

An Excerpting of Corona Covering number of Cartesian product of path graphs

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Abstract Recently the authors initiated the concept of corona covering number of graphs. A vertex cover set $S \subseteq V(G)$ is a corona cover set if every vertex $v \in S$ such that $d_{\langle S \rangle}(v) = 1$ or there exist a vertex $u \in S$ with $d_{\langle S \rangle}(u) = 1$ and $uv \in E$. The least cardinality of a corona cover set is the corona covering number of a graph and it is expressed as τ_C . In this paper, we enhance the investigation of this parameter for cartesian product of paths.

1 Introduction

We consider simple, finite and undirected graphs, vertex and edge set of G is expressed as V and E respectively. Motivated from vertex covering number [1] and corona domination number [2] the authors have introduced a concept of corona covering number [3]. A vertex cover set $S \subseteq V(G)$ is a corona cover set if every vertex $v \in S$ such that $d_{\langle S \rangle}(v) = 1$ or there exist a vertex $u \in S$ with $d_{\langle S \rangle}(u) = 1$ and $uv \in E$. The least cardinality of a corona cover set is the corona covering number of a graph and it is expressed as τ_C . The corona covering number for some standard graphs and some special types of graphs has been discussed in [3] and The parameter has several real-life applications, including military zone allocation and the allocation of medical facilities, as discussed in [4] and [5]. Let $P_n = (u_1, u_2, \dots, u_n)$ and $P_m = (v_1, v_2, \dots, v_m)$ be the paths. Then the vertex set of cartesian product of paths P_n and P_m , $V(P_n \times P_m) = V(P_n) \times V(P_m)$ with $|V(P_n \times P_m)| = mn$ and any two vertices in $P_n \times P_m$ are adjacent say (u_p, v_q) is adjacent to (u_r, v_s) if $d(u_p, u_r) = 1$ and $d(v_q, v_s) = 0$ or $d(u_p, u_r) = 0$ and $d(v_q, v_s) = 1$ [7]. The corona covering number for cartesian product of two paths will be discussed in the section 2. In all graphical illustrations considered in this work, vertices colored white correspond to the corona cover set of the graph.

2 Corona Covering number of graphs

Let $P_n \times P_m$ be the cartesian product of paths P_n and P_m , $V(P_n \times P_m) = \{v_{i,j} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(P_n \times P_m) = \{v_{i,j}v_{i+1,j} : 1 \leq i \leq m-1, 1 \leq j \leq n\} \cup \{v_{i,j}v_{i,j+1} : 1 \leq j \leq n-1, 1 \leq i \leq m\}$. Assume $a = \lfloor \frac{m}{n+1} \rfloor$ and $b = \lceil \frac{m}{n+1} \rceil$ throughout this section.

2.1 Corona covering number of $P_n \times P_m$, n is odd

Theorem 2.1. If n is odd and $m \equiv 1$ or $3 \pmod{4}$, then $\tau_C(P_n \times P_m) = \lceil \frac{n}{4} \rceil (m - \lceil \frac{m-2}{2} \rceil) + \lfloor \frac{m-2}{4} \rfloor + \lfloor \frac{n}{2} \rfloor (m - \lceil \frac{m-2}{2} \rceil) + \lceil \frac{n-2}{4} \rceil (m - \lceil \frac{m-2}{4} \rceil)$

Proof. Assume $S = \{\{v_{i,j} : i \equiv 0$ or 2 or $3 \pmod{4}, j \equiv 1 \pmod{4}\} \cup \{v_{i,j} : i \equiv 1 \pmod{2}, j \equiv 0 \pmod{2}\} \cup \{v_{i,j} : i \equiv 0$ or 1 or $2 \pmod{4}, j \equiv 3 \pmod{4}\}, 3 \leq i \leq m, 3 \leq j \leq n\}$. Then S is a corona cover set of $P_n \times P_m$ and hence

$\tau_C(P_n \times P_m) \leq |S| = \lceil \frac{n}{4} \rceil (m - \lceil \frac{m-2}{2} \rceil + \lfloor \frac{m-2}{4} \rfloor) + \lfloor \frac{n}{2} \rfloor (m - \lceil \frac{m-2}{2} \rceil) + \lceil \frac{n-2}{4} \rceil (m - \lceil \frac{m-2}{4} \rceil)$
 Let S' be a corona cover set of $P_n \times P_m$. Suppose D is a vertex cover set of cardinality at most $N = \lceil \frac{n}{4} \rceil (m - \lceil \frac{m-2}{2} \rceil + \lfloor \frac{m-2}{4} \rfloor) + \lfloor \frac{n}{2} \rfloor (m - \lceil \frac{m-2}{2} \rceil) + \lceil \frac{n-2}{4} \rceil (m - \lceil \frac{m-2}{4} \rceil) - 1$
 then either $\langle D \rangle$ has a vertex with degree zero or there exist an edge e in $\langle V - D \rangle$, Thus we have $|S'| \geq N + 1 = \lceil \frac{n}{4} \rceil (m - \lceil \frac{m-2}{2} \rceil + \lfloor \frac{m-2}{4} \rfloor) + \lfloor \frac{n}{2} \rfloor (m - \lceil \frac{m-2}{2} \rceil) + \lceil \frac{n-2}{4} \rceil (m - \lceil \frac{m-2}{4} \rceil)$
 Hence the result follows $\square$

Example 2.2.

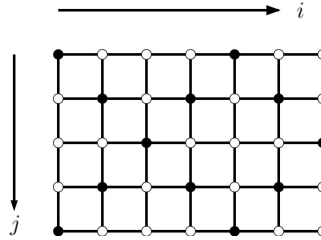


Figure 1. $P_5 \times P_7$

From Figure 1 it is clear that $\tau_C(P_5 \times P_7) = 20$

Theorem 2.3. If n is odd, $m \equiv 0$ or $2 \pmod{4}$ then $\tau_C(P_n \times P_m) = \frac{2mn}{3}$

Proof. Let $S_1 = \{v_{i,j} : i \equiv 0 \pmod{2}, j \equiv 1 \pmod{3}\} \cup \{v_{i,j} : j \equiv 2 \pmod{3}\} \cup \{v_{i,j} : i \equiv 1 \pmod{2}, j \equiv 0 \pmod{3}\}$ $S_2 = \{v_{i,j} : i \equiv 0$ or $2 \pmod{3}, j \equiv 1 \pmod{2}\} \cup \{v_{i,j} : i \equiv 1$ or $2 \pmod{3}, j \equiv 0 \pmod{2}\}$

Assume $S = \begin{cases} S_1 & \text{if } n \equiv 0 \pmod{3}, \\ S_2 & \text{if } n \not\equiv 0 \pmod{3} \text{ and } m \equiv 0 \text{ or } 6 \pmod{12}. \end{cases}$

Then S is a corona cover set of $P_n \times P_m$ and hence

$\tau_C(P_n \times P_m) \leq |S| = \frac{2mn}{3}$ if either $n \equiv 0 \pmod{3}$ or $m \equiv 0$ or $6 \pmod{12}$. Let S' be a corona cover set of $P_n \times P_m$. Suppose D is a vertex cover set of cardinality at most $N = \frac{2mn}{3} - 1$ if either $n \equiv 0 \pmod{3}$ or $m \equiv 0$ or $6 \pmod{12}$

then either $\langle D \rangle$ has a vertex with degree zero or there exist an edge e in $\langle V - D \rangle$, Thus we have $|S'| \geq N + 1 = \frac{2mn}{3}$

Hence the result follows. $\square$

Remark

Let n be odd and $m \equiv 0$ or $2 \pmod{4}$. Then the graph $P_n \times P_m$ has no corona cover set if neither $n \equiv 0 \pmod{3}$ nor $m \equiv 0$ or $6 \pmod{12}$

Example 2.4.

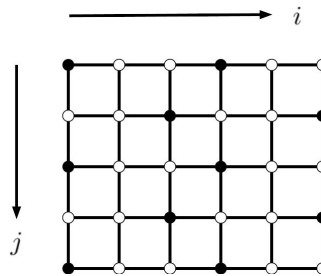


Figure 2. $P_5 \times P_6$

From Figure 2 it is clear that $\tau_C(P_5 \times P_6) = 20$

2.2 Corona covering number of $P_n \times P_m$, n is even

Throughout this section we assume S_i , $1 \leq i \leq 10$ as follows $S_1 = \{v_{i,j} : j \equiv 0 \pmod{4}, i \equiv k \pmod{2n+2}, \text{ where } k \equiv 1 \pmod{2} \text{ if } 1 \leq k \leq 3 + 4(\lceil \frac{n-(j-2)}{4} \rceil - 1) \text{ or } (2j+5) +$

$$\begin{aligned}
& 4(\lceil \frac{n-(j-2)}{4} \rceil - 1) \leq k \leq 2n+1, k \equiv 0 \text{ or } 1 \text{ or } 3 \pmod{4} \text{ if } 4 + 4(\lceil \frac{n-(j-2)}{4} \rceil - 1) \leq k \leq \\
& (2j+3) + 4(\lceil \frac{n-(j-2)}{4} \rceil - 1) \} \\
& S_2 = \{v_{i,j} : j \equiv 1 \pmod{4}, i \equiv k \pmod{2n+2}, \text{ where } k \equiv 0 \pmod{2} \text{ and } k = \\
& 2n+1 \text{ if } 0 \leq k \leq 2n \text{ and if } n = j+1, k \equiv 0 \text{ or } 2 \text{ or } 3 \pmod{4} \text{ if } 0 \leq k \leq \\
& 6 + 4(\lceil \frac{n-(j+1)}{4} \rceil - 1), k \equiv 0 \pmod{2} \text{ if } 8 + 4(\lceil \frac{n-(j+1)}{4} \rceil - 1) \leq k \leq (2j+6) + 4(\lceil \frac{n-(j+1)}{4} \rceil - \\
& 1), k \equiv 0 \text{ or } 1 \text{ or } 2 \pmod{4} \text{ if } 7 + 4(\lceil \frac{n-(j+1)}{4} \rceil - 1) \leq k \leq 2n+1 \} \\
& S_3 = \{v_{i,j} : j \equiv 2 \pmod{4}, i \equiv k \pmod{2n+2} \text{ where } k \equiv 1 \pmod{2} \text{ if } 1 \leq k \leq \\
& 1 + 4(\lceil \frac{n-(j-4)}{4} \rceil - 1) \text{ or } (2j+2) + 4(\lceil \frac{n-(j-4)}{4} \rceil - 1) \leq k \leq 2n+1, k \equiv 1 \text{ or } 2 \text{ or } 3 \pmod{4} \text{ if } 2 + \\
& 4(\lceil \frac{n-(j-4)}{4} \rceil - 1) \leq k \leq (2j+1) + 4(\lceil \frac{n-(j-4)}{4} \rceil - 1) \} \\
& S_4 = \{v_{i,j} : j \equiv 3 \pmod{4}, i \equiv k \pmod{2n+2} \text{ where } k \equiv 0 \text{ or } 1 \text{ or } 2 \pmod{4} \text{ if } 0 \leq k \leq \\
& 4 + 4(\lceil \frac{n-(j-1)}{4} \rceil - 1), k \equiv 0 \pmod{2} \text{ if } 6 + 4(\lceil \frac{n-(j-1)}{4} \rceil - 1) \leq k \leq (2j+4) + 4(\lceil \frac{n-(j-1)}{4} \rceil - \\
& 1), k \equiv 0 \text{ or } 2 \text{ or } 3 \pmod{4} \text{ if } (2j+5) + 4(\lceil \frac{n-(j-1)}{4} \rceil - 1) \leq k \leq 2n \} \\
& S_5 = \{v_{i,j} : j \equiv 0 \pmod{4}, i \equiv k \pmod{2n+2}, \text{ where } k \equiv 1 \pmod{2} \text{ if } 1 \leq k \leq \\
& 4\lceil \frac{n}{4} \rceil - (j-1) \text{ or } (j+3) + 4\lceil \frac{n}{4} \rceil \leq k \leq 2n+1, k \equiv 1 \text{ or } 2 \text{ or } 3 \pmod{4} \text{ if } 4\lceil \frac{n}{4} \rceil - (j-2) \leq \\
& k \leq (j+1) + 4\lceil \frac{n}{4} \rceil \} \\
& S_6 = \{v_{i,j} : j \equiv 1 \pmod{4}, i \equiv k \pmod{2n+2} \text{ where } k \equiv 0 \text{ or } 1 \text{ or } 2 \pmod{4} \text{ if } 0 \leq \\
& k \leq 4\lceil \frac{n}{4} \rceil - (j-1), k \equiv 0 \pmod{2} \text{ if } 4\lceil \frac{n}{4} \rceil - (j-3) \leq k \leq (j+1) + 4\lceil \frac{n}{4} \rceil \leq \\
& 2n, k \equiv 0 \text{ or } 2 \text{ or } 3 \pmod{4} \text{ if } (j+2) + 4\lceil \frac{n}{4} \rceil \leq k \leq 2n \} \\
& S_7 = \{v_{i,j} : j \equiv 2 \pmod{4}, i \equiv k \pmod{2n+2}, \text{ where } k \equiv 1 \pmod{2} \text{ if } 1 \leq k \leq \\
& 4\lceil \frac{n}{4} \rceil - (j-1) \text{ or } (j+3) + 4\lceil \frac{n}{4} \rceil \leq k \leq 2n+1, k \equiv 0 \text{ or } 1 \text{ or } 3 \pmod{4} \text{ if } 4\lceil \frac{n}{4} \rceil - (j+2) \leq \\
& k \leq (j+1) + 4\lceil \frac{n}{4} \rceil \leq 2n+1 \} \\
& S_8 = \{v_{i,j} : j \equiv 3 \pmod{4}, i \equiv k \pmod{2n+2} \text{ where } k \equiv 0 \text{ or } 2 \text{ or } 3 \pmod{4} \text{ if } 0 \leq k \leq \\
& 4\lceil \frac{n}{4} \rceil - (j-1), k \equiv 0 \pmod{2} \text{ if } 4\lceil \frac{n}{4} \rceil - (j-3) \leq k \leq (j+1) + 4\lceil \frac{n}{4} \rceil, k \equiv 0 \text{ or } 1 \text{ or } 2 \pmod{4} \text{ if } \\
& (j+2) + 4\lceil \frac{n}{4} \rceil \leq k \leq 2n+1 \} \\
& S_9 = \{\{v_{i,j} : i \equiv 0 \pmod{3}, j \equiv 1 \pmod{2}\} \cup \{v_{i,j}, i \equiv 1 \pmod{3}, j \equiv 0 \pmod{2}\} \cup \{v_{i,j} : \\
& i \equiv 2 \pmod{3}, 1 \leq j \leq n\} \} \\
& S_{10} = \{\{v_{i,j} : j \equiv 0 \pmod{3}, i \equiv 1 \pmod{2}\} \cup \{v_{i,j} : j \equiv 1 \pmod{3}, i \equiv 0 \pmod{2}\} \cup \\
& \{v_{i,j}, j \equiv 2 \pmod{3}, 1 \leq i \leq m\} \}
\end{aligned}$$

Also we assume $\zeta_i, 1 \leq i \leq 18$ as follows $\zeta_1 = \sum_{p=1}^{\lceil \frac{n}{4} \rceil} (m - (\frac{n-2p+4}{2}))a + p\lfloor \frac{a}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} (m - (\frac{n}{4} + \lceil \frac{4q-3}{4} \rceil))a + q(\lfloor \frac{a}{2} \rfloor - 1) + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} ((\frac{n+4r-2}{2}))a + r + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} (\frac{3n}{4} + \lfloor \frac{4s-1}{4} \rfloor)a + s$

$$\begin{aligned}
\zeta_2 &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n-2p+4}{2})b - 2) + p\lfloor \frac{a}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + \lceil \frac{4q-3}{4} \rceil)b - 2) + q\lfloor \frac{a}{2} \rfloor + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} m - \\
& ((\frac{n}{2} - (\lfloor \frac{2r-1}{2} \rfloor - 2))b - 2) + 2r\lfloor \frac{a}{2} \rfloor + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + \lceil \frac{4s-1}{4} \rceil + 1)b - 2) + 2s\lfloor \frac{a}{2} \rfloor \\
\zeta_3 &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n-4p+6}{2})b - 1) + p\lfloor \frac{b}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + \lceil \frac{4q-3}{4} \rceil)b - 1) + q\lfloor \frac{a}{2} \rfloor + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} m - \\
& ((\frac{n}{2} - (\lfloor \frac{2r-1}{2} \rfloor - 1))b - 1) + 2r\lfloor \frac{a}{2} \rfloor + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + (\lceil \frac{4s-1}{4} \rceil + 1))b - 2) + 2s\lfloor \frac{a}{2} \rfloor \\
\zeta_4 &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n-2p+4}{2})a) + p\lfloor \frac{a}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} (n - (3\lfloor \frac{n}{12} \rfloor + \lfloor \frac{4q-3}{4} \rfloor))a + q + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} (n - (6\lfloor \frac{n}{12} \rfloor + \\
& \lfloor \frac{4r-2}{4} \rfloor))a + r + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} m - ((\lfloor \frac{n}{4} \rfloor + \lceil \frac{4s-1}{4} \rceil)a + 1) + s\lfloor \frac{a}{2} \rfloor \\
\zeta_5 &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n-2p+4}{2})b - 2) + p\lfloor \frac{a}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} m - ((\lfloor \frac{n}{4} \rfloor + \lceil \frac{4q-3}{4} \rceil)b - 2) + 2q\lfloor \frac{a}{2} \rfloor + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} m - \\
& ((\frac{n}{2} - (\lfloor \frac{4r-2}{4} \rfloor - 1))b - 2) + 2r\lfloor \frac{a}{2} \rfloor + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} m - ((\lfloor \frac{n}{4} \rfloor + \lceil \frac{4s-1}{4} \rceil)b - 2) + s\lfloor \frac{a}{2} \rfloor \\
\zeta_6 &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} (\frac{n+2(p-1)}{2})b + p\lfloor \frac{a}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} (\lfloor \frac{3n}{4} \rfloor - \lfloor \frac{4q-3}{4} \rfloor)b - q + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} (\frac{n}{2} + \lceil \frac{2r-1}{2} \rceil)b + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} (\lfloor \frac{3n}{4} \rfloor - \\
& \lfloor \frac{4s-1}{4} \rfloor)b + s\lfloor \frac{a}{2} \rfloor \\
\zeta_7 &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - (\frac{n-2(p-2)}{2})a + p\lfloor \frac{a}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} m - (\frac{n}{4} + \lceil \frac{4q-3}{4} \rceil)a + q\lfloor \frac{a}{2} \rfloor + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} (\frac{n}{2} + \lceil \frac{2r-1}{2} \rceil)b + \\
& (\frac{n}{2} + \lfloor \frac{2r-1}{2} \rfloor) + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} (\frac{3n}{4} - \lfloor \frac{4s-1}{4} \rfloor)a - (\frac{3n}{4} - \lfloor \frac{4s-1}{4} \rfloor) \\
\zeta_8 &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - (\frac{n-2(p-2)}{2})b - 2 + p\lfloor \frac{a}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + \lceil \frac{4q-3}{4} \rceil)b - 2) + q\lfloor \frac{a}{2} \rfloor + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} m - \\
& ((\frac{n}{2} - (\lfloor \frac{2r-1}{2} \rfloor - 1))b - 2) + 2r\lfloor \frac{a}{2} \rfloor + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + (\lceil \frac{4s-1}{4} \rceil + 1))b - 2) + 2s\lfloor \frac{a}{2} \rfloor \\
\zeta_9 &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - (\frac{n-2(p-2)}{2})b - 1 + p\lfloor \frac{a}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + \lceil \frac{4q-3}{4} \rceil)b - 1) + q\lfloor \frac{a}{2} \rfloor + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} m - \\
& ((\frac{n}{2} - (\lfloor \frac{2r-1}{2} \rfloor - 1))b - 1) + 2r\lfloor \frac{a}{2} \rfloor + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + (\lceil \frac{4s-1}{4} \rceil + 1))b - 2) + 2s\lfloor \frac{a}{2} \rfloor \\
\zeta_{10} &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n-2(p-2)}{2})a) + p\lfloor \frac{a}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} (\lfloor \frac{3n}{4} \rfloor - \lfloor \frac{4q-3}{4} \rfloor)a + q + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} (\frac{n}{2} + \lceil \frac{2r-1}{2} \rceil)a + \\
& r + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} m - ((\lfloor \frac{n}{4} \rfloor + \lceil \frac{4s-1}{4} \rceil)a + 1) + s\lfloor \frac{a}{2} \rfloor
\end{aligned}$$

$$\begin{aligned}
\zeta_{11} &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n-2(p-2)}{2})a - 2) + p\lceil \frac{a}{2} \rceil + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} m - ((\lceil \frac{n}{4} \rceil + \lceil \frac{4q-3}{4} \rceil)b - 2) + 2q\lfloor \frac{a}{2} \rfloor + \\
&\quad \sum_{r=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{2} - (\lfloor \frac{2r-1}{2} \rfloor - 1))b - 2) + 2r\lceil \frac{a}{2} \rceil + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} m - ((\lceil \frac{n}{4} \rceil + \lceil \frac{4s-1}{4} \rceil)b - 2) + s\lfloor \frac{a}{2} \rfloor \\
\zeta_{12} &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n-2(p-2)}{2})b - 1) + p\lceil \frac{a}{2} \rceil + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} (\lceil \frac{3n}{4} \rceil - \lfloor \frac{4q-3}{4} \rfloor)b - q + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} (\frac{n}{2} + \lceil \frac{2r-1}{2} \rceil)b + \\
&\quad \sum_{s=1}^{\lceil \frac{n}{4} \rceil} m - ((\lceil \frac{n}{4} \rceil + \lceil \frac{4s-1}{4} \rceil)b - 1) + s\lfloor \frac{a}{2} \rfloor \\
\zeta_{13} &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n-2(p-2)}{2})a) + p\lfloor \frac{a}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + \lceil \frac{4q-3}{4} \rceil)a + 1) + q\lceil \frac{a}{2} \rceil + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} (\frac{n}{2} + \\
&\quad \lceil \frac{2r-1}{2} \rceil)b + r + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} (\frac{3n}{4} - \lfloor \frac{s}{4} \rfloor)a + s \\
\zeta_{14} &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n-2(p-1)}{2})b - 2) + p\lceil \frac{a}{2} \rceil + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + \lceil \frac{4q-3}{4} \rceil)b - 2) + q\lfloor \frac{a}{2} \rfloor + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} m - \\
&\quad ((\frac{n}{2} - (\lfloor \frac{2r-1}{2} \rfloor - 1))b - 2) + 2r\lceil \frac{a}{2} \rceil + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + (\lceil \frac{4s-1}{4} \rceil + 1))b - 2) + 2s\lfloor \frac{a}{2} \rfloor \\
\zeta_{15} &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n-2(p-1)}{2})b - 1) + p\lceil \frac{a}{2} \rceil + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + \lceil \frac{4q-3}{4} \rceil)b - 1) + q\lfloor \frac{a}{2} \rfloor + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} m - \\
&\quad ((\frac{n}{2} - (\lfloor \frac{2r-1}{2} \rfloor - 1))b - 1) + 2r\lceil \frac{a}{2} \rceil + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{4} + (\lceil \frac{4s-1}{4} \rceil + 1))b - 2) + 2s\lfloor \frac{a}{2} \rfloor \\
\zeta_{16} &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} (\frac{n+2(p-1)}{2})b + p + p\lfloor \frac{b}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} (\lceil \frac{3n}{4} \rceil - \lfloor \frac{4q-3}{4} \rfloor)b + q + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} (\frac{n}{2} + \lceil \frac{2r-1}{2} \rceil)b + r + \\
&\quad \sum_{s=1}^{\lceil \frac{n}{4} \rceil} (\lfloor \frac{3n}{4} \rfloor - \lfloor \frac{4s-1}{4} \rfloor)b + s + s\lfloor \frac{a}{2} \rfloor \\
\zeta_{17} &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n-2(p-2)}{2})b - 2) + p\lceil \frac{a}{2} \rceil + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} m - ((\lceil \frac{n}{4} \rceil + \lceil \frac{4q-3}{4} \rceil)b - 2) + 2q\lfloor \frac{a}{2} \rfloor + \\
&\quad \sum_{r=1}^{\lceil \frac{n}{4} \rceil} m - ((\frac{n}{2} - (\lfloor \frac{2r-1}{2} \rfloor - 1))b - 2) + 2r\lceil \frac{a}{2} \rceil + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} m - ((\lceil \frac{n}{4} \rceil + \lceil \frac{4s-1}{4} \rceil)b - 2) + s\lfloor \frac{a}{2} \rfloor \\
\zeta_{18} &= \sum_{p=1}^{\lceil \frac{n}{4} \rceil} (\frac{n+2(p-1)}{2})b + p\lfloor \frac{b}{2} \rfloor + \sum_{q=1}^{\lceil \frac{n}{4} \rceil} (\lceil \frac{3n}{4} \rceil - \lfloor \frac{4q-3}{4} \rfloor)b - q + \sum_{r=1}^{\lceil \frac{n}{4} \rceil} (\frac{n}{2} + \lceil \frac{2r-1}{2} \rceil)b + \sum_{s=1}^{\lceil \frac{n}{4} \rceil} (\lfloor \frac{3n}{4} \rfloor - \\
&\quad \lfloor \frac{4s-1}{4} \rfloor)b + s\lfloor \frac{a}{2} \rfloor
\end{aligned}$$

Theorem 2.5. If $n \equiv 0 \pmod{12}$, then

$$\tau_c(P_n \times P_m) = \begin{cases} \zeta_1 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_2 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_3 & \text{for } m \equiv n \pmod{n+1} \\ \frac{2mn}{3} & \text{otherwise.} \end{cases}$$

Proof. Assume $S = \begin{cases} \cup_{l=1}^4 S_l & \text{for } m \equiv 1 \text{ or } n \pmod{n+1}, \\ \cup_{l=5}^8 S_l & \text{for } m \equiv (n-2) \pmod{n+1}, \\ S_{10} & \text{otherwise.} \end{cases}$

Then S is a corona cover set of $P_n \times P_m$ and hence

$$\tau_C(P_n \times P_m) \leq |S| = \begin{cases} \zeta_1 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_2 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_3 & \text{for } m \equiv n \pmod{n+1} \\ \frac{2mn}{3} & \text{otherwise.} \end{cases}$$

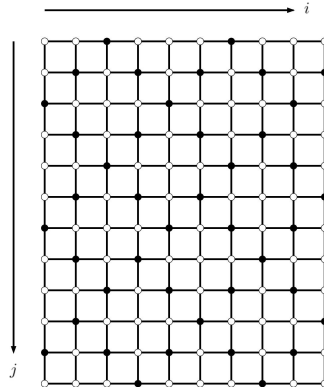
Let S' be a corona cover set of $P_n \times P_m$. Suppose D is a vertex cover set of cardinality at most

$$N = \begin{cases} \zeta_1 - 1 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_2 - 1 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_3 - 1 & \text{for } m \equiv n \pmod{n+1} \\ \frac{2mn}{3} - 1 & \text{otherwise.} \end{cases}$$

then either $\langle D \rangle$ has a vertex with degree zero or there exist an edge e in $\langle V - D \rangle$, Thus we

$$\text{have } |S'| \geq N + 1 = \begin{cases} \zeta_1 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_2 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_3 & \text{for } m \equiv n \pmod{n+1} \\ \frac{2mn}{3} & \text{otherwise.} \end{cases}$$

Hence the result follows $\square$

Example 2.6.**Figure 3.** $P_{12} \times P_{10}$

From Figure 3 it is clear that $\tau_C(P_{12} \times P_{10}) = 78$

Theorem 2.7. If $n \equiv 2 \pmod{12}$, then

$$\tau_c(P_n \times P_m) = \begin{cases} \zeta_4 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_5 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_6 & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{for } m \equiv 3i \pmod{n+1}, 0 \leq i \leq \lfloor \frac{n}{3} \rfloor - 1. \end{cases}$$

Proof. Assume $S = \begin{cases} \cup_{l=1}^4 S_l & \text{for } m \equiv (n-2) \text{ or } (n-1) \pmod{n+1}, \\ \cup_{l=5}^8 S_l & \text{for } m \equiv 1 \pmod{n+1}, \\ S_9 & \text{for } m \equiv 3i \pmod{n+1}, 0 \leq i \leq \lfloor \frac{n}{3} \rfloor - 1. \end{cases}$

Then S is a corona cover set of $P_n \times P_m$ and hence

$$\tau_C(P_n \times P_m) \leq |S| = \begin{cases} \zeta_4 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_5 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_6 & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{for } m \equiv 3i \pmod{n+1}, 0 \leq i \leq \lfloor \frac{n}{3} \rfloor - 1. \end{cases}$$

Let S' be a corona cover set of $P_n \times P_m$. Suppose D is a vertex cover set of cardinality at most

$$N = \begin{cases} \zeta_4 - 1 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_5 - 1 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_6 - 1 & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} - 1 & \text{for } m \equiv 3i \pmod{n+1}, 0 \leq i \leq \lfloor \frac{n}{3} \rfloor - 2. \end{cases}$$

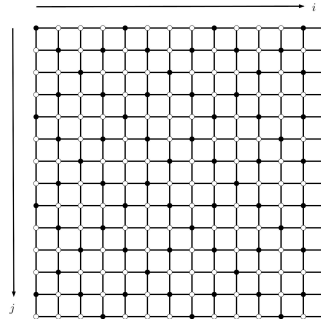
then either $\langle D \rangle$ has a vertex with degree zero or there exist an edge e in $\langle V - D \rangle$, Thus we

$$\text{have } |S'| \geq N + 1 = \begin{cases} \zeta_4 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_5 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_6 & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{for } m \equiv 3i \pmod{n+1}, 0 \leq i \leq \lfloor \frac{n}{3} \rfloor - 1. \end{cases}$$

Hence the result follows $\square$

Remark

If $n \equiv 2 \pmod{12}$ and $m \not\equiv 1$ or $(n-2)$ or n or $3i \pmod{n+1}, 0 \leq i \leq \lfloor \frac{n}{3} \rfloor - 1$ then the corona cover set of $P_n \times P_m$ does not exist.

Example 2.8.**Figure 4.** $P_{14} \times P_{14}$

From Figure 4 it is clear that $\tau_C(P_{14} \times P_{14}) = 126$

Theorem 2.9. If $n \equiv 4 \pmod{12}$, then

$$\tau_c(P_n \times P_m) = \begin{cases} \zeta_7 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_8 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_9 & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{for } m \not\equiv 1 \text{ or } (n-2) \text{ or } n \pmod{n+1} \text{ and } m \equiv 0 \pmod{3}. \end{cases}$$

Proof. Assume

$$S = \begin{cases} \cup_{l=1}^4 S_l & \text{for } m \equiv 1 \text{ or } n \pmod{n+1}, \\ \cup_{l=5}^8 S_l & \text{for } m \equiv (n-2) \pmod{n+1}, \\ S_9 & \text{for } m \not\equiv 1 \text{ or } (n-2) \text{ or } n \pmod{n+1} \text{ and } m \equiv 0 \pmod{3}. \end{cases}$$

Then S is a corona cover set of $P_n \times P_m$ and hence

$$\tau_C(P_n \times P_m) \leq |S| = \begin{cases} \zeta_7 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_8 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_9 & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{for } m \not\equiv 1 \text{ or } (n-2) \text{ or } n \pmod{n+1} \text{ and } m \equiv 0 \pmod{3}. \end{cases}$$

Let S' be a corona cover set of $P_n \times P_m$. Suppose D is a vertex cover set of cardinality at most

$$N = \begin{cases} \zeta_7 - 1 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_8 - 1 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_9 - 1 & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} - 1 & \text{for } m \not\equiv 1 \text{ or } (n-2) \text{ or } n \pmod{n+1} \text{ and } m \equiv 0 \pmod{3}. \end{cases}$$

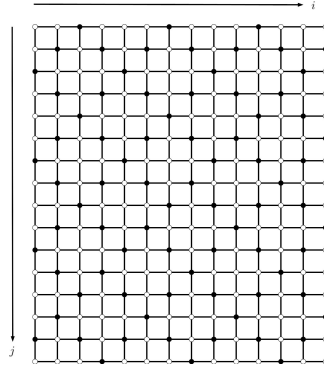
then either $|D| > N$ has a vertex with degree zero or there exist an edge $e \in \langle V - D \rangle$, Thus we have

$$|S'| \geq N + 1 = \begin{cases} \zeta_7 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_8 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_9 & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{for } m \not\equiv 1 \text{ or } (n-2) \text{ or } n \pmod{n+1} \text{ and } m \equiv 0 \pmod{3}. \end{cases}$$

Hence the result follows $\square$

Remark

If $n \equiv 4 \pmod{12}$ and $m \not\equiv 1 \text{ or } (n-2) \text{ or } n \pmod{n+1}$ and $m \not\equiv 0 \pmod{3}$ then the corona cover set of $P_n \times P_m$ does not exist.

Example 2.10.**Figure 5.** $P_{16} \times P_{14}$

From Figure 5 it is clear that $\tau_C(P_{16} \times P_{14}) = 145$

Theorem 2.11. If $n \equiv 6 \pmod{12}$, then

$$\tau_C(P_n \times P_m) = \begin{cases} \zeta_{10} & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_{11} & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_{12} & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{otherwise.} \end{cases}$$

Proof. Assume $S = \begin{cases} \cup_{l=1}^4 S_l & \text{for } m \equiv (n-2) \text{ or } n \pmod{n+1}, \\ \cup_{l=5}^8 S_l & \text{for } m \equiv 1 \pmod{n+1}, \\ S_{10} & \text{otherwise.} \end{cases}$

Then S is a corona cover set of $P_n \times P_m$ and hence

$$\tau_C(P_n \times P_m) \leq |S| = \begin{cases} \zeta_{10} & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_{11} & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_{12} & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{otherwise.} \end{cases}$$

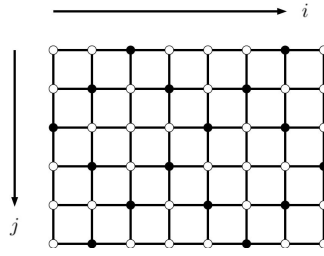
Let S' be a corona cover set of $P_n \times P_m$. Suppose D is a vertex cover set of cardinality at most

$$N = \begin{cases} \zeta_{10} - 1 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_{11} - 1 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_{12} - 1 & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} - 1 & \text{otherwise.} \end{cases}$$

then either $\langle D \rangle$ has a vertex with degree zero or there exist an edge e in $\langle V - D \rangle$. Thus we have

$$|S'| \geq N + 1 = \begin{cases} \zeta_{10} & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_{11} & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_{12} & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{otherwise.} \end{cases}$$

Hence the result follows □

Example 2.12.**Figure 6.** $P_6 \times P_8$

From Figure 6 it is clear that $\tau_C(P_6 \times P_8) = 32$

Theorem 2.13. If $n \equiv 8 \pmod{12}$, then

$$\tau_C(P_n \times P_m) = \begin{cases} \zeta_{13} & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_{14} & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_{15} & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{for } m \equiv 3i \pmod{n+1}, 0 \leq i \leq \lfloor \frac{n}{3} \rfloor - 1. \end{cases}$$

Proof. Assume $S = \begin{cases} \cup_{l=1}^4 S_l & \text{for } m \equiv 1 \text{ or } n \pmod{n+1}, \\ \cup_{l=5}^8 S_l & \text{for } m \equiv (n-2) \pmod{n+1}, \\ S_9 & \text{for } m \equiv 3i \pmod{n+1}, 0 \leq i \leq \lfloor \frac{n}{3} \rfloor - 1. \end{cases}$

Then S is a corona cover set of $P_n \times P_m$ and hence

$$\tau_C(P_n \times P_m) \leq |S| = \begin{cases} \zeta_{13} & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_{14} & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_{15} & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{for } m \equiv 3i \pmod{n+1}, 0 \leq i \leq \lfloor \frac{n}{3} \rfloor - 1. \end{cases}$$

Let S' be a corona cover set of $P_n \times P_m$. Suppose D is a vertex cover set of cardinality at most

$$N = \begin{cases} \zeta_{13} - 1 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_{14} - 1 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_{15} - 1 & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} - 1 & \text{for } m \equiv 3i \pmod{n+1}, 0 \leq i \leq \lfloor \frac{n}{3} \rfloor - 1. \end{cases}$$

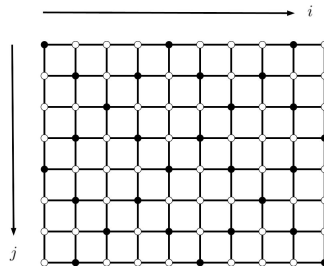
then either $\langle D \rangle$ has a vertex with degree zero or there exist an edge e in $\langle V - D \rangle$, Thus we have

$$|S'| \geq N + 1 = \begin{cases} \zeta_{13} & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_{14} & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_{15} & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{for } m \equiv 3i \pmod{n+1}, 0 \leq i \leq \lfloor \frac{n}{3} \rfloor - 1. \end{cases}$$

Hence the result follows $\square$

Remark

If $n \equiv 8 \pmod{12}$ and $m \not\equiv 1 \text{ or } (n-2) \text{ or } n \text{ or } 3i \pmod{n+1}, 0 \leq i \leq \lfloor \frac{n}{3} \rfloor - 1$ then the corona cover set of $P_n \times P_m$ does not exist.

Example 2.14.**Figure 7.** $P_8 \times P_{10}$

From Figure 7 it is clear that $\tau_C(P_8 \times P_{10}) = 52$

Theorem 2.15. If $n \equiv 10 \pmod{12}$, then

$$\tau_C(P_n \times P_m) = \begin{cases} \zeta_{16} & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_{17} & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_{18} & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{for } m \not\equiv 1 \text{ or } (n-2) \text{ or } n \pmod{n+1} \text{ and } m \equiv 0 \pmod{3}. \end{cases}$$

Proof. Assume

$$S = \begin{cases} \cup_{l=1}^4 S_l & \text{for } m \equiv (n-2) \text{ or } n \pmod{n+1}, \\ \cup_{l=5}^8 S_l & \text{for } m \equiv 1 \pmod{n+1}, \\ S_9 & \text{for } m \not\equiv 1 \text{ or } (n-2) \text{ or } n \pmod{n+1} \text{ and } m \equiv 0 \pmod{3}. \end{cases}$$

Then S is a corona cover set of $P_n \times P_m$ and hence

$$\tau_C(P_n \times P_m) \leq |S| = \begin{cases} \zeta_{16} & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_{17} & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_{18} & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{for } m \not\equiv 1 \text{ or } (n-2) \text{ or } n \pmod{n+1} \text{ and } m \equiv 0 \pmod{3}. \end{cases}$$

Let S' be a corona cover set of $P_n \times P_m$. Suppose D is a vertex cover set of cardinality at most

$$N = \begin{cases} \zeta_{16} - 1 & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_{17} - 1 & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_{18} - 1 & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} - 1 & \text{for } m \not\equiv 1 \text{ or } (n-2) \text{ or } n \pmod{n+1} \text{ and } m \equiv 0 \pmod{3}. \end{cases}$$

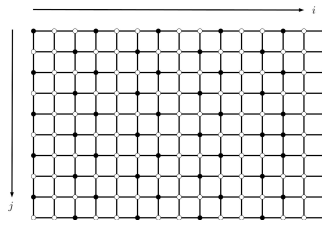
then either $\langle D \rangle$ has a vertex with degree zero or there exist an edge e in $\langle V - D \rangle$, Thus we have

$$|S'| \geq N + 1 = \begin{cases} \zeta_{16} & \text{for } m \equiv 1 \pmod{n+1}, \\ \zeta_{17} & \text{for } m \equiv (n-2) \pmod{n+1}, \\ \zeta_{18} & \text{for } m \equiv n \pmod{n+1}, \\ \frac{2mn}{3} & \text{for } m \not\equiv 1 \text{ or } (n-2) \text{ or } n \pmod{n+1} \text{ and } m \equiv 0 \pmod{3}. \end{cases}$$

Hence the result follows $\square$

Remark

If $n \equiv 10 \pmod{12}$ and $m \not\equiv 1 \text{ or } (n-2) \text{ or } n \pmod{n+1}$ and $m \not\equiv 0 \pmod{3}$ then the corona cover set of $P_n \times P_m$ does not exist.

Example 2.16.**Figure 8.** $P_{10} \times P_{15}$

From Figure 8 it is clear that $\tau_C(P_{10} \times P_{15}) = 99$

3 Conclusion

This study concludes by characterizing the corona covering number of Cartesian path products and presenting an optimal covering configuration. It is evident from the analysis that the structure of the corona cover set changes systematically with the order of P_n .

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