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On r-Dynamic coloring of R-graphs

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Abstract Let us considered the graph G be simple, undirected and finite graph with maximum (Δ) and minimum (δ) degree. Proper vertex coloring k of an graph G is $c : V(G) \to S$ where $|S| = k$ and it is k- colorable and $\chi(G)$ denotes the chromatic number of graph G. In this paper, the results on r-dynamic coloring $\chi_r(G)$ on R - graphs of some graphs are extracted. i.e., the R - graphs of Path, Cycle, Complete graph and some cycle related graphs such as Sunlet graph, Wheel graph and Parachute graph.

1 Introduction

D. M. Cvetkovic et. al.[3] was the first to introduce the concept of an R-graph. Let $R(G)$ be the graph obtained from a graph G by joining new vertices u_e to the end vertices of an edge e for each $e \, \epsilon \, E(G)$. The vertices $I(G) = V(R(G))/V(G)$ is the newly added vertices. It is also stated as an edge corona of $R(G)$ to a singleton graph. The concept $R(G)$ is widely used in the spectra of graph theory which deals with the spectra of matrices that associates with graph and the properties of graph. Several researchers have also find the results of R-graph with many operations such as R-vertex corona, R-edge corona, the R-vertex neighbourhood corona, the R-edge neighbourhood corona and so on. The results for the above operations can be seen in the following papers [2], [4], [10]. Now, in this article we are going to combine R- graph with r-coloring.

In this paper, we concern with r-coloring of cycle related R-graphs. Once we recall the definition of r-dynamic coloring which was introduced by Montgomery [12]. It is a proper vertex coloring that extends from dynamic coloring. An r-dynamic coloring is defined by a map $c : V(G) \to \{1, 2, \cdots , k\}$ such that

$$c(u) \neq c(v) \tag{1.1}$$

$$|c(N(v))| \geqslant min \{r, d_G(v)\} \tag{1.2}$$

Where, $N(v)$ is the neighborhood of v and $d(v)$ denote the degree of the vertex v. At $r = 1$ the 1-dynamic chromatic number is equal to the chromatic number of the graph and at $r = 2$ it is called as dynamic chromatic number or dynamic coloring $\chi_2(G)$. The r-values can be extended upto to the maximum degree Δ. Montogomery have also shows that $\chi_2(G) - \chi(G) \leq 2$, for r-regular graph. Also, the bounds of r-dynamic coloring was given minimum and maximum degree. The r-dynamic chromatic number of some graphs and their bounds are studied in [1], [5], [6], [7], [8], [11], [13].

In the next section, we study the basic prelimiaries of graph theory and some preliminary lemmas which can be used in the section 3. In the section 3 the exact values of r-coloring of R graphs of path, cycle, complete graph and some cycle related graphs such as sunlet graph, wheel graph and parachute graph are given.

2 Preliminaries

In this section, we examine some basic definitions and some preliminary lemmas which helps for the next section. A graph G consist of a pair $(V(G), E(G))$ where $V(G)$ is the set of vertices of G and $E(G)$ is the set of edges of G. In a graph, *loop* is an edge between a vertex and itself. An undirected graph without loops or parallel edges is called a *simple graph*. If the order and size of the graph are finite, then a graph is *finite*. In this work, the graphs are simple, connected and undirected graph and the minimum and maximum degrees of G are $\delta(G)$ and $\Delta(G)$.

A *Complete* graph is a simple graph if any two vertices are adjacent. It is denoted as K_n with order n.

Graph $W_n = (V_n, E_n)$ is an *Wheel* graph which is obtained by joining $n-1$ vertices of cycle graph C_{n-1} to a singleton graph K_1.

Sunlet graph $S_p = (V_p, E_p)$ is also obtained from cycle by including an singleton edge to each vertex which are denoted as S_p.

A *Parachute* graph $P_{b,l}$ is given by $P_{b,l} = (V_{b,l}, E_{b,l})$ is a combination of $(K_1 + P_b) \cup C_{b+l}$, which is obtained from the union of graph join of singleton graph K_1 to path graph P_b and cycle of order $b + l$ such that the intersection $(K_1 + P_b) \cap C_{b+l}$ is equal to P_b. It is noted that $b \geq 3$ and $b, l \in N$.

An r-proper coloring is a map of $c : V(G) \to \{1, 2, \cdots, k\}$ that allots k-colors to the vertices. An r-dynamic chromatic number is the minimal coloring of a graph G which is r-dynamic k-colorable. The following lemma and theorems are much needed for the next section.

Lemma 2.1. *[9] Let us consider a finite and a connected graph G. Then the following conditions hold:*

* $\chi_r(G) \leq \chi_{r+1}(G)$
* $\chi_r(G) \geq min\{r, \Delta(G)\} + 1$
* $\chi(G) = \chi_1(G) \leq \chi_2(G) \leq \cdots \leq \chi_{\Delta(G)}(G)$.
* *At* $r \geq \Delta(G)$, *then* $\chi_r(G) = \chi_{\Delta(G)}(G)$.

Theorem 2.2. *[9] For any positive integer* $n \geq 1$*, then* $\chi_r[K_n] = n$.

Theorem 2.3. *[9] Let* $r \geq 2$ *and* p *be any positive integers. Then*

$$\chi_r[(C_p)] = \begin{cases} 5, & for\ p = 5 \\ 3, & for\ p = 0(\text{mod}3) \\ 4, & otherwise \end{cases}$$

3 Main Results

In this section, the r-dynamic coloring concern with cycle related R-graphs such as path, cycle, complete graph and some cycle related graphs such as sunlet graph, wheel graph and parachute graph. Then the cardinality of graphs such as order and size of the graphs has been found which helps to find the exact result and the minimum and maximum degrees are also be mentioned in the proof. Then the bounds are also mentioned in the form of lemmas. Through the cardinality and the bounds the exact results of R-graphs are investigated. It is noted that for any cycle related graphs at $r \leq \delta + 1$ the chromatic number $\chi_{r \leq \delta+1}(G) = 3$, since it forms an cycle graph of order 3.

Theorem 3.1. *Let* r *and* $n \geq 2$ *be any positive integers. Then* r*- dynamic chromatic number of* R*-graph of path* $R(P_n)$ *are*

$$\chi_r[R(P_n)] = \begin{cases} 3, & for\ 1 \leq r \leq 2 \\ r + 2, & otherwise \end{cases}$$

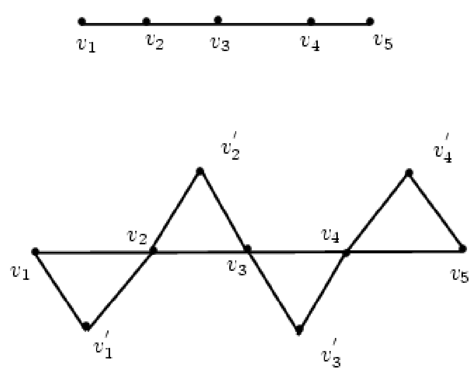

Figure 1. path P_5 and $R(P_5)$

Proof. Let us suppose that

$$V[R(P_n)] = \{t_i : 1 \leq i \leq n\} \cup \{t_i' : 1 \leq i \leq n-1\}$$
$$E[R(P_n)] = \{t_i t_{i+1}, t_i t_i', t_i' t_{i+1} : 1 \leq i \leq n-1\}$$

Here, t_i be the vertices of path graph and t_i' be the vertices obtained from the operation $R(P_n)$. The order and the size of the graph $R(P_n)$ are $|V[R(P_n)]| = 2n-1$ and $|E[R(P_n)]| = 3n-3$. The minimum and maximum degree are $\delta(R(P_n)) = 2$ and $\Delta(R(P_n)) = 4$.

Case : 1 $1 \leq r \leq 2$
Color the vertices t_i with color $\{1, 2\}$ for $1 \leq i \leq n$. Then color the vertices t_i' with color 3 for $1 \leq i \leq n$. From the condition (1.1) and Lemma 2.1 we get, $\chi_r[R(P_n)] = 3$.

Case : 2 Otherwise

* When $r = 3$, color the vertices t_i with color $\{1, 3\}$ for $1 \leq i \leq n$. Then color the vertices t_i' with color $\{2, 4\}$ for $1 \leq i \leq n$. Thus, from the condition (1.2) and Lemma 2.1 we get, $\chi_r[R(P_n)] = 4$. Hence, $\chi_r[R(P_n)] = r + 1$.
* When $r = 4$, color the vertices t_i with color $\{1, 3, 5\}$ for $1 \leq i \leq n$. Next the vertices t_i' are colored with $\{2, 4\}$ colors for $1 \leq i \leq n$. From the condition (1.2) and Lemma 2.1 the result is, $\chi_r[R(P_n)] = 5$. Therefore, $\chi_r[R(P_n)] = r + 1$.

□

Theorem 3.2. *Let* r *and* $m \geq 3$ *be any positive integers. Then* r*- dynamic chromatic number of* R*-graph of cycle* $R(C_m)$ *are*

$$\chi_r[R(C_m)] = \begin{cases} 3, & \textit{for } 1 \leq r \leq 2 \\ r+1, & \textit{for } r = 3 \\ r+1, & \textit{for } r \geq \Delta,\ m \equiv 0(\text{mod} 2, 3) \\ r+2, & \textit{for } r \geq \Delta,\ m \not\equiv 0(\text{mod} 2, 3) \end{cases}$$

Proof. Let us suppose that

$$V[R(C_m)] = \{s_i, s_i' : 1 \leq i \leq m\}$$
$$E[R(C_m)] = \{s_m s_1, s_i s_{i+1} : 1 \leq i \leq m-1\} \cup \{s_i s_i' : 1 \leq i \leq m\}$$
$$\cup \{s_m' s_1, s_i' s_{i+1} : 1 \leq i \leq m-1\}.$$

Here, s_i be the vertices of cycle graph and s_i' be the vertices obtained from the operation $R(C_m)$. The order and the size of the graph $R(C_m)$ are $|V[R(C_m)]| = 2m$ and $|E[R(C_m)]| = 3m$. The minimum and maximum degree are $\delta(R(C_m)) = 2$ and $\Delta(R(C_m)) = 4$.

Case :1 $1 \leq r \leq 2$

* When m is odd, color the vertices s_i with color $\{1, 3\}$ for $1 \leq i \leq m-1$ and color the vertex s_m with color 2. Then color the vertices s'_i with color 2 for $1 \leq i \leq m-2$. The vertex s'_{m-1} is colored with color 1 and s'_m vertex is colored with color 3. Thus, from Lemma 2.1 and Theorem 2.3, we conclude that $\chi_r[R(C_m)] = 3$.
* When m is even, color the vertices s_i with color $\{1, 3\}$ for $1 \leq i \leq m$. Next the vertices s'_i are colored with 2 colors for $1 \leq i \leq m$. Thus, from Lemma 2.1 and Theorem 2.3, we conclude that $\chi_r[R(C_m)] = 3$.

Case :2 $r = 3$

* When m is odd, color the vertices s_i as given in case-1. Next, color the vertices s'_i with color $\{2, 4\}$ for $1 \leq i \leq m-2$. Then the vertices s'_{m-1} and s'_m are with color 4. Thus, from the condition (1.2) and Lemma 2.1, the result is $\chi_r[R(C_m)] = 4$. Therefore, $\chi_r[R(C_m)] = r + 1$.
* When m is even, color the vertices s_i as given in case-1. Then color the vertices s'_i with color $\{2, 4\}$ for all $1 \leq i \leq m$. Thus, by the condition (1.2) and Lemma 2.1, we get the result as $\chi_r[R(C_m)] = 4$. Therefore, $\chi_r[R(C_m)] = r + 1$.

Case :3 $r \geq \Delta$

* When $m \equiv 0(\text{mod } 2, 3)$, color the vertices s_i with color $\{1, 3, 5\}$ for all $1 \leq i \leq m$. Then color the leftover vertices s'_i with the color $\{2, 4\}$ for all $1 \leq i \leq m$. Thus, from the condition (1.2) and Lemma 2.1, the result is $\chi_r[R(C_m)] = 5$. Therefore, $\chi_r[R(C_m)] = r + 1$.
* When $m \equiv 0(\text{mod } 3)$, color the vertices s_i with color $\{1, 3, 5\}$ for all $1 \leq i \leq m$. Then color the vertices s'_i with the color $\{2, 4\}$ for all $1 \leq i \leq m-1$ and color the leftover vertex s'_m with color 6. Thus, from the condition (1.2) and Lemma 2.1, the result obtained as $\chi_r[R(C_m)] = 6$. Therefore, $\chi_r[R(C_m)] = r + 2$.
* When $m \equiv 1(\text{mod } 3)$, if m is odd, color the vertices s_i with color $\{1, 3, 5, 6\}$ for $1 \leq i \leq 4$ and the other vertices s_i with color $\{1, 3, 5\}$ for all $5 \leq i \leq m$. Then, color the vertex s'_1 with color 6 and the leftover vertices of s'_i are with color $\{2, 4\}$ for $2 \leq i \leq m$, else color the vertices of s_i as same as given above and color s'_i with color $\{2, 4\}$ for $1 \leq i \leq m$. Therefore, from the condition (1.2) and Lemma 2.1, the result occurs as $\chi_r[R(C_m)] = 6$. Hence, $\chi_r[R(C_m)] = r + 2$.
* When $m \equiv 2(\text{mod } 3)$, if m is odd, color the vertices of s_i with color from the set $\{1, 3, 4, 5, 6\}$ for $1 \leq i \leq m$. Then color the vertices of s'_i with the color from the set $\{2, 3, 4, 5\}$ for $1 \leq i \leq m$. If m is even, color the vertices s_i with the colors $\{1, 3, 5, 6\}$ for $1 \leq i \leq m$. Next, color the vertices of s'_i with the color from the set $\{2, 4\}$ for $1 \leq i \leq m$. Therefore, from the condition (1.2) and Lemma 2.1, the result occur as $\chi_r[R(C_m)] = 6$. Hence, $\chi_r[R(C_m)] = r + 2$.

□

Theorem 3.3. *Let r and $p \geq 3$ be any positive integers. Then r-dynamic chromatic number of R-graph of Sunlet graph $R(S_p)$ are*

$$\chi_r[R(S_p)] = \begin{cases} 3, & \text{for } 1 \leq r \leq 3 \\ r + 2, & \text{otherwise} \end{cases}$$

Proof. The proof is similar as given in above theorem. Since, Sunlet graph (S_p) graph is an graph extended from cycle (C_m) with an pendant edge on each vertices. Thus, the order of a graph $R(S_p)$ are $|V[R(S_p)]| = 4p$ and the size of a graph $R(S_p)$ are $|E[R(S_p)]| = 6p$. □

Lemma 3.4. *Let $R(K_n)$ be the complete graph. The lower bound for r-dynamic chromatic number of R-graph of complete graph $R(K_n)$ are*

$$\chi_r[R(K_n)] \geq \begin{cases} n, & \text{for } 1 \leq r < n \\ r + 1, & \text{for } n \leq r \leq \Delta \end{cases}$$

Proof. The vertices of $R(K_n)$ are $\{q_i, q_i' : 1 \leq i \leq n\} \cup \{q_{jk} : 1 \leq j, k \leq n, j \neq k\}$, where the vertices q_i' are corresponding to the edges $q_i q_{i+1}$ and q_{jk} are the vertices where q_j are adjacent. The minimum and maximum degree of $R(K_n)$ are $\delta = 2$ and $\Delta = 2n - 2$. For $1 \leq r \leq n-1$, the vertices $V = q_i$ persuade a clique of order n in $R(K_n)$. Thus, $\chi_r[R(K_n)] \geq n$. Thus for $n \leq r \leq \Delta$,we have $\chi_r[R(K_n)] \geq min\{r, \Delta[R(K_n)]\} + 1 = r + 1$, based on Lemma 2.1. Thus, it completes the proof. □

Theorem 3.5. *Let r and $n \geq 4$ be any positive integers. Then r-dynamic chromatic number of R-graph of complete graph $R(K_n)$ are*

$$\chi_r[R(K_n)] = \begin{cases} n, & for\ 1 \leq r < n \\ n+1, & for\ n \leq r \leq \Delta,\ n\ is\ even \\ r+1, & for\ n \leq r \leq \Delta - 3,\ n\ is\ odd \\ n+1, & for\ r \geq \Delta - 2,\ n\ is\ odd \end{cases}$$

Proof. The r- coloring for $R(K_n)$ are as follows:

Case :1 $1 \leq r < n$

Since, the vertices q_i forms an clique of order n, color the vertices q_i with color $\{1, 2, \cdots, n\}$ for all $1 \leq i \leq n$. Next, color the vertices q_i' with the color $\{3, 4, \cdots, 1, 2\}$ for all $1 \leq i \leq n$. Then, color the remaining vertices q_{jk} with the color from the set $\{1, 2, \cdots, n\}$. Thus, from Lemma 2.1 and Lemma 3.4 the result obtained as, $\chi_r[R(K_n)] = n$.

Case :2 $n \leq r \leq \Delta,\ n\ is\ even$

- ∗ When $n \leq r \leq \Delta - 2$, color the vertices q_i with the color $\{1, 2, \cdots, n\}$. Then color the vertices, q_i' with the color $\{3, 4, \cdots, 1, 2\}$ for all $1 \leq i \leq n$. When $r = n$, color the remaining vertices q_{jk} with the color $n+1$ for all $1 \leq j \leq n$ and $1 \leq k \leq n$. Next, when $r = n + 1$, color the vertices q_{jk} with the color $\{n+1, n+2\}$ for $1 \leq j \leq n$ and for $1 \leq k \leq n$. Continuing the process, when $r = \Delta - 2$, color the vertices q_{jk} with the colors from the set $\{n+1, n+2, \cdots, r+1\}$. Hence, from Lemma 2.1 and Lemma 3.4 $\chi_r[R(K_n)] = n + 1$.
- ∗ When $\Delta - 1 \leq r \leq \Delta$, color the vertices q_i with the colors $\{1, 2, \cdots, n\}$. Next, color the vertices q_{jk} with the colors from the set $\{n+1, n+2, \cdots, r\}$ for $1 \leq j \leq n$ and for $1 \leq k \leq n$. But also we need one more color, so color the vertices q_i' for $1 \leq i \leq n$ with color $r + 1$, for $r = \Delta - 1$. In the case of $r = \Delta$, color q_{jk} with the color from the set $\{n+1, n+2, \cdots, r-1\}$ and color the vertices q_i' for $1 \leq i \leq n$ with color $\{r, r+1\}$ respectively. Hence, from Lemma 2.1 and Lemma 3.4 it is clear check that $\chi_r[R(K_n)] = n + 1$.

Case :3 $n \leq r \leq \Delta - 3,\ n\ is\ odd$

- ∗ Color the vertices q_i with the colors $\{1, 2, \cdots, n\}$ for all $1 \leq i \leq n$. Then, color the vertices q_i' with the colors $\{3, 4, \cdots, 1, 2\}$ for all $1 \leq i \leq n$.
- ∗ When $r = n$, color the vertices q_{jk} with color $n + 1$ for all $1 \leq j \leq n$ and $1 \leq k \leq n$. Next, at $r = n+1$, color q_{jk} with color $\{n+1, n+2\}$ for all $1 \leq j \leq n$ and $1 \leq k \leq n$. By proceeding this way, at $r = \Delta - 3$, color the vertices q_{jk} with the colors from the set $\{n+1, n+2, \cdots, r+1\}$. Hence, from Lemma 2.1 and Lemma 3.4 it is clear check that $\chi_r[R(K_n)] = r + 1$.

Case :4 $r \geq \Delta - 2,\ n\ is\ odd$

- ∗ Color the vertices q_i with the colors $\{1, 2, \cdots, n\}$ for all $1 \leq i \leq n$. When $r = \Delta - 2$, color the vertices, q_i' with the colors $\{3, 4, \cdots, 1, 2\}$ for all $1 \leq i \leq n$ and the remaining vertices q_{jk} are colored with color $\{n+1, n+2, \cdots, r+1, r+2\}$ for all $1 \leq j \leq n$ and $1 \leq k \leq n$.
- ∗ When $r = \Delta - 1$, color the vertices q_{jk} with color from the set $\{n+1, n+2, \cdots, r\}$ for all $1 \leq j \leq n$ and $1 \leq k \leq n$ and color the vertices q_i' with the color $r + 1$ for all $1 \leq i \leq n$. Next, at $r = \Delta$, color q_{jk} for all $1 \leq j \leq n$ and $1 \leq k \leq n$ with colors from the set $\{n+1, n+2, \cdots, r-1, r\}$ with the condition that the colors of

the vertices q_{1k} and q_{nk} should not be the same. So that color the vertices q'_i with the color $\{r+1, r+2\}$ for all $1 \leq i \leq n-1$ and finally, color the vertex q'_n with the color r. Hence, from Lemma 2.1 and Lemma 3.4 the result obtained as $\chi_r[R(K_n)] = r+2$.

□

Remark 3.6. In $R(K_n)$, at $n = 2$ the degrees $\delta = \Delta = 2$, so the r-coloring is 3. At $n = 3$, the graphs $R(K_3) = (C_3)$ thus, the coloring are $\chi_{1,2}[R(K_n)] = 3$, $\chi_3[R(K_n)] = 4$ and $\chi_4[R(K_n)] = 6$.

Theorem 3.7. *Let r and $m \geq 4$ be any positive integers. Then r-dynamic chromatic number of R-graph of wheel graph $R(W_m)$ are*

$$\chi_r[R(W_m)] = \begin{cases} 3, & \text{for } 1 \leq r \leq 2, \ m \text{ is odd} \\ 4, & \text{for } 1 \leq r \leq 2, \ m \text{ is even} \\ r+1, & \text{for } 3 \leq r \leq \Delta - 3, \ m = 5 \\ r+2, & \text{for } \Delta - 2 \leq r \leq \Delta, \ m = 5 \\ r+1, & \text{otherwise} \end{cases}$$

Proof. Let us suppose that

$$\begin{aligned} V[R(W_m)] &= \{q_m, q_i, q'_i, u_i : 1 \leq i \leq m-1\} \\ E[R(W_m)] &= \{q_{m-1}q_1, q_iq_{i+1}, q'_{m-1}q_1, q'_iq_{i+1} : 1 \leq i \leq m-2\} \\ &\quad \cup \{q_iq_m, q_iq'_i : 1 \leq i \leq m-1\} \\ &\quad \cup \{u_iq_i, u_iq_m : 1 \leq i \leq m-1\}. \end{aligned}$$

Where q_m is the hub vertex. Here u_i and q'_i are obtained from the R-graph of wheel graph. The degrees of $R(W_m)$ are $\delta = 2$ and $\Delta = 2m-2$. Then the order of the graph $R(W_m)$ are $|V[R(W_m)]| = 3m-2$ and the size of the graph are $|E[R(W_m)]| = \frac{2(m-1)}{2}$.

Case :1 $1 \leq r \leq 2, \ m$ *is odd*

- ∗ Color the vertices q_i with the color $\{1, 2\}$ for $1 \leq i \leq m-1$ respectively. Color the vertices u_i for $1 \leq i \leq m-1$ with the color $\{2, 1\}$ respectively.
- ∗ If the vertices q'_i for $1 \leq i \leq m-1$ and q_m are colored with the same set of colors $\{1, 2\}$ then the condition (1.1) does not holds. Hence, color the vertices q'_i for $1 \leq i \leq m-1$ and q_m with color 3. Thus, from the condition (1.2) and Lemma 2.1 we conclude the result as $\chi_r[R(W_m)] = 3$.

Case :2 $1 \leq r \leq 2, \ m$ *is even*

- ∗ Color the vertices q_i for $1 \leq i \leq m-2$ with color $\{1, 2\}$ and the vertex q_{m-1} are with color 3. Next, color the vertices u_i for $1 \leq i \leq m-1$ with color $\{2, 1\}$ respectively.
- ∗ Color the remaining vertices q'_i for $1 \leq i \leq m-3$ with color 3, color the vertices q'_{m-2} with color 1 and q'_{m-1} with color 2. Finally, color the vertex q'_m with color 4. Thus, from the condition (1.2) and Lemma 2.1 we conclude the result as $\chi_r[R(W_m)] = 4$.

Case :3 $3 \leq r \leq \Delta - 3, \ m = 5$
Color the vertices q_i, q'_i and u_i with the color from the set $\{1, 2, 3, \cdots, r\}$ for $1 \leq i \leq m-1$ in accordance with the r-values. Then color the vertex q_m with the color $r+1$. Therefore the condition (1.2) and Lemma 2.1 holds, to obtain the result as $\chi_r[R(W_m)] = r+1$.

Case :4 $\Delta - 2 \leq r \leq \Delta, \ m = 5$
Color the vertices q_i with the colors $\{1, 2, 3, 4\}$ for all $1 \leq i \leq m-1$ and next color the vertices q'_i for $1 \leq i \leq m-1$ with the colors from the set $\{5, 6, 7\}$. Then, the remaining vertices u_i for $1 \leq i \leq m-1$ are colored from the set $\{6, 7, \cdots, r+1\}$. Atlast, the vertex q_m are colored with $r+2$ color. Therefore, the condition (1.2) and Lemma 2.1 holds, to obtain the result as $\chi_r[R(W_m)] = r+2$.

Case :5 $otherwise$

Color the vertices q_i with the color from the set $\{1, 2, \cdots, r\}$ for all $1 \leq i \leq m-1$. Next, color the remaining vertices u_i and q_i' for all $1 \leq i \leq m-1$ from the set $\{1, 2, \cdots, r\}$ in order to the condition (1.1). Atlast, color the vertices q_m with the color $r+1$ such that the condition (1.2) and Lemma 2.1 are verified. Thence, $\chi_r[R(W_m)] = r+1$.

□

Theorem 3.8. *Let l, r, $b \geq 3$ and $b = l$ be any positive integers. Then r- dynamic coloring of R-graph of parachute graph $R(P_{b,l})$ are*

$$\chi_r[R(P_{b,l})] = \begin{cases} 3, & \text{for } 1 \leq r \leq 2 \\ r+2, & \text{for } 3 \leq r \leq \Delta \end{cases}$$

Proof. The vertices of $R(P_{b,l})$ are $\{v, u_i, v_i, v_i' : 1 \leq i \leq b\} \cup \{p_j, u_k' : 1 \leq j \leq b-1, 1 \leq k \leq b+1\}$. The vertices v_i' corresponding to the edges vv_i, then p_j are corresponding to the edges $v_i v_{i+1}$ and u_k' are corresponding to the edges $\{v, u_i, u_i u_{i+1}, u_b v_n\}$.

Case :1 $1 \leq r \leq 2$

Color the vertices v_i with the color $\{1, 2\}$ for all $1 \leq i \leq b$. Next color the vertices u_i and v_i' with the color $\{2, 1\}$ for all $1 \leq i \leq b$. Color the vertices v, u_k' and p_j with the color 3. Then, from the condition (1.2) and Lemma 2.1 it is clear check that $\chi_r[R(P_{b,l})] = 3$.

Case :2 $3 \leq r \leq \Delta$

* When $3 \leq r \leq 6$, the r- colors in the set $\{1, 2, \cdots, r\}$ are assigned to all the vertices $\{v_i, v_i', u_i, p_j, u_k'\}/v$ in accordance with r- condition. Finally, color the vertex v with the color $r+1$. Then, from the condition (1.2) and Lemma 2.1 the result obtained as $\chi_r[R(P_{b,l})] = r+1$.
* When $6 < r \leq \Delta$, the v is the only vertex with the maximum degree Δ, so color the all other vertices except (v_i') with the color from the set $\{1, 2, \cdots, r\}$. Then, color the vertex v_1' with the color $r+1$ and the remaining vertices of v_i' with the color from the set $\{1, 2, \cdots, r\}$ for all $2 \leq i \leq b$, at $r = 7$. When $r = 8$, color the all other vertices as given above except v_i'. Now, color the vertices v_1' and v_2' with the color $\{r, r+1\}$ and the remaining vertices of v_i' with the color from the set $\{1, 2, \cdots, r\}$ for all $3 \leq i \leq b$. By proceeding this way: At $r \geq \Delta$, color the vertices of v_i' with the colors $\{1, 2, \cdots, r, r+1\}/c(v_i)$. Thus, from the condition (1.2) and Lemma 2.1 the result obtained as $\chi_r[R(P_{b,l})] = r+1$.

□

Conclusion

In this paper, we have obtained the exact results of r-dynamic coloring of some cycle related R-graphs such as, path, cycle, complete graph, sunlet graph, wheel graph and parachute graph. Further we try to extend r-dynamic coloring for any cycle related R-graphs with generalized lower and upper bounds.

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