Palestine Journal of Mathematics

Vol 15(Special Issue II)(2026) , 233–244

PERFORMANCE ANALYSIS FOR AN $M/M/1$ QUEUE WITH WORKING BREAKDOWNS AND INTERRUPTIONS

V. Vijayalakshmi, J. Vimal Andrew and U.R. Ramakrishnan

2020 Mathematics Subject Classification: 60K25, 60J27, 90B22, 68M20.

Keywords: Queueing theory, $M/M/1$ queue, Working breakdowns, Service interruptions, Steady-state analysis, Stochastic decomposition, Waiting time distribution, Performance measures.

The authors would like to thank the reviewers and editor for their constructive comments and valuable suggestions that improved the quality of our paper.

Corresponding Author: V.Vijayalakshmi

Abstract *The unlimited capacity Markovian queueing model with working breakdowns and interruptions is investigated in this article. The model's steady-state solution is reached, stochastic decomposition, waiting time distribution and several performance measurements are provided. The numerical examples of the model are carried out.*

1 Introduction

Queueing theory studies systems with congestion and delays, often affected by service interruptions such as server breakdowns [8], catastrophic arrivals [4], or preemptions [3]. Krishnamoorthy et al. [8] surveyed models with both server- and customer-induced disruptions.

Unlike classical assumptions where service stops during failures, practical systems may operate at a reduced rate, termed working breakdowns. Kalidass and Kasturi [5] initiated such models, later extended by Kim and Lee [7] for disaster queues, Kalidass and Pavithra [6] for finite-capacity feedback systems, and Ezeagu et al. [13] for transient analysis. Related studies include Deepa and Kalidass [2], Srinivas et al. [18], Gupta and Kumar [19], and Rathore and Shrivastava [20].

Models with combined breakdowns and vacations are relevant in manufacturing and services. Medhi [11] developed the stochastic basis; Vijayalakshmi and Kalidass [14] examined vacation queues with abandonments and feedback, while Vijayalakshmi, Kalidass, and Pavitha [15] analyzed $M/M/1/N$ queues with breakdowns and two-phase service. These works emphasize the role of unreliability and vacations in system design.

This paper studies a system where the server, under breakdown, continues service at a reduced rate before repair. Section 2 presents the model, Section 3 stability, Section 4 steady-state analysis, Sections 5–6 decomposition and waiting times, Section 7 performance measures, Section 8 numerical examples, and Section 9 conclusions.

2 Model Formulation

We consider a single-server infinite-capacity queue with Poisson arrivals at rate λ and exponential service times with rate μ. The server experiences partial failures, occurring exponentially with rate α. During a failure, it completes the current service at a reduced rate $\mu_b < \mu$, then undergoes repair, exponentially distributed with rate γ, before resuming normal operation. All processes are independent.

Let $C(t)$ represent the state of the server at time t, defined as

$$C(t) = \begin{cases} 0, & \textit{if the server is under repair at time } t, \\ 1, & \textit{if the server is idle or providing normal service at time } t, \\ 2, & \textit{if the server is serving at a reduced rate (working breakdown) at time } t. \end{cases}$$

Let $N(t)$ denote the number of customers present in the system at time t. Then, the joint process $\{(C(t), N(t)), t \geq 0\}$ forms a continuous-time Markov chain. We define

$$P_{i,n}(t) = \Pr\{C(t) = i,\, N(t) = n\}, \quad i = 0, 1, 2,\, n \geq 0.$$

3 Stability Condition

The system's stability is analyzed via the matrix-geometric method. The infinitesimal generator of $\{X(t), C(t), t \geq 0\}$ is represented as a block Jacobi matrix with states in lexicographical order:

$$\mathbf{R} = \begin{bmatrix} \mathbf{A_0} & \mathbf{B_0} & \cdots & & & & & \\ \mathbf{C_1} & \mathbf{A_2} & \mathbf{B_2} & \cdots & & & & \\ & \mathbf{C_2} & \mathbf{A_2} & \mathbf{B_2} & \cdots & & & \\ \vdots & \vdots & \ddots & \vdots & \vdots & & & \\ & & & & & \mathbf{C_2} & \mathbf{A_2} & \mathbf{B_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & & & \end{bmatrix}$$

In this study, A_2, B_2, and C_2 are $m \times m$ matrices, with $m = 3$. The environmental process is dictated by A_2, whereas B_2 and C_2 are diagonal matrices. The elements of the matrix $\mathbf{R}$ are defined as

$$\begin{aligned} \mathbf{A_0} &= \begin{bmatrix} -\gamma & \gamma \\ 0 & 0 \end{bmatrix}, \\ \mathbf{A_1} &= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \end{bmatrix}, \\ \mathbf{B_0} &= \begin{bmatrix} 0 & 0 \\ 0 & \mu \\ \mu_b & 0 \end{bmatrix}, \\ \mathbf{C_1} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mu & 0 \\ \mu_b & 0 & 0 \end{bmatrix}, \\ \mathbf{B_2} &= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}. \end{aligned}$$

According to Neuts' theorem [12], the steady-state probability vector exists if and only if

$$X\mathbf{B_2}\mathbf{e} < X\mathbf{C_2}\mathbf{e}, \tag{3.1}$$

where X represents the stationary distribution of the matrix $\mathbf{F} = C_2 + A_2 + B_2$.

$$\mathbf{F} = \begin{bmatrix} \lambda & \gamma & -\gamma \\ -\alpha & (\lambda + \mu) & \alpha \\ \mu_b & 0 & \lambda \end{bmatrix}.$$

The vector X satisfies $X\mathbf{F} = 0$ and $Xe = 1$, where e is a 3×1 column vector of ones. Solving these equations yields the stationary vector:

$$X = \big(-(\lambda+\mu)\mu_b,\ \gamma\mu_b,\ \lambda^2+\mu\lambda+\alpha\gamma\big)\,. \tag{3.2}$$

By substituting into (3.1) and performing simplification, we obtain the stability condition:

$$\lambda^3 + (\mu-\mu_b)\lambda^2 + (\alpha\gamma+\mu_b\gamma-\mu\mu_b)\lambda < 0. \tag{3.3}$$

Therefore, the system is stable if and only if the parameters λ, α, γ, μ, and μ_b satisfy the inequality derived above.

4 Steady-state analysis

The balance equations for the steady-state probabilities of the model are given by

$$\begin{aligned}
(\lambda+\gamma)P_{0,0} &= \mu_b P_{2,1}, \quad n=0, &(4.1)\\
(\lambda+\gamma)P_{0,n} &= \lambda P_{0,n-1} + \mu_b P_{2,n+1}, \quad n>0, &(4.2)\\
\lambda P_{1,0} &= \gamma P_{0,0} + \mu P_{1,1}, \quad n=0, &(4.3)\\
(\lambda+\mu+\alpha)P_{1,n} &= \gamma P_{0,n} + \mu P_{1,n+1} + \lambda P_{1,n-1}, \quad n>0, &(4.4)\\
(\lambda+\mu_b)P_{2,1} &= \alpha P_{1,1}, \quad n=1, &(4.5)\\
(\lambda+\mu_b)P_{2,n} &= \alpha P_{1,n} + \lambda P_{2,n-1}, \quad n>1. &(4.6)
\end{aligned}$$

To facilitate the analysis of the system, we define the partial probability generating functions (PGFs) for each server state as

$$P_0(z) = \sum_{n=0}^{\infty} P_{0,n} z^n, \qquad P_1(z) = \sum_{n=0}^{\infty} P_{1,n} z^n, \qquad P_2(z) = \sum_{n=1}^{\infty} P_{2,n} z^n.$$

By multiplying the balance equations by suitable powers of z and summing over all n, we obtain the following relationships among the generating functions:

$$\begin{aligned}
z\big[\lambda(1-z)+\gamma\big]P_0(z) - \mu_b\, P_2(z) &= 0, &(4.7)\\
-\gamma P_0(z) + \Big(\lambda(1-z) + \mu\big(1-\tfrac{1}{z}\big) + \alpha\Big)P_1(z) &= \Big(\mu\big(1-\tfrac{1}{z}\big)+\alpha\Big)P_{1,0}, &(4.8)\\
\big(\lambda(1-z)+\mu_b\big)P_2(z) - \alpha P_1(z) &= -\alpha P_{1,0}. &(4.9)
\end{aligned}$$

Solving the linear system (4.11)–(4.13) and setting $z=1$ provides the marginal probabilities of the server states expressed in terms of $P_{1,0}$.

$$\begin{aligned}
P_0(1) &= \frac{\mu_b\,\lambda\alpha\,P_{1,0}}{\gamma\mu\mu_b+\alpha\gamma\mu_b-\alpha\lambda\mu_b-\alpha\gamma\lambda-\mu_b\lambda\gamma}, &(4.10)\\
P_1(1) &= \frac{\big(\gamma\mu\mu_b+\alpha\gamma\mu_b-\alpha\lambda\mu_b-\alpha\gamma\lambda\big)\,P_{1,0}}{\gamma\mu\mu_b+\alpha\gamma\mu_b-\alpha\lambda\mu_b-\alpha\gamma\lambda-\mu_b\lambda\gamma}, &(4.11)\\
P_2(1) &= \frac{\lambda\gamma\alpha\,P_{1,0}}{\gamma\mu\mu_b+\alpha\gamma\mu_b-\alpha\lambda\mu_b-\alpha\gamma\lambda-\mu_b\lambda\gamma}. &(4.12)
\end{aligned}$$

Applying the normalization condition $P_0(1)+P_1(1)+P_2(1)=1$ allows us to derive an explicit expression for $P_{1,0}$.

$$P_{1,0} = \frac{\gamma\mu\mu_b+\alpha\gamma\mu_b-\alpha\lambda\mu_b-\alpha\gamma\lambda-\mu_b\lambda\gamma}{\gamma\mu\mu_b+\alpha\gamma\mu_b}. \tag{4.13}$$

The probability generating function (PGF) for the total number of customers in the system is given by

$$P(z) \quad = \quad P_0(z) + P_1(z) + P_2(z) \tag{4.14}$$

By eliminating P_0, P_1, and P_2 from the linear relations, the PGF can be expressed in a compact rational form:

$$P(z) = \frac{Az^2 - Bz + C}{Dz^3 + Ez^2 - Fz + G}\, P_{1,0}, \tag{4.15}$$

where the coefficients of the polynomial are defined as

$$\begin{aligned}
A &= 3\lambda^2\mu,\\
B &= 6\lambda^2\mu + 2\lambda(\alpha\mu_b + \mu\mu_b + \mu\gamma),\\
C &= 3\lambda^2\mu + \lambda(2\alpha\mu_b + \mu\mu_b + \mu\gamma) + \mu\mu_b(\mu + \alpha),\\
D &= -4\lambda^3,\\
E &= 9\lambda^3 + 3\lambda^2(\mu + \alpha + \mu_b + \gamma),\\
F &= 3\lambda^3 + 2\lambda^2(3\mu + 2\alpha + 2\mu_b + 2\gamma) + 2\lambda(\alpha\mu_b + \alpha\gamma + \mu_b\gamma),\\
G &= \lambda^3 + \lambda(2\mu\mu_b + 2\mu\gamma + \alpha\mu_b + \alpha\gamma + \mu_b\gamma)\\
&\qquad + \gamma\mu_b(\mu + \gamma) + 3\mu + \alpha + \gamma + \mu_b.
\end{aligned}$$

Finally, by combining (4.5), (4.7), and (4.9), the boundary probability $P_{0,0}$ can be expressed in closed form as:

$$P_{0,0} \quad = \quad \frac{\alpha\lambda\big(\gamma\mu\mu_b + \alpha\gamma\mu_b - \alpha\lambda\mu_b - \alpha\gamma\lambda - \mu_b\lambda\gamma\big)}{\gamma(\mu + \alpha)\big[(\lambda\mu + \gamma\mu)(\lambda + \mu_b) + \alpha\gamma\mu_b\big]}. \tag{4.16}$$

5 Stochastic Decomposition of System Size

This section examines the stochastic decomposition of the system size, a property commonly seen in vacation-type queues. The concept was first introduced for retrial queues by Yang and Templeton [16] and later extended by Artalejo and Falin [1] and Yang [17]. In this framework, the system size is expressed as the sum of two independent parts: one for a standard queue without breakdowns and another accounting for the extra load caused by working breakdowns.

$$\begin{aligned}
P(z) &= P_c(z) \times P_{wb}(z),\\
&\textit{where}\\
P_{wb}(z) &= \left(\frac{Az^2 - Bz + C}{Dz^3 + Ez^2 - Fz + G}\right) \times \left(\frac{\alpha + \mu - \lambda - \lambda z}{\mu - \lambda} - \frac{\alpha\lambda}{\gamma(\mu - \lambda)}\right.\\
&\qquad \left. - \frac{\alpha\lambda\big(\mu - (\mu - (\mu_b + \lambda)z)\big)}{\mu\mu_b(\mu - \lambda)} + \frac{\lambda z(\alpha + \lambda\gamma)}{\gamma\mu(\mu - \lambda)}\right),\\
P_c(z) &= \left(\frac{1 - \rho}{1 - \rho z}\right).
\end{aligned}$$

In a standard $M/M/1$ queue, the system-size PGF is $P_c(z)$ [10], while in the present model $P_{wb}(z)$ captures the additional load from working breakdowns.

6 Waiting time distributions

In steady state, let W be the waiting time of a typical customer with Laplace–Stieltjes transform $W^*(s)$. Keilson and Servi [9] established the relation

$$P(z) = W^*(\lambda(1-z)), \tag{6.1}$$

which holds under four conditions: (i) arrivals follow a Poisson process, (ii) customers finding the server busy must wait, (iii) service follows the FIFO rule, and (iv) post-t arrivals are independent of earlier waiting times.

As all four conditions are satisfied in the current model, equation (6.20) holds. Employing (4.18), the Laplace–Stieltjes transform of the waiting-time distribution can be expressed as

$$W^*(s) = \frac{3\mu(\lambda-s)^2 - H(\lambda-s) + I}{-4(\lambda-s)^3 + J(\lambda-s)^2 - K(\lambda-s) + L}, \tag{6.2}$$

where

$$\begin{aligned}
H &= 6\mu + 2(\alpha\mu_b + \mu\mu_b + \mu\gamma),\\
I &= 3\lambda^2\mu + (2\alpha\mu_b + \mu\mu_b + \mu\gamma)\lambda + \mu\mu_b(\mu+\alpha),\\
J &= 9\lambda + 3(\mu + \alpha + \mu_b + \gamma),\\
K &= 3\lambda^2 + 2\lambda(3\mu + 2\alpha + 2\mu_b + 2\gamma) + 2(\alpha\mu_b + \alpha\gamma + \mu_b\gamma),\\
L &= \lambda^3 + \lambda(2\mu\mu_b + 2\mu\gamma + \alpha\mu_b + \alpha\gamma + \mu_b\gamma) + \gamma\mu_b(\mu+\gamma) + 3\mu + \alpha + \gamma + \mu_b.
\end{aligned}$$

7 Performance Measures

In this section, we present derivations for several key performance metrics of the system.

(i) ***Expected number of customers in the system:***

$$E(L) = \sum_{u=0}^{2} P_u^{'}(1).$$

From equations (4.14)–(4.16), the derivatives of the generating functions yield

$$\begin{aligned}
P_0^{'}(1) &= \frac{-\mu_b\lambda^2\alpha P_{1,0}\left(\lambda(\alpha+\mu_b+\gamma) - (\mu\mu_b + \mu\gamma + \alpha\mu_b + \alpha\gamma + \mu_b\gamma)\right)}{(\gamma\mu\mu_b + \alpha\gamma\mu_b - \alpha\lambda\mu_b - \alpha\gamma\lambda - \mu_b\lambda\gamma)^2},\\
P_1^{'}(1) &= \frac{\left(\alpha\lambda^2 - \lambda(\mu\mu_b + \mu\gamma + \alpha\mu_b + \alpha\gamma)\right)P_{1,0}}{\gamma\mu\mu_b + \alpha\gamma\mu_b - \alpha\lambda\mu_b - \alpha\gamma\lambda - \mu_b\lambda\gamma} - \frac{\mathcal{S}}{\mathcal{T}},\\
P_2^{'}(1) &= \frac{\alpha\lambda P_{1,0}(\gamma-\lambda)}{\gamma\mu\mu_b + \alpha\gamma\mu_b - \alpha\lambda\mu_b - \alpha\gamma\lambda - \mu_b\lambda\gamma}\\
&\quad - \frac{\left[\lambda(\alpha+\mu_b+\gamma) - (\mu\mu_b + \mu\gamma + \alpha\mu_b + \alpha\gamma + \mu_b\gamma)\right]\lambda^2\gamma\alpha P_{1,0}}{(\gamma\mu\mu_b + \alpha\gamma\mu_b - \alpha\lambda\mu_b - \alpha\gamma\lambda - \mu_b\lambda\gamma)^2}.
\end{aligned} \tag{7.1}$$

where

$$\mathcal{S} = (\gamma\mu\mu_b + \alpha\gamma\mu_b - \alpha\lambda\mu_b - \alpha\gamma\lambda)\left(\lambda^2(\alpha+\mu_b+\gamma) - \lambda(\mu\mu_b + \mu\gamma + \alpha\mu_b + \alpha\gamma + \mu_b\gamma)\right)P_{1,0},$$

$$\mathcal{T} = (\gamma\mu\mu_b + \alpha\gamma\mu_b - \alpha\lambda\mu_b - \alpha\gamma\lambda - \mu_b\lambda\gamma)^2.$$

Hence, the expected number of customers in the system is

$$E(L) = \sum_{u=0}^{2} P_u^{'}(1) = \frac{-\lambda\mu\mu_b - \lambda\mu\gamma - \alpha\lambda\mu_b}{\gamma\mu\mu_b + \alpha\gamma\mu_b} - \frac{\lambda^2(\alpha+\mu_b+\gamma) - \lambda(\mu\mu_b + \mu\gamma + \alpha\mu_b + \alpha\gamma + \mu_b\gamma)}{\gamma\mu\mu_b + \alpha\gamma\mu_b - \alpha\lambda\mu_b - \alpha\gamma\lambda - \mu_b\lambda\gamma}.$$

(ii) ***Expected waiting time:***

$$E(W) = \frac{E(L)}{\lambda}.$$

(iii) ***Probability that the server is undergoing repair:*** $P_0(1)$.

(iv) ***Probability that the server is busy:*** $P_1(1)$.

(v) ***Probability that the server is operating under a working breakdown:*** $P_2(1)$.

8 Illustrative Numerical Examples

In this section, we provide several examples to illustrate the behavior of the proposed queueing model. Nine cases are analyzed in total.

Example 1. *We first analyze the effect of varying λ and μ on the probability $P_{0,0}$ by fixing $\alpha = 2$, $\gamma = 3$, and $\mu_b = 50$. As shown in Figure 1, $P_{0,0}$ increases with μ when λ is fixed, whereas it decreases with λ when μ is fixed.*

Example 2. *Next, we study the dependence of $P_0(1)$ on λ and μ (Figure 2). For a constant μ, the probability $P_0(1)$ increases with λ. Conversely, for a constant λ, the probability decreases as μ grows.*

Example 3. *Figure 3 illustrates the impact of λ and γ on $P_0(1)$. When λ is fixed, $P_0(1)$ decreases as γ increases. For constant γ, however, $P_0(1)$ rises with λ.*

Example 4. *We examine the effect of λ and μ on $P_1(1)$ (Figure 4). For fixed μ, $P_1(1)$ decreases with λ. On the other hand, for fixed λ, $P_1(1)$ increases with μ.*

Example 5. *In this case, the influence of λ and α on $P_1(1)$ is shown in Figure 5. For a constant λ, $P_1(1)$ decreases with α. Furthermore, for constant α, the probability decreases as λ grows.*

Example 6. *Figure 6 demonstrates the joint effect of μ and α on $P_1(1)$. For fixed μ, the probability $P_1(1)$ decreases as α increases. Conversely, for fixed α, $P_1(1)$ rises with μ.*

Example 7. *The effect of λ and μ on $P_2(1)$ is depicted in Figure 7. For fixed λ, the probability $P_2(1)$ decreases as μ increases. However, for constant μ, it increases with λ.*

Example 8. *Figure 8 presents the effect of λ and α on $P_2(1)$. For a fixed λ, $P_2(1)$ increases as α grows. Similarly, for a fixed α, $P_2(1)$ increases with λ.*

Example 9. *Finally, we investigate the dependence of the expected system size $E(X)$ on μ and μ_b (Figure 9). For fixed μ_b, $E(X)$ decreases as μ increases. Conversely, for fixed μ, $E(X)$ decreases as μ_b increases.*

9 Conclusion

This paper examined a single-server Markovian queueing model subject to working breakdowns and interruptions. We derived several important performance indicators and presented analytical results. To validate and illustrate the theoretical findings, multiple numerical experiments were carried out, highlighting how different system parameters influence overall performance and reliability.

Figure 1. $P_{0,0}$ versus λ and μ

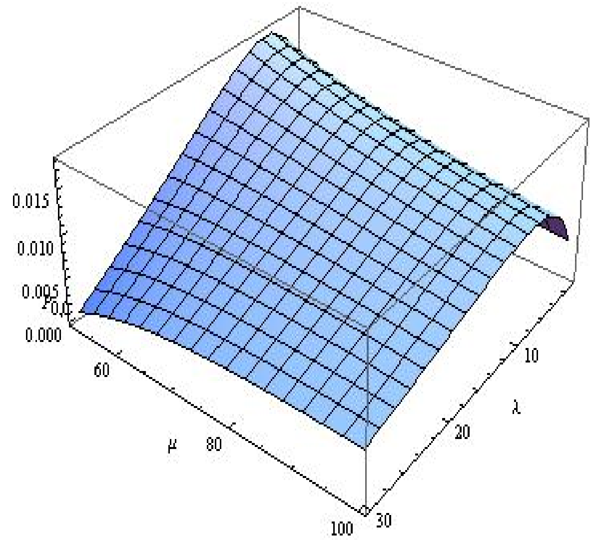


Figure 2. $P_0(1)$ versus λ and μ

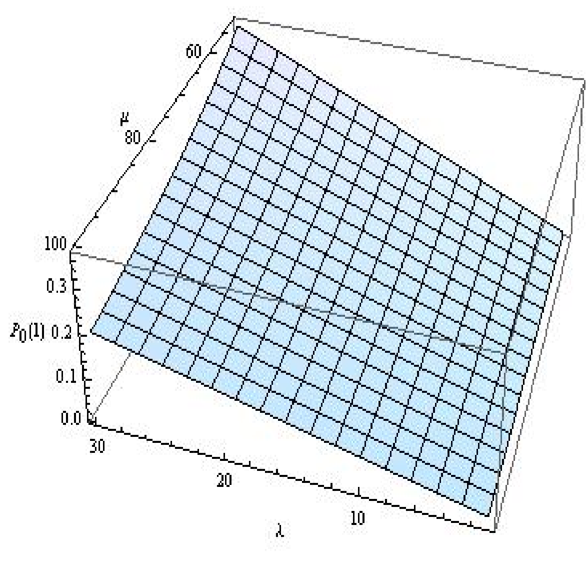


Figure 3. $P_0(1)$ versus λ and γ

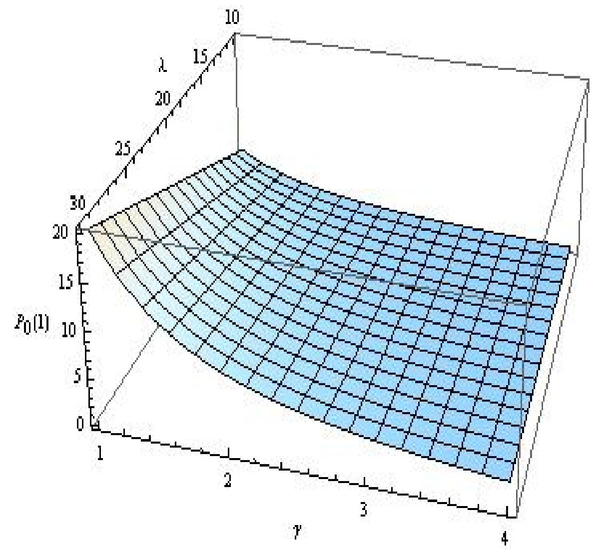


Figure 4. $P_1(1)$ versus λ and μ

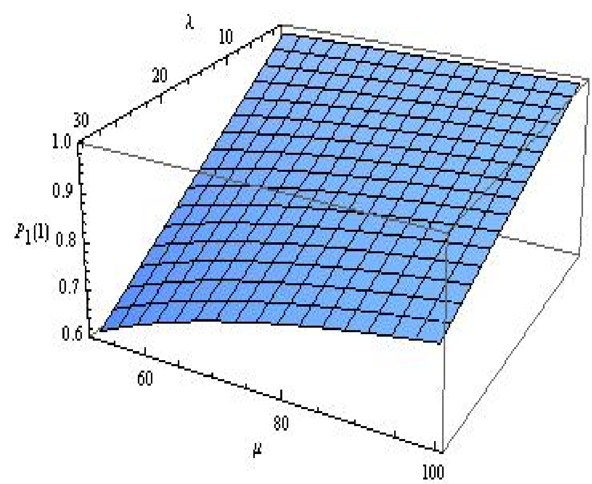


Figure 5. $P_1(1)$ versus λ and α

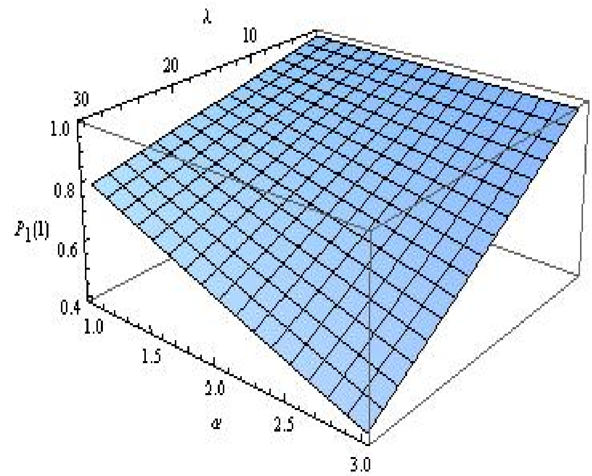


Figure 6. $P_1(1)$ versus μ and α

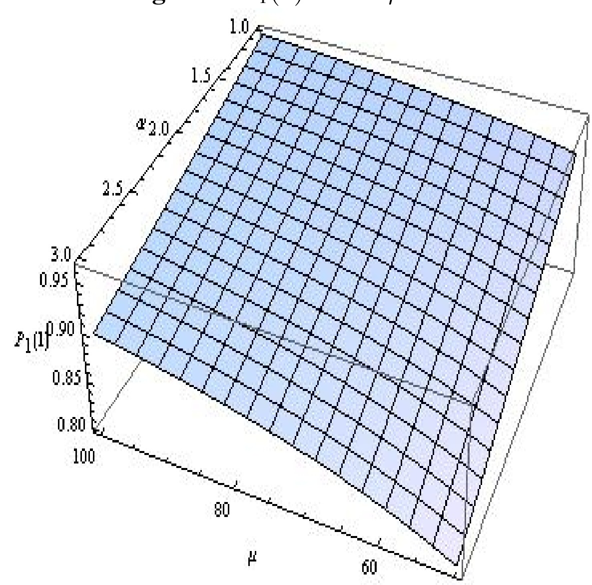


Figure 7. $P_2(1)$ versus λ and μ

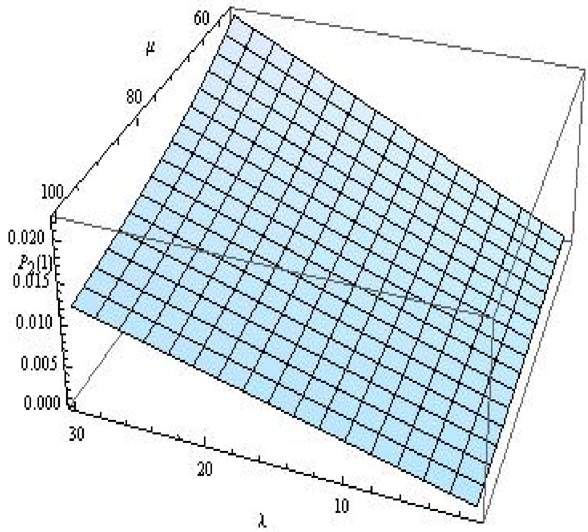


Figure 8. $P_2(1)$ versus λ and α

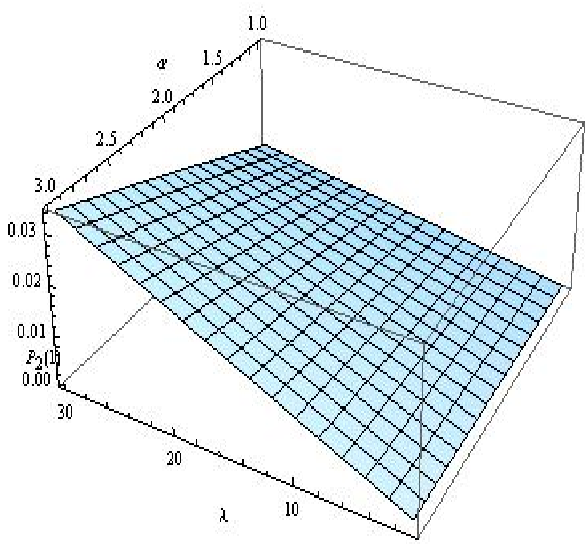
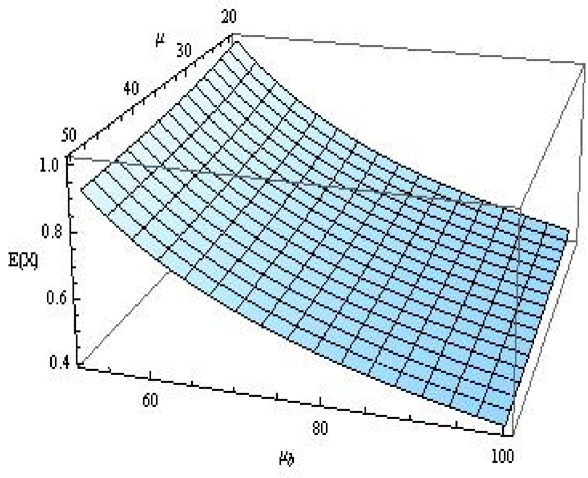


Figure 9. $E(X)$ versus μ and μ_b



References

[1] *J. R. Artalejo and G. I. Falin,* Stochastic decomposition for retrial queues, *Top,* ***2*** *(1994), 329–342.*

[2] *B. Deepa and K. Kalidass,* Markovian queue with finite capacity, working breakdowns and server vacations, *International Journal of Pure Applied Mathematics,* ***10*** *(2018), 859–873.*

[3] *N. K. Jaiswal,* Preemptive resume priority queue, *Operations Research Perspectives,* ***9*** *(1961), 732–770.*

[4] *V. I. Klimenok and A. N. Dudin,* A $BMAP/PH/N$ queue with negative customers and partial protection of service, *Communications in Statistics - Simulation and Computation,* ***41*** *(2012), 1062–1082.*

[5] *K. Kalidass and R. Kasturi,* A queue with working breakdowns, *Computers and Industrial Engineering,* ***63*** *(2012), 779–783.*

[6] *K. Kalidass and K. Pavithra,* An $M/M/1/N$ queue with working breakdowns and Bernoulli feedbacks, *International Journal of Applied Mathematics,* ***6****(2) (2016).*

[7] *B. K. Kim and D. H. Lee,* The $M/G/1$ queue with disasters and working breakdowns, *Applied Mathematical Modelling,* ***38*** *(2014), 1788–1798.*

[8] *A. Krishnamoorthy, P. K. Pramod and S. R. Chakravarthy,* Queues with interruptions: a survey, *Theory of Operations of Production Systems,* ***22*** *(2014), 290–320.*

[9] *J. Keilson and L. D. Servi,* A distributional form of Little's law, *Operations Research Letters,* ***7*** *(1988), 223–227. (see also: M. F. Neuts, Matrix-Geometric Solutions in Stochastic Models: An Algorithmic Approach, Dover, New York, 1994).*

[10] *J. Medhi, Stochastic Models in Queueing Theory, 2nd ed., Academic Press, San Diego, 2002.*

[11] *J. Medhi, Stochastic Processes, 2nd ed., Wiley Eastern, New Delhi, 1994.*

[12] *M. F. Neuts and D. M. Lucantoni,* A Markovian queue with N servers subject to breakdowns and repairs, *Management Science,* ***25****(9) (1979), 849–861.*

[13] *N. J. Ezeagu, G. O. Orwa and M. J. Winckler,* Transient analysis of a finite capacity $M/M/1$ queueing system with working breakdowns and recovery policies, *Global Journal of Pure and Applied Mathematics,* ***14****(8) (2018), 1049–1065.*

[14] *V. Vijayalakshmi and K. Kalidass,* The single server vacation queue with geometric abandonments and Bernoulli's feedbacks, *International Journal of Engineering and Technology,* ***7****(2.21) (2018), 172–179.*

[15] *V. Vijayalakshmi, K. Kalidass and K. Pavitha,* An $M/M/1/N$ queue with working breakdowns and a two phase service, *International Journal of Pure and Applied Mathematics,* ***119****(15) (2018), 2285–2292.*

[16] *T. Yang and J. G. C. Templeton,* A survey on retrial queues, *Queueing Systems,* ***2*** *(1987), 201–233.*

[17] *T. Yang and H. Li,* The $M/G/1$ retrial queue with the server subject to starting failures, *Queueing Systems,* ***16*** *(1994), 83–96.*

[18] *S. R. Chakravarthy, S. Shruti and R. Kulshrestha,* A queueing model with server breakdowns, repairs, vacations and backup server, *Operations Research Perspective,* ***7*** *(2020), 100131.*

[19] *P. Gupta and N. Kumar,* Performance analysis of retrial queuing model with working vacation, interruption, waiting server, breakdown and repair, *Journal of Scientific Research,* ***13****(3) (2021), 833–844.*

[20] *R. Rathore and K. Shrivastava,* Analysis of $M/M/1$ queuing model with removable and unreliable server, partial breakdowns during working vacation, setup with repair, *Journal of Mathematical Problems, Equations and Statistics,* ***5****(2) (2024), 1–7.*

Author information

V. Vijayalakshmi, Department of Science and Humanities(Mathematics), Faculty of Engineering, Karpagam Academy of Higher Education, Coimbatore, India.
E-mail: viji.sanmugam@gmail.com

J. Vimal Andrew, Department of Mathematics, Research Scholar, Karpagam Academy of Higher Education, Coimbatore, India.
E-mail: vimalandmat@gmail.com

U.R. Ramakrishnan, Department of Mathematics, Faculty of Arts and Science, Karpagam Academy of Higher Education, Coimbatore, India.
E-mail: ur.rama@gmail.com