

THE ROLE OF AWARENESS PROGRAMS IN MANAGING WHITEFLY POPULATIONS IN COCONUT FARMS: DDE AND SDE MODEL

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MSC 2010 Classifications: Primary 92D45; Secondary 37H10.

Keywords and phrases: Pest dynamics; Awareness-based intervention; Time-delay system; Stochastic perturbations; Numerical simulation.

The authors would like to thank the reviewers and editor for their constructive comments and valuable suggestions that improved the quality of our paper.

The authors gratefully acknowledge the financial support provided by the Selective Excellence Research Initiative (SRMIST/R/AR(A)/SERI2023/174/31). We sincerely thank the management of SRM Institute of Science and Technology for their constant encouragement and support. The authors also express their gratitude to Dr. B. R. Ambedkar National Institute of Technology, Jalandhar, for providing the opportunity to publish this research work through the conference MAMM-2024.

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Abstract Coconut (*Cocos nucifera*) trees are known to be seriously affected by Rugose spiraling whiteflies (RSWs), causing significant crop losses. Pest control is an important factor in obtaining a healthy crop, but farming awareness is equally important in agricultural practice, as people involved in cultivation may take time to become aware of the problem and respond effectively. In this article, a delay-induced pest control model and a stochastic model were proposed. The model exhibits two equilibria: pest-free and coexistence equilibrium. The pest-free equilibrium was stable if the basic reproduction number was less than unity, while the coexistence equilibrium exhibits hopf bifurcation whenever it exists. A stochastic system was derived from the deterministic model and compared with the delay model. Whitefly spreading probabilities depending on changing parameter values were investigated. Numerical simulations were performed using MATLAB software. Our findings suggest that increasing public awareness about implementing different control measures can effectively decrease the rate of contact and thereby control the spread of the pest. These results provide practical guidance for policymakers and agricultural extension services, indicating that targeted awareness campaigns and farmer training programs can enhance early response and support sustainable coconut farming practices.

1 Introduction

The coconut tree plantation covers approximately 12 million hectares across 93 countries worldwide. India holds a prominent position in the global market for coconut product production. The uses of coconut and its by-products span various fields, from health and skincare to bio-farming techniques and building materials. Nearly half of our yearly coconut production is used for domestic consumption. The major states for coconut tree plantations include Tamil Nadu, Assam, Kerala, West Bengal, Odisha, and Karnataka. Farmers have encountered numerous challenges in cultivating coconut trees. RSW, an invasive species of whitefly, poses a major threat to coconut trees by hindering their growth and affecting the yield. It was first identified in 2004 on coconut trees in Belize, Central America [1]. In India, RSW has become a significant catastrophe, severely impacting coconut plantations. In August 2016, the Pollachi region of Tamil Nadu became the first to report the invasion of the RSW pest on coconut trees [2], [3]. The spread of the RSW threatens coconut cultivation in India by lowering production levels, compromising the quality of the fruit, and increasing the financial burden of pest management [4]. This whitefly feeds on coconut tree leaves, stripping away essential nutrients and water. Moreover, it encourages the development of sooty mold on the leaf surface, which can hinder photosynthesis and

stunt growth and yield [5]. Studies on the population dynamics and host range of this pest have been conducted [6], [7].

The formulation of mathematical models in epidemiology has been examined in [8], [9], [10], [11], [12] and an investigation into the population dynamics of RSW infesting coconut trees was presented in [13]. The introduction of awareness programs aimed at educating farmers contributes to better development, benefiting both cultivators and farmers. The role of agricultural awareness in managing pests were examined in [14]. The effects of farming awareness, time delays, and pest control on mosaic disease dynamics in *Jatropha curcas* plantations were studied in [15], [16]. A delay differential equation (DDE) differs from an ordinary differential equation (ODE) in that it includes delayed terms, meaning the rate of change at a given time is influenced by both the present and previous states of the system. Since many real-world systems do not respond instantaneously to changes, DDEs are employed to model systems where the rate of change is influenced by the past states of the system [17], [18], [19]. Stochastic differential equations (SDEs) incorporate randomness or noise into the system, differentiating them from ordinary differential equations (ODEs), which represent deterministic systems. SDEs are particularly useful for modeling processes affected by random factors [20], [21]. The study in [22] examined tumor virotherapy dynamics through both deterministic and stochastic frameworks, while [23] analyzed the progression of drug-resistant tuberculosis using a similar approach with deterministic and stochastic models. In epidemiological studies, the basic reproduction number plays a key role in understanding the likelihood of disease transmission in a population. This number is often analyzed using the next generation matrix method, as discussed in [24].

In this study, we investigate DDEs and SDEs to manage RSW infestations in coconut trees, specifically in the Pollachi region of Tamil Nadu. MATLAB software was used to analyze the model's dynamics, with parameter values selected based on the local context. Local stability at the equilibrium points was assessed, and the reproduction number was computed using the next generation matrix. A sensitivity analysis was also conducted to examine the parameters affecting the system.

The study begins with the formulation of the mathematical problem in section 2, where the ODE and DDE models are introduced. Positive invariance and system boundedness are covered in sections 3 and 4 to ensure that the solutions remain biologically meaningful and finite. Section 5 the stability analysis of the delayed system followed by the reproduction number. In section 6, the analysis of the system stability in the presence of delay and in section 7 SDE model is used to account for random factors. Section 8 uses numerical simulations for model validation, while section 9 concludes the study with the main findings.

2 Mathematical formulation of the problem

To assess the impact of whiteflies on coconut trees, the total coconut tree population is divided into healthy trees P and infected trees I . The variable C denotes the whitefly population, while A represents the accumulated awareness level over time t . The mathematical model is outlined below:

2.1 ODE Model

$$\frac{dP}{dt} = aP \left(1 - \frac{P+I}{r} \right) - \beta PC, \quad (2.1)$$

$$\frac{dI}{dt} = \beta PC - sI - bAI, \quad (2.2)$$

$$\frac{dC}{dt} = hI - \alpha C - gAC, \quad (2.3)$$

$$\frac{dA}{dt} = k + \phi I - \xi A. \quad (2.4)$$

with the initial values

$$P(0) = l > 0, \quad I(0) = m > 0, \quad C(0) = n > 0, \quad A(0) = q > 0. \quad (2.5)$$

where l, m, n and q are the positive real numbers.

2.2 DDE model

A time delay is incorporated into the awareness accumulated dynamics, thereby affecting the infected trees together with the whiteflies as part of the effort to control the spread of the pest. The delayed mathematical model is presented below:

$$\frac{dP(t)}{dt} = aP(t) \left(1 - \frac{P(t) + I(t)}{r} \right) - \beta P(t)C(t), \quad (2.6)$$

$$\frac{dI(t)}{dt} = \beta P(t)C(t) - sI(t) - bA(t - \tau)I(t), \quad (2.7)$$

$$\frac{dC(t)}{dt} = hI(t) - \alpha C(t) - gA(t - \tau)C(t), \quad (2.8)$$

$$\frac{dA(t)}{dt} = k + \phi I(t) - \xi A(t). \quad (2.9)$$

with the initial values

$$P(\theta) = \kappa_1, \quad I(\theta) = \kappa_2, \quad C(\theta) = \kappa_3, \quad A(\theta) = \kappa_4, \quad \theta \in [-\tau, 0]. \quad (2.10)$$

The parameters related to public awareness, namely k , b , g , ϕ , and ξ , are quantified based on empirical studies and expert recommendations to reflect real-world intervention efforts. Specifically, k represents the baseline generation of awareness through active community engagement, while ϕ captures the recruitment of individuals into awareness programs via media campaigns and extension services. The parameters b and g quantify the effectiveness of aware individuals in reducing infected trees and controlling whitefly populations, respectively. Finally, ξ represents the natural fading of awareness over time. The assigned ranges and default values in Table 1 are derived from previous studies [14] and reflect realistic operational scenarios. By explicitly incorporating these parameters, the model allows for the evaluation of different awareness campaign strategies, providing actionable insights for designing timely and effective interventions to mitigate whitefly infestations.

Table 1: The parameters, their interpretations, and associated values with ranges.

Parameters	Interpretation	Default values and source [14]	Range
a	Rate of healthy tree growth	0.05 day^{-1}	$0.01 - 0.1 \text{ day}^{-1}$
r	Maximum healthy tree biomass capacity	50 kg plant^{-1}	$20 - 80 \text{ kg plant}^{-1}$
k	Rate of awareness generation from active individuals	0.03 day^{-1}	$0.01 - 0.1 \text{ day}^{-1}$
β	Contact rate between healthy trees and infected whiteflies	$0.0005 \text{ kg vector}^{-1} \text{ day}^{-1}$	$0.0001 - 0.001 \text{ kg vector}^{-1} \text{ day}^{-1}$
s	Combined natural and disease-induced reduction rate	0.03 day^{-1}	$0.01 - 0.05 \text{ day}^{-1}$
b	Rate of infected tree removal by aware individuals	0.005 day^{-1}	$0.001 - 0.01 \text{ day}^{-1}$
h	Rate of infected whitefly emergence	0.8 day^{-1}	$0.5 - 1.0 \text{ day}^{-1}$
α	Whitefly mortality rate	0.12 day^{-1}	$0.05 - 0.2 \text{ day}^{-1}$
g	Rate of whitefly elimination due to human control efforts	0.005 day^{-1}	$0.001 - 0.01 \text{ day}^{-1}$
ϕ	Rate of recruitment into awareness programs via media campaigns	0.05 day^{-1}	$0.01 - 0.1 \text{ day}^{-1}$
ξ	Rate at which awareness programs lose influence over time	0.015 day^{-1}	$0.01 - 0.05 \text{ day}^{-1}$

3 Positive invariance

A unified matrix formulation is obtained for the system given in Eqs. (2.1)–(2.4).

$$\frac{d\mathbf{Z}}{dt} = \mathcal{F}(\mathbf{Z}). \quad (3.1)$$

Let $\mathbf{Z}(\theta) \in \mathcal{B}([-\tau, 0], \mathbb{R}_+^4)$, be given by $\mathbf{Z}(\theta) = (P(\theta), I(\theta), C(\theta), A(\theta))^T$. Here, $\mathcal{F} = (F_1, F_2, F_3, F_4)^T$ denotes the collection of nonlinear terms appearing on the right-hand side of Eqs. (2.1)–(2.4). By applying standard results from the theory of functional differential equations [25], the existence and uniqueness of a solution $(P(t), I(t), C(t), A(t))$ are guaranteed for the initial data given in Eq. (2.5).

Theorem 3.1. *The system in Eqs. (2.1) – (2.4) preserves positivity for the initial conditions in Eq. (2.5).*

Proof. The result follows from the approach presented in [26], [27]. Specifically, it is easy to verify from Eqs. (2.1) – (2.4) that whenever $P(\theta) \in R_+$ such that $P = 0, I = 0, C = 0, A = 0$,

$$F_i(\mathbf{Z})|_{\mathbf{Z}_i=0, \mathbf{Z} \in \mathbb{R}_+^4} \geq 0.$$

Combining Lemma 2 in [27] with Theorem 1.1 of [26], it is ensuring every trajectory $\varphi(t) = \varphi(t, \varphi(\theta))$ of Eqs. (2.1) – (2.4), in the case of $\varphi(\theta) \in \mathcal{B}$, remains in $\mathbf{Z}(t) \in \mathbb{R}_+^4$ for every $t \geq 0$. Moreover, each state variables remain non-negative for $t > 0$, implying that the system trajectories evolve entirely within $\mathbb{R}_+^4$. Consequently, the set $\mathbb{R}_+^4$ is a positively invariant region. $\square$

4 Boundedness

The boundedness of a system implies that it behaves in a stable manner, ensuring that no interacting component grows without bound in finite time.

Let $N = P + I$ represent the total tree biomass at any given time t . By summing the first two Eqs. (2.1) and (2.2), we obtain the following expression:

$$\frac{dN}{dt} = aP \left[1 - \frac{P+I}{r} \right] - sI - bA(t-\tau)I \leq aN \left(1 - \frac{N}{r} \right),$$

which leads to the expression

$$\limsup_{t \rightarrow \infty} N \leq M := \max\{N(0), r\}.$$

In a similar manner, by considering the aware population, we obtain

$$\frac{dA}{dt} = k + \phi I - \xi A \leq k + \phi r - \xi A,$$

giving again an upper bound

$$\limsup_{t \rightarrow \infty} A \leq \frac{k + \phi r}{\xi}.$$

Again,

$$\frac{dC}{dt} = hI - \alpha C - gA(t-\tau)C \leq hr - \alpha C,$$

from which it follows

$$\limsup_{t \rightarrow \infty} C \leq \frac{hr}{\alpha}.$$

Therefore, the region of attraction is defined by the set

$$\mathcal{D} = \left\{ (P, I, C, A) \in \mathbb{R}_+^4 : 0 \leq P+I \leq M, 0 \leq C \leq \frac{hr}{\alpha}, 0 \leq A \leq \frac{k + \phi r}{\xi} \right\}$$

constituting a positively invariant domain that globally attracts all trajectories starting within the positive cone.

5 Analysis of equilibrium points and stability in the absence of delay

5.1 Equilibrium point existence and analysis

Model Eqs. (2.1) – (2.4) possesses precisely three equilibria.

- The pest-free equilibrium $E^0 = \left(r, 0, 0, \frac{k}{\xi}\right)$ which is always feasible,
- The coexistence equilibrium $E^* = (P^*, I^*, C^*, A^*)$,

where

$$P^* = \frac{\phi(\alpha + gA^*)(s + bA^*)}{\beta h \phi}, \quad I^* = \frac{\xi A^* - k}{\phi}, \quad C^* = \frac{h(\xi A^* - k)}{\phi(\alpha + gA^*)}.$$

Furthermore, A^* represents the positive root of the equation given below

$$F(A) = c_1 A^3 + c_2 A^2 + c_3 A + c_4 = 0. \quad (5.1)$$

where $c_1 = g^2 b a \phi > 0$, $c_3 = a \phi (2 g s \alpha + \alpha^2 b) + r \beta^2 h^2 \xi - a r \phi h \beta g + h \beta \phi (\xi \alpha - k s)$,
 $c_2 = a \phi (2 g \alpha b + s g^2) + h \beta \phi g \alpha$, $c_4 = a \phi s \alpha^2 - a \phi r \alpha h \beta - h \beta \phi k \alpha - r \beta^2 h^2 k$.

Since Eq. (5.1) represents a cubic equation with a positive leading term, the existence of at least one real solution is guaranteed. In particular, this solution is positive when $c_4 < 0$. This condition is sufficient to ensure that the E^* lies within the region $\mathcal{D}$, provided that

$$a \phi s \alpha^2 < h \beta k \alpha + a r h \beta \phi \alpha + r \beta^2 h^2 k. \quad (5.2)$$

5.2 Stability of equilibria

Initially, we analyze the system in the absence of delay. For $\tau = 0$, the Jacobian matrix corresponding to Eqs. (2.6) – (2.9) evaluated at an arbitrary equilibrium point E is given by:

$$J(E) = \begin{pmatrix} a \left(1 - \frac{2P+I}{r}\right) - \beta C & -\frac{aP}{r} & -\beta P & 0 \\ \beta C & -s - bA & \beta P & -bI \\ 0 & h & -\alpha - gA - gC & 0 \\ 0 & \phi & 0 & -\xi \end{pmatrix}. \quad (5.3)$$

Substituting E^0 into Eq. (5.3) yields two explicit negative eigenvalues, namely $-a$ and $-\xi$, together with the quadratic equation

$$\zeta^2 + (s + \alpha)\zeta + (s\alpha - h\beta r) = 0.$$

Hence, the equilibrium point E^0 is stable provided that the constant term is positive, that is, $s\alpha > \beta hr$.

5.3 Reproduction Number

At the pest-free equilibrium, we analyze the matrices representing new infections and transfers. The reproduction number is obtained using the next-generation matrix method [28].

$$R_0 = \frac{\beta hr \xi^2}{(s\xi + bk)(\alpha\xi + gk)}. \quad (5.4)$$

Lemma: Then E^0 is stable if and only if

$$R_0 < 1. \quad (5.5)$$

A transcritical bifurcation occurs at $R_0 = 1$, where the system transitions from a pest-free state to an endemic state [15].

Theorem 5.1. *The coexistence equilibrium point E^* is asymptotically stable if the following Routh–Hurwitz conditions are satisfied:*

$$A_4 > 0, \quad A_1 A_2 - A_3 > 0, \quad (A_1 A_2 - A_3) A_3 - A_1^2 A_4 > 0. \quad (5.6)$$

Proof. At E^* , the corresponding characteristic equation is given by

$$\sigma^4 + A_1 \sigma^3 + A_2 \sigma^2 + A_3 \sigma + A_4 = 0, \quad (5.7)$$

$$\begin{aligned} A_1 &= \frac{aP^*}{r} + (s + bA^*) + (\alpha + gA^*) + \xi > 0, \\ A_2 &= \frac{aP^*}{r} \beta C^* + \frac{aP^*}{r} (s + bA^*) + \frac{aP^*}{r} (\alpha + gA^*) + (s + bA^*) (\alpha + gA^*) \\ &\quad + bI^* \phi + \frac{aP^*}{r} \xi + (g + hA^*) \xi + (\alpha + gA^*) \xi - \beta P^* h, \\ A_3 &= \frac{aP^*}{r} \alpha C^* (\alpha + gA^*) + \frac{aP^*}{r} (s + bA^*) (\alpha + gA^*) + \frac{aP^*}{r} bI^* \phi \\ &\quad + bI^* (\alpha + gA^*) \phi + \beta P^* gC^* \phi + \frac{aP^*}{r} \beta C^* \xi \\ &\quad + \frac{aP^*}{r} (s + bA^*) \xi + \frac{aP^*}{r} (\alpha + gA^*) \xi + (s + bA^*) (\alpha + gA^*) \xi \\ &\quad - \frac{aP^*}{r} \beta P^* h + \beta^2 P^* C^* h - \beta P^* \xi h \\ A_4 &= \frac{aP^*}{r} bI^* (\alpha + gA^*) \phi + \frac{aP^*}{r} \beta P^* gC^* \phi \\ &\quad - \beta^2 P^* C^* gC^* \phi + \frac{aP^*}{r} \beta C^* (\alpha + gA^*) \xi + \frac{aP^*}{r} (s + bA^*) (\alpha + gA^*) \xi \\ &\quad - \frac{aP^*}{r} \beta P^* \xi h + \beta^2 P^* C^* \xi h. \end{aligned} \quad (5.8)$$

Assuming the Routh–Hurwitz conditions specified in Eq. (5.6) are satisfied, then Eq. (5.7) has roots with negative real parts (real or complex), guaranteeing the stability of the system. $\square$

6 Analysis of system stability in the presence of delay.

We analyze the local stability of the E^* is studied for the system with time delay described by Eqs. (2.6) – (2.9).

The characteristic equation of system Eqs. (2.6) – (2.9) at equilibrium E^* is expressed as:

$$\Upsilon(\xi) = |\xi I - F - e^{-\xi \tau} G| = 0. \quad (6.1)$$

with F and G given by:

$$\begin{aligned} F &= \begin{bmatrix} -\frac{aP^*}{r} & -\frac{aP^*}{r} & -\beta P^* & 0 \\ \beta C^* & -s - bA^* & \beta P^* & 0 \\ 0 & h & -\alpha - gA^* & 0 \\ 0 & \phi & 0 & -\eta \end{bmatrix}, \\ G &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -bI^* \\ 0 & 0 & 0 & -gC^* \\ 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

This gives the following transcendental equation in ξ :

$$\psi(\xi, \tau) = \xi^4 + D_1 \xi^3 + D_2 \xi^2 + D_3 \xi + D_4 + e^{-\xi \tau} [B_1 \xi^2 + B_2 \xi + B_3] = 0, \quad (6.2)$$

with coefficients:

$$\begin{aligned}
 D_1 &= \frac{aP^*}{r} + (s + bA^*) + (\alpha + gA^*) + \xi, \\
 D_2 &= \frac{aP^*}{r} \beta C^* + \frac{aP^*}{r} (s + bA^*) + \frac{aP^*}{r} (\alpha + gA^*) + (s - bA^*) (\alpha + gA^*) \\
 &\quad + \frac{aP^*}{r} \xi + (s + bA^*) \xi + (\alpha + gA^*) \xi - \beta P^* h, \\
 D_3 &= \frac{aP^*}{r} \beta C^* (\alpha + gA^*) + \frac{aP^*}{r} (s + bA^*) (\alpha + gA^*) + \frac{aP^*}{r} \beta C^* \xi \\
 &\quad + \frac{aP^*}{r} (s + bA^*) \xi + \frac{aP^*}{r} (\alpha + gA^*) \xi + (s + bA^*) (\alpha + gA^*) \xi \\
 &\quad - \frac{aP^*}{r} \beta P^* h + \beta P^* \beta C^* h - \beta P^* \xi h, \\
 D_4 &= \frac{aP^*}{r} \beta C^* (\alpha + gA^*) \xi + \frac{aP^*}{r} (s + bA^*) (\alpha + gA^*) \xi \\
 &\quad - \frac{aP^*}{r} \beta P^* \xi h + \beta^2 P^* C^* \xi h. \\
 B_1 &= bI^* \phi, \\
 B_2 &= \frac{aP^*}{r} bI^* \phi + bI^* (\alpha + gA^*) \phi + \beta P^* gC^* \phi, \\
 B_3 &= \frac{aP^*}{r} bI^* (\alpha + gA^*) \phi + \frac{aP^*}{r} \beta P^* gC^* \phi - \beta^2 P^* C^* gC^* \phi.
 \end{aligned}$$

The characteristic Eq. (6.2) is transcendental because it contains the exponential term $e^{-\xi\tau}$, which originates from the time delay τ in the system. This term makes the equation non-polynomial, and as a result, it possesses an infinite number of complex roots. Specifically, when $\xi = i\omega$, the exponential term can be expressed as $e^{-i\omega\tau} = \cos(\omega\tau) - i\sin(\omega\tau)$, introducing periodicity in the complex plane. Consequently, for each frequency that satisfies the real and imaginary parts of Eq. (6.2), there exists an infinite sequence of characteristic roots that repeat periodically. This is a typical feature of delay differential equations, as the inclusion of delay extends the system to an infinite-dimensional phase space. The local stability of the equilibrium E^* is determined by the root with the largest real part if all roots have negative real parts, the equilibrium is locally asymptotically stable (LAS); otherwise, if any root crosses into the right half of the complex plane, instability occurs. Therefore, we first analyze the system with zero delay and then derive the requirements ensuring stability for $\tau > 0$.

Case I: System with zero delay ($\tau = 0$)

Assuming no time delay, Eq. (6.2) becomes:

$$\xi^4 + D_1 \xi^3 + (D_2 + B_1) \xi^2 + (D_3 + B_2) \xi + (D_4 + B_3) = 0 \quad (6.3)$$

and the Routh-Hurwitz criterion for stability remains identical to that derived for the non-delayed system analysis Eq. (5.6).

Case II: System with positive delay ($\tau > 0$)

Under these conditions, Eq. (6.2) admits infinite number of roots. For the system to be stable, all roots of the characteristic Eq. (6.2) must have negative real components. The equilibrium E^* changes when the characteristic equation (6.2) has roots lying on the imaginary axis. Suppose that $i\omega$ is a root of Eq. (6.2). Then,

$$\omega^4 - D_2 \omega^2 + D_4 = [\omega^2 B_1 - B_3] \cos \omega\tau - [\omega B_2] \sin \omega\tau, \quad (6.4)$$

$$D_1 \omega^3 - D_3 \omega = [\omega^2 B_1 - B_3] \sin \omega\tau + [\omega B_2] \cos \omega\tau. \quad (6.5)$$

By squaring and adding the above two equations

$$\omega^8 + \alpha_1 \omega^6 + \alpha_2 \omega^4 + \alpha_3 \omega^2 + \alpha_4 = 0. \quad (6.6)$$

By introducing the substitution $\omega^2 = \ell$ to Eq. (6.6) reduces the complexity of the stability study with respect to τ . The substitution converts the original eighth-degree equation in ω into a quartic equation in ℓ (Eq. (6.7)), which is more tractable for analytical treatment. Since ℓ represents the square of a real frequency, it is non-negative, eliminating the distinction between positive and negative roots of ω , and enabling the Routh–Hurwitz criterion to identify stability transitions and appearance of imaginary-axis roots.

$$\ell^4 + \gamma_1 \ell^3 + \gamma_2 \ell^2 + \gamma_3 \ell + \gamma_4 = 0, \quad (6.7)$$

where

$$\begin{aligned} \gamma_1 &= D_1^2 - 2D_2, & \gamma_2 &= 2D_4 + D_2^2 - 2D_1D_3 - B_1^2, \\ \gamma_3 &= -2D_4D_2 + D_3^2 + 2B_1B_3 - B_2^2, & \gamma_4 &= D_4^2 - B_3^2. \end{aligned}$$

Eq. (6.7) has roots with negative real parts if and only if the Routh–Hurwitz condition Eq. (6.8) holds. Under these conditions, the roots satisfy $\phi_k^2 = \ell_k < 0$, meaning the roots of Eq. (6.2) are real, with $i\phi_k = \pm\sqrt{\ell_k}$, and not purely imaginary. This leads to the following result.

Theorem 6.1. *For the delayed model Eq. (6.2), the endemic equilibrium E^* is LAS for all $\tau > 0$, if the following conditions are satisfied:*

$$\gamma_1 > 0, \quad \gamma_4 > 0, \quad \gamma_1\gamma_2 - \gamma_3 > 0, \quad (\gamma_1\gamma_2 - \gamma_3)\gamma_3 - \gamma_1^2\gamma_4 > 0. \quad (6.8)$$

Proof. Alternatively, if $\gamma_4 < 0$ Eq. (6.7) admits at least one positive root $m_0 > 0$. Consequently, the roots $\pm i\sqrt{m_0}$ to be a root of Eq. (6.2) correspond to a critical delay value τ^* . According to Butler’s lemma [29], E^* remains stable as long as $\tau < \tau^*$, where τ^* denotes the critical delay value below which the delayed system Eqs. (2.6)–(2.9) retains stability. To compute this, we use Eqs. (6.4) and (6.5) to obtain:

$$\tau^* = \frac{1}{m_0} \cos^{-1} \left[\frac{m_0^2 B_3 [D_1 m_0^2 - D_3] + (B_1 m_0^2 - B_3) [m_0^4 - D_2 m_0^2 + D_4]}{(B_1 m_0^2 - B_3)^2 + B_2^2 m_0^2} \right] \quad (6.9)$$

In summary, we have the following result. If $\gamma_4 < 0$ is satisfied the steady-state E^* is LAS for $\tau < \tau^*$ and becomes unstable for $\tau > \tau^*$. Furthermore, the system will undergo a Hopf bifurcation at E^* when $\tau = \tau^*$.

Now $\text{sgn} \left[\frac{d}{d\tau} \text{Re}(\xi) \right]_{\tau=\tau^*} > 0$ when the numerator is positive, satisfying the transversality condition, the system experiences a Hopf bifurcation at $\tau = \tau^*$. $\square$

7 SDE Model

Deterministic ODE models may fail to capture the inherent variability present in biological systems, which are influenced by factors that cannot always be predicted or quantified. Therefore, it is necessary to develop models that incorporate stochasticity to better reflect the dynamics of such systems. Here, we construct an SDE model by adding stochastic processes to the previously derived ODE system. The procedure for this derivation is based on the methodology proposed by Yuvar et al. [30].

Consider the random vector $X(t) = (X_1(t), X_2(t), X_3(t), X_4(t))^T$ representing $(P(t), I(t), C(t), A(t))^T$, with T denoting the transpose. We introduce a vector of random changes describing how $X(t)$ evolves over a small time interval Δt . Each possible transition in the stochastic model is specified by a transition mappings. From the ODE system in Eqs. (2.1) – (2.4), there are 11 distinct state changes that may occur in Δt , summarized in Table 2 with their probabilities. For example, if a healthy tree becomes infected due to whitefly interaction, the corresponding state change is $\Delta X = (-1, 1, 0, 0)$, with probability given in Table 2.

Based on the stochastic framework of Allen et al. [31], the stochastic system is obtained and given by the equations below.

$$\begin{aligned} d\vec{X} &= \vec{f}(t, \vec{X}(t)) dt + B(t, \vec{X}(t)) d\vec{W}(t), \\ \vec{X}(0) &= [X_1(0), X_2(0), X_3(0), X_4(0)]^T. \end{aligned}$$

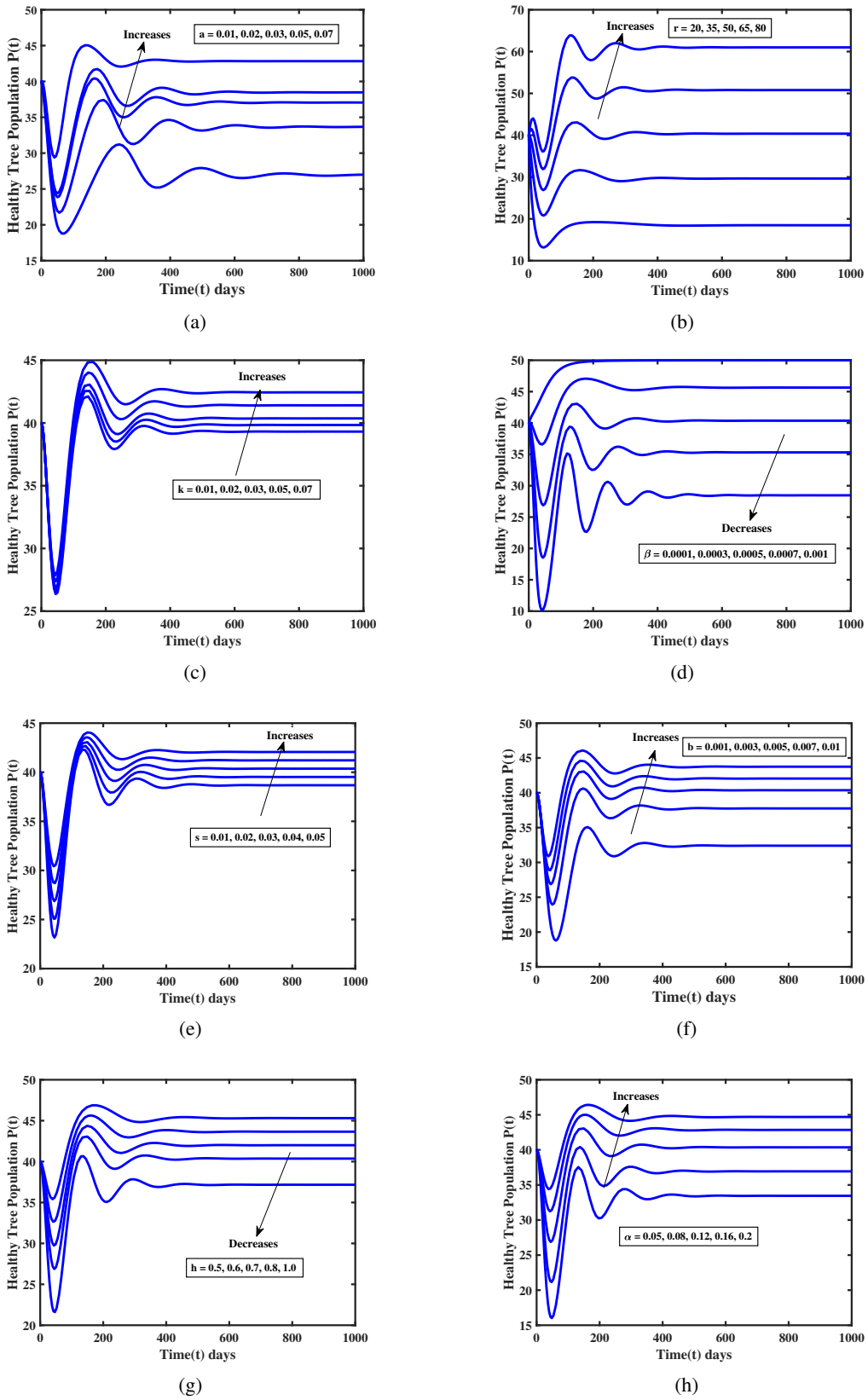


Figure 1: Healthy tree population $P(t)$ over time (1a) – (1h) for varying values of $a, r, k, \beta, s, b, h, \alpha$ for $\tau = 0.5$ with the remaining parameters as given in Table 1.

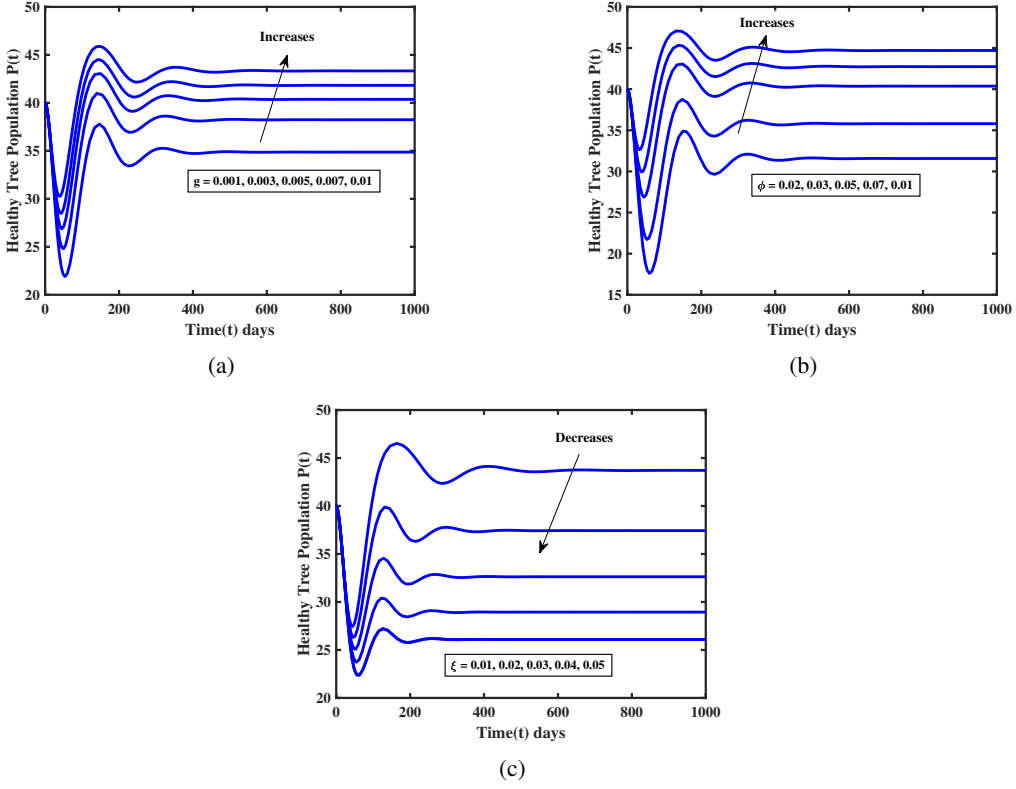


Figure 2: Healthy tree population $P(t)$ over time (2a) – (2c) for varying values of g, ϕ, ξ for $\tau = 0.5$ with the remaining parameters as given in Table 1.

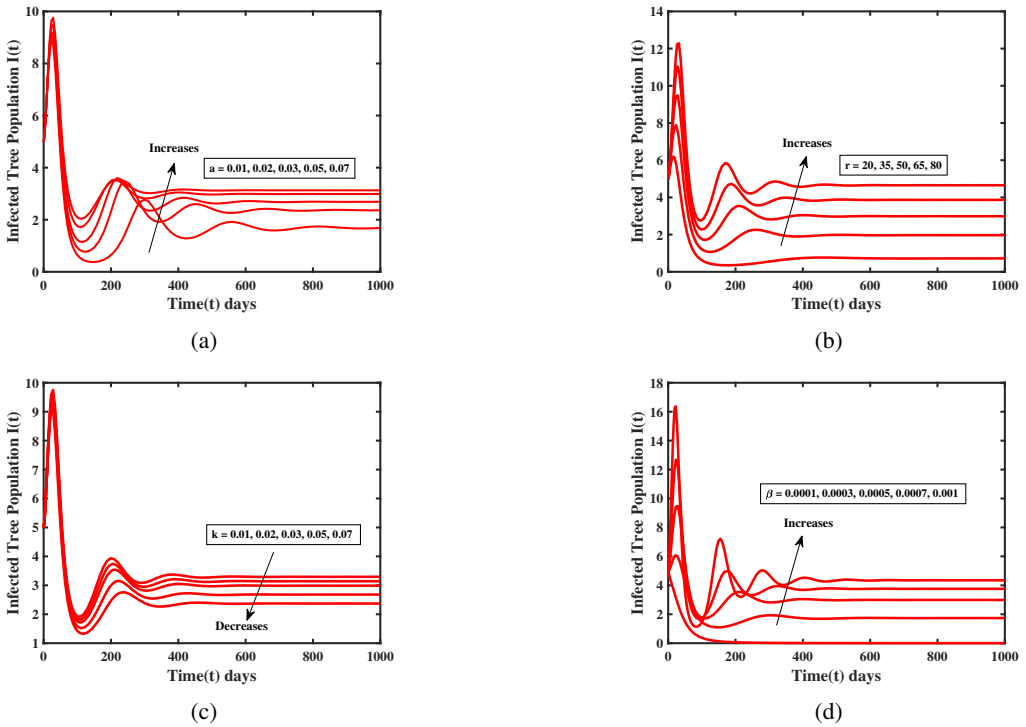


Figure 3: Infected tree population $I(t)$ over time (3a) – (3d) for varying values of a, r, k, β for $\tau = 0.5$ with the remaining parameters as given in Table 1.

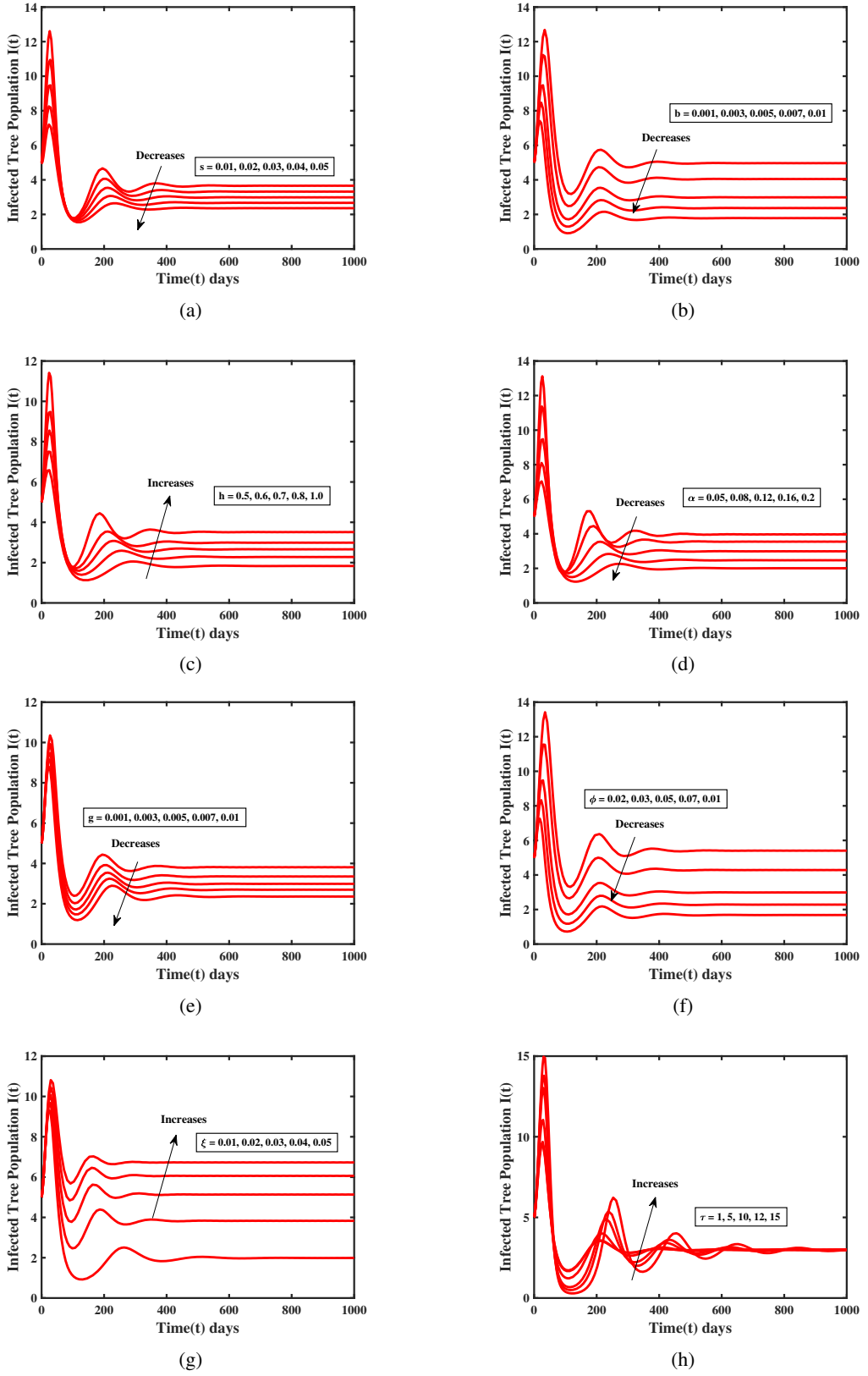


Figure 4: Infected tree population $I(t)$ over time (4a) – (4h) for varying values of $s, b, h, \alpha, g, \phi, \xi, \tau$ for $\tau = 0.5$ with the remaining parameters as given in Table 1.

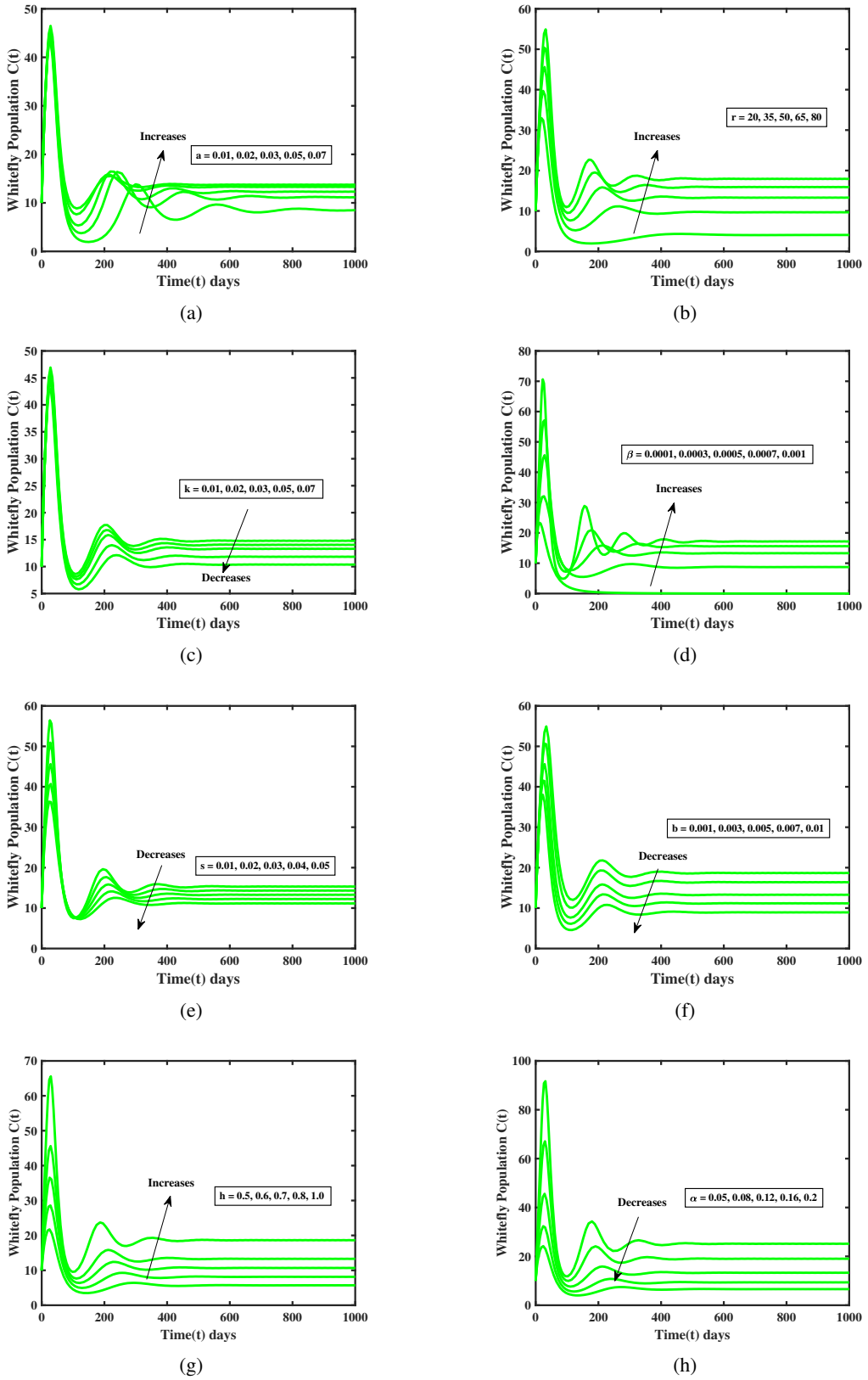


Figure 5: Whitefly population $C(t)$ over time (5a) – (5h) for varying values of $a, r, k, \beta, s, b, h, \alpha$ for $\tau = 0.5$ with the remaining parameters as given in Table 1.

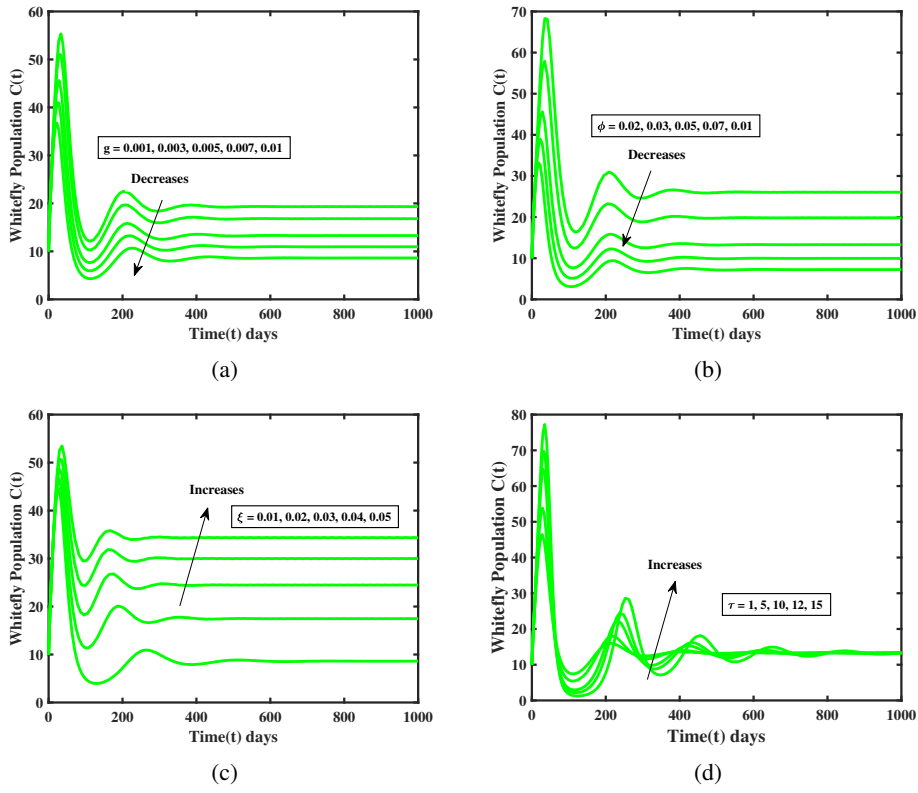


Figure 6: Whitefly population $C(t)$ over time (6a) – (6d) for varying values of g, ϕ, ξ, τ for $\tau = 0.5$ with the remaining parameters as given in Table 1.

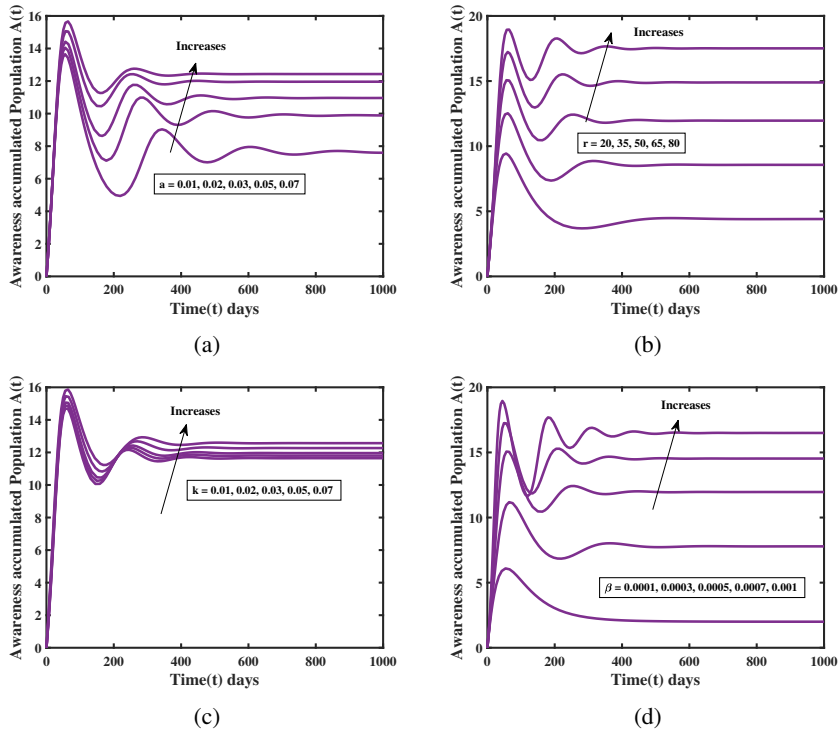


Figure 7: Awareness accumulated population $A(t)$ over time (7a) – (7d) for varying values of a, r, k, β for $\tau = 0.5$ with the remaining parameters as given in Table 1.

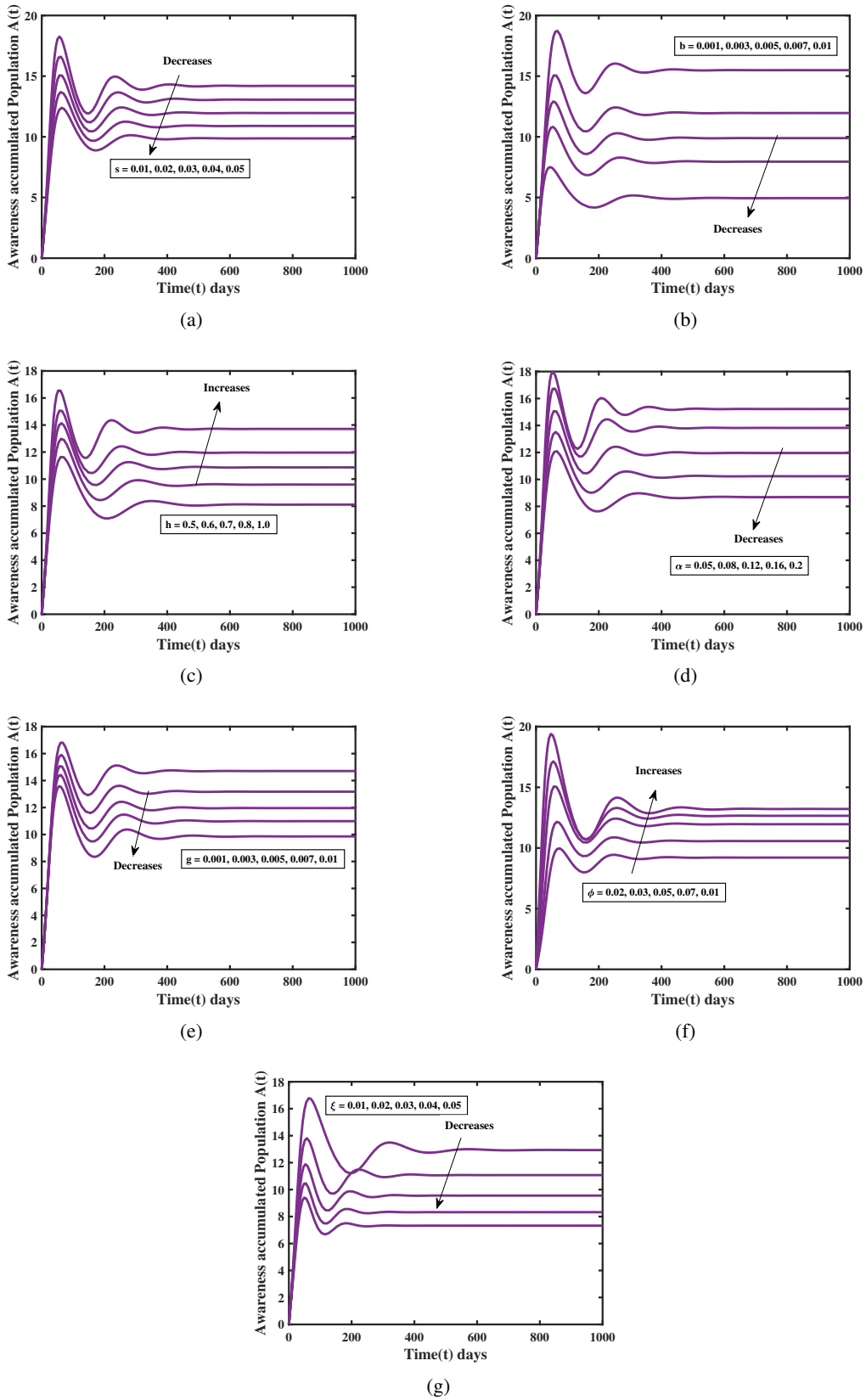


Figure 8: Awareness accumulated population $A(t)$ over time (8a) – (8g) for varying values of $s, b, h, \alpha, g, \phi, \xi$ for $\tau = 0.5$ with the remaining parameters as given in Table 1.

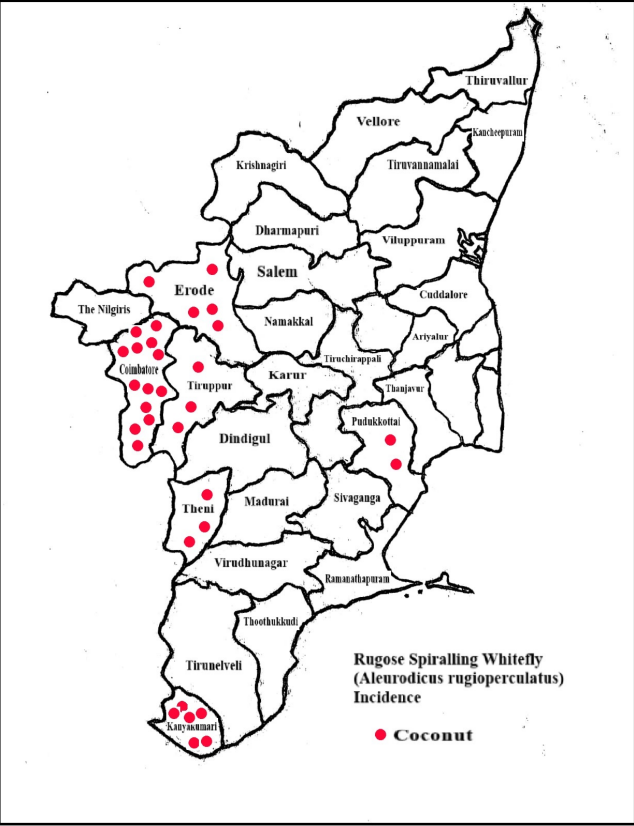


Figure 9: Incidence of RSW on coconut tree in Tamil Nadu were reported in 2023 [32].

Table 2: The possible transitions between states and their associated probabilities in the modified model.

State Change	Description	Transition Probability
$(\Delta X)_1 = (-1, -1, 0, 0)^T$	Healthy trees contact with infected RSWs, reducing their number and increasing infected trees.	$P_1 = \beta PC\Delta t + o(\Delta t)$
$(\Delta X)_2 = (0, -1, 0, 0)^T$	Natural mortality of the infected tree population.	$P_2 = sI\Delta t + o(\Delta t)$
$(\Delta X)_3 = (0, 0, -1, 0)^T$	RSWs die naturally or due to environmental changes.	$P_3 = \alpha C\Delta t + o(\Delta t)$
$(\Delta X)_4 = (0, 0, -1, -1)^T$	RSWs removed by awareness programs.	$P_4 = gAC\Delta t + o(\Delta t)$
$(\Delta X)_5 = (1, 0, 0, 0)^T$	Growth of healthy trees under optimal conditions.	$P_5 = aP\left(1 - \frac{P+I}{r}\right)\Delta t + o(\Delta t)$
$(\Delta X)_6 = (0, 0, 0, 1)^T$	Increase in awareness programs due to external campaigns.	$P_6 = k\Delta t + o(\Delta t)$
$(\Delta X)_7 = (0, 0, 0, -1)^T$	Awareness programs fade away over time.	$P_7 = \xi A\Delta t + o(\Delta t)$
$(\Delta X)_8 = (0, -1, 0, 1)^T$	Infected trees eradicated directly due to awareness campaigns.	$P_8 = bAI\Delta t + o(\Delta t)$
$(\Delta X)_9 = (0, 0, 1, 0)^T$	RSW population increases due to new infected trees.	$P_9 = hI\Delta t + o(\Delta t)$
$(\Delta X)_{10} = (0, 1, 0, 0)^T$	Variation in infected tree growth resulting from a delay in disease transmission	$P_{10} = \beta PC\Delta t + o(\Delta t)$
$(\Delta X)_{11} = (0, 0, 0, 0)^T$	System state remains unchanged.	$P_{11} = \left(1 - \sum_{i=1}^{10} P_i\right) + o(\Delta t)$

Here we define the drift vector as

$$\vec{f} = \sum_{j=1}^{10} P_j \vec{\lambda}_j. \tag{7.1}$$

In this context, $\vec{\lambda}_j$ corresponds to the stochastic state changes, while P_j indicates their respective probabilities of occurrence (as summarized in Table 2).

We define the drift vector $\vec{f}$ by

$$\vec{f} = P_1 \vec{\lambda}_1 + P_2 \vec{\lambda}_2 + P_3 \vec{\lambda}_3 + P_4 \vec{\lambda}_4 + P_5 \vec{\lambda}_5 + P_6 \vec{\lambda}_6 + P_7 \vec{\lambda}_7 + P_8 \vec{\lambda}_8 + P_9 \vec{\lambda}_9 + P_{10} \vec{\lambda}_{10} \tag{7.2}$$

The covariance matrix,

$$\vec{B} = \sum_{j=1}^{10} P_j \lambda_j \lambda_j^T. \quad (7.3)$$

Computing the square root of a covariance matrix is necessary, but for positive semi-definite matrices of order above two, no explicit formula is available. Consequently, we employ a second modeling technique that constructs a diffusion matrix is formulated in such a way that computation of the square root is unnecessary. Based on the approach of Allen et al. [31], the corresponding stochastic system is expressed as:

$$d\vec{X} = \vec{f}(t, \vec{X}(t))dt + G(t, \vec{X}(t))d\vec{W}(t). \quad (7.4)$$

$$\vec{X}(0) = [X_1(0), X_2(0), X_3(0), X_4(0)]^T. \quad (7.5)$$

Drift vector $\vec{f}$ is consistent with the deterministic formulation.

We define the diffusion matrix G is constructed as

$$G = \lambda_{i,j} P_j^{1/2}, \quad j = 1, 2, \dots, 11, \quad i = 1, 2, \dots, 4 \quad (7.6)$$

$$G = \begin{bmatrix} -\sqrt{\beta X_1 X_3} & 0 & 0 & 0 & \sqrt{a X_1 \left(1 - \frac{X_1 + X_2}{r}\right)} & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{\beta X_1 X_3} & -\sqrt{s X_2} & 0 & 0 & 0 & 0 & 0 & -\sqrt{b X_4 X_2} & 0 & \sqrt{\beta X_1 X_3} \\ 0 & 0 & -\sqrt{\alpha X_3} & -\sqrt{g X_4 X_3} & 0 & 0 & 0 & 0 & \sqrt{h X_2} & 0 \\ 0 & 0 & 0 & -\sqrt{g X_4 X_3} & 0 & \sqrt{k} & -\sqrt{\xi X_4} & \sqrt{b X_4 X_2} & 0 & 0 \end{bmatrix}$$

Consequently, the Itô stochastic differential formulation is given by:

$$dX(t) = f(X_1, X_2, X_3, X_4)dt + G.dW(t), \quad (7.7)$$

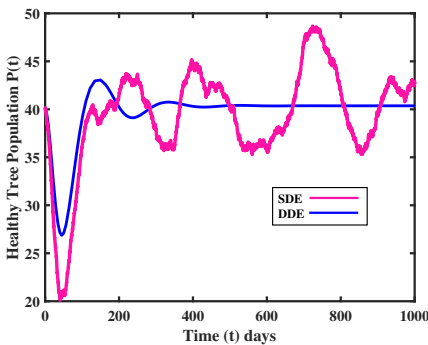
with initial conditions $X(0) = (X_1(0), X_2(0), X_3(0), X_4(0))^T$ and a wiener process, $W(t) = (W_1(t), W_2(t), W_3(t), W_4(t))^T$. The SDE model is constructed in the following form:

$$dP = \left(aP \left(1 - \frac{P+I}{r} \right) - \beta PC \right) dt - \sqrt{\beta X_1 X_3} + \sqrt{a X_1 \left(1 - \frac{X_1 + X_2}{r} \right)}, \quad (7.8)$$

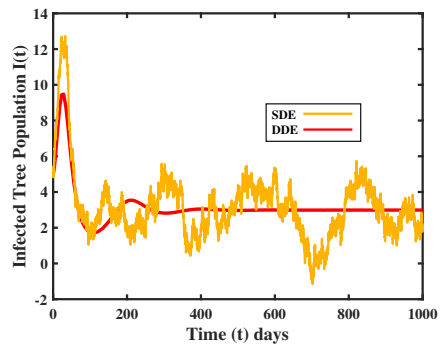
$$dI = \left(\beta PC - sI - bAI - \sqrt{\beta X_1 X_3} \right) dt - \sqrt{s X_2} - \sqrt{b X_4 X_2} + \sqrt{\beta X_1 X_2}, \quad (7.9)$$

$$dC = (hI - \alpha C - gAC)dt - \sqrt{\alpha X_3} - \sqrt{g X_4 X_3} + \sqrt{h X_2}, \quad (7.10)$$

$$dA = (k + \phi I - \xi A)dt - \sqrt{g X_2 X_3} + \sqrt{k} - \sqrt{\xi X_4} + \sqrt{b X_4 X_2}. \quad (7.11)$$



(a)



(b)

Figure 10: Graph shows dynamics of (10a) healthy tree population under the DDE and SDE models. (10b) infected tree population under the DDE and SDE models.

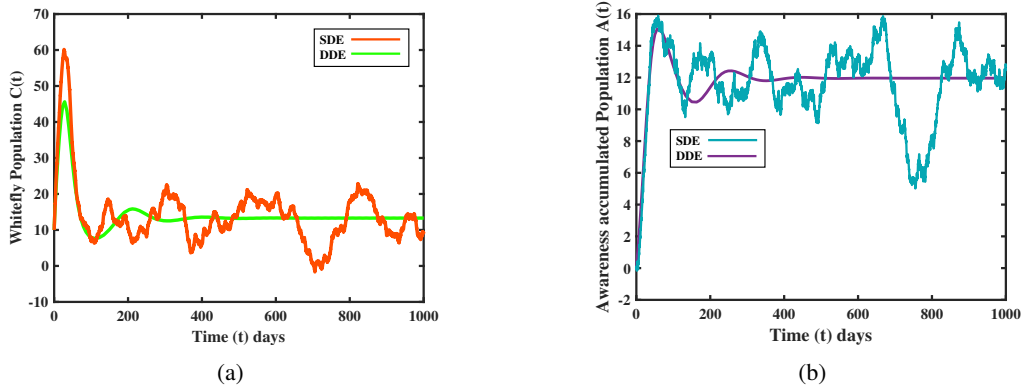


Figure 11: Graph shows dynamics of (11a) whitefly population under the DDE and SDE models. (11b) accumulated awareness population under the DDE and SDE models.

8 Numerical Simulation

We analyze the impact of various parameters on the population dynamics of healthy trees, infected trees, whiteflies, and awareness programs over time, incorporating the time delay τ and using parameter values provided in Table 1. Starting with initial values of 40 healthy trees, 5 infected trees, 10 whiteflies per tree, and no awareness programs, numerical simulations were performed to illustrate these effects.

Figures (1a) – (1h) highlights how key parameters influence the healthy tree population. An increase in the rate of healthy tree growth (a) and the maximum healthy tree biomass capacity (r) enhances the healthy tree population, while greater awareness generation (k) also aids recovery. However, a higher contact rate with infected whiteflies (β) reduces healthy trees. The natural and disease-induced reduction rate (s) and infected tree removal rate (b) positively affect healthy trees, whereas higher infected whitefly emergence (h) diminishes healthy trees. Conversely, whitefly mortality (α) support healthy tree growth. Figures (2a) – (2c) represent human control efforts (g), and awareness recruitment (ϕ) also support healthy tree growth. In contrast, the fading influence of awareness programs (ξ) reduces the healthy tree population.

Figures (3a) – (3d) illustrates trends in the infected tree population. Growth rates a and r increase infected trees, while awareness generation (k) reduce infections and (β) amplify infection spread. Figures (4a) – (4h) illustrates natural and disease-induced reduction rate (s), tree removal efforts (b), whitefly mortality (α), human control (g), awareness recruitment (ϕ) reduce infections. Whitefly emergence (h), delayed awareness (ξ) and time delays (τ) escalate infections.

Figures (5a) – (5h) examines the whitefly population. Healthy tree growth rate a , and maximum healthy tree biomass capacity r , contact rate (β) and whitefly emergence (h) promote whitefly proliferation, while awareness efforts (k), tree removal (b), natural and disease-induced reduction rate (s), mortality (α), suppress whiteflies. Figures (6a) – (6d) shows a human interventions (g), awareness recruitment (ϕ) reduces whitefly numbers, but fading awareness programs (ξ) and time delays (τ) contribute to population persistence.

Finally, Figures (7a) – (7d) analyzes awareness program dynamics. Increased tree growth (a), biomass capacity (r), awareness generation (k) and whitefly contact (β) enhance awareness accumulation. Figures (8a) – (8e) shows a natural and disease-induced reduction rate (s), tree removal (b), whitefly mortality (α), and human interventions (g) lower awareness levels. emergence (h) and awareness recruitment (ϕ) strengthen awareness efforts, while fading awareness (ξ) weakens their impact over time.

Figure 9 displays the incidence of RSW on coconut tree in Tamil Nadu were reported in 2023. Figure 10 & 11 compares the delay and stochastic models, showcasing their effects on system dynamics. In addition to deterministic trends captured by the ODE and DDE models, the stochastic component of our framework reflects the inherent variability and unpredictability of real farming systems. Environmental fluctuations, random pest outbreaks, and varying human responses are represented through stochastic terms, which allow the model to capture deviations from the

average dynamics observed in deterministic simulations. Comparison of the delay and stochastic models (Figures 10a – 11b) illustrates how stochasticity can amplify or dampen fluctuations in tree health, infection levels, whitefly populations, and awareness accumulation. Incorporating these stochastic effects provides a more realistic assessment of intervention strategies and highlights the importance of adaptive management in addressing the uncertainty present in real - world agricultural systems. The model parameters are provided in Table 1. Table 2 details the different state transitions and their probabilities.

9 Conclusion

In this study, we developed a mathematical framework incorporating DDE and SDE to investigate the population dynamics of whitefly infestations on coconut trees. The DDE model enabled us to analyze the impact of time delays, species interactions, and system parameters on the overall dynamics, while identifying equilibrium points and establishing conditions for LAS. Through numerical simulations and sensitivity analysis, we demonstrated that time delay τ significantly influences the system's behavior, particularly in determining the range of days during which coconut trees become infected. This insight is critical for designing targeted farming awareness campaign that educate farmers on sustainable agricultural practices, including the adoption of eco-friendly methods, reduced chemical usage, and improved soil health. By addressing these delays, such campaign can provide farmers with practical strategies to manage whitefly infestations, ultimately improving tree health and ecosystem sustainability. Moreover, real-world systems often exhibit stochastic effects that cannot be captured by purely deterministic models. To address this, we incorporated an SDE model to account for the inherent randomness and variability in pest dynamics. Simulations of the combined DDE-SDE system under various conditions revealed optimal intervention strategies to mitigate the spread of whitefly infestations. The integration of DDE, which captures natural delays, and SDE, which introduces randomness, provides a robust and comprehensive approach for understanding and managing whitefly populations. This combined modeling framework offers valuable insights into pest control, balancing immediate interventions with long-term ecological sustainability.

Future research should focus on expanding the current framework to incorporate spatial heterogeneity, climate variability, and interactions with multiple pest species, which would improve the predictive accuracy and applicability of the model. Integrating optimal control strategies can help identify cost-effective and timely interventions, while calibration with field data will enhance real-world relevance. Practically, our findings suggest that targeted awareness campaigns, aligned with predicted infection delays, and the promotion of integrated pest management practices combining biological control, eco-friendly methods, and minimal chemical usage can provide effective and sustainable strategies for managing whitefly infestations in coconut plantations.

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