

EFFECT OF RADIATION ON MHD FLOW AND HEAT TRANSFER IN A MOVING FLUID OVER A MOVING FLAT SURFACE ALONG WITH VISCOUS DISSIPATION, INTERNAL HEAT SOURCE/SINK AND PRANDTL NUMBER

Ajaykumar M and Ajay C K

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Corresponding Author: Ajaykumar M

Abstract. The purpose of this research is to determine the natural convection MHD flow of an incompressible moving fluid across a moving flat surface is impacted by radiation, heat sources, and sinks in addition to viscous dissipation. Additional momentum and energy equations are transformed into nonlinear ordinary differential equations using PDE similarity analysis. A finite difference system that yields the intended outcomes is the Keller box technique. The velocity and temperature profiles clearly show the final result of radiation, MHD, Eckert number, Prandtl number and internal heat source/sink. It is establish that radiation has a significant influence on the thermal boundary layer thickness.

1 Introduction

For many engineering processes, the behaviour of the boundary layer on a moving, flat surface is essential. These processes involve materials created through extrusion, the manufacture of paper and metallic sheets, glass fiber, the cooling of electronic chips, and the growth of crystals. In these situations, the cooling ratio in the process determines the final product with the desired qualities. Since the author Sakiadis's creative endeavor, a number of authors have looked at various facets of the issue Sakiadis, B.C [1]. Boundary layer theory on viscoelastic fluids has been identified by numerous authors [2, 3, 4, 5, 6, 7]. Chen [8] observed the flow down a non-isothermal flat plate under natural stream circumstances. Some of the authors [9, 10] considered the stretching example of an incompressible fluid. Klemp and Acrivos [12] acknowledged a very time-consuming operation, In the event of an opposing flow, Siekman [11] noted the difficulty of transferring momentum across the moving wall boundary layer with an external stream. The flow and heat transmission from a moving sheet in the incidence of a co-flowing fluid were noted by Sparrow and Abraham [13]. The author Rafael Cortell [14] studied the transport of heat across a flat surface by a flowing fluid. Lately, the author R. N. Jat [15] investigated transfer of heat and magnetohydrodynamic flow boundary layer across a moving flat plate surface that is not isothermal. It has persisted a specialized common study subject for a number of years, and some authors have looked at boundary layer flow brought on by forced convection alongside different geometrical contours like vertical flat plates, spheres, cylinders, and so forth. Viscous evaporation is an energy source that alters temperature disseminations and, in turn, heat transfer rates. B. Gebhart[16] considered how viscous dissipation affected free convection flow. After that, forced convection and heat sink/ source connections were studied by other authors [17, 18, 19, 20].

Radiation is energy that originates from heat generation and travels across space or an object.

Intensity, sound, and light are types of radiation. Warm radiation influence is thought about in his examination since it might assume a critical part in controlling the intensity move process in the polymer planning producing perseverance. Various new designing cycles for instance oil based commodity start gas turbines, energy measures and the different impulse devises plane, rockets, satellites, space vehicle reappearance occur at maximum temperature. In the energy condition, the radiative intensity transition is addressed utilizing the Rosseland strategy. Centered radiation creates a force convection stream of thick liquid down a porous vertical plate, as established by the S. Siddiqua et al. [21]. Kohilavani Naganthran [22] and D. Srinivasacharya [23] as of late thought to be the result of radiation on a porous surface. B.S. Goud et al. [24] examined the radiation outcome on magnetohydrodynamics boundary layer flow induced by an exponentially stretched sheet. S. Matta et al. [25] examined the upshot of viscous dissipation on magnetohydrodynamics free convection flow across a semi-infinite passing porous vertical plate collected with a heat absorption and chemical reaction. In numerous uses, such as reactor confirmation evaluation, industry metal waste, radioactive material strength, fire and start investigations, and many more, radiation can alter the temperature factor. Due to its importance in several design applications, the intensity stream and movement in a passing sheet surface placed in a liquid moving have attracted more attention.

2 Mathematical Formulation

Analyse the movement of an incompressible viscous fluid at a persistent velocity U_w along a flat, impermeable sheet $y = 0$, either in the similar direction as the free stream U_∞ or in the opposite direction

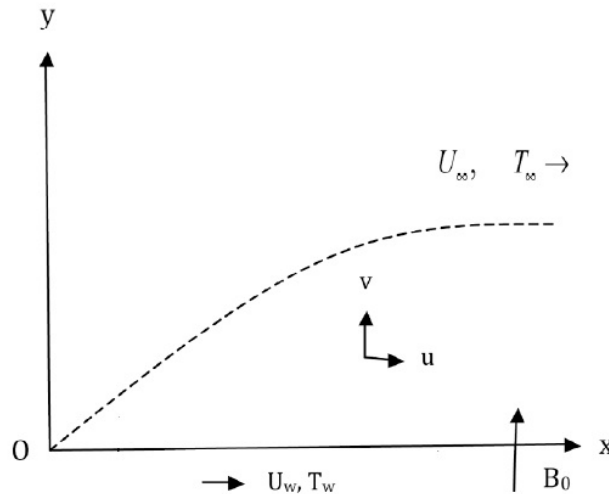


Figure 2.1. The coordinate system and the physical method

Despite the y axis being vertical to the surface and the x axis is comparable to it (Figure 2.1). Normal to the surface, a (B_0) magneto hydrodynamic field of constant intensity is applied. Since the Reynolds magnetic number is assumed to be insignificant, the induced magnetic field is deserted. For a flat surface, the two-dimensional flow boundary layer regulating equations are mentioned as

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2.1)$$

Momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho}, \quad (2.2)$$

Energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q_o}{\rho C_p} (T - T_\infty) + \frac{\nu}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} \quad (2.3)$$

The starting circumstances are

$$\begin{aligned} \text{at } y = 0; \quad u &= U_w \quad v = 0, \\ \text{as } y \rightarrow \infty \quad u &= U_\infty. \end{aligned} \quad (2.4)$$

When the surface temperature remains constant, the boundary circumstances are

$$\begin{aligned} T &= T_w \quad \text{at } y = 0, \\ T &\rightarrow T_\infty \quad \text{as } y \rightarrow \infty. \end{aligned} \quad (2.5)$$

Where U_∞ and U_w are presumed to be constants. They describe the wall and free stream velocities, respectively. The writer Afzal et al. [26] established the similarity modification known on the shared velocity $U (= U_\infty + U_w)$.

Here the stream function ψ , which is symbolized

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \quad (2.6)$$

The comparison modifications are presented as follows

$$\left. \begin{aligned} \psi &= \sqrt{2\nu x} U f(\eta), \quad G = \frac{T - T_\infty}{T_w - T_\infty}, \\ u &= U f'(\eta), \quad v = U \left(\frac{\eta f' - f}{\sqrt{2Re_x}} \right), \quad \eta = y \sqrt{\frac{U}{2\nu x}}. \end{aligned} \right\} \quad (2.7)$$

The radiative heat flux is mentioned by employed the Rosseland definition for radiation [27]

$$q_r = -\frac{4\sigma}{3\alpha^*} \frac{\partial T^4}{\partial y}. \quad (2.8)$$

The term T^4 can be mention as a temperature linear function if it is acknowledged that the temperature inside the flows varies. Consequently, we have a Taylor sequence concerning the T_∞ expanding T^4 expansion and neglect of higher level terms.

$$T^4 = 4T_\infty^3 T - 3T_\infty^4. \quad (2.9)$$

Using (2.8) and (2.9), (2.3) becomes

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q_o}{\rho C_p} (T - T_\infty) + \frac{\nu}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{16\sigma T_\infty^3}{3\alpha^* \rho C_p} \frac{\partial^2 T}{\partial y^2} \quad (2.10)$$

With the aid of Equations (2.6) – (2.9), Eqns. (2.2) and (2.10) condense to the subsequent method:

$$F'' - MF + fF' = 0, \quad (2.11)$$

$$(1 + N_R)G'' + PrEc(F')^2 + PrfG' + PrQG = 0. \quad (2.12)$$

Consequently, the following boundary conditions apply:

$$\left. \begin{aligned} f(0) &= 0, F(0) = 1 - r, G(0) = 1, \\ F(\infty) &= r, G(\infty) = 0. \end{aligned} \right\} \quad (2.13)$$

Here $r = \frac{U_\infty}{U} = \frac{U_\infty}{U_w + U_\infty}$, is the moving parameter ($0 \leq r \leq 1$) and the prime (') represents derivative with respect to the η . Where f is the dimensionless stream function and Pr be the Prandtl number. The skin friction (C_f) and the Nusselt number (Nu) are specified by

$$R_e^{\frac{1}{2}} C_f = \frac{2\tau_w}{\rho U^2} = \frac{2\mu \left(\frac{\partial u}{\partial y} \right)_{y=0}}{\rho U^2} = \sqrt{2} F'(0), \quad (2.14)$$

the wall-shear stress τ_w and the heat flux q_w are known by

$$R_e^{-\frac{1}{2}} Nu = \frac{x q_w}{(k(T_w - T_\infty))} = -\frac{x k \left(\frac{\partial T}{\partial y} \right)_{y=0}}{(k(T_w - T_\infty))} = -\sqrt{\frac{1}{2}} G'(0), \quad (2.15)$$

here $R_e = \frac{Ux}{\nu}$, is the local Reynolds number.

3 Outcomes And Discussion

Since the idea is defined in Bradshaw P et.al [28], Ajaykumar M et al. [29], and Keller, H. B [30], the Keller-box technique, an implicit finite differences concept, has been used to mathematically resolve the coupled nonlinear PDEs (2.11) and (2.12) together with the boundary circumstances (2.13). To degree the consistency of precise examination results were contrasted and the previous investigation of Rafael Cortell [14]. The result shows exceptional understanding when associated by Rafael Cortell [14] in Table 1. Figures 3.1 to 3.5 shows temperature aspects in changed MHD parameter (M), Prandtl number (Pr), Eckert number (E_c) and heat sink /source (Q).

The conventional velocity and temperature of altered radiation (N_R) data in the incidence of a heat source, magnetic field and viscous dissipation are presented in figures 3.1 (a) and 3.1 (b). Considering fixed data of $Pr = 0.72, r = 0.2, Q = M = E_c = 0.5$ and is selected to capture realistic magnetic effects without overwhelming the flow. It is apparent that for static moving and Pr , the velocity drops and the heat upsurges through the increasing data of radiation parameter (N_R). The ratio of reduced velocity factor is 0.02 and the augmented temperature factor is 16.97. When thermal radiation becomes stronger, more energy is transferred into the fluid in the form of radiative heat.

It is ideal that when the magneto hydrodynamics (M) increases, the temperature upsurges and velocity drops with fixed data of $Pr = 0.72, r = 0.2, \text{ and } Q = N_R = E_c = 0.5$. This denotes a main level of viscous heating. The enlarged temperature aspect is 7.44 and the let down velocity aspect is 20.14. The magnetic field tends to speed smooth movement, whereas the Lorentz force reduces boundary layer escalation. As a outcome, the stream is disturbed by the electrically leading liquid, which lowers energy and thickens the thermal boundary layer.

Figure 3.3 mention the typical temperature of specific data of E_c within the sort of range ($0.0 \leq E_c \leq 2.0$). It is plainly seen that temperature outline climbs with the increasing upsides of (E_c) utilizing consistent upsides of $M = N_R = Q = 0.5, r = 0.2 \text{ and } Pr = 0.72$ corresponds to moderate radiative heat flux. The level of expanded temperature factor is 5.86. The E_c mention how dissipation occurs in the flow region due to fluid viscosity. It is perceived that the liquid temperature upsurges as thick dissemination rises. Surface cooling is the reason for this, and the E_C has improved. The temperature of the fluid increases as heat transfers from its surface to it. The thickness of the thermal boundary layer grows as the E_C increases.

Figure 3.4 indications the standard temperature curve for specific (Pr). The temperature contour lowers as the (Pr) increases with steady increases of $r = 0.2, N_R = E_c = Q = M = 0.5$. The level of declined temperature coefficient is 75.31. More pronounced Pr values indicate a smaller thermal boundary layer thickness. A slow pace of warm dispersion is also shown by the expansion in Pr . Consequently,, the fluid does not conduct heat far from the surface.

Figure 3.5 illustrates how different heat sink/source (Q) characteristics impact the temperature (G) frame in the example of $E_c = 0.5, M = 0.5, r = 0.2$ with $N_R = 0.5$. The factor of lowered temperature is 25.95. It is perfect that neither the temperature drop nor the thicknesses of the thermal boundary layer are affected by the incidence of a linked heat absorption and generation. It is recalled that Q has additional heat at the surface, permitting the liquid to move speedier and subsequently diminishing the liquid temperature inside the limit layer area.

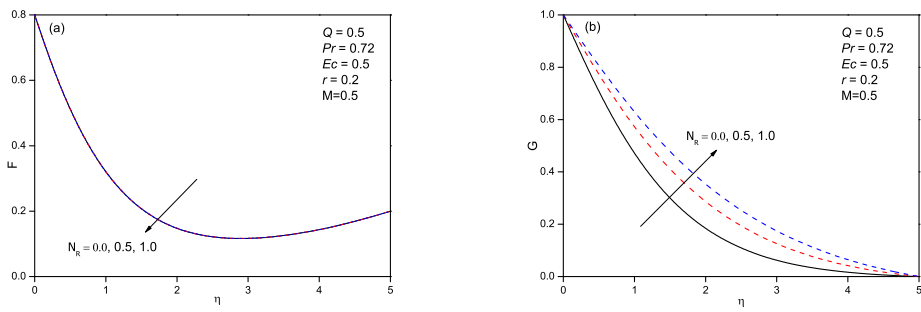
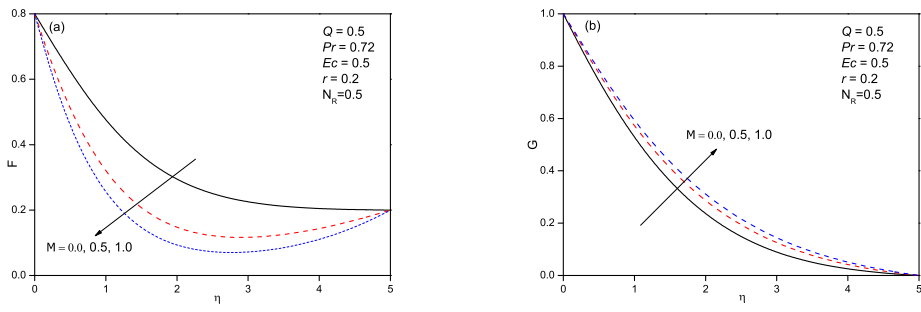


Figure 3.1 (a) and 3.1(b) signifies the velocity and temperature for the changed N_R numbers.



Figures 3.2 (a) and 3.2(b) represents the velocity and temperature for altered data of M .

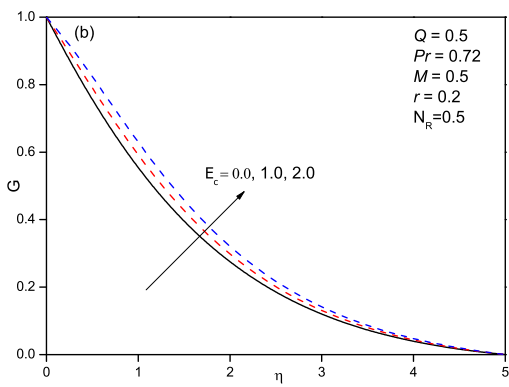


Figure 3.3 denotes the temperature profile, for different data of E_c

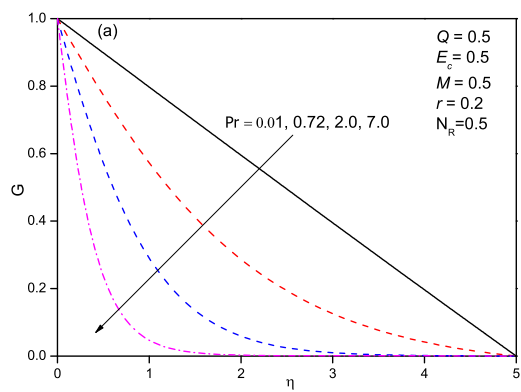


Figure 3.4 mentions the temperature instant for stable values of Pr .

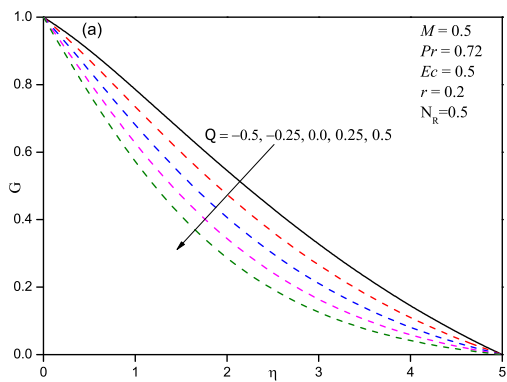


Figure 3.5 indications the influence of heat sink/source(Q) on temperature contours.

Table 1. Illustrates the velocity gradient by the side of the wall f'' in the mentioned domain $0 \leq r \leq 1, M = 0.0, Q = 0.0, Ec = 0.0, N_R = 0.0$ and $Pr = 0.7$

<i>Present Results</i>		<i>Previous Results</i> [14]
r	f''	f''
0.0	-0.6274	-0.6275
0.1	-0.4935	-0.4937
0.2	-0.3632	-0.3633
0.25	-0.2994	-0.2995
0.3	-0.2370	-0.2371
0.4	-0.1156	-0.1157
0.5	0.0	0.0
0.6	0.1095	0.1096
0.7	0.2122	0.2123
0.75	0.2610	0.2610
0.8	0.3072	0.3073
0.9	0.3934	0.3935
1.0	0.4695	0.4696

4 Conclusions

The present effort has resolute the effect of radiation factor, ensuing in the subsequent conclusions

1. The temperature rises and the velocity falls due to radiation parameter rises.
2. The temperature rises and velocity falls with respect to increase of magnetic parameter.
3. The temperature rises as an upshot of increasing the Ec.
4. The temperature is reduced as the Pr increases.

5. The thickness of the thermal boundary layer has considerably decreased as an end result of the combined heat absorption and generation.

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Author information

Ajaykumar M, Department of Mathematics, Maharaja Institute of Technology Mysore, India.
E-mail: ajayinfo2k20@gmail.com

Ajay C K, Department of Mathematics, Maharaja Institute of Technology Mysore, India.
E-mail: ajayarkesh@gmail.com