

INSIGHTS ABOUT THE NUTRITIONAL IMPACT OF MID-DAY MEAL SCHEME USING MATHEMATICAL MODELLING

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Abstract The children are the future and backbone of our nation. Nevertheless, India had encountered a difficult challenge, as one-third of the world's malnourished children are Indian. Several measures have been taken by the Tamil Nadu government to address this problem, including the implementation of the student Mid-day meal program. This study evaluated the scheme's effectiveness in enhancing the nutritional status of a single child by employing a mathematical model. The model had explicitly examined the role of daily Mid-day meals in the progression of a malnourished child to better-nourished states. It had employed non-linear differential equations to describe the dynamics of this transition. Stability analysis elucidated the efficacy of the Mid-day meal scheme in improving the health of an individual child. We used the basic reproduction number (R_0) in the context of nutritional modelling as a threshold parameter to determine the effectiveness of the scheme in transforming a malnourished child into a moderately or well-nourished state. The results highlighted the potential of Mid-day meal programs to significantly improve nutritional status while also highlighting the limitations of this scheme.

1 Introduction

The prevalence of malnutrition among children is an alarming problem to be addressed in India. Inadequate nutrition during critical growth periods severely impacts children's physical, cognitive, and emotional development, often leading to stunted growth, weakened immunity, and increased susceptibility to infectious diseases[1]. This early disadvantage hinders children's ability to perform well in school, diminishing educational outcomes and limiting future opportunities for productive employment. For a country, this translates to a loss in human capital, reduced economic productivity, and increased burden on health and social services. Investing in proper nutrition for children is, therefore, not just a humanitarian priority but an essential strategy for sustainable development. Ensuring that children receive adequate nutrition equips them with the foundation for a healthier, more capable adulthood, fostering a workforce that can actively contribute to and benefit from the nation's growth and prosperity [2].

India accounts for over 20 percent of child deaths worldwide, highlighting the critical importance of focusing on child survival within broader discussions on child welfare of the country. The Comprehensive National Nutrition Survey (CNNS) India 2016–18 report found that 10% of school-age children (5–9 years) and 47% of late adolescent girls (15–19 years) in India were underweight. Under-nutrition poses a serious barrier to children's optimal growth and development, increasing their susceptibility to infectious diseases, impairing learning ability, and reducing school performance [3]. To tackle this problem, school feeding programs have been

implemented. On November 28, 2001, the Supreme Court mandated that all State Governments and Union Territories to implement the Mid-day Meal (MDM) Scheme.

This directive required that every child in Government and Government-aided schools receive a prepared meal containing at least 300 kilocalories and 8–12 grams of protein daily, for a minimum of 200 days each year [4].

Tamil Nadu was the first state to implement school feeding initiatives in order to address the critical issue of malnutrition among school students. In 1925, the Madras Municipal Corporation initiated the "Mid-day Meal Programme" to provide assistance to underprivileged children. The program was expanded to include institutions in 1956. The "Puratchi Thalaivar MGR Nutritious Meal Programme" was implemented in 1982 with the objective of providing nutritious meals to pre-school children aged 2 to 5 years and primary school children aged 5 to 9 years in rural Tamil Nadu. In the same year, the program was broadened to encompass students residing in urban areas. The scheme had been further expanded to include school pupils aged 10 to 15 by 1984.

The Mid-day meal program in Tamil Nadu is generally operational on all school working days, providing sustenance for approximately 220 days annually, including holidays, weekends, and school vacations [5]. The meals are designed to meet specific caloric and nutritional standards:

- Primary School Children: 450 calories and 12 grams of protein per meal.
- Upper Primary School Children: 700 calories and 20 grams of protein per meal.
- Iron and Folic Acid Supplements: Provided to address micronutrient deficiencies.
- Fortified Foods: In some regions, fortified foods (with added vitamins and minerals) are included to combat specific nutritional deficiencies.

Several states, including Kerala, Karnataka, Andhra Pradesh, Maharashtra, Gujarat, and Uttar Pradesh, followed Tamil Nadu's lead by introducing a Mid-day meal program in primary schools [6]. The program seeks to boost school attendance and retention rates while enhancing the nutritional health of school children [7].

A study by Deka [8] found a substantial improvement in school attendance rates following the introduction of the Mid-day meal scheme. Prior to the scheme, only 16.26% of students maintained good attendance (60% and above), which rose significantly to 89.77% after the scheme's implementation. The study also highlighted the program's strong acceptance among both children and parents, particularly in rural areas with limited economic resources. Many parents, are in a situation to leave for work early, had previously been unable to prioritize adequate meals for their children. As a result, students often arrived at school with only a light breakfast or an empty stomach, sometimes skipping lunch entirely, which contributed to high dropout rates and low attendance. The Mid-day meal scheme has successfully addressed these issues, leading to notable improvements in attendance and retention[9]. The provision of kids with at least one lunch per day was found to have a favorable correlation with higher academic outcomes, increased food and nutrition security, and better diet quality, according to a systematic analysis that investigated universal free school meal programs [10]. The program aims to enhance school attendance and retention rates while improving the nutritional status of the children[11].

A number of methodologies have been used to examine the effects of the Mid-day Meal (MDM) Scheme on children's nutrition, such as observational studies [12], randomised controlled trials (RCTs)[13] , statistical analyses (pre- and post-Intervention)[14]. Developmental markers like height, weight, and body mass index are commonly used in observational studies to assess changes in dietary position. Although statistical analyses comparing nutritional outcomes before and after program implementation are popular, it is important to note that these approaches tend to concentrate on overall trends rather than the intricacies of individual nutritional advancement. Comparatively, a compartmental modelling strategy provides a fresh framework by classifying child's nutritional state into three separate nutritional states namely, severely malnourished, moderately nourished, and well-nourished. To better understand the scheme's efficacy, this segmentation allows for a targeted evaluation of the rate at which children go from severely malnourished to moderately or well-nourished states. Using nonlinear differential equations, this method accounts for realistic dynamics, unlike static or linear models, and takes into consideration things like saturation effects and declining benefits as child get closer to better nutritional states. This approach permits long-term simulations to assess the stability and sustainability of

dietary improvements and offers a more detailed perspective of individual progress. If the model can forecast the rates of state transitions, it can determine if the plan will produce sustained nutritional benefits or if supplementary nutrients or other forms of support will be necessary. Consequently, this method goes beyond the insights given by traditional assessment tools by providing a dynamic, personalised viewpoint that improves our knowledge of the Mid-day Meal Scheme's effect on child nutrition.

2 Materials and Methods

2.1 Model description

The model depicts the nutritional development of a malnourished upper primary school student in Tamil Nadu who benefits from the Mid-day Meal Scheme. It is assumed that the child's nutrient absorption rate is typical and that no other external factors influence the outcome. The nutritional status of the child is classified into three distinct states: Malnourished (N_0), Moderately Nourished (N_1), and Well-Nourished (N_2). The primary objective of this model is to assess how the Mid-day Meal Scheme influences transitions between these nutritional states over time.

Accordingly, the total nutritional status at any given time t is defined as

$$N(t) = N_0(t) + N_1(t) + N_2(t). \quad (2.1)$$

Based on the above assumptions, the dynamics of the system are governed by the following set of nonlinear ordinary differential equations:

$$\frac{dN_0}{dt} = -\delta N_0 + \eta N_0 \left(1 - \frac{N_0}{K}\right), \quad (2.2)$$

$$\frac{dN_1}{dt} = \delta N_0 - \epsilon N_1 + \xi N_1 \left(1 - \frac{N_1}{L}\right), \quad (2.3)$$

$$\frac{dN_2}{dt} = \epsilon N_1 - \sigma N_2. \quad (2.4)$$

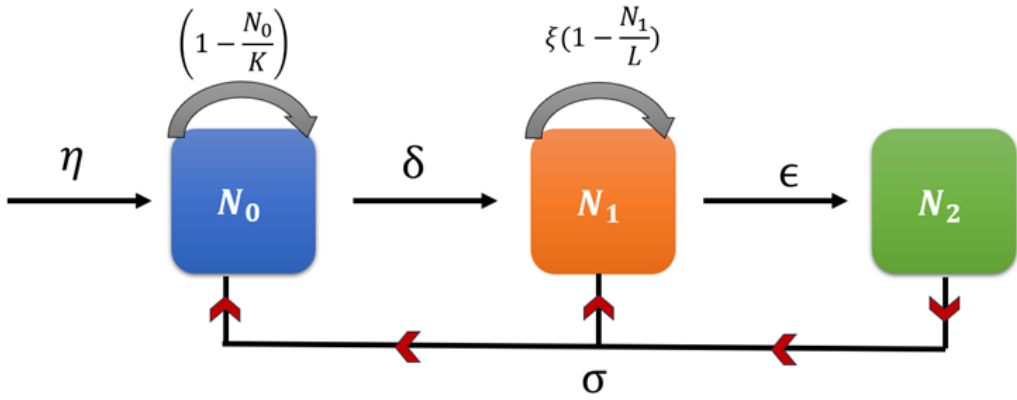


Figure 1: Schematic representation of a Mid-day meal intervention model for combating child malnutrition

The parameter δ denotes the rate at which a child transitions from the malnourished state due to improvements or nutritional interventions. The parameter η represents the effectiveness of interventions (such as nutritional supplementation) in enhancing the child's nutritional status. The parameter ϵ denotes the rate at which a child transitions to the well-nourished state, reflecting the effectiveness of continued nutritional support. The parameter ξ represents the effectiveness of additional interventions in further improving the nutritional status of the child.

The parameters K and L denote the carrying capacities associated with the malnourished and moderately nourished states, respectively. In this within-host framework, these carrying capacity parameters represent physiological saturation limits on nutritional absorption and recovery.

Their inclusion prevents unrealistic unbounded growth in nutritional status. Finally, the parameter σ represents the growth rate of the child's nutritional status in the well-nourished state due to sustained nutritional interventions.

3 Model Analysis

3.1 Invariant region

Theorem 3.1 (Positivity and Boundedness). *The solution set of the model system defined by the equations (2.2)–(2.4) with initial conditions $N_0(0) \geq 0$, $N_1(0) \geq 0$, $N_2(0) \geq 0$ defined in the region ϑ in $\mathbb{R}_+^3$, where*

$$\vartheta = \{(N_0(t), N_1(t), N_2(t)) \in \mathbb{R}_+^3 : N(t) \leq \max(K, L)\}, \quad (3.1)$$

is positively bounded.

Proof. We first analyse the dynamics of $N(t)$. From equation (2) we have that if N_0 approaches K , the term $-\frac{\eta N_0^2}{K}$ ensures that if $N_0 > K$, the growth rate $\frac{dN_0}{dt}$ becomes negative, leading to a decline in N_0 . Therefore, N_0 is bounded by K .

Similarly, from equation (3) we have if N_1 approaches L , the term $-\frac{\xi N_1^2}{L}$ ensures that if $N_1 > L$, the growth rate $\frac{dN_1}{dt}$ becomes negative, leading to a decline in N_1 . Therefore, N_1 is bounded by L .

Considering equation (4), since ϵN_1 is a positive term when $N_1 \geq 0$ and $-\sigma N_2$ is a negative term, we see that if $N_2 \rightarrow \infty$, the term $-\sigma N_2$ will dominate. Thus, N_2 cannot grow indefinitely and will be positively bounded based on the dynamics of N_1 .

Thus, by analysing each component N_0 , N_1 , and N_2 we conclude that the solution set is positively bounded in the region ϑ . Each component remains within a maximum determined by K , L and the model parameters, ensuring that

$$N(t) \leq \max(K, L), \quad \text{for all } t \geq 0. \quad (3.2)$$

□

3.2 Equilibrium solutions

We now solve the model equations to obtain the equilibrium states [15]. At the equilibrium state

$$\frac{dN_0}{dt} = \frac{dN_1}{dt} = \frac{dN_2}{dt} = 0. \quad (3.3)$$

Thus, by equating the equations (2–4) to zero we obtain equilibrium points as follows:

$$(N_0^*, N_1^*, N_2^*) = \left(\frac{K(\eta - \delta)}{\eta}, \frac{(\xi - \epsilon) \pm \sqrt{(\xi - \epsilon)^2 - 4\frac{\xi}{L}\delta N_0^*}}{2\frac{\xi}{L}}, \frac{\epsilon N_1^*}{\sigma} \right). \quad (3.4)$$

Since the proposed model is being formulated as a within-host system, the coexistence state refers to the amount of malnourishment, the partial recovery state and completely nourished state. Hence, these are not exclusive categorical states. Therefore, coexistence state refers to a physiological steady state of nutritional status that corresponds to a steady nutritional balance within a single child, where the nutritional progression coexists rather than exclusive nutritional classes.

3.3 Stability Analysis

In this section, we perform local stability analysis about the interior equilibrium points that represents sustained nutritional transitions. Now we construct Jacobian Matrix,

$$J = \begin{pmatrix} -\delta + \eta \left(1 - \frac{2N_0^*}{K}\right) & 0 & 0 \\ \delta & -\epsilon + \xi \left(1 - \frac{2N_1^*}{L}\right) & 0 \\ 0 & \epsilon & -\sigma \end{pmatrix}. \quad (3.5)$$

Calculating the eigen values $\det(J - \lambda I) = 0$, we get,

$$J - \lambda I = \begin{pmatrix} -\delta + \eta \left(1 - \frac{2N_0^*}{K}\right) - \lambda & 0 & 0 \\ \delta & -\epsilon + \xi \left(1 - \frac{2N_1^*}{L}\right) - \lambda & 0 \\ 0 & \epsilon & -\sigma - \lambda \end{pmatrix}. \quad (3.6)$$

Since it is a triangular matrix, the eigen values of the matrix are its diagonal elements. Therefore,

$$\lambda_1 = -\delta + \eta \left(1 - \frac{2N_0^*}{K}\right), \quad \lambda_2 = -\epsilon + \xi \left(1 - \frac{2N_1^*}{L}\right), \quad \lambda_3 = -\sigma. \quad (3.7)$$

3.4 Stability Conditions

To ensure the stability the system all the eigen values should be negative. Thus, we have below conditions:

$$\eta \left(1 - \frac{2N_0^*}{K}\right) < \delta. \quad (3.8)$$

i.e. the effect of interventions on reducing malnourishment must be strong but not overly aggressive, so that natural recovery remains dominant.

$$\xi \left(1 - \frac{2N_1^*}{L}\right) < \epsilon. \quad (3.9)$$

i.e. the interventions improving children's nutrition to moderate levels must be balanced so that natural transitions to moderate nourishment remain effective.

$$-\sigma < 0. \quad (3.10)$$

The rate at which children remain well-nourished must always be positive, ensuring the well-nourished population does not collapse.

The conditions described are not only mathematically sound but also reflect a practical understanding of how public health and nutritional interventions should be applied. Interventions need to be carefully calibrated to improve outcomes while allowing natural processes to function, ensuring long-term stability and sustainability. Thus, the framework aligns well with the practical considerations of policy-makers and public health experts.

3.5 Computation of basic reproduction number

In this section, we have computed basic reproduction number by using next generation matrix method [16] and have provided the comparative analysis by varying with different values of effectiveness parameter η .

Reproduction number in the context of Nutritional Intervention model

In the context of assessing interventions for children, such as Mid-day meal programs, we seek to compute the fundamental reproductive number R_0 . This statistic indicates the efficacy of Mid-day meal in enhancing the nutritional status of children, especially those progressing from malnourished (N_0) to moderately nourished (N_1).

Interpretation of R_0

- $R_0 > 1$: The intervention is sufficiently successful to guarantee a favourable nutritional change for the child.
- $R_0 < 1$: The intervention lacks adequate strength to effectuate notable nutritional enhancement for the child. A result below 1 suggests that the child is prone to persistent malnutrition unless further interventions are implemented.

Calculating R_0 by using next generation matrix method we get, the F matrix contains the contributions of new entries into each state due to transitions from other states and the V matrix captures the outflow from each state, representing how children leave each state due to transitions or natural decay.

$$\begin{aligned} F &= \begin{pmatrix} 0 & 0 \\ \delta & 0 \end{pmatrix}, \\ V &= \begin{pmatrix} \eta & 0 \\ -\delta & \epsilon \end{pmatrix}, \\ FV^{-1} &= \begin{pmatrix} 0 & 0 \\ \frac{\delta}{\eta} & 0 \end{pmatrix}, \\ R_0 &= \frac{\delta}{\eta}. \end{aligned} \tag{3.11}$$

3.6 Numerical Simulations

The parametric values can be obtained as follows.

Table 1: Parametric values used for numerical simulation

Parameter	Formula	Description	Value
δ	$\delta = \frac{C_{mm}}{C_{Mod}}$	C_{mm} = calories provided by Mid-day meal (700 calories). C_{Mod} = caloric requirement per day for an upper primary to become moderately nourished (2000 calories).	0.35
ϵ	$\epsilon = \frac{C_{mm}}{C_{well}}$	C_{mm} = calories provided by Mid-day Meal (700 calories). C_{well} = caloric requirement per day for an upper primary to become moderately nourished (2200 calories).	0.32
σ	$\sigma = k \left(\frac{\text{caloric intake}}{\text{caloric requirement}} \right) + k \left(\frac{\text{Protein intake}}{\text{Protein requirement}} \right)$	Caloric Intake = Energy provided by the Mid-day meal (700 kcal/day). Daily caloric requirement = 2200 kcal/day (well nourished). Protein intake = protein provided by the Mid-day meal (20 g). Protein required = 34 g.	0.1
ξ	$\xi = \frac{\text{Effective Nutritional Contribution}}{\text{Threshold requirement}}$	Effective nutritional contribution = 780 kcal/day. Threshold requirement = 1800 kcal/day (minimum threshold).	0.43
K	$K = \frac{\delta + \epsilon}{\delta}$	K is the carrying capacitor for malnourished state.	1.9
L	$L = \frac{\delta + \epsilon}{\epsilon}$	L is the carrying capacitor for moderately nourished state.	2.1

Note that these parameters are formulated in a daily time scale consistent with nutritional intake practices. These coefficients represent proportional nutritional influence per day.

Here, k is taken as a proportionality constant and is set to $k = 0.1$ for numerical simulations.

$$N_0^* = 0.15; \quad N_1^* = 0.485; \quad N_2^* = 1.552$$

Substituting the values from Table 1 and the equilibrium points in Jacobian matrix, we get

$$J = \begin{pmatrix} -0.031 & 0 & 0 \\ 0.35 & -0.0886 & 0 \\ 0 & 0.32 & -0.1 \end{pmatrix},$$

(3.12)

Eigenvalues: $\lambda_1 = -0.031, \lambda_2 = -0.0886, \lambda_3 = -0.1,$

$$R_0 = \frac{\delta}{\eta} = \frac{0.35}{\eta}.$$

All eigenvalues are negative, indicating the stability of the equilibrium positions. The system will likely revert to these equilibrium states when perturbed. This verifies that the model remains stable within the specified parameters.

The effectiveness parameter (η) plays a crucial role as it quantifies the effectiveness of Mid-day meal in child’s nutritional state. Now we perform sensitivity analysis to find out the threshold below which the intervention is effective in promoting transitions from the malnourished state to moderately nourished state and above which it becomes less effective or even counterproductive.

4 Results and discussions

Sensitivity Analysis of Intervention Effectiveness (η)

Now we vary η from 0.3 to 0.7. For each value of η , the Jacobian matrix J was constructed and the eigenvalues were computed to assess the stability of the system. Negative eigenvalues indicate stable system while positive eigen values suggest the instability.

Table 2: Sensitivity Analysis of Intervention Effectiveness (η)

η	λ_1	λ_2	λ_3	Stability condition
0.30	-0.0974	-0.0886	-0.1	Stable
0.35	-0.0553	-0.0886	-0.1	Stable
0.38	-0.0300	-0.0886	-0.1	Stable
0.40	-0.0132	-0.0886	-0.1	Stable
0.50	0.0289	-0.0886	-0.1	Unstable
0.60	0.1132	-0.0886	-0.1	Unstable
0.70	0.1553	-0.0886	-0.1	Unstable

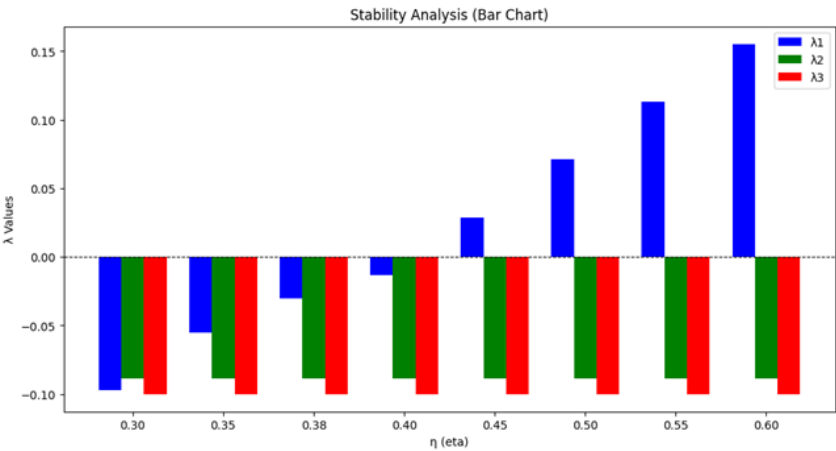


Figure 2: Sensitivity Analysis of Eigen values at the nutritional coexistence equilibrium with respect to the intervention effectiveness parameter η

From the above graph, we can obtain the critical value of the intervention effectiveness pa-

parameter as $\eta = 0.40$ (approximately), after which the system becomes unstable. Hence, the effective range of the intervention parameter is given by

$$0.30 < \eta < 0.40.$$

Table 3: Variation of basic reproduction number R_0 with respect to η

η	0.30	0.35	0.40
R_0	1.667	1	0.875

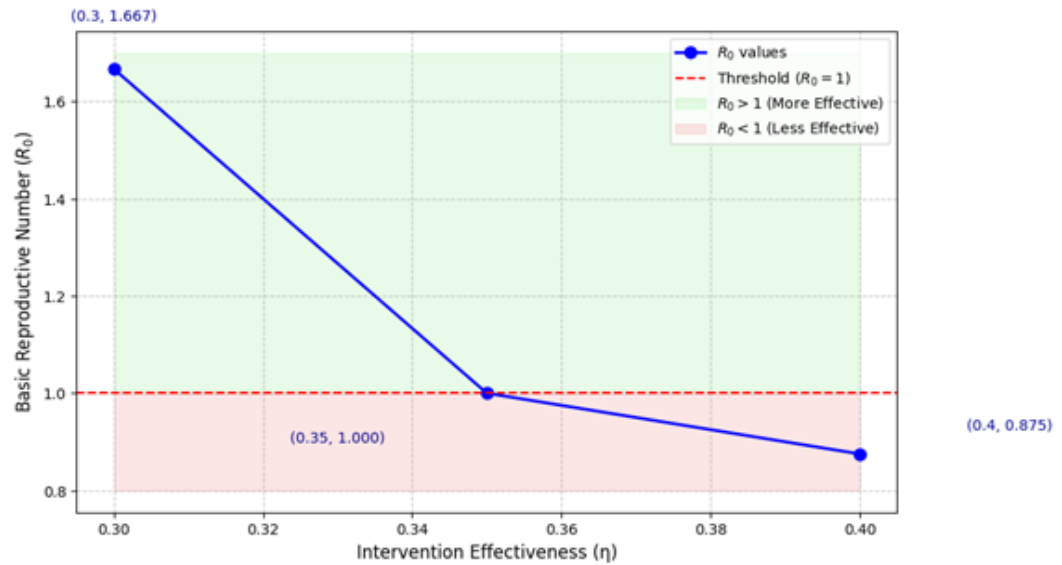


Figure 3: Threshold relationship between intervention Effectiveness vs basic reproductive number

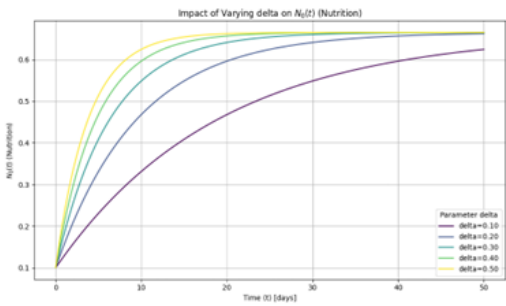
Figure 3 illustrates the following observations:

- **Above the Threshold ($R_0 > 1$):** When η exceeds 0.35, R_0 surpasses 1. This indicates that the intervention is beneficial in enhancing the child’s nutritional status, facilitating the transition from malnourished to better-nourished conditions. A greater value of R_0 in this context indicates a more significant influence on attaining nutritional objectives.
- **At the Threshold ($R_0 = 1$):** At $\eta = 0.35$, R_0 equals 1, signifying a critical threshold where the intervention’s efficacy just sustains the current nutritional status. The intervention has a neutral impact, and there is no improvement or decline.
- **Below the Threshold ($R_0 < 1$):** When η is below 0.35, R_0 falls below 1, indicating that the intervention is ineffective or inadequate for promoting nutritional enhancement.

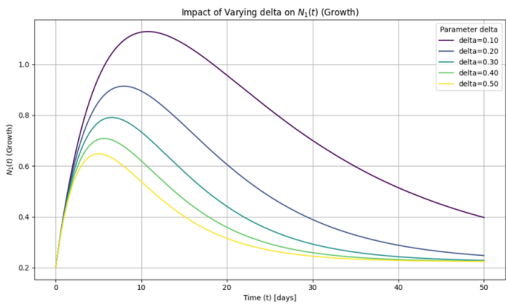
This graph underscores that while the Mid-day Meal program can improve children’s nutritional status, additional interventions may be needed to achieve the well-nourished state.

Figures 4.1–4.3 illustrate the following dynamics:

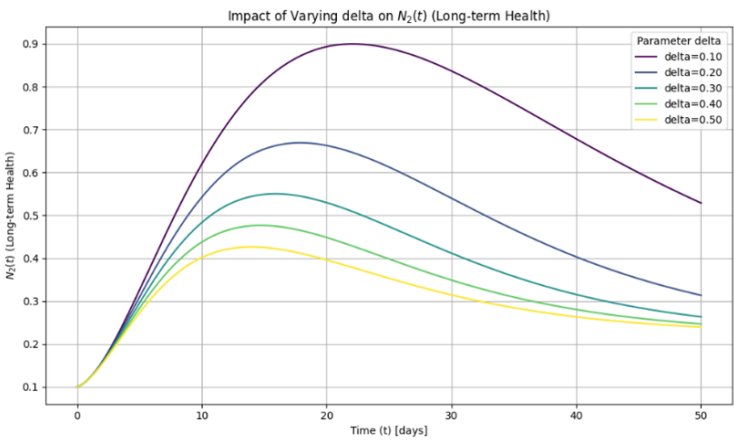
- Figure 4.1 shows that higher values of δ accelerate nutritional recovery, while lower values of δ delay recovery, simulating scenarios with limited resources or inefficient systems.
- Figure 4.2 represents that growth initially increases for all δ values, peaks at different levels, and then declines over time. This indicates the need to balance resource replenishment rates to sustain growth over time without premature depletion.
- Figure 4.3 shows that rapid resource replenishment boosts initial health recovery but may cause earlier depletion of reserves, reducing long-term sustainability.



(a) Figure 4.1

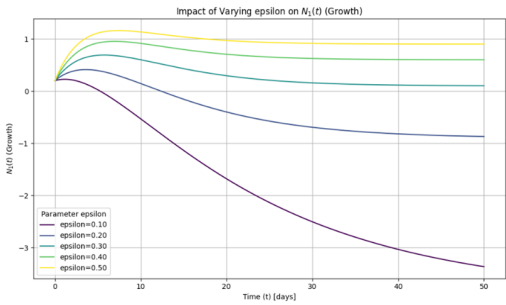


(b) Figure 4.2

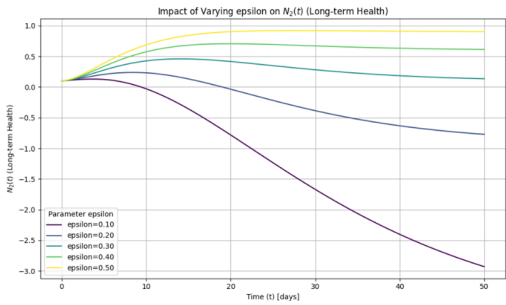


(c) Figure 4.3

Figure 4: Impact of δ on various nutritional states



(a) Figure 5.1



(b) Figure 5.2

Figure 5: Impact of ϵ on various nutritional states

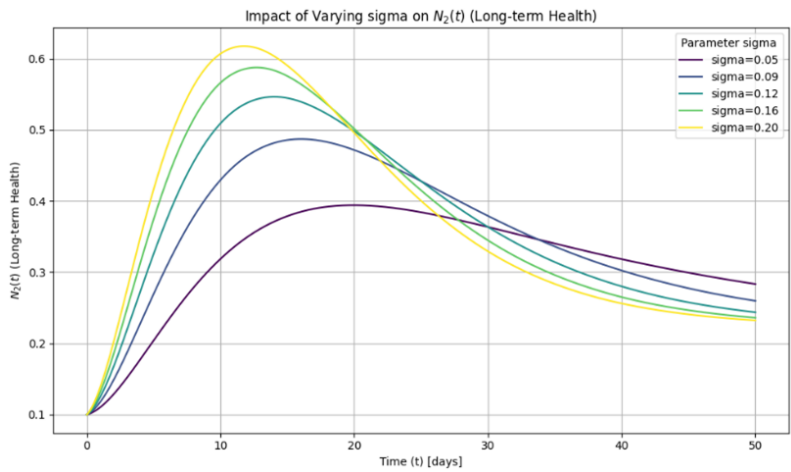
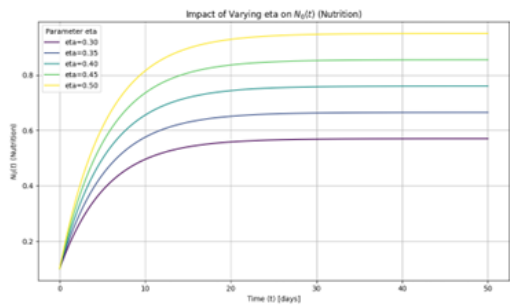
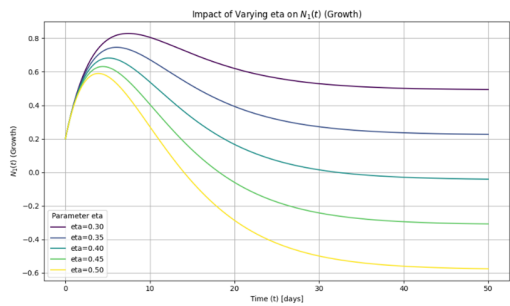


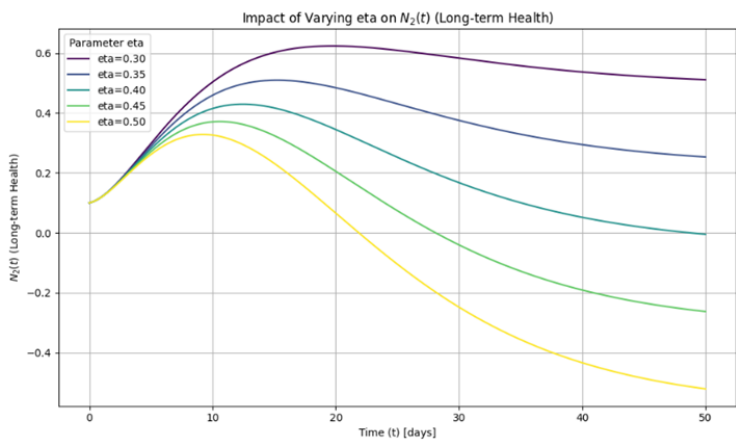
Figure 6: Impact of σ on $N_2(t)$



(a) Figure 7.1



(b) Figure 7.2



(c) Figure 7.3

Figure 7: Impact of η on various nutritional states

Thus, these insights emphasize the need for balanced interventions, where δ should be optimized to achieve both immediate recovery and sustained well-being.

Figures 5.1–5.2 are interpreted as follows:

- Figure 5.1 shows that higher values of ϵ lead to positive growth, stabilizing the nutritional state at higher levels, while lower values of ϵ show a significant decline in $N_1(t)$, indicating poorer growth outcomes. This demonstrates the importance of ϵ in driving improvements in short-term growth.
- Figure 5.2 illustrates that interventions play a critical role in stabilizing and improving long-term health states.

These findings highlight the importance of effective nutritional programs to improve both immediate and sustained health in children.

Figure 6 shows that for all values of σ , $N_2(t)$ starts at a low level and exhibits an initial upward trend, indicating improvement in long-term health. The function reaches a peak before starting to decline, showing that higher values of σ accelerate health benefits but reach saturation over time. Thus, higher values of σ lead to faster and greater initial health improvements but do not sustain these levels indefinitely.

Figures 7.1–7.3 are described as follows:

- Figure 7.1 shows that the nutritional state improves rapidly in the initial phase and stabilizes as time progresses.
- Figure 7.2 illustrates a decline in growth after the peak, indicating diminishing returns or limitations in available resources for sustained growth over time.
- Figure 7.3 shows that the health state $N_2(t)$ experiences a slight decline after its peak and stabilizes at different levels depending on η . This parameter influences nutrition, growth, and long-term health.

Limitations of the Model

Although the proposed within-host model setup is novel in capturing the nutritional status of a child, it is subject to certain limitations that can be addressed in future work. First, the model captures the nutritional dynamics of a single child and hence fails to capture interpersonal variability. Second, the model primarily focuses on caloric-based recovery and does not include other factors such as micronutrient deficiencies, disease interaction, or immune response. Third, the model assumes regular consumption of the intervention with uniform nutritional content. Despite these limitations, the proposed model provides qualitative insights into the role of sustained nutritional intervention in the reduction of malnourishment in India.

5 Conclusion

The analysis of the model, focused on the nutritional transitions of a child, shows that the Mid-day Meal scheme effectively helps children move from a malnourished state to a moderately nourished state, but does not significantly transition the child to a well-nourished state. The model indicates that the Mid-day Meal contributes approximately 30%–40% improvement in the child's nutritional status, helping them transition from a malnourished to a moderately nourished state. This finding aligns with the realistic contribution of a single meal per day, which may only provide partial nutritional requirements, particularly for children receiving no other supplements. The model shows that at an effectiveness level of $\eta = 0.35$ and above, the system remains stable with $R_0 > 1$. This suggests that at least 30%–35% effectiveness is required for the intervention to have a substantial impact in transitioning children out of the malnourished category. For values of $\eta > 0.4$, while the system becomes more effective, it remains insufficient to elevate the nutritional status to the well-nourished state, implying that the Mid-day Meal alone cannot ensure full nutritional development. The sensitivity analysis of η shows that beyond 40%

effectiveness, the intervention reaches a point of diminishing returns. This indicates that enhancing the effectiveness of the Mid-day Meal program alone is not sufficient to ensure the long-term stability required for complete nourishment.

Thus, these results are consistent with real-life observations that Mid-day Meals provide critical but limited nutrition, especially in resource-constrained settings where children may rely on a single meal per day. The model reinforces that while the Mid-day Meal helps children reach moderate nourishment, further interventions or complementary nutritional support are necessary to ensure full nourishment.

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