

FIXED COST TRANSPORTATION PROBLEM UNDER UNCERTAIN CONDITION USING REVISED VOGEL'S APPROXIMATION METHOD

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Abstract An extension of the classical transportation problem is the fixed- cost transportation problem, where a variable expense that varies according to the amount shipped is incurred combined with a fixed cost that is incurred regardless of the amount transported. The objective function becomes a step function with the addition of fixed-costs to the variable expenses. Therefore, advanced analytical or computer software is typically used to tackle fixed- cost problems. This research departs from that strategy. For the resolution of minor fixed- cost problems, it offers a heuristic technique. This manuscript emphasizes the integration of Balinski's relaxation approach, Hungarian matrix reduction, and the Revised Vogel's Approximation Method (RVAM) to obtain an efficient initial feasible solution for the Fixed Cost Transportation Problem under uncertain conditions. The proposed method provides a computationally efficient heuristic approach capable of producing near-optimal solutions for small-scale problems with reduced computational stages compared to existing approaches. To demonstrate the uses of the suggested approach, we provide numerical examples.

1 Introduction

A very important application of linear programming problem is transportation problems and in real-world, the transportation cost come along with some fixed cost and hence call for modeling as fixed-cost shipping costs. For example, freight rates that include both fixed costs (like property taxes or permit fees) and variable costs (like personnel and equipment expense) are frequently used by transportation firms such as railroads and truck companies. Numerous traditional applications of the issue have been recorded in the literature on scheduling and facility location. The two most frequent ones are judgments about where to locate warehouses or plants, where there are charges associated with opening the facility, and second are shipping issues, where there are set expenses associated with moving items between supply and demand points.

Unlike the classical transportation problem, which can often be solved manually even for relatively large instances, the fixed-cost transportation problem is considerably more difficult to handle by hand due to the presence of fixed charges. One of the most accurate approaches for solving such problems involves ranking extreme points, which requires an extensive analysis of possible load distributions. The size of the search domain largely depends on the initial distribution chosen. Murty [10] proposed a method based on the optimal solution of the variable cost component to determine the initial distribution; however, he noted that this starting solution may not perform well when the fixed costs are relatively large. To address this limitation, Sadagopan and Ravindran [21] introduced a dual-based approach in which the fixed-cost component of the problem is first analyzed and then used to guide the selection of the initial solution, replacing Murty's approach whenever it yields better results. Although their method represents a significant improvement over Murty's procedure, it still requires the evaluation of a large number of possible distributions.

A further technique to solving the fixed-cost shipping expense issue is to use a mixed integer programming formulation. In an attempt to arrive at a precise solution, Gray [20] broke down

the issue into a master integer program and several shipping sub-programs. In the meantime, Sandrock [11] presented a simple technique designed for minor fixed-cost issues, while Palekar et al. [22] and Steinberg [3] sought precise solutions using the branch-and-bound approaches. Solving a fixed-cost shipping expense by hand is not feasible with the accurate branch and bound approach. A 4×4 issue involves analysis of many hundred distributions in order to find a solution. There are numerous situations (such in the field of solid-waste management, Walker [23]) where the problem's dimensions are quite small, making a solution using a simple heuristic technique worthwhile. An additional benefit of solving such issues by hand is that the analyst gains a comprehensive understanding of how optimality is attained. An algorithm to identify a full tour at the lowest possible cost that does not go over the total cost and duration of the trip has been explained by Mondal and Srivastava [17, 18]. Keshavarz et.al [5] describes a method for solving a fixed-cost transportation problem with fuzzy shipping costs, using Bellman-Zadeh's max-min method to convert it into a crisp nonlinear mixed-integer programming problem. After reformulating the problem, it can then be solved with the conventional techniques. Singh and Singh [7] presents a hybrid Particle Swarm Optimization (PSO) algorithm which suggests avoiding reducing the objective function, demonstrates superior efficiency and effectiveness compared to existing heuristic methods. Kumar et.al [1] introduced approach, which shows improved accuracy by presenting a new algorithm for solving the bi-criteria fixed-cost transportation problem using a linear ranking function in a fuzzy environment, directly handling fuzzy values without converting to crisp values. Muralidaran and Venkateswarlu [2] presents a linear approximation method for fixed-cost transportation problems, which efficiently delivers optimal solutions for large-scale cases and is more practical than traditional methods. The review article of Kacher and Singh [25] provides a thorough summary of transportation problems and their extensions, using a broad range of scholarly sources. It aims to help future researchers by offering a framework to classify these problems and pinpoint optimization criteria.

This study proposes a method for solving a fixed-cost shipping expense in a relatively straightforward manner. It is attainable for managers who lack a solid mathematics background. While the suggested approach requires more time to solve than standard TP algorithms, it offers a solid basis for manual issue resolution in smaller scenarios. Additionally, an optimality test is introduced. Heuristic approaches created by Cooper and Drebes [13], Drenzier [4], Steinberg [3], Cooper [14], Walker [23], and Sun et al. [19] are some of the aspects of the method that are used. These techniques employ simplex-like iterations in an attempt to reach the optimal. In an attempt to find a better solution, they experiment with substituting other combinations of variables, as there is no guarantee that the achieved minimal solution is global and not local. The original solution is not given much weight by any of these approaches. Because it is complex to investigate multiple local minima, prior efforts to solve this issue have focused mostly on estimating the answer. Balinski [16] made one of the first attempts, noting that the relaxed version of the fixed-cost shipping expense issue had an optimal answer that did not take into account the integer restriction on b_{ij} . Other heuristic techniques are based on boundary point ranking [10, 21], and include those put out by Diaby [15], Cooper and Drebes [13], and Kuhn and Baumol [9]. A simple approach is provided by Sandrock [11] for the resolution of small fixed-cost expense issues in which the supply points rather than the routes under consideration in this paper are the source of the fixed expense.

This study proposes a simple and efficient heuristic approach to solve the fixed-cost transportation problem under uncertain conditions. The proposed method is implemented in two stages. In the first stage, a strong initial feasible solution is obtained by integrating Balinski's relaxation technique with the Revised Vogel's Approximation Method (RVAM) and Hungarian matrix reduction. The suggested approach attempts to obtain the optimal solution during the initial iteration itself. In the second stage, simplex-like iterations are applied either to achieve final convergence or to verify the optimal conditions of the obtained solution. For manual problem solving, this verification process is carried out using a simplified version of Murty's "one step pivot method" [10]. It is worth noting that the second stage may be omitted if the decision maker is satisfied with the solution obtained in the initial stage. Moreover, the first stage of the proposed method can also be applied independently to solve large fixed-cost transportation problems, particularly in situations where other heuristic methods become computationally expensive due to memory or time limitations.

1.1 Research Gap

In the existing methods, there is high computational complexity, difficulty in manual implementation, and limited applicability under uncertain environments.

1.2 Motivation and Contribution

The motivation for this research is to develop a simple and efficient heuristic method that can be implemented manually for small-scale problems. The main contribution of the study is the development of a hybrid approach combining Balinski's relaxation, Hungarian reduction, and RVAM for solving fixed cost transportation problems with uncertain parameters.

2 Preliminaries

In this sections, definition of fuzzy numbers and their membership functions has been defined (Zadeh [12] and Zimmermann [26]).

2.1 Fuzzy Number

An uncertain set $\tilde{P}$, defined on the set of real numbers R is said to be an uncertain number if its membership function $\mu_{\tilde{P}}(a) : R \rightarrow \{0,1\}$ achieve certain requirements as:

- $\exists a \in R$ such that $\mu_{\tilde{P}}(a) = 1$ that means $\tilde{P}$ is normal.
- $\tilde{P}$ is convex. It means that for every $a_1, a_2 \in R$, $\mu_{\tilde{P}}(\lambda a_1 + (1 - \lambda)a_2) \geq \min\{\mu_{\tilde{P}}(a_1), \mu_{\tilde{P}}(a_2)\}$, $\lambda \in [0, 1]$.
- $\sup(\tilde{P})$ is bounded in R .
- $\mu_{\tilde{P}}(a)$ is upper semi- continuous.

2.2 Triangular Fuzzy number

A fuzzy set $\tilde{P} = (r, s, t)$ with membership function $\mu_{\tilde{P}}(a); a \in R$, holds the following properties, is called triangular fuzzy number,

- $\mu_{\tilde{P}}(a) : R \rightarrow \{0, 1\}$ is continuous.
- $\mu_{\tilde{P}}(a) = 0, \forall a \in (\infty, r] \cup [t, \infty)$.
- It is strictly rising on $[r, s]$ and strictly falling on $[s, t]$.
- $\mu_{\tilde{P}}(a) = 1, \forall a = s$.

2.3 Membership function of triangular fuzzy number

Triangular fuzzy numbers $\tilde{P} = (r, s, t)$, where $r \leq s \leq t$ have a membership function $\mu_{\tilde{P}}(a)$ that can be represented as follows,

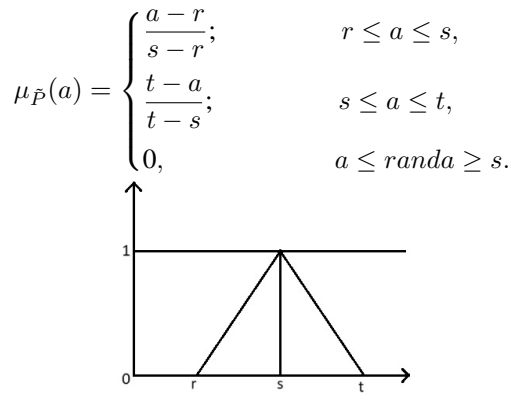


Figure 1: The geometric depiction of a triangular fuzzy number $\tilde{P} = (r, s, t)$

2.4 Defuzzification of triangular fuzzy number

Several approaches have been proposed in the literature to defuzzify the triangular fuzzy number. An efficient approach to defuzzify is by the use of a ranking function based on their graded means [10]. That is, for every $\tilde{P} = (r, s, t) \in \tilde{F}(R)$, set of fuzzy numbers, the ranking function $R : \tilde{F}(R) \rightarrow R$, by graded mean is defined as,

$$\text{Rank of } \tilde{P} = \mathfrak{R}(\tilde{P}) = \frac{r+4s+t}{6}$$

2.5 Comparison of triangular fuzzy numbers

Let $\tilde{P} = (r_1, s_1, t_1)$ and $\tilde{Q} = (r_2, s_2, t_2)$ be two fuzzy triangular numbers, then

- $\tilde{P} \preceq \tilde{Q}$ if and only if $\mathfrak{R}(\tilde{P}) \leq \mathfrak{R}(\tilde{Q})$.
- $\tilde{P} \prec \tilde{Q}$ if and only if $\mathfrak{R}(\tilde{P}) < \mathfrak{R}(\tilde{Q})$.
- $\tilde{P} \approx \tilde{Q}$ if and only if $\mathfrak{R}(\tilde{P}) = \mathfrak{R}(\tilde{Q})$

2.6 Algebraic operations on triangular fuzzy numbers

Let $\tilde{P} = (r_1, s_1, t_1)$ and $\tilde{Q} = (r_2, s_2, t_2)$ be two fuzzy triangular numbers. The algebraic operations on triangular fuzzy numbers $\tilde{P}$ and $\tilde{Q}$ are given under as,

- **Addition:** $\tilde{P} \oplus \tilde{Q} = (r_1 + r_2, s_1 + s_2, t_1 + t_2)$.
- **Subtraction:** $\tilde{P} \ominus \tilde{Q} = (r_1 - t_2, s_1 - s_2, t_1 - r_2)$.
- **Multiplication:** $\tilde{P} \otimes \tilde{Q} = (r, s, t)$,
where $r = \min\{r_1r_2, r_1t_2, t_1r_2, t_1t_2\}$, $s = \{s_1s_2\}$ and $r = \max\{r_1r_2, r_1t_2, t_1r_2, t_1t_2\}$.
- **Scalar Multiplication:**

$$\alpha \times \tilde{P} = \begin{cases} (\alpha r_1, \alpha s_1, \alpha t_1); & \alpha \geq 0, \\ (\alpha t_1, \alpha s_1, \alpha r_1); & \alpha < 0, . \end{cases}$$

3 Fixed-charge transportation problem with uncertain parameters

The fixed- cost transportation problem entails distributing goods from m points of origin to n points of destination. Each supplier at origin can deliver to any customer at destination at a unit shipping expense c_{ij} and a fixed opening expense r_{ij} for the route. Suppliers at origin have $\tilde{u}_i$ units of uncertain capacity, while customers at destination have uncertain demand of $\tilde{v}_j$ units. The goal is to optimize route selection and shipment sizes to minimize total expense while meeting requirement with in capacity constraints.

Even though it is similar to a standard shipping expenditures problem, the discontinuity in the function of objective caused by fixed-costs makes solving the fixed- cost shipping expense problem much more difficult. The feasible region of the fixed- cost shipping expense problem forms a convex bounded set with objective function which is concave, as shown by Hirsch and Dantzig [24]. An end point of the restriction set yields an optimal answer, and every boundary point of the feasible region functions as a local minimum in a non-degenerate problem with all positive fixed-costs. This characteristic will be useful when determining the ideal circumstances for a derived solution.

$$\begin{aligned} \text{Minimize } Z &= \sum_{i=1}^m \sum_{j=1}^n (c_{ij}a_{ij} + r_{ij}b_{ij}) \\ \sum_{j=1}^n a_{ij} &= u_i, \forall i = 1, 2, \dots, m \\ \sum_{i=1}^m a_{ij} &= v_j, \forall j = 1, 2, \dots, n \\ a_{ij} &\geq 0, \forall i, j \\ \text{If } a_{ij} &= 0 \text{ then } b_{ij} = 0 \\ \text{If } a_{ij} &> 0 \text{ then } b_{ij} = 1 \end{aligned}$$

We assume that $\sum_{i=1}^m u_i = \sum_{j=1}^n v_j$ and $u_i, v_j, c_{ij}, r_{ij} \geq 0$.

4 Methodology

There are two components to the suggested algorithm. The method finds a strong initial feasible solution in the first part and provides procedures for the optimal solution's search and verification in the second half. Although it can result in a large increase in work, the process described in the second section can be utilized independently with other crude approximation methods. Furthermore, a minimum for any concave function with transportation-type restrictions can be found using the same process.

4.1 Part I — initial feasible solution

Balinski's approach and Vogel's Approximation Method (VAM) are used to find a robust initial solution for our method. Additionally, we'll "Hungarize" the pre-revised Vogel's approximation matrix, which is believed to enhance this process. Regarding the revised Vogel's approximation technique and classical transportation problem, the reader might consult Hillier and Lieberman [6] and Wagner [8]. By loosening the integer restriction on b_{ij} , Balinski [16] found an optimal answer to the relaxed form of fixed- cost shipping expense problem, which has the following property:

$$b_{ij} = \frac{a_{ij}}{h_{ij}}$$

Where $h_{ij} = \min[u_i, v_j]$

Thus, the relaxed shipping expense problem of a fixed- cost shipping expense problem becomes a standard shipping expense problem with unit shipping expenses as $C_{ij} = c_{ij} + \frac{r_{ij}}{h_{ij}}$. We designate this problem as R, and we derive an initial workable solution using this RTP formulation of the fixed- cost shipping expense. It is simple to convert the ideal solution A'_{ij} to the RTP problem R into a workable solution of $[A'_{ij}, b'_{ij}]$ of R by doing the following:

$$\begin{aligned} b'_{ij} &= 0 \text{ if } A'_{ij} = 0 \\ b'_{ij} &= 1 \text{ if } A'_{ij} > 0 \end{aligned}$$

Balinski demonstrates that an ideal amount of relaxed shipping expense problem offers a lower constraint for the ideal value $Z^*(R)$ of fixed- cost shipping expense problem and

$$\sum_{i=1}^m \sum_{j=1}^n C_{ij} A'_{ij} \leq Z^*(R) \leq \sum_{i=1}^m \sum_{j=1}^n (c_{ij} A'_{ij} + r_{ij} b'_{ij})$$

Remark 1 The lower bound in above equation from the best solution of the RTP can be rounded up to the nearest interval because the fixed- cost shipping expense is a discrete function.

An initial assessment technique

Prior to examining into Balinski's [16] relaxed shipping expense problem formulation of an fixed- expense shipping expense problem, we present an initial assessment technique aimed at identifying all fixed costs r_{st}^* that will consistently occur in each likely circulation (a_{ij}, b_{ij}) for a given fixed- cost shipping expense problem. This occurrence is particularly noticeable in most of the small-scale problems.

Reformulating the objective function given in the problem in Section 3 as,

$$Z = r_{ij}^* + \min \sum_{i=1}^m \sum_{j=1}^n (c_{ij} a_{ij} + (r_{ij} b_{ij} | i \neq r, j \neq s))$$

Where $r_{ij} = r_{st}^*$ if $\sum_{i \neq s} u_i < v_t$ or $\sum_{j \neq t} v_j < u_s$.

The value of r_{st} is set to 0 to solve the fixed- cost shipping expense once a fixed-cost, r_{st}^* , has been identified. This fixed cost is then added to the final solution's value. One well-known optimization technique is hungarization. Row and column reduction, often known as matrix reduction, is a step in the process. The process of hungarization involves deducting each row's smallest element from each row's total elements. The smallest element in each column is then removed from each element in that column using the row-reduced matrix. Since the identical solution will be ideal, the problem this reduced matrix presents is equivalent to the original problem.

4.2 Proposed method for the initial solution

Step 1: Initial assessment of the fixed cost shipping cost to identify and separate the fixed cost R_{st}^* , if any.

Step 2: Formulate Balinski's RTP matrix with $C_{ij} = c_{ij} + \frac{r_{ij}}{h_{ij}}$.

Step 3: Find row reduction of RTP matrix.

Step 4: Find column reduction of RTP matrix, we get reduced (Hungarize) matrix of the RTP matrix.

Step 5: Apply Revised Vogel's Approximation Method as explained under as,

(a) Determine the minimum and next minimum cost for each row and each column and calculate the difference between them. This is called a penalty. Find out penalties (p_i) of rows and columns; $p_i = ||c_{ih} - c_{ik}||$ and $p_j = ||c_{hi} - c_{kj}||$.

(b) If the lowest cost appears twice or more in the row or column, please select the same expense as the lowest cost and next lowest cost, and the penalty is zero i.e. $p_i = ||c_{ih} - c_{ik}|| = 0$ and $p_j = ||c_{hi} - c_{kj}|| = 0$.

(c) Choose the highest penalty (p_i, p_j) and allocate the maximum possible quantity the cell in i^{th} row and j^{th} column i.e. minimum of the quantity at s_i source and d_j destination.

Remarks 2: (i) If highest penalty appears in certain rows or columns are tied, select the row or column having lowest cost.

(ii) If the lowest cost appears in two or more places, select the cell in which we can allocate maximum quantity.

(iii) If maximum allocation is also tied at two or more than two places, select the cell with maximum remainder of supply and demand.

Step 6: Reduce the cost matrix by deleting the i^{th} row and j^{th} column.

Step 7: Repeat the steps from Step - 3 to Step - 6 till all the allocations has been made.

5 Part II—improvement and modification to the optimum

The concavity of the objective function serves as the foundation for the second portion of the suggested strategy. Consider the following sequence of extreme points (sometimes called peaks) of the fixed-charge problem: $a^{(1)}, a^{(2)}, \dots, a^{(k)}$. These points are ranked in decreasing order of value of the objective function $Z(a)$. The procedure recursively explores all nearby peaks of the best solution found up, beginning with the initial solution $a^{(1)}$. If $Z(a^{(K)}) < Z(a)$ for all adjacent peaks of $a^{(k)}$, then solution $a^{(k)}$ is considered as the final solution of the fixed cost shipping problem.

6 Illustrative numerical examples

The proposed two-part algorithm constitutes the first “down to earth” heuristic method for finding a good solution for a fixed cost shipping problem. As demonstrated by the following numerical examples, the minimum cost obtained by the proposed method will most likely be the global solution or optimal solution for small problems. To illustrate steps of the algorithm, two problems ([16],[21]) has been solved given in Table 1 and Table 6.

Example 1: Part I - Finding the initial solution

An optimal solution to this RTP is $A_{12} = 10, A_{22} = 20, A_{23} = 10, A_{31} = 20, A_{32} = 20$ and $A_{43} = 20$ to ship 100 units at a cost $\sum \sum C_{ij}A_{ij} = 321.70$. Since $\sum \sum (c_{ij}A_{ij} + r_{ij}b_{ij}) = 360$, hence $321.70 \leq Z \leq 360$. Note that the minimum interval between the fixed expense value r_{ij} is equal to 10 and the minimum quantity is also a multiple of 10. Therefore, using Remark 1, we can round up the lower bound to the optimal value Z of the fixed- cost shipping problem to 330.

By using Step 2 of the proposed method, Balinski's RTP matrix is given in Table 2,

By using Step 3 and Step 4 of the proposed method, Table 3 and Table 4 has been obtained.

Table 1: Cost matrix

	D_1	D_2	D_3	Capacity
W_1	(10,2)	(30,3)	(20,4)	(5,10,15)
W_2	(10,3)	(30,2)	(20,1)	(20,30,40)
W_3	(10,1)	(30,4)	(20,3)	(20,40,60)
W_4	(10,4)	(30,5)	(20,2)	(10,20,30)
Requirement	(10,20,30)	(30,50,70)	(15,30,45)	

Table 2: Cost matrix

	D_1	D_2	D_3	Capacity
W_1	3	6	6	(5,10,15)
W_2	3.5	3	1.67	(20,30,40)
W_3	1.5	4.75	3.67	(20,40,60)
W_4	4.5	6.5	3	(10,20,30)
Requirement	(10,20,30)	(30,50,70)	(15,30,45)	

Table 3: Row reduction matrix

	D_1	D_2	D_3	Capacity
W_1	0	3	3	10
W_2	1.83	1.33	0	30
W_3	0	3.25	2.17	40
W_4	1.5	3.5	0	20
Requirement	(10,20,30)	(30,50,70)	(15,30,45)	

Table 4: Column reduction matrix

	D_1	D_2	D_3	Capacity
W_1	0	1.67	3	(5,10,15)
W_2	1.83	0	0	(20,30,40)
W_3	0	1.92	2.17	(20,40,60)
W_4	1.5	2.17	0	(10,20,30)
Requirement	(10,20,30)	(30,50,70)	(15,30,45)	

After applying Step 5 to Step 7 of the proposed method, the allocation has been shown in Table 5.

Total cost is equal to $= (4 \times (5, 10, 15) + 2 \times (20, 30, 40) + 1 \times (10, 20, 30) + 4 \times (10, 20, 30) + 2 \times (10, 20, 30)) + (20 \times 1 + 30 \times 1 + 10 \times 1 + 30 \times 1 + 20 \times 1) = 240 + 110 = 350$.

We obtain an initial solution $a^{(1)}$ of the fixed - cost shipping problem of Table 1 as,

$A_{13} = (5, 10, 15), A_{22} = (20, 30, 40), A_{31} = (10, 20, 30), A_{32} = (-10, 20, 50), A_{43} = (10, 20, 30)$ to ship 100 units at an cost of $Z(a^{(1)}) = 350$. Therefore, the solution $330 \leq Z^* \leq 350$.

Part II - Improvement and modification to the optimum

The solution $a^{(1)}$ has been obtained in Part I is degenerate with 5 allocated cells. Now we will find the possibilities in the adjacent cells to check that whether we can improve the initial

Table 5: Table with all allocations

	D_1	D_2	D_3	Capacity
W_1	(10,2)	(30,3)	(20,4) (5,10,15)	(5,10,15)
W_2	(10,3)	(30,2) (20,30,40)	(20,1)	(20,30,40)
W_3	(10,1) (10,20,30)	(30,4) (-10,20,50)	(20,3)	(20,40,60)
W_4	(10,4)	(30,5)	(20,2) (10,20,30)	(10,20,30)
Requirement	(10,20,30)	(30,50,70)	(15,30,45)	

solution.

The obtained possibilities are,

Initial solution (I_1) is $A_{12} = 10, A_{22} = 20, A_{23} = 10, A_{31} = 20, A_{32} = 20, A_{43} = 20$ and $Z(I_1) = 360$.

Initial solution (I_2) is $A_{12} = 10, A_{22} = 30, A_{31} = 20, A_{32} = 10, A_{33} = 10, A_{43} = 20$ and $Z(I_2) = 360$.

Initial solution (I_3) is $A_{11} = 10, A_{22} = 30, A_{31} = 10, A_{32} = 20, A_{33} = 10, A_{43} = 20$ and $Z(I_3) = 360$.

Initial solution (I_4) is $A_{13} = 10, A_{21} = 20, A_{22} = 10, A_{32} = 40, A_{43} = 20$ and $Z(I_4) = 430$.

Initial solution (I_5) is $A_{13} = 10, A_{22} = 20, A_{32} = 20, A_{33} = 20, A_{41} = 20$ and $Z(I_5) = 430$.

Initial solution (I_6) is $A_{13} = 10, A_{22} = 30, A_{31} = 20, A_{33} = 20, A_{42} = 20$ and $Z(I_6) = 390$.

Initial solution (I_7) is $A_{13} = 10, A_{22} = 10, A_{23} = 20, A_{31} = 40, A_{32} = 20, A_{42} = 20$ and $Z(I_4) = 420$.

Since the objective values at the possible initial solutions are higher than the value at the initial solution, we stop the process and consider solution $a^{(1)}$ as the optimum solution as considered by Murty [10] also.

Example 2: (Sadagopan and Ravindran [21], Table 6)

Table 6: Cost matrix - Example 2

	D_1	D_2	D_3	D_4	Capacity
W_1	(900,760)	(1000,71)	(700,283)	(800,594)	(5,10,15)
W_2	(900,594)	(300,64)	(700,170)	(600,564)	(20,30,40)
W_3	(600,594)	(200,69)	(400,79)	(0,202)	(20,40,60)
Requirement	(20,25,30)	(16,20,24)	(14,15,16)	(7,10,13)	

Table 7: Allocated table - Example 2

	D_1	D_2	D_3	D_4	Capacity
W_1	(900,760) (20,25,30)	(1000,71) (16,20,24)	(700,283)	(800,594) (0,5,10)	(5,10,15)
W_2	(900,594)	(300,64)	(700,170) (14,15,16)	(600,564)	(20,30,40)
W_3	(600,594)	(200,69)	(400,79)	(0,202) (3,7,5)	(20,40,60)
Requirement	(20,25,30)	(16,20,24)	(14,15,16)	(7,10,13)	

The obtained fixed cost shipping value from Table 7 is 30350 and hence $29950 \leq Z^* \leq 30350$.

Table 8: Comparison of computational efficiency

Fixed to stage-count ratio for optimal ranking of variable expenses	Number of stages of ranking		
	Murty [10]	Sadagopan and Ravindran [21]	Proposed approach
0.126	21	12	1
0.219	24	14	1
0.312	26	14	1
0.404	26	14	1
0.590	27	16	1

7 Conclusion

We have proposed a simple and effective method to address the fixed-cost shipping problems. In Part I, the proposed approach finds the best initial solution; in Part II, iterations are used to refine and confirm the optimality of the solution. We used the same comparison model as Sadagopan and Ravindran [21] to examine the computing efficiency of the suggested approach. We increased all of the fixed costs by 500, 1000, 1500, and 2500 in order to solve the numerical Example 2. There is value in solving many relatively small-scale problems by a straightforward "hands on" method. This includes the common solid-waste management challenge, where the best possible solutions are looked for in terms of tip sites, transfer stations, and collection depot locations and levels of activity. A further benefit of manually solving such fixed-charge problems is that the analyst gains a thorough understanding of how optimality was attained. The creation of an effective code to apply this strategy for computational and comparative studies should be the focus of future research.

The proposed method demonstrates improved computational efficiency by significantly reducing the number of ranking stages required to reach a near-optimal solution. Unlike many existing approaches that require extensive iterative computations, the proposed algorithm obtains a strong initial solution in the first stage and verifies optimality with minimal additional steps. A comparative table has been included to highlight the reduced computational effort relative to previously reported methods. The proposed method is applicable to small-scale uncertain fixed cost transportation problems, and its computational advantages. Additionally, future scope is to extend the approach to larger-scale problems and implementing the algorithm in computational software for broader practical applications.

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