

MAGNETOHYDRODYNAMIC WILLIAMSON NON-NEWTONIAN FLUID FLOW AND HEAT TRANSFER VIA A POROUS MEDIUM USING AN EXPONENTIALLY STRETCHING SHEET

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Keywords and phrases: Heat transfer, porous medium, exponentially stretching sheet, non-Newtonian fluid, Williamson fluid.

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Abstract The present work examined the heat transfer of steady two-dimensional laminar magnetohydrodynamic Williamson non-Newtonian fluid flow via a porous medium using an exponentially Stretching sheet. To solve differential equations, we first formulate a mathematical issue and then use the Runge-Kutta fourth-order approach. Using tables and graphs, we determine the effects of different factors on two basic profiles (temperature and velocity) in the end as well. It is established that velocity and temperature have a directly proportional nature with some parameter and on the other hand, some parameters have an opposite nature also. The rate of shear stress and rate of heat transfer are determined with the help of the coefficient of skin friction and the Nusselt number. The skin friction coefficient decreases when the magnetic field parameter or suction parameter rise but increases as the velocity slip parameter grows. In conclusion, we also linked the results with published works. The temperature slip parameter causes the fluid temperature to decrease. A rise in the permeability parameter may cause a decrease in fluid temperature while increasing fluid velocity. The skin friction coefficient decreases when the magnetic field parameter or suction parameter rises but increases with the velocity slip parameter grows. When Ec , K and Pr increase, the Nusselt number increases. Conversely, when the M , S_v , S_t , Q and W_e increase, the Nusselt number decreases.

1 Introduction:

Most modern metallurgical and metalworking processes rely on a knowledge of how electrically conducting fluids, such as heat, move through widening porous surfaces under the influence of hydromagnetic fields. This topic is of interest to many academics. Using finite element simulation and MHD equations, sophisticated medical science approaches offer cancer-affected tissues with correct medicine by studying magnetic fluid particle-magnetic field relations. Examples of non-Newtonian fluid flow in porous medium include the movement of liquids in the earth as well as the movement of fluids inside the lungs of humans. A thick, stiff, conductive liquid moving across a smooth, exponentially increasing, nonconducting porous material submerged in a randomly permeable medium is analysed using a two-dimensional laminar flow model. The y-axis is taken orthogonal to the stretching sheet, whereas the x-axis is placed parallel to it.

A fresh approach for fluid dissipation in fluids with pores spanning a variety of permeability numbers provides in [1]. MHD free convective heat and mass movement across an upward porous plane plate immersed in a medium with pores [2]. Following that, the computational modelling of the boundary layer flow across an enormously extended plate with radiant heat [3]. Several years later, boundary layer motion of fluid across an extensible sheet of varied width was also examined and findings presented in [4]. The impact of heat radiation on Williamson fluid circulation owing to an expanding or diminishing conduit carrying tiny particles [5]. explore of the MHD circulation of a second-degree liquid in a porous channel and an approach created to explore the impact of wall characteristics in the MHD fluid mobility with heat exchange

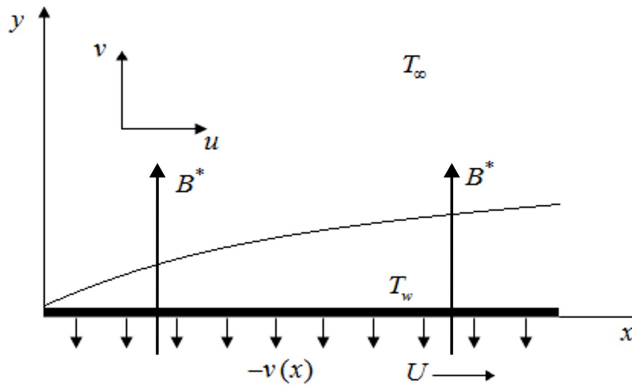


Figure 1: Physical diagram of the problem.([See 36])

and a porous substance [6]. how an inclined magnetic field affects the peristaltic circulation of Williamson fluid in a conduit with a gradient under turbulent circumstances examined in [7]. MHD nanofluid circulation across an evolving perpendicular surface for permeable medium [8]. The MHD flow across the boundary layer owing to an enormously extended plate with radiation effect has been examined in [9]. The effects of the magnetic pull and tiny particle mixture on the circulation of the mixed convective boundary layer and heat exchange in mixed tiny fluids induced by an inclined expanding-contracting surface are investigated in this study [10]. The movement of mixed nanofluid boundary layers over an extensible surface is extensively investigated numerically in this work, considering multiple diffusive events (Soret and Dufour effects), the Marangoni circulation, fluctuating density, a medium that is porous, a field of magnets, generation of heat, and permeability [11]. The irregular MHD flow with the transfer of warmth and mass of a Casson-type nanofluid mixture (GO-TiO₂/SCMC) across a nearly infinitely sliding sheet is analysed in this study, considering thermal energy, Joule heat transfer, electrostatic drag, permeability, velocity and caloric slip, Brownian diffusion, and thermophoresis as variables [12]. A temperature instability technique to evaluate compelled heat convection move in porous medium is numerically employed in [13]. mathematically, a Williamson boundary layer movement that is not Newtonian was expressed [14]. In the circulation of Williamson fluid across an arbitrary expanded area with heat radiation, the consequences of Soret and Dufour were examined [15]. The outcomes of Hall and ion slip effects on an indeterminate MHD-free convective rotating circulation through a highly permeable medium across an exponentially accelerating area have been presented in [16]. The outcome of Hall and ion slip on agitated MHD turbulent rotational circulation in a heat-producing and -absorbing second-grade liquid [17]. Consequences of Hall and ion slip on MHD spinning border layer on the flow of tiny fluids across an endless upward plate put into a medium with pores [18]. The interplay of substances and radiative movement of MHD slip circulation with joule heating and the soret-dufour effect over an exponentially increasing layer were examined mathematically in [19]. The research investigation of the consequence of axisymmetric coupled convective boundary layer circulation and transfer of heat across an enormously spreading surface rooted in a porous material with a heat pump/sink and radiative interaction [20].

Numerous studies addressing different issues have focused on the liquid behaviour of mediums with pores. The consequences of axisymmetric coupled circulation boundary layer circulation and transfer of heat across an extraordinarily stretched surface rooted in a porous medium with radiation and heating/cooling impacts [21]. The influence of extraction and injection on the movement of a magnetohydrodynamic Casson fluid over an upward spreading layer implanted in a medium with pores with a moving heat reservoir or source [22]. The impact of Joule heating on radiated MHD combined convection resting point circulation over an upward extended band submerged in a permeable medium [23]. A hydrophilic tiny fluid conveying heat across a massively spreading material was investigated [24]. It has been studied how conveying heat affects Williamson fluid mobility over the porous coating of a massively spreading material [25]. Research on the flow of Williamson fluid via a growing area [26]. Using Buongiorno's concept, mass as well as heat transport in a tiny liquid delicate layer over an unsteady elastic film

[27]. Contrary to its prior works, this approach accommodates for both the lowest and maximum viscosities needed by a plastic-like liquid [28]. Widening substances have been intensively explored by academics owing of their beneficial applications in production, fabric yarn, vinyl film shaping, etc. [29]. The implications of a generated field of magnets on the peristaltic circulation of Williamson fluid in a bent conduit [30]. The magnetohydrodynamic mobility of a micro-Williamson fluid produced by spreading area with numerous slips was explored in [31]. The impact of heat radiation on the boundary layer flow owing to an exponentially increasing surface was explored [32] in 2008. An algorithmic technique that utilises the MHD flow of the Cattaneo–Christov thermodynamic simulation for Williamson fluid across an elongated sheet with variable widths utilised in [33]. The movement of plastic-like compounds investigated in [34]. Computational algorithms of Williamson fluid with pressure-responsive rigidity [35]. Porous media flow has several practical and technical purposes, such as moving oil or gas, liquid in oil reservoirs or gas fields, cleaning oil, operating gas wells, drilling, processing carbon-based products, and making cosmetics. The objective of this research is to broaden the understanding of transporting energy and activity in the layer that borders over an enormously rising film submerged in a medium with pores with an altering surface heat flux. 3rd-order differential equations for momentum and 2nd-order differential equations for the heat energy equation are created via similarity substitutions. To attain the needed degree of accuracy for different non-dimensional parameters, mathematical calculations are conducted and the findings are visibly shown. Williamson fluid and nanofluid in different forms have been the topic of multiple research projects, discovering unusual fluid features that have been documented owing to their relevance across multiple fields of research. Influenced by these vital and helpful investigations, the current purpose is to identify the features of Williamson fluid circulation along a broad area.. This work is novel because it analyses steady two-dimensional laminar MHD slip flow in detail while combining internal heat source parameters, temperature slip, suction, permeability, Joule heating (Eckert number), internal heat source parameters, Williamson fluid effects, and velocity slip all in one mathematical framework. Thermal management in porous systems, biomedical flows of shear-thinning fluids, metallurgical cooling and continuous casting, polymer extrusion and stretching processes, energy and chemical engineering applications, and electromagnetic flow control for controlling fluid motion and heat transfer are all areas where this study has practical applications. The model, which is limited to a two-dimensional boundary-layer formulation, employs a simplified Darcy porous medium, assumes steady laminar flow, ignores the induced magnetic field, and takes constant fluid characteristics into account.

2 Mathematical Formulation of the Problem:

The surface is extremely elastic, extending in the x-direction and having a velocity of $U = U_0 e^{\frac{x}{l}}$. In y-direction, a non-uniform magnetic field $B^* = B_0 e^{\frac{x}{l}}$ is involved. The temperature of the sheet T_w is meant to be changeable and may be computed using the formula $T_w = T_\infty + T_0 e^{\frac{x}{l}}$. Additionally, a heat source that is not consistent is employed. It is believed that during the motion, all the fluid properties remain the same. Considering these factors, the governing equations for the boundary layer are ([see 22])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho} B^{*2} u - \frac{\nu}{K^*} u, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{(\rho C_p)} \frac{\partial^2 T}{\partial y^2} + \frac{Q^*}{(\rho C_p)} (T - T_\infty) + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma}{(\rho C_p)} B^{*2} u^2 + \sqrt{2} \nu \Gamma \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \quad (3)$$

The boundary conditions are

$$u = U + L^* \frac{\partial u}{\partial y}, v = -V(x), T = T_w + D^* \frac{\partial T}{\partial y} : \text{ at } y = 0, \quad (4)$$

$$u \rightarrow 0, T \rightarrow T_\infty, : \text{ as } y \rightarrow \infty,$$

$Q^* = Q_0 e^{\frac{x}{l}}$ is non-uniform heat source, $K^* = K_0 e^{\frac{-x}{l}}$ is medium's non-uniform permeability. The suction velocity at the sheet is represented by $v(x) = V_0 e^{\frac{x}{2l}}$, the velocity slip factor by $L^* = L_0 e^{\frac{-x}{2l}}$ and the thermal slip factor by $D^* = D_0 e^{\frac{-x}{2l}}$.

3 Solution Method:

Presenting the subsequent similarity transformations

$$\eta = y \sqrt{\frac{U_0}{2\nu l}} e^{\frac{x}{2l}}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \psi = \sqrt{2\nu U_0 l} e^{\frac{x}{2l}} f(\eta) \quad (5)$$

The stream function ψ is defined as

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}, \quad (6)$$

which satisfies the continuity Eq. (1). Using Eq. (5) into the Eq. (6) we get velocity components. We have

$$u = U_0 e^{\frac{x}{l}} f'(\eta), \quad (7)$$

$$v = -\sqrt{\frac{\nu U_0}{2l}} e^{\frac{x}{2l}} (\eta f'(\eta) + f(\eta)), \quad (8)$$

where differentiation with respect to η is denoted by prime. Now with the help of Eqs. 5, 7 and 8

$$f''' + f f'' - 2f'^2 - (M + \frac{1}{K}) f' = 0, \quad (9)$$

$$\theta'' + Pr (f \theta' - \theta f' + 2Q \theta + Ec f''^2 + MEc f'^2 + We f'' f''') = 0, \quad (10)$$

Now boundary conditions are simplified to

$$\eta = 0 : f = S, f' = 1 + S_v f'', \theta = 1 + S_T \theta', \quad (11)$$

$$\eta \rightarrow \infty : f' \rightarrow \infty, \theta \rightarrow 0, \quad (12)$$

where $M = \frac{2\sigma B_0^2 l}{\rho U_0}$ is Magnetic field parameter, $Pr = \frac{\mu C_P}{k}$ is Prandtl number, $Q = \frac{2Q_0 l}{\rho C_P U_0}$ is heat source parameter, $K = \frac{U_0 K^*}{2\nu l}$ is the permeability parameter, $S = V_0 \sqrt{\frac{2l}{\nu U_0}}$ is suction parameter, $S_v = L_0 \sqrt{\frac{U_0}{2\nu l}}$ is velocity slip parameter, $S_T = D_0 \sqrt{\frac{U_0}{2\nu l}}$ is thermal slip parameter and $We = \Gamma e^{\frac{2x}{l}} \sqrt{\frac{U_0 \nu}{T_0}} U_0^2$ is Williamson number.

4 Coefficient of Skin friction and Nusselt number

In terms of the sheet's coefficient of skin friction, the rate of shear stress is determined by

$$C_f = \frac{2\tau_w}{\rho U_0} = \left(\frac{Re_x}{2} e^{\frac{x}{l}} \right)^{-\frac{1}{2}} f''(0), \quad (13)$$

Rate of heat transfer at sheet is measured by Nusselt number Which is

$$Nu = \frac{l q_w}{k(T_w - T_\infty)} = - \left(\frac{Re_x}{2} e^{\frac{x}{l}} \right)^{\frac{1}{2}} \theta'(0), \quad (14)$$

where $\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{(y=0)}$ is wall's shear stress, $q_w = -k \left(\frac{\partial T}{\partial y} \right)_{(y=0)}$ is rate of heat transfer.

5 Numerical Solution

MATLAB software is used to numerically solve the nonlinear second-order ODEs (9) and (10) as well as the boundary conditions (11) and (12). First, as shown below, we convert the BVP into a set of IVP.

$$f = f_1 ; f' = f_2 ; f'' = f_3 ; f''' = f_3' ; \theta = f_4 ; \theta' = f_5 ; \quad (15)$$

$$f_1' = f_2 , \quad (16)$$

$$f_2' = f_3 , \quad (17)$$

$$f_3' = -f_1 f_3 + 2f_2^2 + \left(M + \frac{1}{K}\right) f_2 , \quad (18)$$

$$f_4' = f_5 , \quad (19)$$

$$\begin{aligned} f_5' = & Pr(f_2 f_4 - f_5 f_1) - 2PrQf_4 - PrEc f_3^2 - PrMEc f_2^2 \\ & - PrWe f_3 \left(-f_1 f_3 + 2f_2^2 + \left(M + \frac{1}{K}\right) f_2 \right) , \end{aligned} \quad (20)$$

The boundary conditions are

$$f_1(0) = S, f_2(0) = 1 + S_v f_3(0), f_4(0) = 1 + S_T f_5(0), \quad (21)$$

$$f_2(\infty) = 0, f_4(\infty) = 0 \quad (22)$$

Now use the shooting technique with the Runge-Kutta fourth order method to clarify the system of IVP.

6 Results and Discussion

The direction in which the magnetic parameter, also known as the magnetic field parameter, influences the velocity of the fluid is seen in Figure 2. Because of the magnetic field, the flow creates force that is known as the Lorentz force, which is a resistive force. It was because of this force that the fluid velocity decreased. Consequently, a drop in fluid velocity occurs whenever M is increased. When M is larger, the Lorentz force is stronger than it would be otherwise. A fluid temperature rises because of the presence of this resistive force. One example of this influence may be found in Figure 3. There has been evidence of velocity increasing in tandem with the K (permeability parameter) in Figure 4. The porosity characteristic of the medium rises as the permeability parameter increases. Given that there is more room for the fluid to go through and fewer obstacles to overcome, it is evident that the porosity characteristic of the medium causes an increase the fluid velocity as it travels through it. It has been found that temperature falls as the permeability parameter increases. Fluids can move more readily through a material with a high permeability. This increased flow enhances the efficiency of heat transmission. This influence is represented in Figure 5. Figure 6 displays the temperature profile for a variety of values for the heat source parameters. Because it signifies that heat is entering the fluid from the outside, a rise in the heat source parameter obviously raises the fluid temperature. Figure 7 shows how fluid velocity is affected by S_v . It is noticed that velocity falls as the velocity slip parameter is raised. Figure 8 illustrates the impact of S_v and shows that temperature of fluid increases with increasing S_v .

Figure 9 shows how fluid temperature and the thermal slip parameter are related, showing that fluid temperature drops as the thermal slip value increases. To determine the Prandtl number, momentum diffusivity is divided by thermal diffusivity. Figure 10 demonstrates how the Prandtl number impact on temperature. As Pr grows, the fluid thermal conductivity drops, causing its temperature to fall. Figure 11 depicts how temperature declines as the Ec grows. Because more heat is transferred convectively and by viscous dissipation, the total temperature falls as the Eckert number rises. The shear thinning characteristic of the fluid, which flows more freely in high

Table 1: For the parameters $M, Pr, K, S, S_v, S_t, Ec, Q, W_e$ the numerical values of $-\theta'(0)$ and $f''(0)$.

M	K	S	S_v	S_t	Q	Pr	Ec	W_e	$-\theta'(0)$	$f''(0)$
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	1.08765128	-1.63842015
1.5	2.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	0.98765128	-1.733228991
2.0	2.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	0.88765128	-1.819654091
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	1.08765128	-1.63842015
1.0	4.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	1.28763828	-1.58719139
1.0	6.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	1.48701528	-1.56946068
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	1.08765128	-1.63842015
1.0	2.0	0.6	0.1	0.1	0.5	5.71	0.5	0.9	1.98765128	-1.67658001
1.0	2.0	0.7	0.1	0.1	0.5	5.71	0.5	0.9	2.48765128	-1.71540121
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	1.08765128	-1.63842015
1.0	2.0	0.5	0.15	0.1	0.5	5.71	0.5	0.9	1.08764901	-1.49689977
1.0	2.0	0.5	0.2	0.1	0.5	5.71	0.5	0.9	1.08764702	-1.37957203
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	1.08765128	-1.63842015
1.0	2.0	0.5	0.1	0.2	0.5	5.71	0.5	0.9	1.08728651	-1.63842015
1.0	2.0	0.5	0.1	0.3	0.5	5.71	0.5	0.9	1.08708165	-1.63842015
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	1.08765128	-1.63842015
1.0	2.0	0.5	0.1	0.1	0.55	5.71	0.5	0.9	1.08765028	-1.63842015
1.0	2.0	0.5	0.1	0.1	0.6	5.71	0.5	0.9	1.08765008	-1.63842015
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	1.08765128	-1.63842015
1.0	2.0	0.5	0.1	0.1	0.5	6.71	0.5	0.9	1.78765128	-1.63842015
1.0	2.0	0.5	0.1	0.1	0.5	7.71	0.5	0.9	2.48765128	-1.63842015
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	1.08765128	-1.63842015
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.6	0.9	1.28708561	-1.63842015
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.7	0.9	1.38708615	-1.63842015
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.5	0.9	1.08765128	-1.63842015
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.5	1.0	0.88765128	-1.63842015
1.0	2.0	0.5	0.1	0.1	0.5	5.71	0.5	1.1	0.68765128	-1.63842015

Table 2: Comparison of $f''(0)$ for different values of physical parameters when $W_e = 0, Ec = 0$.

K	M	Pr	Q	S	S_v	S_t	$f''(0)$ [Sharma et al. 36]	$f''(0)$ [Present work]
1.0	0.5	0.71	0.5	3	0.1	0.1	-2.7051005	-2.7046255
3.0	0.5	0.71	0.5	3	0.1	0.1	2.6221080	-2.6211080
1.0	2	0.71	0.5	3	0.1	0.1	-2.8681380	-2.8680330
1.0	0.5	0.71	0.5	4	0.1	0.1	-3.1313106	-3.1313290
1.0	0.5	0.71	0.5	3	0.3	0.1	1.7351530	-1.9112030

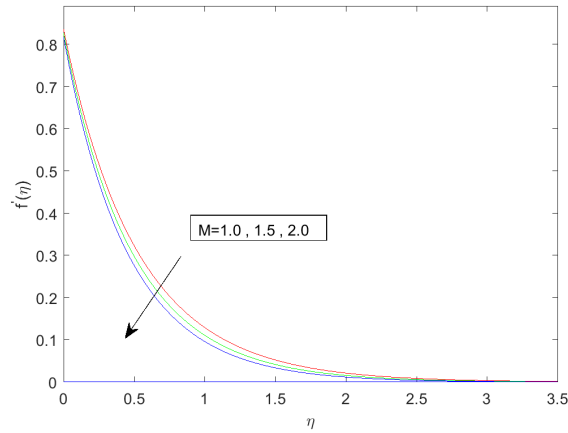


Figure 2: Graph performance between $f'(\eta)$ and η for $M=1.0, 1.5, 2.0$

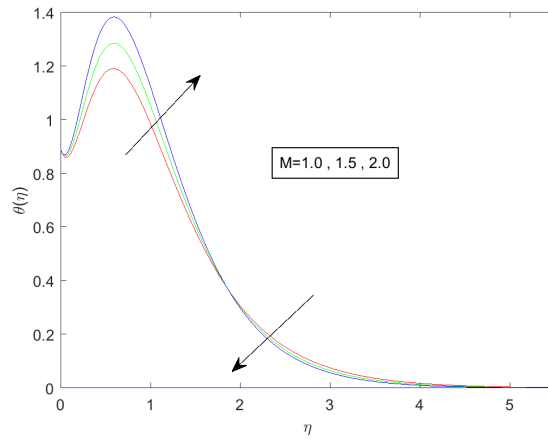


Figure 3: Graph performance between $\theta(\eta)$ and η for $M=1.0, 1.5, 2.0$.

shear zones, can also improve convective heat transfer inside the flow with higher Williamson values. This convective movement can more efficiently distribute heat throughout the fluid at areas of vigorous flow, raising the fluid total temperature. This influence is represented in Figure 12. As the suction parameter increases, velocity decreases, as seen in Figure 13. The porosity of the sheet is reflected by the suction parameter. The flow through the sheet also increases with increasing pore size, which raises the flow resistance throughout the sheet. This explains why the amount of fluid flow down the sheet has decreased. Figure 14 demonstrates the connection between the suction parameter and fluid temperature, illustrating that the fluid temperature drops as suction parameter rises.

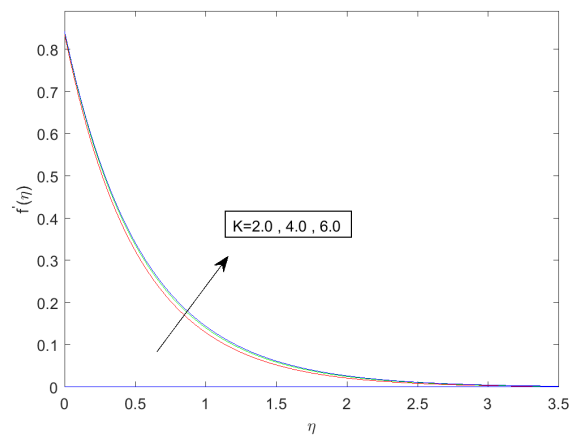


Figure 4: Graph performance between $f'(\eta)$ and η for $K=2.0,4.0,6.0$.

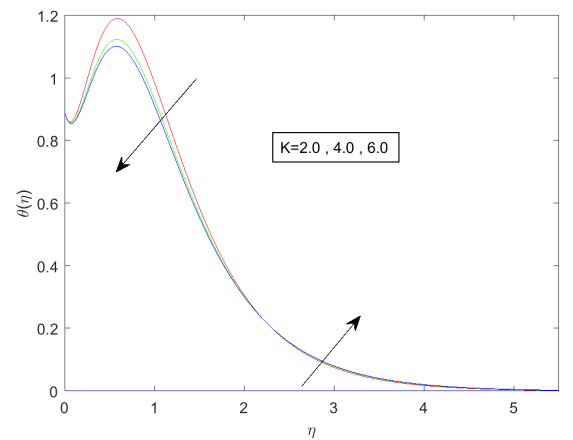


Figure 5: Graph performance between $\theta(\eta)$ and η for $K=2.0,4.0,6.0$.

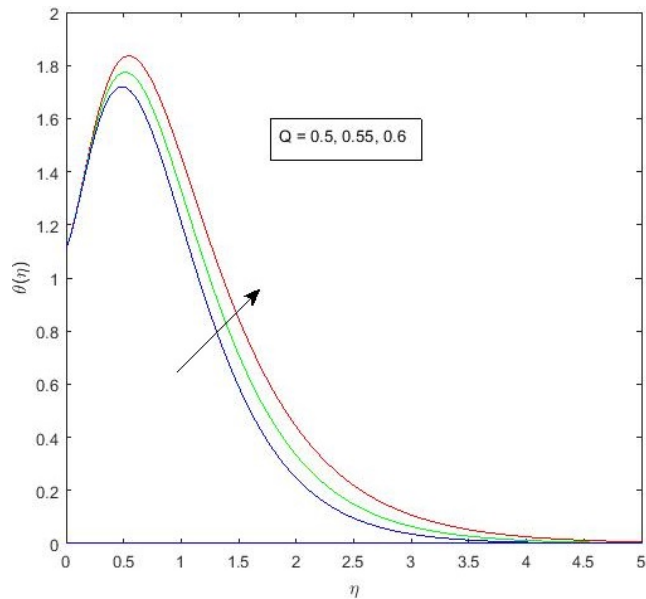


Figure 6: Graph performance between $\theta(\eta)$ and η for $Q=0.5,0.55,0.6$.

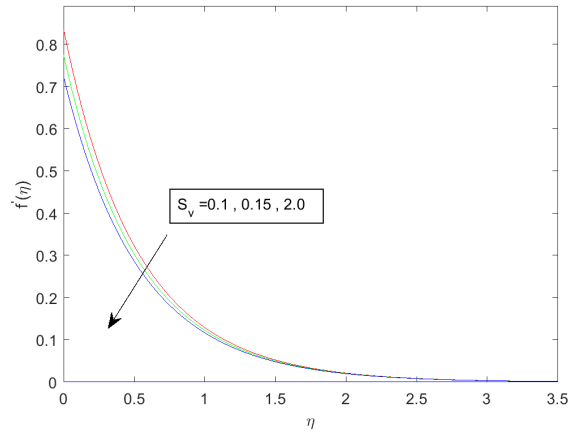


Figure 7: Graph performance between $f'(\eta)$ and η for $S_v=0.1, 0.15, 2.0$

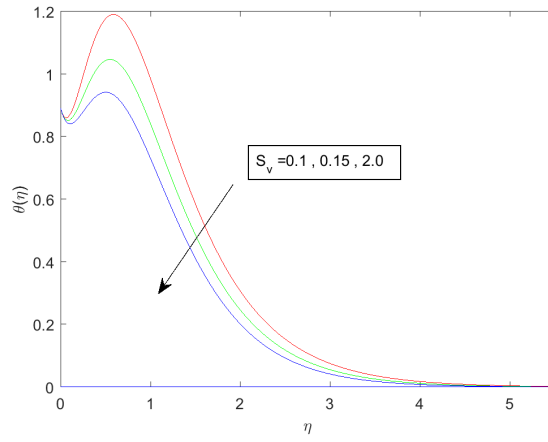


Figure 8: Graph performance between $\theta(\eta)$ and η for $S_v=0.1, 0.15, 2.0$.

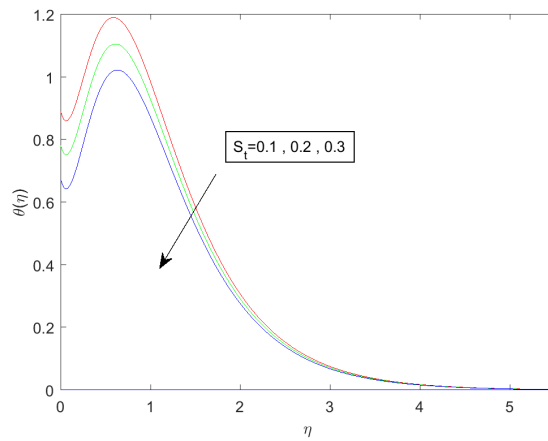


Figure 9: Graph performance between $\theta(\eta)$ and η for $S_T=0.1, 0.2, 0.3$.

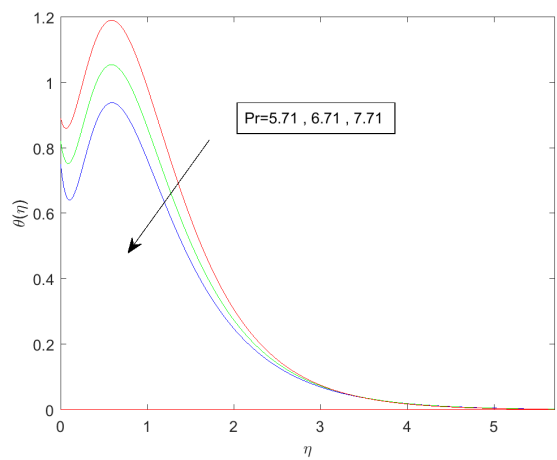


Figure 10: Graph performance between $\theta(\eta)$ and η for $Pr=5.71,6.71,7.71$.

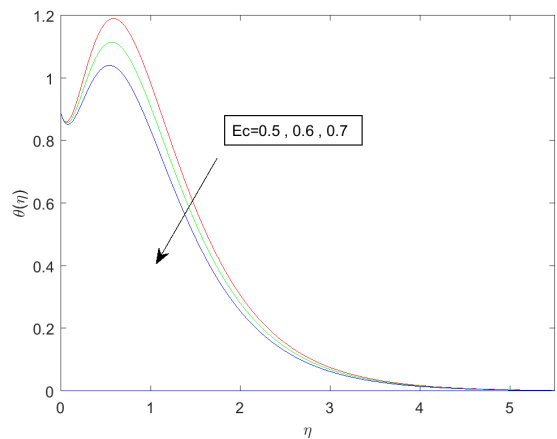


Figure 11: Graph performance between $\theta(\eta)$ and η for $Ec=0.5,0.6,0.7$.

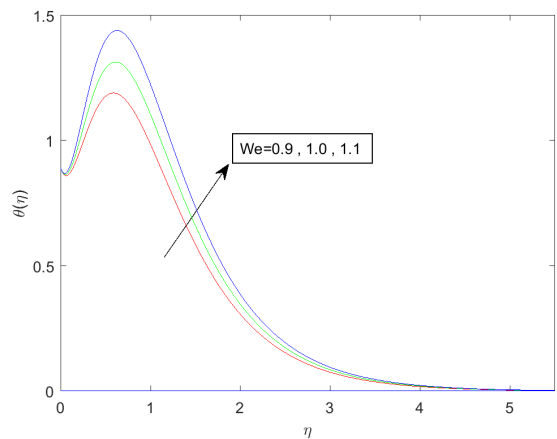


Figure 12: Graph performance between $\theta(\eta)$ and η for $We_e=0.9,1.0,1.1$.

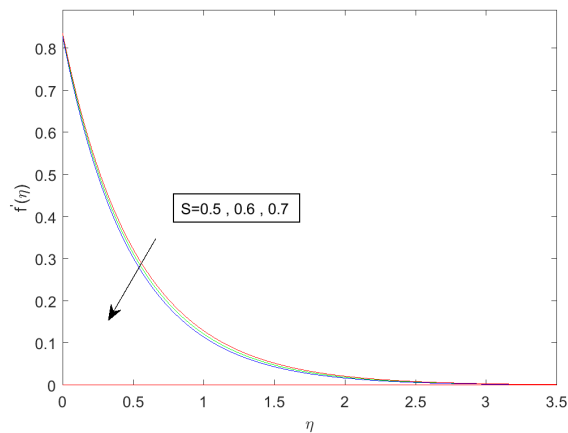


Figure 13: Graph performance between $f'(\eta)$ and η for $S=0.5, 0.6, 0.7$.

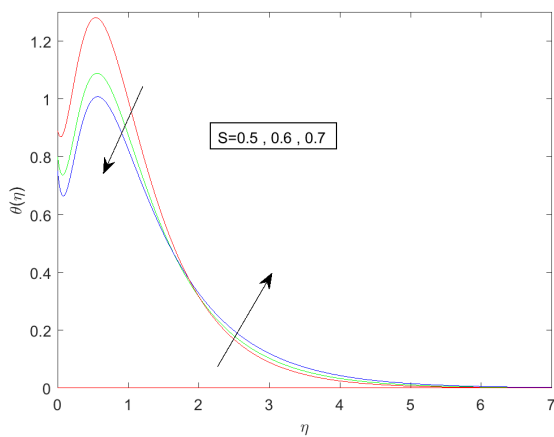


Figure 14: Graph performance between $\theta(\eta)$ and η for $S=0.5, 0.6, 0.7$.

7 Conclusions

Using tables and graphs, determine the effects of different factors on two fundamental profiles: temperature and velocity. The skin friction coefficient rises as the velocity slip parameter increases but falls when the magnetic field parameter or suction parameter increases. It has been shown that although temperature and velocity are inversely proportional to certain parameters, they are directly connected to others. The rate of heat transfer and shear stress are determined using the coefficient of skin friction and the Nusselt number, respectively. The following are some significant conclusions from the preceding work:

- Increasing the decreases fluid velocity but raises fluid temperature.
- Increasing the heat source parameter increases the fluid temperature.
- The fluid temperature falls as the Eckert number increases, but it increases in proportion to the Prandtl number.
- Fluid temperature and velocity decrease as the velocity slip parameter rises.
- The fluid temperature increases as the Williamson number rises.
- The temperature slip parameter causes the fluid temperature to decrease.
- A reduction in fluid temperature as an increase in the permeability parameter.
- Raising the magnetic field parameter or suction parameter causes a drop in the skin friction coefficient, whereas increasing the velocity slip parameter causes it to rise.
- When Ec, K and Pr increase, the Nusselt number increases. Conversely, when the M, S_v, S_v, Q and W_e increase, the Nusselt number decreases.

8 References

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