (e)$ Let $Q_{\theta(1)}$ be any pn-OS of $\Omega^{\delta_{[}2]}$. Since $P^{\delta_{[}2]}/pnCl(Q_{\theta(1)})$ is pn-OS in $\Omega^{\delta_{[}2]}$, then
$\Omega^{\delta_{[}1]}\rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)}))) = \rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(\Omega^{\delta_{[}2]}/pnCl(Q_{\theta(1)})))$
$\subset \tilde{\varpi}^{-1}(pnCl(\Omega^{\delta_{[}2]}/pnCl(Q_{\theta(1)}))) \subset \Omega^{\delta_{[}1]}/\tilde{\varpi}$ inverse of $(Q_{\theta(1)})$.
Hence, $\tilde{\varpi}$ inverse of $(Q_{\theta(1)}) \subset \rho n^{\#I}_{\beta}$ of $\tilde{\varpi}$ inverse of $(pnCl(Q_{\theta(1)})))$.
$(e) \Longrightarrow (a)$ Let $e \in \Omega^{\delta_{[}1]}$ and $Q_{\theta(1)}$ be any pn-OS subset of $\Omega^{\delta_{[}2]}$ containing $\tilde{\varpi}(e)$. Then $e \in \tilde{\varpi}^{-1}(Q_{\theta(1)}) \subset \rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)})))$. Take $Q_{\theta(3)} = \rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)})))$. Thus $\tilde{\varpi}(Q_{\theta(3)}) \subset pnCl(Q_{\theta(1)})$ and hence $\tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ at e in $\Omega^{\delta_{[}1]}$.
$(a) \Longrightarrow (f)$ Let $Q_{\theta(1)}$ be any pn-POS of $\Omega^{\delta_{[}2]}$ and $e \in \Omega^{\delta_{[}1]}/\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)}))$. $\exists$ an pn-OS, $Q^{\#}$ containing $\tilde{\varpi}(e)$ such that $Q^{\#} \cap Q_{\theta(1)} = \phi$. We've $pnCl(Q^{\#} \cap Q_{\theta(1)}) = \phi$. Since $Q_{\theta(1)}$ is pn-POS, then $Q_{\theta(1)} \cap pnCl(Q^{\#}) \subset pnInt(pnCl(Q_{\theta(1)})) \cap pnCl(Q^{\#}) \subset pnCl(pnInt(pnCl(Q_{\theta(1)})) \cap Q^{\#}) \subset pnCl(pnInt(pnCl(Q_{\theta(1)}) \cap Q^{\#})) \subset pnCl(pnInt(pnCl(Q_{\theta(1)} \cap Q^{\#}))) \subset pnCl(Q_{\theta(1)} \cap Q^{\#}) = \phi$. Since $\tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ and $Q^{\#}$ is an pn-OS containing $\tilde{\varpi}(e)$, $\exists$ a $(\beta_{\rho n})$-OS, $Q_{\theta(3)}$ in $\Omega^{\delta_{[}1]}$ containing e such that $\tilde{\varpi}(Q_{\theta(3)}) \subset pnCl(Q^{\#})$. Then $\tilde{\varpi}(Q_{\theta(3)}) \cap Q_{\theta(1)} = \phi$ and $Q_{\theta(3)} \cap \tilde{\varpi}^{-1}(Q_{\theta(1)}) = \phi \Longrightarrow e \in \Omega^{\delta_{[}1]}/\rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(Q_{\theta(1)}))$ and then $\rho n^{\#C}_{\beta}$ of $\tilde{\varpi}$ inverse of $(Q_{\theta(1)})$is a component of $\tilde{\varpi}$ inverse of $(pnCl(Q_{\theta(1)}))$.
$(f) \Longrightarrow (g)$ Let $Q_{\theta(1)}$ be any pn-POS of $\Omega^{\delta_{[}2]}$. Since $\Omega^{\delta_{[}2]}/pnCl(Q_{\theta(1)})$ is pn-OS in $\Omega^{\delta_{[}2]}$, then
$\Omega^{\delta_{[}1]}/\rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)}))) = \rho n^{\#C}_{\beta}(\tilde{\varpi}^{-1}(\Omega^{\delta_{[}2]}/pnCl(Q_{\theta(1)})))$
$\subset \tilde{\varpi}^{-1}(pnCl(\Omega^{\delta_{[}2]}/pnCl(Q_{\theta(1)}))) \subset \Omega^{\delta_{[}1]}/\tilde{\varpi}^{-1}(Q_{\theta(1)})$. This shows that $\tilde{\varpi}^{-1}(Q_{\theta(1)}) \subset \rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1}$
$(g) \Longrightarrow (a)$ Let $e \in \Omega^{\delta_{[}1]}$ and $Q_{\theta(1)}$ any pn-OS of $\Omega^{\delta_{[}2]}$ containing $\tilde{\varpi}(e)$. We've $e \in \tilde{\varpi}^{-1}(Q_{\theta(1)}) \subset \rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)})))$. Take $Q_{\theta(3)} = \rho n^{\#I}_{\beta}(\tilde{\varpi}^{-1}(pnCl(Q_{\theta(1)})))$. Then $\tilde{\varpi}(Q_{\theta(3)}) \subset pnCl(Q_{\theta(1)})$ and hence $\tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ at e in $\Omega^{\delta_{[}1]}$. □

Definition 2.6. A function $\delta : (J_m, \tau_{PN}) \longrightarrow (J_n, \tau_{PN})$ is called
(a) an $\beta_{\rho n}\#CF$ if $\delta^{-1}(K)$ is $(\beta_{\rho n})$-CS in (J_m, τ_{PN}) $\forall CS,\ K$ of a (J_n, τ_{PN}).
(b) $\beta_{\rho n} \prec IF$ if $\delta^{-1}(K)$ is $(\beta_{\rho n})$-CS in (J_m, τ_{PN}) $\forall$ $(\beta_{\rho n})$-CS, K of (J_n, τ_{PN}).

Example 2.7. Let $J_m = \{\alpha^1, \alpha^2\}$ and $J_n = \{\beta^1, \beta^2\}$. Then, $\tau(1)_{PN} = \{0_N,\ 1_N,\ \xi_1,\ \xi_2\}$ and $\tau(2)_{PN} = \{0_N,\ 1_N,\ \zeta_1\}$ is a Pythagorean neutrosophic topological spaces on J_m and J_n respectively, where
$\xi_1 = \left\langle p, (\frac{7}{40}^{\delta_1}, \frac{13}{40}^{\delta_2}, \frac{24}{40}^{\delta_3}), (\frac{15}{40}^{\delta_1}, \frac{12}{40}^{\delta_2}, \frac{17}{40}^{\delta_3}) \right\rangle$,
$\xi_2 = \left\langle p, (\frac{11}{40}^{\delta_1}, \frac{15}{40}^{\delta_2}, \frac{12}{40}^{\delta_3}), (\frac{24}{40}^{\delta_1}, \frac{11}{40}^{\delta_2}, \frac{17}{40}^{\delta_3}) \right\rangle$ and
$\zeta_1 = \left\langle q, (\frac{11}{40}^{\delta_1}, \frac{24}{40}^{\delta_2}, \frac{12}{40}^{\delta_3}), (\frac{15}{40}^{\delta_1}, \frac{12}{40}^{\delta_2}, \frac{17}{40}^{\delta_3}) \right\rangle$. Then, the function $\delta : (J_m, \tau(1)_{PN}) \longrightarrow (J_n, \tau(2)_{PN})$ is a $\beta_{\rho n}\#CF$ is defined as $\delta(\alpha^1) = \beta^1$ and $\delta(\alpha^2) = \beta^2$, since $\delta^{-1}(K)$ is $(\beta_{\rho n})$-CS in $(J_m, \tau(1)_{PN})$ $\forall CS,\ K$ of a $(J_n, \tau(2)_{PN})$.

Example 2.8. Let $Py_1 = \{\alpha^1, \alpha^2\}$ and $Py_2 = \{\beta^1, \beta^2\}$. Then, $TS(1)_{PN} = \{0_N,\ 1_N,\ \xi_1,\ \xi_2\}$ and $TS(2)_{PN} = \{0_N,\ 1_N,\ \zeta_1\}$ is a Pythagorean neutrosophic topological spaces on Py_1 and Py_2 respectively, where
$\xi_1 = \left\langle p, ({\frac{17}{40}}^{\delta_1}, {\frac{17}{40}}^{\delta_2}, {\frac{21}{40}}^{\delta_3}), ({\frac{21}{40}}^{\delta_1}, {\frac{9}{40}}^{\delta_2}, {\frac{5}{8}}^{\delta_3})\right\rangle$,
$\xi_2 = \left\langle p, ({\frac{21}{40}}^{\delta_1}, {\frac{21}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3}), ({\frac{17}{40}}^{\delta_1}, {\frac{17}{40}}^{\delta_2}, {\frac{21}{40}}^{\delta_3})\right\rangle$ and
$\zeta_1 = \left\langle q, ({\frac{17}{40}}^{\delta_1}, {\frac{21}{40}}^{\delta_2}, {\frac{5}{8}}^{\delta_3}), ({\frac{21}{40}}^{\delta_1}, {\frac{29}{40}}^{\delta_2}, {\frac{29}{40}}^{\delta_3})\right\rangle$. Then, we define the function $\delta : (Py_1, TS(1)_{PN}) \longrightarrow (Py_1, TS(2)_{PN})$ as $\delta(\alpha^1) = \beta^1$ and $\delta(\alpha^2) = \beta^2$ We can see that δ^{-1} are $\beta_{\rho n} \prec IF$ since $\delta^{-1}(Py_2)$ is $(\beta_{\rho n})$-CS in $(Py_1, TS(1)_{PN})$ $\forall$ $(\beta_{\rho n})$-CS of a $(Py_2, TS(2)_{PN})$.

Definition 2.9. A map $\delta : (J_m, \tau_{PN}) \longrightarrow (J_n, \tau_{PN})$ is said to be
(a) $\lfloor C\#, \beta_{\rho n}\rfloor$-map if $\delta(V)$ is $(\beta_{\rho n})$-CS in (J_n, τ_{PN}) for every PN-CS, V of (J_m, τ_{PN}).
(b) $\lfloor O*, \beta_{\rho n}\rfloor$-map if $\delta(V)$ is $(\beta_{\rho n})$-OS in (J_n, τ_{PN}) for every PN-OS, V of (J_m, τ_{PN}).

Example 2.10. Let $J_m = \{\alpha^1, \alpha^2\}$ and $J_n = \{\beta^1, \beta^2\}$. Then, $\tau(1)_{PN} = \{0_N,\ 1_N,\ \xi_1,\ \xi_2\}$ and $\tau(2)_{PN} = \{0_N,\ 1_N,\ \zeta_1\}$ is a Pythagorean neutrosophic topological spaces on J_m and J_n respectively, where
$\xi_1 = \left\langle p, ({\frac{5}{40}}^{\delta_1}, {\frac{7}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3}), ({\frac{24}{40}}^{\delta_1}, {\frac{12}{40}}^{\delta_2}, {\frac{7}{40}}^{\delta_3})\right\rangle$,
$\xi_2 = \left\langle p, ({\frac{15}{40}}^{\delta_1}, {\frac{7}{40}}^{\delta_2}, {\frac{6}{40}}^{\delta_3}), ({\frac{7}{40}}^{\delta_1}, {\frac{6}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3})\right\rangle$ and
$\zeta_1 = \left\langle q, ({\frac{7}{40}}^{\delta_1}, {\frac{17}{40}}^{\delta_2}, {\frac{15}{40}}^{\delta_3}), ({\frac{24}{40}}^{\delta_1}, {\frac{7}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3})\right\rangle$. Then, the map $\delta : (J_m, \tau(1)_{PN}) \longrightarrow (J_n, \tau(2)_{PN})$ is a $\lfloor C\#, \beta_{\rho n}\rfloor$-map and it is defined as $\delta(\alpha^1) = \beta^1$ and $\delta(\alpha^2) = \beta^2$, since $\delta(V)$ is $(\beta_{\rho n})$-CS in $(J_n, \tau(2)_{PN})$ for every PN-CS, V of $(J_m, \tau(1)_{PN})$.

Example 2.11. Let $J_m = \{\alpha^1, \alpha^2\}$ and $J_n = \{\beta^1, \beta^2\}$. Then, $\tau(1)_{PN} = \{0_N,\ 1_N,\ \xi_1,\ \xi_2\}$ and $\tau(2)_{PN} = \{0_N,\ 1_N,\ \zeta_1\}$ is a Pythagorean neutrosophic topological spaces on J_m and J_n respectively, where
$\xi_1 = \left\langle p, ({\frac{7}{40}}^{\delta_1}, {\frac{13}{40}}^{\delta_2}, {\frac{24}{40}}^{\delta_3}), ({\frac{15}{40}}^{\delta_1}, {\frac{12}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3})\right\rangle$,
$\xi_2 = \left\langle p, ({\frac{11}{40}}^{\delta_1}, {\frac{15}{40}}^{\delta_2}, {\frac{12}{40}}^{\delta_3}), ({\frac{24}{40}}^{\delta_1}, {\frac{11}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3})\right\rangle$ and
$\zeta_1 = \left\langle q, ({\frac{11}{40}}^{\delta_1}, {\frac{24}{40}}^{\delta_2}, {\frac{12}{40}}^{\delta_3}), ({\frac{15}{40}}^{\delta_1}, {\frac{12}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3})\right\rangle$. Then, the map $\delta : (J_m, \tau(1)_{PN}) \longrightarrow (J_n, \tau(2)_{PN})$ is a $\lfloor O*, \beta_{\rho n}\rfloor$-map and it is defined as $\delta(\alpha^1) = \beta^1$ and $\delta(\alpha^2) = \beta^2$, since $\delta(V)$ is $(\beta_{\rho n})$-OS in (J_n, τ_{PN}) for every PN-OS, V of (J_m, τ_{PN}).

Example 2.12. Let $J_m = \{\alpha^1, \alpha^2\}$ and $J_n = \{\beta^1, \beta^2\}$. Then, $\tau(1)_{PN} = \{0_N,\ 1_N,\ \xi_1,\ \xi_2\}$ and $\tau(2)_{PN} = \{0_N,\ 1_N,\ \zeta_1\}$ is a Pythagorean neutrosophic topological spaces on J_m and J_n respectively, where
$\xi_1 = \left\langle p, ({\frac{17}{40}}^{\delta_1}, {\frac{17}{40}}^{\delta_2}, {\frac{6}{40}}^{\delta_3}), ({\frac{7}{40}}^{\delta_1}, {\frac{6}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3})\right\rangle$,
$\xi_2 = \left\langle p, ({\frac{17}{40}}^{\delta_1}, {\frac{6}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3}), ({\frac{6}{40}}^{\delta_1}, {\frac{17}{40}}^{\delta_2}, {\frac{15}{40}}^{\delta_3})\right\rangle$ and
$\zeta_1 = \left\langle q, ({\frac{15}{40}}^{\delta_1}, {\frac{24}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3}), ({\frac{6}{40}}^{\delta_1}, {\frac{17}{40}}^{\delta_2}, {\frac{6}{40}}^{\delta_3})\right\rangle$. Then, the map $\delta : (J_m, \tau(1)_{PN}) \longrightarrow (J_n, \tau(2)_{PN})$ is a $\lfloor C\#, \beta_{\rho n}\rfloor$-map and it is defined as $\delta(\alpha^1) = \beta^1$ and $\delta(\alpha^2) = \beta^2$, since $\delta(V)$ is $(\beta_{\rho n})$-CS in $(J_n, \tau(2)_{PN})$ for every PN-CS, V of $(J_m, \tau(1)_{PN})$.

Example 2.13. Let $J_m = \{\alpha^1, \alpha^2\}$ and $J_n = \{\beta^1, \beta^2\}$. Then, $\tau(1)_{PN} = \{0_N,\ 1_N,\ \xi_1,\ \xi_2\}$ and $\tau(2)_{PN} = \{0_N,\ 1_N,\ \zeta_1\}$ is a Pythagorean neutrosophic topological spaces on J_m and J_n respectively, where
$\xi_1 = \left\langle q, ({\frac{15}{40}}^{\delta_1}, {\frac{24}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3}), ({\frac{6}{40}}^{\delta_1}, {\frac{17}{40}}^{\delta_2}, {\frac{6}{40}}^{\delta_3})\right\rangle$,
$\xi_2 = \left\langle p, ({\frac{17}{40}}^{\delta_1}, {\frac{17}{40}}^{\delta_2}, {\frac{6}{40}}^{\delta_3}), ({\frac{7}{40}}^{\delta_1}, {\frac{6}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3})\right\rangle$ and
$\zeta_1 = \left\langle p, ({\frac{17}{40}}^{\delta_1}, {\frac{6}{40}}^{\delta_2}, {\frac{17}{40}}^{\delta_3}), ({\frac{6}{40}}^{\delta_1}, {\frac{17}{40}}^{\delta_2}, {\frac{15}{40}}^{\delta_3})\right\rangle$. Then, the map $\delta : (J_m, \tau(1)_{PN}) \longrightarrow (J_n, \tau(2)_{PN})$ is a $\lfloor O*, \beta_{\rho n}\rfloor$-map and it is defined as $\delta(\alpha^1) = \beta^1$ and $\delta(\alpha^2) = \beta^2$, since $\delta(V)$ is $(\beta_{\rho n})$-OS in (J_n, τ_{PN}) for every PN-OS, V of (J_m, τ_{PN}).

Proposition 2.14. *If $\delta : (J_m, \tau_{PN}) \longrightarrow (J_n, \tau_{PN})$ be an $\beta_{\rho n}\#CF$ and (J_m, τ_{PN}) be a $\Omega^{\langle\beta_{\rho n}}$-space. Then δ is continuous.*

Proof. Here K be CS in (J_n, τ_{PN}). As for $\delta \Longrightarrow \beta_{\rho n}\#CF$, $\delta^{-1}(K)$ is an $(\beta_{\rho n})$-CS in (J_m, τ_{PN}). Since (J_m, τ_{PN}) is a $\Omega^{\langle \beta_{\rho n}}$-space, $\delta^{-1}(K)$ is closed set in (J_m, τ_{PN}). Hence δ is continuous. □

Let $\delta : (J_m, \tau_{PN}) \longrightarrow (J_n, \tau_{PN})$ be a mapping and (J_m, τ_{PN}) be a $\Omega^{\langle \beta_{\rho n}}$-space, then δ is continuous if one of the two condition holds true.
(a) δ is $\beta_{\rho n}\#CF$.
(b) δ is $\beta_{\rho n} \prec IF$.

Theorem 2.15. *A map $\delta : (J_m, \tau_{PN}) \longrightarrow (J_n, \tau_{PN})$ is an $\beta_{\rho n}\#CF \Longleftrightarrow$ inverse image of all OS in $(J_n, \tau_{PN}) \Longrightarrow (\beta_{\rho n})$-$OS$ in a (J_m, τ_{PN}).*

Proof. $Necessity$: Let $\delta : (J_m, \tau_{PN}) \longrightarrow (J_n, \tau_{PN})$ be an $\beta_{\rho n}\#CF$ and U be an OS in (J_n, τ_{PN}), $J_n - U$ is CS in a (J_n, τ_{PN}). Since δ is a $\beta_{\rho n}\#CF$, $\delta^{-1}(J_n - U) = J_m - \delta^{-1}(U)$ is a $(\beta_{\rho n})$-CS in (J_m, τ_{PN}) and hence $\delta^{-1}(U)$ is an $(\beta_{\rho n})$-OS in (J_m, τ_{PN}).
$Sufficiency$: Assume that $\delta^{-1}(K)$ is a $(\beta_{\rho n})$-OS in (J_m, τ_{PN}) for each open set K in (J_n, τ_{PN}). Let K be a CS in (J_n, τ_{PN}), $J_n - K$ is an OS in (J_n, τ_{PN}). By assumption, $\delta^{-1}(J_n - K) = J_m - \delta^{-1}(K)$ is an $(\beta_{\rho n})$-OS in (J_m, τ_{PN}), which implies that $\delta^{-1}(K)$ is a $(\beta_{\rho n})$-CS in (J_m, τ_{PN}). Hence δ is a $\beta_{\rho n}\#CF$. □

Theorem 2.16. *If $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$ is a $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ and $\Omega^{\delta_{[2]}}$ is Hausdorff, then $\tilde{\varpi}$ has $(\beta_{\rho n})$-CS point inverses.*

Proof. Let $e2 \in \Omega^{\delta_{[2]}}$ and $e1$ is in $\{e1 \in \Omega^{\delta_{[1]}} : \tilde{\varpi}(e1) \neq e2\}$. Since $\tilde{\varpi}(e1) \neq e2$ and $\Omega^{\delta_{[2]}}$ is Hausdorff, $\exists$ disjoint OS Gz_1, Gz_2 such that $\tilde{\varpi}(e1) \in Gz_1$ and $e2 \in Gz_2$. Since $Gz_1 \cap Gz_2 = \phi$ then $Cl(Gz_1) \cap Gz_2 = \phi$. We have $e2 \neq Cl(Gz_1)$. Since $\tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$, $\exists$ a $(\beta_{\rho n})$-OS $Q_{\theta(1)}$ containing $e1$ such that $\tilde{\varpi}(Q_{\theta(1)}) \subset Cl(Gz_1)$. Assume that $Q_{\theta(1)}$ is not contained in $\{e1 \in \Omega^{\delta_{[1]}} : \tilde{\varpi}(e1) \neq e2\}$. $\exists$ a point $r1 \in Q_{\theta(1)}$ such that $\tilde{\varpi}(r1) = e2$. Since $\tilde{\varpi}(Q_{\theta(1)}) \subset Cl(Gz_1)$, we have $e2 = \tilde{\varpi}(r1) \in Cl(Gz_1)$. This is a contradiction. Hence, $Q_{\theta(1)} \subset \{e1 \in \Omega^{\delta_{[1]}} : \tilde{\varpi}(e1) \neq e2\}$ and $Q_{\theta(1)}$ is $(\beta_{\rho n})$-OS in $\Omega^{\delta_{[1]}}$. This shows that $\{e1 \in \Omega^{\delta_{[1]}} : \tilde{\varpi}(e1) \neq e2\}$ is $(\beta_{\rho n})$-OS in $\Omega^{\delta_{[1]}}$, equivalently $\tilde{\varpi}^{-1}(e2) = \{e1 \in \Omega^{\delta_{[1]}} : \tilde{\varpi}(e1) = e2\}$ is $(\beta_{\rho n})$-CS in $\Omega^{\delta_{[1]}}$. □

A space $\Omega^{\delta_{[1]}}$ is locally indiscrete $\Longleftrightarrow \forall(\beta_{\rho n})$-$OS$ of $\Omega^{\delta_{[1]}}$ is open in $\Omega^{\delta_{[1]}}$.

Remark 2.17. Let $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$ a function, $\Omega^{\delta_{[1]}}$ is locally indiscrete $\Longrightarrow \tilde{\varpi}$ is *pn-wc* $\Longleftrightarrow \tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$.

Theorem 2.18. *If $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ and $\tilde{\Theta} : (\Omega^{\delta_{[2]}}, \theta(2)_{PN}) \longrightarrow (\Omega^{\delta_{[3]}}, \theta(3)_{PN})$ is continuous, then the composition $\tilde{\Theta} \circ \tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[3]}}, \theta(3)_{PN})$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$.*

Proof. Let $e1 \in \Omega^{\delta_{[1]}}$ and Ai be an OS of $\Omega^{\delta_{[3]}}$ containing $\tilde{\Theta}(\tilde{\varpi}(e1))$. We've $\tilde{\Theta}^{-1}(Ai)$ is an OS of $\Omega^{\delta_{[2]}}$ containing $\tilde{\varpi}(e1)$. Then $\exists$ a $(\beta_{\rho n})$-OS Aj containing $e1$ such that $\tilde{\varpi}(Aj) \subset Cl(\tilde{\Theta}^{-1}(Ai))$. Since $\tilde{\Theta}$ is continuous, then $(\tilde{\Theta} \circ \tilde{\varpi})(Aj) \subset \tilde{\Theta}(Cl(\tilde{\Theta}^{-1}(Ai))) \subset Cl(Ai)$. Thus, $\tilde{\Theta} \circ \tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$. □

Remark 2.19. Let $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$ be $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ and $\Omega^{\delta_{[2]}}$ be Urysohn. If $(\beta_{\rho n})$-$O(\Omega^{\delta_{[1]}})$ is closed under the finite intersections, then the set $\{e1 \in \Omega^{\delta_{[1]}} : \tilde{\varpi}(e1) = \tilde{\Theta}(e1)\}$ is $(\beta_{\rho n})$-CS in $\Omega^{\delta_{[1]}}$.

For a function $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$, the graph function $\tilde{\Theta} : \Omega^{\delta_{[1]}} \longrightarrow \Omega^{\delta_{[1]}} \times \Omega^{\delta_{[2]}}$ of $\tilde{\varpi}$ is defined by $\tilde{\Theta}(e1) = (e1,\ \tilde{\varpi}(e1))$ for each $e1 \in \Omega^{\delta_{[1]}}$.

Theorem 2.20. *If the graph function $\tilde{\Theta}$ of a function $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$, then $\tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$.*

Proof. Let $\tilde{\Theta}$ be $(\beta_{\rho n})$-$W^{\theta}C_{\#}$ and $e1 \in \Omega^{\delta_{[1]}}$ and $Q_{\theta(1)}$ be an OS of $\Omega^{\delta_{[2]}}$ containing $\tilde{\varpi}(e1)$. Then $\Omega^{\delta_{[1]}} \times Q_{\theta(1)}$ is an OS containing $\tilde{\Theta}(e1)$. $\exists$ a $(\beta_{\rho n})$-OS $Q_{\theta(2)}$ containing $e1$ such that $\tilde{\Theta}(Q_{\theta(2)}) \subset Cl(\Omega^{\delta_{[1]}} \times Q_{\theta(1)}) = \Omega^{\delta_{[1]}} \times Cl(Q_{\theta(1)})$. This implies that $\tilde{\varpi}(Q_{\theta(2)}) \times Cl(Q_{\theta(1)})$ and hence $\tilde{\varpi}$ is $(\beta_{\rho n})$-$W^{\theta}C_{\#}$. □

Definition 2.21. A function $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$ is said to be a $\langle\Psi,\ c\rangle$-map then $\rho n_{\beta}^{\#C}(\tilde{\varpi}^{-1}(Z)) \subseteq \tilde{\varpi}^{-1}(\rho n_{\beta}^{\#C}(Z))$.

Definition 2.22. Two non-empty subsets Y and Z of $PNTS(J, \tau_{PN})$ have been referred to as $(\beta_{\rho n})$-segmented if $Y \cap \rho n_{\beta}^{\#C}(Z) = \rho n_{\beta}^{\#C}(Y) \cap Z = \phi$.

Definition 2.23. Two non-empty subsets Y and Z of $PNTS(J, \tau_{PN})$ have been referred to as $\frac{1}{2}$-$(\beta_{\rho n})$-segmented if $Y \cap \rho n_{\beta}^{\#C}(Z) = \phi$ or $\rho n_{\beta}^{\#C}(Y) \cap Z = \phi$.

Definition 2.24. A set $U_{\theta(1)} \subset P^{\delta_{[1]}}$ have been referred to as $\frac{1}{2}$-$(\beta_{\rho n})$-Linked, if $U_{\theta(1)}$ doesn't consist of two non-empty $\frac{1}{2}$-$(\beta_{\rho n})$-segmented sets in $P^{\delta_{[1]}}$.

Definition 2.25. A subset L of $PNTS(J, \tau_{PN})$ have been referred to as $(\beta_{\rho n})$-Linked if there are no two $(\beta_{\rho n})$-segmented subsets Y and Z such that $L = Y \cap Z$.

Definition 2.26. ($(\beta_{\rho n}) \approx (\beta_{\rho n})$ segmented sets). Let Y and Z are two non-empty subsets of the $PNTS(J, \tau_{PN})$ are called $(\beta_{\rho n}) \approx (\beta_{\rho n})$ segmented sets if $\rho n_{\beta}^{\#C}(Y) \cap \rho n_{\beta}^{\#C}(Z) = \phi$.

Definition 2.27. A subset Y of $PNTS(J, \tau_{PN})$ have been referred to as $(\beta_{\rho n}) \approx (\beta_{\rho n})$-Linked, if Y not able to be shown as the union of two $(\beta_{\rho n}) \approx (\beta_{\rho n})$ segmented sets in (J, τ_{PN}).

Theorem 2.28. *A space $\Omega^{\delta_{[1]}}$ is $(\beta_{\rho n}) \approx (\beta_{\rho n})$-Linked $\Longleftrightarrow$ it is not possible to express $\Omega^{\delta_{[1]}}$ as union of two non-empty and disjoint $(\beta_{\rho n})$-$CLOS$.*

Proof. Firstly, assume that $\Omega^{\delta_{[1]}}$ be $(\beta_{\rho n}) \approx (\beta_{\rho n})$-Linked. If possible, let us assume $\Omega^{\delta_{[1]}}$ is equal to $Gz_{\theta(1)}$ union $Gz_{\theta(2)}$ such that $Gz_{\theta(1)}$ and $Gz_{\theta(2)}$ are non-empty disjoint $(\beta_{\rho n})$-$CLOS$. Therefore $\rho n_{\beta}^{\#C}(Gz_{\theta(1)})$ & $Gz_{\theta(1)}$ are equal and $\rho n_{\beta}^{\#C}(Gz_{\theta(2)})$ & $Gz_{\theta(2)}$ are equal, then the juncture of $\rho n_{\beta}^{\#C}(Gz_{\theta(1)})$ & $\rho n_{\beta}^{\#C}(Gz_{\theta(2)})$ are equal to ϕ, which is contradiction to $\Omega^{\delta_{[1]}}$ is $(\beta_{\rho n}) \approx (\beta_{\rho n})$-Linked. Because of this, it cannot be stated as the disjoint union of two nonempty $(\beta_{\rho n})$-$CLOS$. Conversely, let us assume $\Omega^{\delta_{[1]}}$ is not $(\beta_{\rho n}) \approx (\beta_{\rho n})$-Linked space. Therefore $\exists$ two non empty sets $Gz_{\theta(1)}$ and $Gz_{\theta(2)}$ such that $\Omega^{\delta_{[1]}} = Gz_{\theta(1)} \cup Gz_{\theta(2)}$ and $\rho n_{\beta}^{\#C}(Gz_{\theta(1)}) \cap \rho n_{\beta}^{\#C}(Gz_{\theta(2)}) = \phi$. Then $\Omega^{\delta_{[1]}} = \rho n_{\beta}^{\#C}(Gz_{\theta(1)}) \cup \rho n_{\beta}^{\#C}(Gz_{\theta(2)})$. Both sets $Q_{\theta(1)} = \rho n_{\beta}^{\#C}(Gz_{\theta(1)})$ and $Q_{\theta(2)} = \rho n_{\beta}^{\#C}(Gz_{\theta(2)})$ are non-empty disjoint $(\beta_{\rho n})$-$CLOS$ and $\Omega^{\delta_{[1]}} = Q_{\theta(1)} \cup Q_{\theta(2)}$, this is a contradiction. Hence $\Omega^{\delta_{[1]}}$ is $(\beta_{\rho n}) \approx (\beta_{\rho n})$-Linked. □

Theorem 2.29. *Every $(\beta_{\rho n})$-Linked space is constantly $(\beta_{\rho n}) \approx (\beta_{\rho n})$-Linked.*

Proof. Let $\Omega^{\delta_{[1]}}$ be a $(\beta_{\rho n})$-Linked space, then there does not exists two $(\beta_{\rho n})$-segmented subsets $U_{\theta(1)}$ and $U_{\theta(2)}$ such that $\Omega^{\delta_{[1]}} = U_{\theta(1)} \cup U_{\theta(2)}$, there are no two $(\beta_{\rho n}) \approx (\beta_{\rho n})$ segmented subsets $U_{\theta(1)}$ and $U_{\theta(2)}$ such that $\Omega^{\delta_{[1]}} = U_{\theta(1)} \cup U_{\theta(2)}$. Hence $\Omega^{\delta_{[1]}}$ is $(\beta_{\rho n}) \approx (\beta_{\rho n})$-Linked. □

Theorem 2.30. *The representation of $(\beta_{\rho n})$-Linked space next to a $\langle\Psi,\ c\rangle$-map is $(\beta_{\rho n})$-Linked.*

Proof. Given $\Omega^{\delta_{[1]}}$ is $(\beta_{\rho n})$-Linked and $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$ is a $\langle\Psi,\ c\rangle$-map. If possible, assume $\tilde{\varpi}(\Omega^{\delta_{[1]}})$ is not $(\beta_{\rho n})$-Linked, $\exists$ two non-void sets $Gz_{\theta(1)}$ and $Gz_{\theta(2)}$ such that $\tilde{\varpi}(\Omega^{\delta_{[1]}}) = Gz_{\theta(1)} \cup Gz_{\theta(2)}$ and $Gz_{\theta(1)} \cap \rho n_{\beta}^{\#C}(Gz_{\theta(2)}) = \phi = \rho n_{\beta}^{\#C}(Gz_{\theta(1)}) \cap Gz_{\theta(2)}$. Then $\Omega^{\delta_{[1]}} = \tilde{\varpi}^{-1}(Gz_{\theta(1)}) \cup \tilde{\varpi}^{-1}(Gz_{\theta(2)})$. Firstly, $Gz_{\theta(1)} \cap \rho n_{\beta}^{\#C}(Gz_{\theta(2)}) = \phi$ implies $\tilde{\varpi}^{-1}(Gz_{\theta(1)} \cap \rho n_{\beta}^{\#C}(Gz_{\theta(2)})) = \phi$ and since $\tilde{\varpi}$ is a $\langle\Psi,\ c\rangle$-map, so we get $\tilde{\varpi}^{-1}(Gz_{\theta(1)}) \cap \rho n_{\beta}^{\#C}\tilde{\varpi}^{-1}(Gz_{\theta(2)}) \subset \tilde{\varpi}^{-1}(Gz_{\theta(1)} \cap \rho n_{\beta}^{\#C}(Gz_{\theta(2)})) = \phi$ Similarly, $\rho n_{\beta}^{\#C}(Gz_{\theta(1)}) \cap \tilde{\varpi}^{-1}(Gz_{\theta(2)}) \subset \tilde{\varpi}^{-1}(\rho n_{\beta}^{\#C}(Gz_{\theta(1)}) \cap Gz_{\theta(2)}) = \phi$. This contradicts the $(\beta_{\rho n})$-Linked of $\Omega^{\delta_{[1]}}$. Hence $\tilde{\varpi}(\Omega^{\delta_{[1]}})$ must be $(\beta_{\rho n})$-Linked. □

Theorem 2.31. *The image of $\frac{1}{2}$-$(\beta_{\rho n})$-Linked space under a $\langle\Psi,\ c\rangle$-map is $\frac{1}{2}$-$(\beta_{\rho n})$-Linked.*

Proof. Given $P^{\delta_{[1]}}$ is $\frac{1}{2}$-$(\beta_{\rho n})$-Linked and $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$ is a $\langle\Psi,\ c\rangle$-map. Let us assume that $\tilde{\varpi}(\Omega^{\delta_{[1]}})$ is not $\frac{1}{2}$-$(\beta_{\rho n})$-Linked $\exists$ two non-empty sets $Gc_{\theta(1)}$ and $Gc_{\theta(2)}$ such that $\tilde{\varpi}(\Omega^{\delta_{[1]}}) = Gc_{\theta(1)} \cup Gc_{\theta(2)}$ and $Gc_{\theta(1)} \cap \rho n_{\beta}^{\#C}(Gc_{\theta(2)}) = \phi$ or $\rho n_{\beta}^{\#C}(Gc_{\theta(1)}) \cap Gc_{\theta(2)} =$

ϕ. By hypotheses $\Omega^{\delta_{[1]}} = \tilde{\varpi}^{-1}(Gc_{\theta(1)}) \cup \tilde{\varpi}^{-1}(Gc_{\theta(2)})$. If $Gc_{\theta(1)} \cap \rho n_{\beta}^{\#C}(Gc_{\theta(2)}) = \phi$ implies $\tilde{\varpi}^{-1}(Gc_{\theta(1)} \cap \rho n_{\beta}^{\#C}(Gc_{\theta(2)})) = \phi$ and since $\tilde{\varpi}$ is a $\langle \Psi,\ c \rangle$-map so we get $\tilde{\varpi}^{-1}(Gc_{\theta(1)}) \cap \rho n_{\beta}^{\#C} \tilde{\varpi}^{-1}(Gc_{\theta(2)}) \subset \tilde{\varpi}^{-1}(Gc_{\theta(1)} \cap \rho n_{\beta}^{\#C}(Gc_{\theta(2)})) = \phi$ On the other hand, if $\rho n_{\beta}^{\#C}(Gc_{\theta(1)}) \cap Gc_{\theta(2)} = \phi$ implies $\rho n_{\beta}^{\#C} \tilde{\varpi}^{-1}(Gc_{\theta(1)}) \cap \tilde{\varpi}^{-1}(Gc_{\theta(2)}) \subset \tilde{\varpi}^{-1}(\rho n_{\beta}^{\#C}(Gc_{\theta(1)}) \cap Gc_{\theta(2)}) = \phi$. This contradicts the $\frac{1}{2}$-$(\beta_{\rho n})$-Linked of $\Omega^{\delta_{[1]}}$. Hence $\tilde{\varpi}(\Omega^{\delta_{[1]}})$ must be $\frac{1}{2}$-$(\beta_{\rho n})$-Linked. □

Theorem 2.32. *The image of* $(\beta_{\rho n}) \approx (\beta_{\rho n})$*-Linked space under a* $\langle \Psi,\ c \rangle$*-map is* $(\beta_{\rho n}) \approx (\beta_{\rho n})$*-Linked.*

Proof. Given $\tilde{\varpi} : (\Omega^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (\Omega^{\delta_{[2]}}, \theta(2)_{PN})$ be a $\langle \Psi,\ c \rangle$-map, $\Omega^{\delta_{[1]}}$ be $(\beta_{\rho n}) \approx (\beta_{\rho n})$-Linked and $\Omega^{\delta_{[2]}}$ be any pythogoren neutrosophic topological space. If possible, assume that $\tilde{\varpi}(\Omega^{\delta_{[1]}})$ is not $(\beta_{\rho n}) \approx (\beta_{\rho n})$-Linked, $\exists$ two non-empty disjoint subsets of $\tilde{\varpi}(\Omega^{\delta_{[1]}})$ such that $\tilde{\varpi}(\Omega^{\delta_{[1]}}) = Gc_{\theta(1)} \cup Gc_{\theta(2)}$ and $n_{\beta}^{\#C}(Gc_{\theta(1)}) \cap n_{\beta}^{\#C}(Gc_{\theta(2)}) = \phi$. Then $\Omega^{\delta_{[1]}} = \tilde{\varpi}^{-1}(Gc_{\theta(1)}) \cup \tilde{\varpi}^{-1}(Gc_{\theta(2)})$. As $n_{\beta}^{\#C}(Gc_{\theta(1)}) \cap n_{\beta}^{\#C}(Gc_{\theta(2)}) = \phi$ implies
$\tilde{\varpi}^{-1}(n_{\beta}^{\#C}(Gc_{\theta(1)}) \cap n_{\beta}^{\#C}(Gc_{\theta(2)})) = \phi$
$\tilde{\varpi}^{-1}(n_{\beta}^{\#C}(Gc_{\theta(1)})) \cap \tilde{\varpi}^{-1}(n_{\beta}^{\#C}(Gc_{\theta(2)})) = \phi$ which implies
$n_{\beta}^{\#C}(\tilde{\varpi}^{-1}(Gc_{\theta(1)})) \cap n_{\beta}^{\#C}(\tilde{\varpi}^{-1}(Gc_{\theta(2)})) = \phi$ therefore the sets $\tilde{\varpi}^{-1}(Gc_{\theta(1)})$ and $\tilde{\varpi}^{-1}(Gc_{\theta(2)})$ a form $(\beta_{\rho n}) \approx (\beta_{\rho n})$-separation which contradicts that $\Omega^{\delta_{[1]}}$ is $(\beta_{\rho n}) \approx (\beta_{\rho n})$-Linked. Hence $\tilde{\varpi}(Gc_{\theta(1)})$ is $(\beta_{\rho n}) \approx (\beta_{\rho n})$-Linked. □

Theorem 2.33. *Let* $P^{\delta_{[1]}}$ *be a* $(\beta_{\rho n})$*-connected space,* $P^{\delta_{[2]}}$ *is an ordered PNTS and* $\tilde{\varpi} : (P^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (P^{\delta_{[2]}}, \theta(2)_{PN})$ *is a* $\langle \Psi,\ c \rangle$*-map. If* $q_1,\ q_2 \in P^{\delta_{[1]}}$ *and* $r \in P^{\delta_{[2]}}$ *s.t.* $r \in (\tilde{\varpi}(q_1),\ \tilde{\varpi}(q_2))$*. Then* $\exists$ $q_3 \in P^{\delta_{[1]}}$ *such that* $\tilde{\varpi}(q_3) = r$*.*

Proof. Given $P^{\delta_{[1]}}$ is a $(\beta_{\rho n})$-connected space, $P^{\delta_{[2]}}$ is an ordered PNTS and $\tilde{\varpi} : (P^{\delta_{[1]}}, \theta(1)_{PN}) \longrightarrow (P^{\delta_{[2]}}, \theta(2)_{PN})$ be a $\langle \Psi,\ c \rangle$-map. The sets $U_{\theta(1)} = \tilde{\varpi}(P^{\delta_{[1]}})/(-\infty,\ r)$ and $U_{\theta(2)} = \tilde{\varpi}(P^{\delta_{[1]}})/(r,\ \infty)$ are non-empty and disconnected due to $\tilde{\varpi}(q_1) \in U_{\theta(1)}$ and $\tilde{\varpi}(q_2) \in U_{\theta(2)}$. Each set are open $\tilde{\varpi}(P^{\delta_{[1]}})$ due to the fact both rays are open in $P^{\delta_{[2]}}$. $\exists$ no $q_3 \in P^{\delta_{[1]}}$ such that $\tilde{\varpi}(q_3) = r$, then $\tilde{\varpi}(P^{\delta_{[1]}})$ could represent a union of $U_{\theta(1)}$ and $U_{\theta(2)}$. Then $U_{\theta(1)}$ and $U_{\theta(2)}$ established a separation of $\tilde{\varpi}(P^{\delta_{[1]}})$, this infractions the image of $(\beta_{\rho n})$-connected space under $\langle \Psi,\ c \rangle$-map is connected. Hence $\exists$ a $q_3 \in P^{\delta_{[1]}}$ such that $\tilde{\varpi}(q_3) = r$. □

3 Conclusion remarks

A pythogorean neutrosophic topological space is by definition an unusually surprising phenomenon. In some locations, the entire space is an open set, as is the empty set. An intersection of a non-empty set and its closure is empty in a space with separated and connected spaces were studied in this research work. Furthermore, the we've created and deduced several outcomes for various sorts of functions we have used in the current functions.

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Author information

P. Basker, Department of Mathematics, Chandigarh University, Mohali-140413, Punjab., India.
E-mail: `mcpdbasker@gmail.com, basker.e11236@cumail.in`