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WAVE PROPAGATION OF NANO RING UNDER THE INFLUENCE OF THERMAL FIELD

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Abstract

In this paper the thermal effects in wave propagation of a vibrating nano ring are analyzed using Euler-Bernoulli beam theory. The circular vibrating nano ring under thermal effect is designed based on the relationships among shear force, bending moment, displacement stiffness and the radius of nano ring. The system of governing equations for a vibrating nano ring under thermal field and phase velocity equations has been established. The dispersion equation are derived for vibrating nano ring in the presence of thermal field. The numerical values for the dimensionless frequency and phase velocity of thermally vibrating nano rings were obtained for lower and higher modes of vibration.

1 Introduction

A nano ring is a minuscule ring-shaped structure typically composed of metals or carbon nano tubes at the nano scale. Scientists at the institute of physics and center for condensed matter physics in Beijing created nano rings composed of gallium nitride in 2000 [1]. Zhou et al. [2] discussed 2D and 3D thermo elastic damping models for circular cross-sectional micro/nano ring resonators undergoing out-of-plane vibration. Moosavi et al. [3] investigated the equations of Erigen's differential constitutive relation for nonlocal elasticity and the motion equations from shear deformable ring theory are obtained. This outcome is beneficial for the design of Micro-ElectroMechanical System (MEMS) and Nano-ElectroMechanical System (NEMS)devices incorporating nano rings. The engineers developed nano rings and nano arches for use in MEMS and NEMS devices [4]. Duan et al.[5] examined that results from the molecular dynamics simulation indicate that the nano wheel can operate at very low temperatures and its rotational speed can be modified by verifying the temperature. Kim et al. [6] studied Q-factor thermo elastic damping in the rotating thin ring under the in-plan vibration. Alghamdi and Youssef [7] discussed thermo elastic damping in, vibration of rotating nano ring using the dual – phase lagging thermo elastic. Using Green's function technique in the nearest neighbour tight -binding approximation the results show that the electronic transport quantities of a system consisting of a nano ring are strongly sensitive to electron energy, magnetic flux and contact hopping energies by Rabani and Mardaani [8].

Hao [9] analyzed thermo elastic damping in the contour- mode vibrations of a free-edged circular thin plate resonator. Using Timoshenko beam theory, Lalani and Amuthalakshmi [10] investigated the effects of magnetic, thermal and rotational effects on the propagation of flexural waves in graphene tubule. Using the Lord-Shulman heat conduction equation, the thermoelastic damping effect for a toroidal micro/nano-ring model is examined with different temperature and modes of vibration [11].

In this paper, the thermal effect in the wave propagation of carbon nano rings is analyzed using the Euler-Bernoulli beam theory. In this results the non dimensional frequency and non dimensional phase velocity are calculated for different temperature for both modes of vibrations . The dispersion relations are derived for thermo ring. The numerical values of non-dimensional frequency and dimensionless phase velocity is calculated and it is plotted to make dispersion curves.

2 Formulation of the Problem

Consider the carbon nano ring influenced by the thermal effect along the directions w and radius R, as shown in Figure 1. A schematic representation of the nano ring is illustrated in Figure 2.

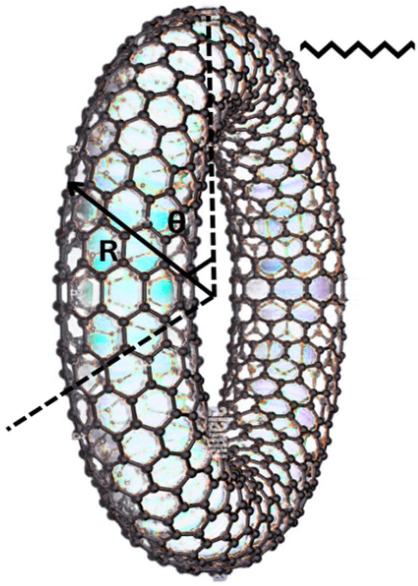

Figure 1. Vibrations of thermal nano ring

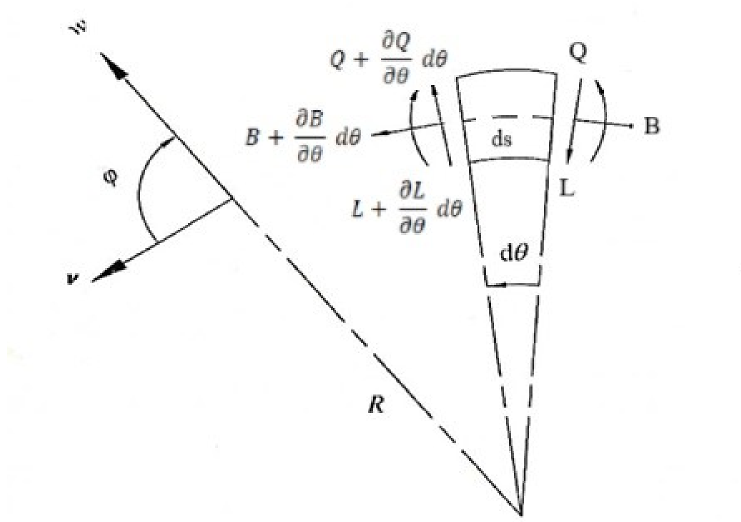

Figure 2. Variable components from the impact of nano ring

In the radial and tangential direction, the centroidal axis displacements are presented by w and v respectively. The equation of the motion of nano ring using shear force L, tensile load B, bending moment Q and mass density ρ are as follows

$$-L + \left(L + \frac{\partial L}{\partial \theta} d\theta\right) - \left(B + \frac{\partial B}{\partial \theta} d\theta\right) d\theta = \rho A R d\theta \frac{\partial^2 w}{\partial t^2} \tag{2.1}$$

$$-B + \left(B + \frac{\partial B}{\partial \theta} d\theta\right) + \left(L + \frac{\partial L}{\partial \theta} d\theta\right) d\theta = \rho A R d\theta \frac{\partial^2 v}{\partial t^2} \tag{2.2}$$

$$Q - \left(Q + \frac{\partial Q}{\partial \theta} d\theta\right) - \left(L + \frac{\partial L}{\partial \theta} d\theta\right) R d\theta = 0 \tag{2.3}$$

Reducing the Eqs. 2.1 - 2.3 the following differential equations are obtained

$$\frac{\partial L}{\partial \theta} - B = \rho A R \frac{\partial^2 w}{\partial t^2} \tag{2.4}$$

$$\frac{\partial B}{\partial \theta} + L = \rho A R \frac{\partial^2 v}{\partial t^2} \tag{2.5}$$

$$\frac{\partial Q}{\partial \theta} + RL = 0 \tag{2.6}$$

The shearing force exerted on the vibrating nano ring in the tangential direction is determined using Eq. 2.6 as

$$L = -\frac{1}{R} \frac{\partial Q}{\partial \theta} \tag{2.7}$$

By utilizing shear force given in Eq. 2.7 into Eqs. 2.4 - 2.5, the equations for displacement can be expressed as

$$-\frac{1}{R} \frac{\partial^2 Q}{\partial \theta^2} - B = \rho A R \frac{\partial^2 w}{\partial t^2} \tag{2.8}$$

$$\frac{\partial B}{\partial \theta} - \frac{1}{R} \frac{\partial Q}{\partial \theta} = \rho A R \frac{\partial^2 v}{\partial t^2} \tag{2.9}$$

The tensile and bending forces are addressed by Graff[12] as

$$B = \int_A \sigma dA \tag{2.10}$$

$$Q = -\int_A \sigma z dA \tag{2.11}$$

According to Hooke's law $\sigma = E\varepsilon$, where E is the Young's modulus, σ is the axial stress and ε the strain of the wave propagation of the ring is expressed as

$$\varepsilon = \frac{1}{R} \left\{ w + \frac{\partial v}{\partial \theta} + \frac{z}{R} \left(v - \frac{\partial w}{\partial \theta} \right) \right\} \tag{2.12}$$

By substituting Eq.2.12 into Eqs. 2.10 and 2.11, the following equations are derived for tensile and bending forces with thermal effect [6] of different modulus as

$$B = \frac{EA}{R} \left(w + \frac{\partial v}{\partial \theta} \right) \tag{2.13}$$

$$Q = -\frac{E_\omega A k^2}{R^2} \frac{\partial}{\partial \theta} \left(v - \frac{\partial w}{\partial \theta} \right) \tag{2.14}$$

where $E_\omega = E + \Delta_E[1 + f(w)]$ is the elastic modulus, $\Delta_E = \frac{E\alpha^2 T_a}{C_v}$ is the relaxation strength in terms of Young's modulus and $f(\omega)$ is the complex function expressed as $f(w) =$

$\frac{24}{b_r^3 k_\omega^3}\left\{\frac{k_\omega b_r}{2} - \tan\left(\frac{k_\omega b_r}{2}\right)\right\}$ in which b_r is the radial thickness of the ring and $k_\omega = (1-i)\sqrt{\frac{\omega_0}{2\chi}}$ is the natural frequency.

Substituting Eqs 2.13 - 2.14 in Eqs 2.8 - 2.9, the resulting equations are

$$\frac{E_\omega A k^2}{R^3}\frac{\partial^3}{\partial\theta^3}\left(v - \frac{\partial w}{\partial\theta}\right) - \frac{EA}{R}\left(w + \frac{\partial v}{\partial\theta}\right) = \rho AR\frac{\partial^2 w}{\partial t^2} \tag{2.15}$$

$$\frac{E_\omega A k^2}{R^3}\frac{\partial^2}{\partial\theta^2}\left(v - \frac{\partial w}{\partial\theta}\right) + \frac{EA}{R}\frac{\partial}{\partial\theta}\left(w + \frac{\partial v}{\partial\theta}\right) = \rho AR\frac{\partial^2 v}{\partial t^2} \tag{2.16}$$

Eq.2.15 and Eq.2.16 are the displacement equations of vibrating thermal nano ring with the help of Euler-Bernoulli model.

3 Solution of the Problem

To examine the diffusion of the vibration of carbon nano ring under thermal effects, the harmonic wave equation in terms of w and v are structured from Graff [12] as

$$w(x,t) = A_1 e^{i(\gamma R\theta - \omega t)} \tag{3.1}$$

$$v(x,t) = A_2 e^{i(\gamma R\theta - \omega t)} \tag{3.2}$$

Where γ is a wave length and ω is a frequency with the radius of the thermal nano ring R. Substituting Eqs. 3.1 - 3.2 into Eqs. 2.15 - 2.16 and solving the equation, it is obtained as

$$\left(\frac{R^2\omega^2}{c_0^2} - 1 - \frac{E_\omega k^2}{E}\gamma^4 R^2\right)A_1 + \left(-i\left(\frac{E_\omega k^2}{E}(\gamma^3 R) + \gamma R\right)\right)A_2 = 0 \tag{3.3}$$

$$i\left(\frac{E_\omega k^2}{E}\gamma^3 R + \gamma R\right)A_1 + \left(\frac{R^2\omega^2}{c_0^2} - \frac{E_\omega k^2}{E}\gamma^2 - \gamma^2 R^2\right)A_2 = 0 \tag{3.4}$$

The equations 3.3 and 3.4 are rewritten in the matrix form as

$$\begin{bmatrix} \frac{R^2\omega^2}{c_0^2} - 1 - \frac{E_\omega k^2}{E}\gamma^4 R^2 & -i\left(\frac{E_\omega k^2}{E}(\gamma^3 R) + \gamma R\right) \\ i\left(\frac{E_\omega k^2}{E}\gamma^3 R + \gamma R\right) & \frac{R^2\omega^2}{c_0^2} - \frac{E_\omega k^2}{E}\gamma^2 - \gamma^2 R^2 \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0 \tag{3.5}$$

On solving Eq. 3.5, a trivial solution is obtained. The determinant of the coefficient matrix is set to zero in the following manner in order to examine non-trivial solutions as

$$\begin{vmatrix} \frac{R^2\omega^2}{c_0^2} - 1 - \frac{E_\omega k^2}{E}\gamma^4 R^2 & -i\left(\frac{E_\omega k^2}{E}(\gamma^3 R) + \gamma R\right) \\ i\left(\frac{E_\omega k^2}{E}\gamma^3 R + \gamma R\right) & \frac{R^2\omega^2}{c_0^2} - \frac{E_\omega k^2}{E}\gamma^2 - \gamma^2 R^2 \end{vmatrix} = 0 \tag{3.6}$$

To express the governing equations in the dimensionless form, the following non-dimensional parameters are introduced:

$$\bar\gamma = k\gamma, \quad \bar k = \frac{k}{R}, \quad \bar\omega = \frac{\omega k}{C_0}, \quad \bar c = \frac{c}{c_0}, \quad \bar T = \frac{E_\omega}{E} \tag{3.7}$$

Substituting the non-dimensional parameters in Eq. 3.6 and estimating the determinant, a fourth-order dimensionless frequency equation is obtained and is given by

$$\bar\omega^4 - \left(\bar T\bar\gamma^4 + (\bar T\bar k^2 + 1)\bar\gamma^2 + \bar k^2\right)\bar\omega^2 + \bar\gamma^2\bar T(\bar k^2 - \bar\gamma^2)^2 = 0 \tag{3.8}$$

Using the dispersion relation $\bar\omega = \bar\gamma\bar c$ to rewriting Eq. 3.8 in terms of phase velocity, we get

$$\bar c^4 - \left(\bar T\bar\gamma^2 + (\bar T\bar k^2 + 1) + \frac{\bar k^2}{\bar\gamma^2}\right)\bar c^2 + \bar T\bar\gamma^2\left(1 - \frac{\bar k^2}{\bar\gamma^2}\right)^2 = 0 \tag{3.9}$$

The dispersion equation of a vibrating nano ring under thermal effect is represented by Eq. 3.8 and Eq. 3.9 respectively, in terms of dimensionless frequency and phase velocity.

4 Numerical Result

This paper uses the Euler–Bernoulli beam model to study the wave propagation of vibrating carbon nano ring under temperature action. The considered material parameters [6] are as follows:

Parameter	**Value**
Young's modulus, E	$E = 165$ GPa
Thermal expansion coefficient, α	$\alpha = 2.6 \times 10^{-6}\ \mathrm{K}^{-1}$
Heat capacity per unit volume, C_v	$C_v = 1.64 \times 10^{6}\ \mathrm{Jm}^{-3}\mathrm{K}^{-1}$
Thermal diffusivity, χ	$\chi = 8.6 \times 10^{-5}\ \mathrm{m}^2\mathrm{s}^{-1}$
Radial thickness of the ring, b_r	$b_r = 20$ nm

Scattered curves for non-dimensional frequency and non dimensional phase velocity of vibrating nano ring under thermal effect at $\bar{k}^2 = 0.05$ is respectively drawn and is shown in Figure 3 and 4. From figures 3 and 4, it is observed that as the non-dimensional wave number increases the non- dimensional frequency and non-dimensional phase velocity respectively increases for higher modes of vibrations at different temperature. Further, it is observed that as the temperature increases its corresponding non-dimensional frequency and non-dimensional phase velocity also increases.

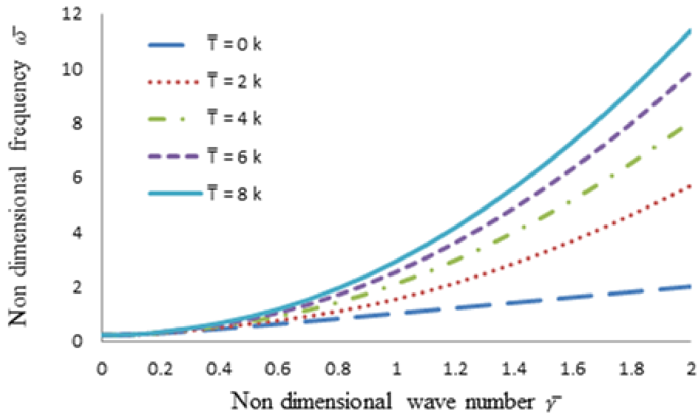

Figure 3. Scattered curves for non-dimensional frequency of carbon nano ring under thermal effect at $\bar{k}^2 = 0.05$

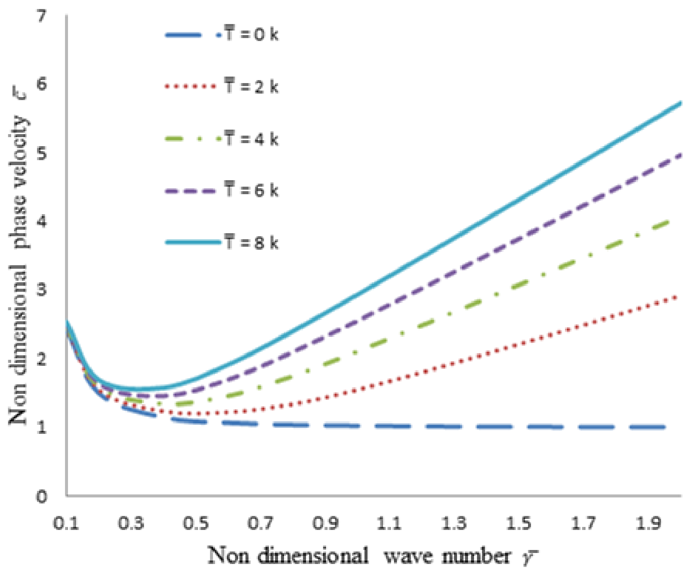

Figure 4. Scattered curves for non-dimensional phase velocity of carbon nano ring under thermal effect at $\bar{k}^2 = 0.05$

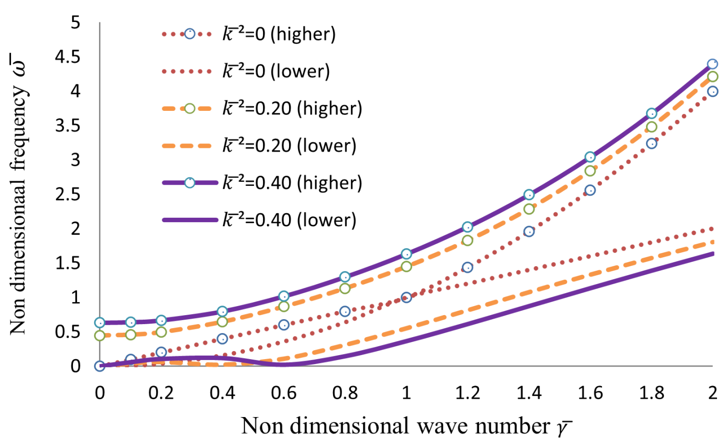

Figure 5. Scattered curves for non-dimensional frequency of carbon nano ring under thermal effect for both lower and higher modes of vibrations.

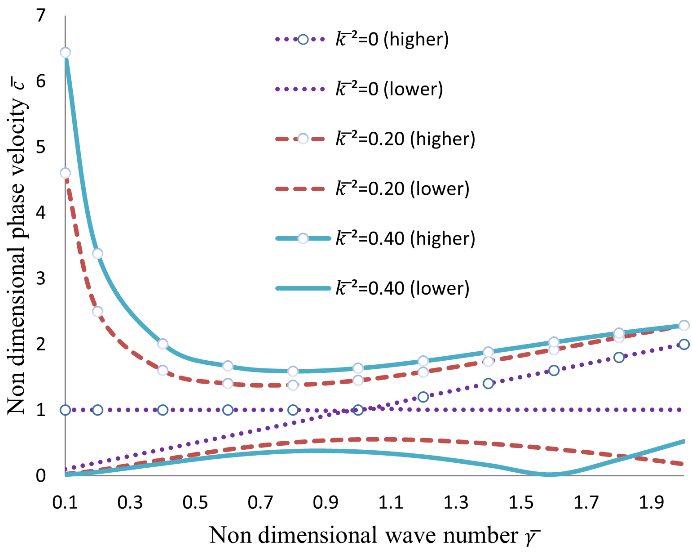

Figure 6. Scattered curves for non-dimensional phase velocity of carbon nano ring under thermal effect for both lower and higher modes of vibrations.

Scattered curves for non-dimensional frequency and non-dimensional phase velocity of a vibrating nano ring respectively under the influence of thermal field is drawn is shown in Figure 5 and 6. From Figures 5 and 6, it is observed that the non dimensional wave number increases the non dimensional frequency and non dimensional phase velocity increases for higher and lower modes of vibrations at different radius of curvature. Further, it is observed that as the radius increases its corresponding dimensionless frequency and dimensionless phase velocity also increases for higher modes of vibrations decreases lower modes of vibrations.

5 Conclusion

In this paper, the wave propagation of thermal influenced nano ring is examined using Euler-Bernoulli model. The governing displacement equations of motion for, a vibrating nano ring under the effect of thermal field is derived. The harmonic solutions of the nano ring is obtained using the variable separable method. Dispersion curves are drawn for vibrating nano ring at different temperature and radius of gyration respectively. The numerical results revealed that at higher temperatures the vibrations of nanao ring is greater. Further the impact of radius of gyration on the dimensionless frequency and dimensionless phase velocity is analyzed.

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