

On $S(\beta)$ -summability in neutrosophic soft- n -normed linear spaces

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Abstract For $0 < \beta \leq 1$, the aim of the present paper is to define $S(\beta)$ -summability of sequences in neutrosophic soft- n -normed linear spaces, briefly called NS - n - NLS . We also define $S(\beta)$ -Cauchy sequences, $S(\beta)$ -completeness and establish the Cauchy convergence criteria for $S(\beta)$ -summability in neutrosophic soft- n -normed linear spaces.

1 Introduction

The concept of statistical convergence was given by Fast [1] and further developed by different author including Fridy [2], Schoenberg [3], Salat [4], Connor [5] and many others. The notion was defined using natural density of subsets of $\mathbb{N}$, the set of positive integers. For any set $A \subseteq \mathbb{N}$, the natural density of A is defined by

$$\psi(\mathbf{A}) = \lim_n \frac{1}{n} |\{a \leq n : a \in \mathbf{A}\}|$$

provided the limit exists. Further, a sequence $w = (w_k)$ is said to be statistical convergent to w_0 , if for each $\varepsilon > 0$

$$\lim_n \frac{1}{n} |\{k \leq n : |w_k - w_0| \geq \varepsilon\}| = 0,$$

that is $\psi(\mathbf{A}_\varepsilon) = 0$, where $\mathbf{A}_\varepsilon = \{k \leq n : |w_k - w_0| \geq \varepsilon\}$.

For $0 < \beta \leq 1$ and any set $\mathbf{A} \subseteq \mathbb{N}$ the natural density of order β is defined by

$$\psi_\beta(\mathbf{A}) = \lim_{n \rightarrow \infty} \frac{1}{n^\beta} |\{k \leq n : k \in \mathbf{A}\}|,$$

provide the limit exists.

For $0 < \beta \leq 1$, Colak [6] defined statistical convergence of order β as follows.

A sequence $w = (w_k)$ is said to be statistical convergent of order β to w_0 if for each $\varepsilon > 0$

$$\psi_\beta(\mathbf{A}) = \lim_{n \rightarrow \infty} \frac{1}{n^\beta} |\{k \leq n : |w_k - w_0| \geq \varepsilon\}| = 0.$$

In this case, we will write $S_\beta - \lim w_k = w_0$.

For more work on statistically convergence and its generalizations, we refers to the readers [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

2 Preliminaries

In this section we review a few definitions and results required in present paper. For $\mathfrak{J} = [0, 1]$, by a t -norm we mean a binary operation $\circ : \mathfrak{J} \times \mathfrak{J} \rightarrow \mathfrak{J}$ such that for every $t_1, t_2, t_3, t_4 \in \mathfrak{J}$, we have (i) $\circ$ is continuous; (ii) $\circ$ is commutative; (iii) $\circ$ is associative; (iv) $t_1 \circ 1 = t_1$ and (v) $t_1 \circ t_2 \leq t_3 \circ t_4$ whenever $t_1 \leq t_3$ and $t_2 \leq t_4$. If we define (i) $t_1 \circ t_2 = t_1 t_2$; (ii) $t_1 \circ t_2 = \min\{t_1, t_2\}$; (iii) $t_1 \circ t_2 = \max\{t_1 + t_2 - 1, 0\}$, then it is easy to see that $\circ$ becomes

t -norm in each cases. Further, a binary operation $\star : \mathfrak{J} \times \mathfrak{J} \rightarrow \mathfrak{J}$ is said to be t -conorm if for every $t_1, t_2, t_3, t_4 \in \mathfrak{J}$ we have (i) $\star$ is continuous; (ii) $\star$ is commutative; (iii) $\star$ is associative; (iv) $t_1 \star 0 = t_1$ and (v) $t_1 \star t_2 \leq t_3 \star t_4$ whenever $t_1 \leq t_3$ and $t_2 \leq t_4$. If we define (i) $t_1 \star t_2 = t_1 + t_2 - t_1 t_2$, (ii) $t_1 \star t_2 = \max\{t_1, t_2\}$, (iii) $t_1 \star t_2 = \min\{t_1 + t_2, 1\}$, then it is easy to see that $\star$ is a t -conorm in all the cases.

Molodtsov [10] defined the concept of soft sets to deal with the uncertainty, fuzziness and objects which are not well defined. He used the set of parameters to define soft sets. For any universe set $\mathcal{U}$ and parameter set $\mathbf{E}$, the soft set is defined as follows:

Definition 2.1. [10] If a pair $(\mathbf{H}, \mathbf{E})$ satisfies the necessary and sufficient condition $\mathbf{H} : \mathbf{E} \rightarrow \mathcal{P}(\mathcal{U})$ where $\mathcal{P}(\mathcal{U})$ is the power set of $\mathcal{U}$ then it is called a soft set over $\mathcal{U}$. i.e., the soft set is a parameterized family of subsets of the set $\mathcal{U}$. Moreover, every set $\mathbf{H}(\varepsilon), \varepsilon \in \mathbf{E}$, from this family may be considered as the set of ε -elements of the soft set $(\mathbf{H}, \mathbf{E})$, or as the set of ε -approximate elements of the set.

Definition 2.2. [10] If a soft set $(\mathbf{H}, \mathbf{E})$ over $\mathcal{U}$ follows the condition $\forall \varepsilon \in \mathbf{E}, \mathbf{H}(\varepsilon) = w$ then the soft set is called absolute soft set and it is denoted by $\tilde{\mathcal{U}}$.

Definition 2.3. [23] Let $\mathbb{R}$ be the set of real numbers, $\mathfrak{B}(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$ and $\mathbf{E}$ taken as a set of parameters. Then a mapping $\mathbf{F} : \mathbf{E} \rightarrow \mathfrak{B}(\mathbb{R})$ is called a soft real set. If a soft real set is a singleton soft set, then it is called a soft real number and denoted by $\tilde{r}, \tilde{s}, \tilde{t}$, etc. $\tilde{0}, \tilde{1}$ are the soft real numbers where $\tilde{0}(e) = 0, \tilde{1}(e) = 1$ for all $e \in \mathbf{E}$ respectively.

Let $\mathbb{R}(\mathbf{E})$ and $\mathbb{R}^+(\mathbf{E})$ respectively denote the sets of all soft real numbers and all positive soft real numbers.

Definition 2.4. [24] Let $(\mathbf{H}, \mathbf{E})$ be a soft set over w . The set $(\mathbf{H}, \mathbf{E})$ is said to be a soft point, denoted by $\mathbf{H}_e^w$ if there is exactly one $e \in \mathbf{E}$ s.t $\mathbf{H}(e) = \{w\}$ for some $w \in \mathcal{U}$ and $\mathbf{H}(e') = \phi$ for all $e' \in \mathbf{E} - \{e\}$.

Two soft points $\mathbf{H}_e^w, \mathbf{H}_{e'}^v$ are said to be equal if $e = e'$ and $w = v$. Let $\Delta_{\tilde{\mathcal{U}}}$ denotes the set of all soft points on $\tilde{\mathcal{U}}$.

In case $\mathcal{U}$ is a vector space over $\mathbb{R}$ and the parameter set $\mathbf{E} = \mathbb{R}$, the soft point is called a soft vector.

Das, Majumdar and Samanta [22] defined soft normed linear spaces as follows.

Definition 2.5. [22] Let $\tilde{\mathcal{U}}$ be a absolute soft vector space. Then a mapping $\|\cdot\| : \tilde{\mathcal{U}} \rightarrow \mathbb{R}^+(\mathbf{E})$ is said to be a soft norm on $\tilde{\mathcal{U}}$, if $\|\cdot\|$ satisfies the following conditions:

- (i) $\|w_e\| \geq \tilde{0}$ for all $w_e \in \tilde{\mathcal{U}}$ and $\|w_e\| = \tilde{0} \Leftrightarrow w_e = \tilde{\theta}_0$ where $\tilde{\theta}_0$ denotes the zero element of $\tilde{\mathcal{U}}$.
- (ii) $\|\tilde{\alpha}w_e\| = |\tilde{\alpha}|\|w_e\|$ for all $w_e \in \tilde{\mathcal{U}}$ and for every soft scalar $\tilde{\alpha}$.
- (iii) $\|w_e + w_{e'}\| \leq \|w_e\| + \|w_{e'}\|$ for all $w_e, w_{e'} \in \tilde{\mathcal{U}}$.
- (iv) $\|w_e \cdot w_{e'}\| = \|w_e\| \|w_{e'}\|, \forall w_e, w_{e'} \in \tilde{\mathcal{U}}$.

The soft vector space $\tilde{\mathcal{U}}$ with a soft norm $\|\cdot\|$ on $\tilde{\mathcal{U}}$ is said to be a soft normed linear space and is denoted by $(\tilde{\mathcal{U}}, \|\cdot\|)$.

Bera and Mahapatra [26] defined neutrosophic normed linear spaces and studied some of their properties.

Definition 2.6. [26] Let $\tilde{w}$ be a soft linear space over the field $\mathbf{F}$ and $\mathbb{R}(\mathbf{E}), \Delta_{\tilde{\mathcal{U}}}$ denote respectively, the set of all soft real numbers and the set of all soft points on $\tilde{\mathcal{U}}$. Then a neutrosophic subset N over $\Delta_{\tilde{\mathcal{U}}} \times \mathbb{R}(\mathbf{E})$ is called a neutrosophic soft norm on $\tilde{\mathcal{U}}$ if for $w_e, w_{e'} \in \tilde{\mathcal{U}}$ and $\tilde{\alpha} \in \mathbf{F}$ ($\tilde{\alpha}$ being soft scalar), the following conditions hold.

- (i) $0 \leq G_N(w_e, \tilde{\xi}_1), B_N(w_e, \tilde{\xi}_1), Y_N(w_e, \tilde{\xi}_1) \leq 1, \forall \tilde{\xi}_1 \in \mathbb{R}(\mathbf{E})$.
- (ii) $0 \leq G_N(w_e, \tilde{\xi}_1) + B_N(w_e, \tilde{\xi}_1) + Y_N(w_e, \tilde{\xi}_1) \leq 3, \forall \tilde{\xi}_1 \in \mathbb{R}(\mathbf{E})$.
- (iii) $G_N(w_e, \tilde{\xi}_1) = 0$ with $\tilde{\xi}_1 \leq \tilde{0}$.
- (iv) $G_N(w_e, \tilde{\xi}_1) = 1$, with $\tilde{\xi}_1 > \tilde{0}$ iff $w_e = \tilde{\theta}_0$, the null soft vector.

- (v) $G_N(\tilde{\alpha}w_e, \tilde{\xi}_1) = G_N\left(w_e, \frac{\tilde{\xi}_1}{|\tilde{\alpha}|}\right), \forall \tilde{\alpha}(\neq \tilde{0}), \tilde{\xi}_1 > \tilde{0}$.
- (vi) $G_N(w_e, \tilde{\xi}_1) \circ G_N(w_{e'}, \tilde{\xi}_2) \leq G_N(w_e \oplus w_{e'}, \tilde{\xi}_1 \oplus \tilde{\xi}_2), \forall \tilde{\xi}_1, \tilde{\xi}_2 \in \mathbb{R}(\mathbf{E})$
- (vii) $G_N(w_e, \cdot)$ is continuous non-decreasing function for $\tilde{\xi}_1 > \tilde{0}$ and $\lim_{\tilde{\xi}_1 \rightarrow \infty} G_N(w_e, \tilde{\xi}_1) = 1$.
- (viii) $B_N(w_e, \tilde{\xi}_1) = 1$ with $\tilde{\xi}_1 \leq \tilde{0}$.
- (ix) $B_N(w_e, \tilde{\xi}_1) = 0$, with $\tilde{\xi}_1 > \tilde{0}$ iff $w_e = \tilde{\theta}_0$, the null soft vector.
- (x) $B_N(\tilde{\alpha}w_e, \tilde{\xi}_1) = B_N\left(w_e, \frac{\tilde{\xi}_1}{|\tilde{\alpha}|}\right), \forall \tilde{\alpha}(\neq \tilde{0}), \tilde{\xi}_1 > \tilde{0}$.
- (xi) $B_N(w_e, \tilde{\xi}_1) \star B_N(w_{e'}, \tilde{\xi}_2) \geq B_N(w_e \oplus w_{e'}, \tilde{\xi}_1 \oplus \tilde{\xi}_2) \quad \forall \tilde{\xi}_1, \tilde{\xi}_2 \in \mathbb{R}(\mathbf{E})$.
- (xii) $B_N(w_e, \cdot)$ is continuous non-increasing function for $\tilde{\xi}_1 > \tilde{0}$ and $\lim_{\tilde{\xi}_1 \rightarrow \infty} B_N(w_e, \tilde{\xi}_1) = 0$.
- (xiii) $Y_N(w_e, \tilde{\xi}_1) = 0$ with $\tilde{\xi}_1 \leq \tilde{0}$.
- (xiv) $Y_N(w_e, \tilde{\xi}_1) = 0$, with $\tilde{\xi}_1 > \tilde{0}$ iff $w_e = \tilde{\theta}_0$, the null soft vector.
- (xv) $Y_N(\tilde{\alpha}w_e, \tilde{\xi}_1) = Y_N\left(w_e, \frac{\tilde{\xi}_1}{|\tilde{\alpha}|}\right), \forall \tilde{\alpha}(\neq \tilde{0}), \tilde{\xi}_1 > \tilde{0}$.
- (xvi) $Y_N(w_e, \tilde{\xi}_1) \star Y_N(w_{e'}, \tilde{\xi}_2) \geq Y_N(w_e \oplus w_{e'}, \tilde{\xi}_1 \oplus \tilde{\xi}_2) \quad \forall \tilde{\xi}_1, \tilde{\xi}_2 \in \mathbb{R}(\mathbf{E})$.
- (xvii) $Y_N(w_e, \cdot)$ is continuous non-increasing function for $\tilde{\xi}_1 > \tilde{0}$ and $\lim_{\tilde{\xi}_1 \rightarrow \infty} Y_N(w_e, \tilde{\xi}_1) = 0$.

We now give the definition of neutrosophic- n -normed linear spaces.

Definition 2.7. Let $\tilde{\mathcal{U}}$ be a soft linear space over the field $\mathbf{F}$ and $\mathbb{R}(\mathbf{E})$, $\Delta_{\tilde{\mathcal{U}}}$ denote respectively, the set of all soft real numbers and the set of all soft points on $\tilde{\mathcal{U}}$. Then a neutrosophic subset N over $\Delta_{\tilde{\mathcal{U}}} \times \mathbb{R}(\mathbf{E})$ is called a neutrosophic soft n -norm on $\tilde{\mathcal{U}}$ if for $(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1})$,

$(w_{e'}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}) \in \tilde{\mathcal{U}}^n$ and $\tilde{\alpha} \in \mathbf{F}$ ($\tilde{\alpha}$ being soft scalar), the following conditions hold.

- (i) $0 \leq G_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1), B_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1),$
 $Y_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) \leq 1, \forall \tilde{\xi}_1 \in \mathbb{R}(\mathbf{E})$.
- (ii) $0 \leq G_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) + B_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) +$
 $Y_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) \leq 3, \forall \tilde{\xi}_1 \in \mathbb{R}(\mathbf{E})$.
- (iii) $G_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) = 0$ with $\tilde{\xi}_1 \leq \tilde{0}$.
- (iv) $G_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) = 1$, with $\tilde{\xi}_1 > \tilde{0}$ iff $w_e = \tilde{\theta}_0$, the null soft vector.
- (v) $G_N(\tilde{\alpha}w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) = G_N\left(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}_1}{|\tilde{\alpha}|}\right), \forall \tilde{\alpha}(\neq \tilde{0}), \tilde{\xi}_1 > \tilde{0}$.
- (vi) $G_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) \circ G_N(w_{e'}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_2) \leq G_N(w_e \oplus w_{e'}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1 \oplus \tilde{\xi}_2), \forall \tilde{\xi}_1, \tilde{\xi}_2 \in \mathbb{R}(\mathbf{E})$
- (vii) $G_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \cdot)$ is continuous non-decreasing function for $\tilde{\xi}_1 > \tilde{0}$ and $\lim_{\tilde{\xi}_1 \rightarrow \infty} G_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) = 1$.
- (viii) $B_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) = 1$ with $\tilde{\xi}_1 \leq \tilde{0}$.
- (ix) $B_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) = 0$, with $\tilde{\xi}_1 > \tilde{0}$ iff $w_e = \tilde{\theta}_0$, the null soft vector.
- (x) $B_N(\tilde{\alpha}w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) = B_N\left(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}_1}{|\tilde{\alpha}|}\right), \forall \tilde{\alpha}(\neq \tilde{0}), \tilde{\xi}_1 > \tilde{0}$.
- (xi) $B_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) \star B_N(w_{e'}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_2) \geq B_N(w_e \oplus w_{e'}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1 \oplus \tilde{\xi}_2) \quad \forall \tilde{\xi}_1, \tilde{\xi}_2 \in \mathbb{R}(\mathbf{E})$.
- (xii) $B_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \cdot)$ is continuous non-increasing function for $\tilde{\xi}_1 > \tilde{0}$ and $\lim_{\tilde{\xi}_1 \rightarrow \infty} B_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) = 0$.
- (xiii) $Y_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) = 0$ with $\tilde{\xi}_1 \leq \tilde{0}$.
- (xiv) $Y_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) = 0$, with $\tilde{\xi}_1 > \tilde{0}$ iff $w_e = \tilde{\theta}_0$, the null soft vector.
- (xv) $Y_N(\tilde{\alpha}w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) = Y_N\left(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}_1}{|\tilde{\alpha}|}\right), \forall \tilde{\alpha}(\neq \tilde{0}), \tilde{\xi}_1 > \tilde{0}$.

(xvi) $Y_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) \star Y_N(w_{e'}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_2) \geq Y_N(w_e \oplus w_{e'}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1 \oplus \tilde{\xi}_2) \forall \tilde{\xi}_1, \tilde{\xi}_2 \in \mathbb{R}(\mathbf{E})$.

(xvii) $Y_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \cdot)$ is continuous non-increasing function for $\tilde{\xi}_1 > \tilde{0}$ and $\lim_{\tilde{\xi}_1 \rightarrow \infty} Y_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}_1) = 0$.

In this case $N = (G_N, B_N, Y_N)$ is called the neutrosophic soft n -norm and

$(\tilde{\mathcal{U}}(\mathbf{F}), G_N, B_N, Y_N, \circ, \star)$ is an neutrosophic soft n -normed linear space (briefly $NS-n-NLS$).

Let $(\tilde{\mathcal{U}}, \|\cdot\|)$ be a soft normed space. Take the operations $\circ$ and $\star$ as $t_1 \circ t_2 = t_1 t_2$ and $t_2 \star t_3 = t_2 + t_3 - t_2 t_3$. For $\tilde{\xi} > \tilde{0}$, define

$$G_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) = \begin{cases} \frac{\tilde{\xi}}{\tilde{\xi} + \|w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}\|} & \text{if } \tilde{\xi} > \|w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}\| \\ 0 & \text{otherwise.} \end{cases}$$

$$B_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) = \begin{cases} \frac{\|w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}\|}{\tilde{\xi} + \|w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}\|} & \text{if } \tilde{\xi} > \|w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}\| \\ 1 & \text{otherwise.} \end{cases}$$

$$Y_N(w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) = \begin{cases} \frac{\|w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}\|}{\tilde{\xi}} & \text{if } \tilde{\xi} > \|w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}\| \\ 1 & \text{otherwise.} \end{cases}$$

Then $(\tilde{\mathcal{U}}(\mathbf{F}), G_N, B_N, Y_N, \circ, \star)$ is the $NS-n-NLS$.

From now onward, unless otherwise stated, $\tilde{W}$ will denote the $NS-n-NLS$ $(\tilde{\mathcal{U}}(\mathbf{F}), G_N, B_N, Y_N, \circ, \star)$.

A sequence $w = (w_{e_k}^k)$ of soft points in $\tilde{W}$ is said to be convergent to a soft point $w_e \in \tilde{W}$ with respect to the neutrosophic soft n -norm (G_N, B_N, Y_N) if for $0 < \varepsilon < 1$ and $\tilde{\xi} > \tilde{0} \ni n_0 \in \mathbb{N}$ such that

$$G_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) > 1 - \varepsilon,$$

$$B_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) < \varepsilon,$$

$$Y_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) < \varepsilon.$$

In this case, we write $(G_N, B_N, Y_N) - \lim_{k \rightarrow \infty} w_{e_k}^k = w_e$.

A sequence $w = (w_{e_k}^k)$ of soft points in $\tilde{W}$ is said to be Cauchy sequence with respect to neutrosophic n -norm (G_N, B_N, Y_N) if for $0 < \varepsilon < 1$ and $\tilde{\xi} > \tilde{0} \ni n_0 \in \mathbb{N}$ s.t for all $k, \gamma \geq n_0$

$$G_N(w_{e_k}^k \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) > 1 - \varepsilon,$$

$$B_N(w_{e_k}^k \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) < \varepsilon,$$

$$Y_N(w_{e_k}^k \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) < \varepsilon.$$

3 Statistical Summability of order β in $NS-n-NLS$

In this section we introduce and study statistical summability of order β in $NS-n-NLS$ and develop some of its properties. We begin with our main definition.

Definition 3.1. For $0 < \beta \leq 1$, a sequence $w = (w_{e_k}^k)$ of soft points in $\tilde{W}$ is said to statistical convergent of order β (or $S_\beta[G_N, B_N, Y_N]$ -summable) to the soft point w_e in $\tilde{W}$ if for every

$0 < \varepsilon < 1$ and $\tilde{\xi} > \tilde{0}$

$$\lim_{n \rightarrow \infty} \frac{1}{n^\beta} \left| \left\{ k \leq n : G_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \leq 1 - \varepsilon \text{ or } \right. \right. \\ \left. \left. B_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon, \right. \right. \\ \left. \left. Y_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon \right\} \right| = 0,$$

or equivalently

$$\psi_\beta \left(\left\{ k \in \mathbb{N} : G_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \leq 1 - \varepsilon \text{ or } \right. \right. \\ \left. \left. B_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon, \right. \right. \\ \left. \left. Y_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon \right\} \right) = 0.$$

In present case, we write $S_\beta[G_N, B_N, Y_N] - \lim_{n \rightarrow \infty} w_{e_k}^k = w_e$.

$$\text{Let, } S_\beta[G_N, B_N, Y_N] = \left\{ w = (w_{e_k}^k) \in \widetilde{W} : S_\beta[G_N, B_N, Y_N] - \lim_{n \rightarrow \infty} w_{e_k}^k \text{ exists} \right\}$$

denotes the spaces of S_β -summable sequences of soft point in $\widetilde{W}$.

The following lemma is the direct consequences of Definition 3.1

Lemma 3.2. *For any sequence $w = (w_{e_k}^k)$ of soft points in $\widetilde{W}$ and $0 < \beta \leq 1$, the following statements are true:*

$$(i) S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} w_{e_k}^k = w_e;$$

$$(ii) \psi_\beta \left\{ k \in \mathbb{N} : G_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \leq 1 - \varepsilon \right\} = \\ \psi_\beta \{ k \in \mathbb{N} : B_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon \} = \\ \psi_\beta \{ k \in \mathbb{N} : Y_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon \} = 0;$$

$$(iii) \psi_\beta \left\{ k \in \mathbb{N} : G_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) > 1 - \varepsilon \text{ and } \right. \\ \left. B_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) < \varepsilon, \right. \\ \left. Y_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) < \varepsilon \right\} = 1;$$

$$(iv) \psi_\beta \left\{ k \in \mathbb{N} : G_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) > 1 - \varepsilon \right\} = \\ \psi_\beta \{ k \in \mathbb{N} : B_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) < \varepsilon \} = \\ \psi_\beta \{ k \in \mathbb{N} : Y_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) < \varepsilon \} = 1;$$

$$(v) S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} G_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) = 1 \text{ and } \\ S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} B_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) = \\ S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} Y_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) = 0.$$

We now give the following theorems: first is on uniqueness of limit if exists, and the second is from algebraic point of view of limits.

Theorem 3.3. *For any sequence $w=(w_{e_k}^k)$ in $\widetilde{W}$ and $0 < \beta \leq 1$, if $S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} w_{e_k}^k$ exists, then it is unique.*

Proof. If possible, let $S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} w_{e_k}^k = w_{e_1}$ and $S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} w_{e_k}^k = w_{e_2}'$. Given $0 < \varepsilon < 1$, choose $\varepsilon_1 > 0$ such that

$$(1 - \varepsilon_1) \circ (1 - \varepsilon_1) > 1 - \varepsilon \text{ and } \varepsilon_1 \star \varepsilon_1 < \varepsilon \quad (3.1)$$

For any $\tilde{\xi} > 0$ and $w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1} \in \widetilde{W}$

$$\begin{aligned} A_{G_N,1}(\varepsilon_1, \tilde{\xi}) &= \left\{ k \in \mathbb{N} : G_N \left(w_{e_k}^k \ominus w_{e_1}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2} \right) \leq 1 - \varepsilon_1 \right\}, \\ A_{G_N,2}(\varepsilon_1, \tilde{\xi}) &= \left\{ k \in \mathbb{N} : G_N \left(w_{e_k}^k \ominus w_{e_2}', v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2} \right) \leq 1 - \varepsilon_1 \right\}, \\ A_{B_N,1}(\varepsilon_1, \tilde{\xi}) &= \left\{ k \in \mathbb{N} : B_N \left(w_{e_k}^k \ominus w_{e_1}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2} \right) \geq \varepsilon_1 \right\}, \\ A_{B_N,2}(\varepsilon_1, \tilde{\xi}) &= \left\{ k \in \mathbb{N} : B_N \left(w_{e_k}^k \ominus w_{e_2}', v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2} \right) \geq \varepsilon_1 \right\}, \\ A_{Y_N,1}(\varepsilon_1, \tilde{\xi}) &= \left\{ k \in \mathbb{N} : Y_N \left(w_{e_k}^k \ominus w_{e_1}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2} \right) \geq \varepsilon_1 \right\}, \\ A_{Y_N,2}(\varepsilon_1, \tilde{\xi}) &= \left\{ k \in \mathbb{N} : Y_N \left(w_{e_k}^k \ominus w_{e_2}', v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2} \right) \geq \varepsilon_1 \right\}. \end{aligned}$$

Since $S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} w_{e_k}^k = w_{e_1}$, by above lemma, we have $\psi_\beta(A_{G_N,1}(\varepsilon_1, \tilde{\xi})) = \psi_\beta(A_{B_N,1}(\varepsilon_1, \tilde{\xi})) = \psi_\beta(A_{Y_N,1}(\varepsilon_1, \tilde{\xi})) = 0$ and therefore $\psi_\beta(A_{G_N,1}^{\mathbb{G}}(\varepsilon_1, \tilde{\xi})) = \psi_\beta(A_{B_N,1}^{\mathbb{G}}(\varepsilon_1, \tilde{\xi})) = \psi_\beta(A_{Y_N,1}^{\mathbb{G}}(\varepsilon_1, \tilde{\xi})) = 1$. Also using $S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} w_{e_k}^k = w_{e_2}'$, so $\psi_\beta(A_{G_N,2}(\varepsilon_1, \tilde{\xi})) = \psi_\beta(A_{B_N,2}(\varepsilon_1, \tilde{\xi})) = \psi_\beta(A_{Y_N,2}(\varepsilon_1, \tilde{\xi})) = 0$ and therefore $\psi_\beta(A_{G_N,2}^{\mathbb{G}}(\varepsilon_1, \tilde{\xi})) = \psi_\beta(A_{B_N,2}^{\mathbb{G}}(\varepsilon_1, \tilde{\xi})) = \psi_\beta(A_{Y_N,2}^{\mathbb{G}}(\varepsilon_1, \tilde{\xi})) = 1$ for all $\tilde{\xi} > 0$. Define $\mathcal{K}_{G_N, B_N, Y_N}(\varepsilon, \tilde{\xi}) = \{A_{G_N,1}(\varepsilon_1, \tilde{\xi}) \cup A_{G_N,2}(\varepsilon_1, \tilde{\xi})\} \cap \{A_{B_N,1}(\varepsilon_1, \tilde{\xi}) \cup A_{B_N,2}(\varepsilon_1, \tilde{\xi})\} \cap \{A_{Y_N,1}(\varepsilon_1, \tilde{\xi}) \cup A_{Y_N,2}(\varepsilon_1, \tilde{\xi})\}$, then $\psi_\beta\{\mathcal{K}_{G_N, B_N, Y_N}(\varepsilon, \tilde{\xi})\} = 0$ and therefore $\psi_\beta\{\mathcal{K}_{G_N, B_N, Y_N}^{\mathbb{G}}(\varepsilon, \tilde{\xi})\} = 1$. Let $m \in \mathcal{K}_{G_N, B_N, Y_N}^{\mathbb{G}}(\varepsilon, \tilde{\xi})$, then we have following possibilities.

- (i) $m \in \{A_{G_N,1}(\varepsilon_1, \tilde{\xi}) \cup A_{G_N,2}(\varepsilon_1, \tilde{\xi})\}^{\mathbb{G}}$;
- (ii) $m \in \{A_{B_N,1}(\varepsilon_1, \tilde{\xi}) \cup A_{B_N,2}(\varepsilon_1, \tilde{\xi})\}^{\mathbb{G}}$;
- (iii) $m \in \{A_{Y_N,1}(\varepsilon_1, \tilde{\xi}) \cup A_{Y_N,2}(\varepsilon_1, \tilde{\xi})\}^{\mathbb{G}}$.

Case1: Let $m \in \left\{ A_{G_N,1}(\varepsilon_1, \tilde{\xi}) \cup A_{G_N,2}(\varepsilon_1, \tilde{\xi}) \right\}^{\mathbb{G}}$, then $m \in A_{G_N,1}^{\mathbb{G}}(\varepsilon_1, \tilde{\xi})$ and $m \in A_{G_N,2}^{\mathbb{G}}(\varepsilon_1, \tilde{\xi})$ and therefore,

$$\begin{aligned} G_N \left(w_{e_m}^m \ominus w_{e_1}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2} \right) &> 1 - \varepsilon_1 \text{ and} \\ G_N \left(w_{e_m}^m \ominus w_{e_2}', v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2} \right) &> 1 - \varepsilon_1. \end{aligned} \quad (3.2)$$

Now

$$\begin{aligned}
 G_N(w_{e_1} \ominus w'_{e_2}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) &= G_N(w_{e_m}^m \ominus w_{e_m}^m \oplus w_{e_1} \ominus w'_{e_2}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \\
 &\geq G_N\left(w_{e_m}^m \ominus w_{e_1}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) \circ G_N\left(w_{e_m}^m \ominus w'_{e_2}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) \\
 &> (1 - \varepsilon_1) \circ (1 - \varepsilon_1) && \text{by(3.2)} \\
 &> 1 - \varepsilon. && \text{by(3.1)}
 \end{aligned}$$

Given that $\varepsilon > 0$ is arbitrary, we now obtain $G_N(w_{e_1} \ominus w'_{e_2}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) = 1 \forall \tilde{\xi} > \tilde{0}$, which gives $w_{e_1} \ominus w'_{e_2} = \tilde{0}$ i.e., $w_{e_1} = w'_{e_2}$.

Case2: Let $m \in \left\{A_{B_N,1}(\varepsilon_1, \tilde{\xi}) \cup A_{B_N,2}(\varepsilon_1, \tilde{\xi})\right\}^c$, then $m \in A_{B_N,1}^c(\varepsilon_1, \tilde{\xi})$ and $m \in A_{B_N,2}^c(\varepsilon_1, \tilde{\xi})$ and therefore,

$$\begin{aligned}
 B_N\left(w_{e_m}^m \ominus w_{e_1}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) &< \varepsilon_1 \text{ and} \\
 B_N\left(w_{e_m}^m \ominus w'_{e_2}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) &< \varepsilon_1.
 \end{aligned} \tag{3.3}$$

Now

$$\begin{aligned}
 B_N(w_{e_1} \ominus w'_{e_2}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) &= B_N(w_{e_m}^m \ominus w_{e_m}^m \oplus w_{e_1} \ominus w'_{e_2}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \\
 &\leq B_N\left(w_{e_m}^m \ominus w_{e_1}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) \star B_N\left(w_{e_m}^m \ominus w'_{e_2}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) \\
 &< \varepsilon_1 \star \varepsilon_1 && \text{by(3.3)} \\
 &< \varepsilon. && \text{by(3.1)}
 \end{aligned}$$

Given that $\varepsilon > 0$ is arbitrary, we now obtain $B_N(w_{e_1} \ominus w'_{e_2}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) = 1$ for all $\tilde{\xi} > \tilde{0}$, which gives $w_{e_1} \ominus w'_{e_2} = \tilde{0}$ i.e., $w_{e_1} = w'_{e_2}$.

Similarly, $w_{e_1} = w'_{e_2}$ in Case3. Hence, in all cases we get $w_{e_1} = w'_{e_2}$. $\square$

Theorem 3.4. Let $0 < \beta \leq 1$ and $w = (w_{e_k}^k)$ and $u = (u_{e_k}^k)$ be any two sequences in $\widetilde{W}$ such that $S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} (w_{e_k}^k) = w_{e_1}$ and $S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} (u_{e_k}^k) = u_{e_2}$. Then

- (i) $S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} (w_{e_k}^k \oplus u_{e_k}^k) = w_{e_1} \oplus u_{e_2}$
- (ii) $S_\beta[G_N, B_N, Y_N] - \lim_{k \rightarrow \infty} (\tilde{\alpha} w_{e_k}^k) = \tilde{\alpha} w_{e_1}$, where $\tilde{0} \neq \tilde{\alpha} \in \mathbf{F}$.

Proof. The proof of this theorem can be obtained easily analogous to the above Theorem so omitted. $\square$

We now give the following characterization of the S_β -summable.

Theorem 3.5. Let $w = (w_{e_k}^k)$ and $u = (u_{e_k}^k)$ in $\widetilde{W}$ such that $u_{e_k}^k \rightarrow w_{e_1}(G_N, B_N, Y_N)$ and $\psi_\beta(k \in \mathbb{N} : w_{e_k}^k \neq u_{e_k}^k) = 0$. Then $(w_{e_k}^k) \rightarrow w_{e_1} S_\beta[G_N, B_N, Y_N]$.

Proof. Suppose that $(w_{e_k}^k)$ are such that $\psi_\beta(k \in \mathbb{N} : w_{e_k}^k \neq u_{e_k}^k) = 0$ holds and $u_{e_k}^k \rightarrow w_{e_1}(G_N, B_N, Y_N)$. Thus by definition, $\forall 0 < \varepsilon < 1, \tilde{\xi} > 0$ and $v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1} \in \widetilde{W}$, the set

$$\begin{aligned}
 A_\varepsilon &= \left\{k \in \mathbb{N} : G_N(u_{e_k}^k \ominus w_{e_1}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \leq 1 - \varepsilon, \right. \\
 &\quad B_N(u_{e_k}^k \ominus w_{e_1}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon \text{ and} \\
 &\quad \left. Y_N(u_{e_k}^k \ominus w_{e_1}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon \right\}
 \end{aligned}$$

consist of finitely many numbers of elements and as a result, we have $\psi_\beta(A_\varepsilon) = 0$. Now, as the containment

$$\begin{aligned} B_\varepsilon = & \left\{ k \in \mathbb{N} : G_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \leq 1 - \varepsilon, \right. \\ & B_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon \text{ and} \\ & \left. Y_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon \right\} \\ & \subseteq A_\varepsilon \cap \{k \in \mathbb{N} : w_{e_k}^k \neq u_{e_k}^k\} \end{aligned}$$

holds, so we must have $\psi_\beta(B_\varepsilon) = 0$. This shows that $w_{e_k}^k \rightarrow w_{e_1} S_\beta[G_N, B_N, Y_N]$. $\square$

Definition 3.6. A sequence $w = (w_{e_k}^k)$ of soft points in $\widetilde{W}$ is said to be statistical Cauchy sequence if for $0 < \varepsilon < 1$ and $\tilde{\xi} > \tilde{0}$, $\exists \gamma \in \mathbb{N}$ such that

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n^\beta} \left| \{k \leq n : G_N(w_{e_k}^k \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \leq 1 - \varepsilon \text{ or} \right. \\ B_N(w_{e_k}^k \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon, \\ \left. Y_N(w_{e_k}^k \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon\} \right| = 0, \end{aligned}$$

or

$$\begin{aligned} \psi_\beta \left(\{k \in \mathbb{N} : G_N(w_{e_k}^k \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \leq 1 - \varepsilon \text{ or} \right. \\ B_N(w_{e_k}^k \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon, \\ \left. Y_N(w_{e_k}^k \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon\} \right) = 0. \end{aligned}$$

Theorem 3.7. Every statistically summable sequence of order β in $\widetilde{W}$ is statistical Cauchy.

Proof. Let $w = (w_{e_k}^k)$ be any statistical convergent sequence of order β in $\widetilde{W}$ with $S_\beta - \lim_{k \rightarrow \infty} w_{e_k}^k = w_e$.

For $\varepsilon > 0$ and $\tilde{\xi} > \tilde{0}$. Choose $\varepsilon_1 > 0$ such that

$$(1 - \varepsilon_1) \circ (1 - \varepsilon_1) > 1 - \varepsilon \text{ and } \varepsilon_1 \star \varepsilon_1 < \varepsilon \quad (3.4)$$

Define a set,

$$\begin{aligned} K(\varepsilon_1, \tilde{\xi}) = & \{k \in \mathbb{N} : G_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}) \leq 1 - \varepsilon_1 \text{ or} \\ & B_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}) \geq \varepsilon_1, \\ & Y_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}) \geq \varepsilon_1\}, \end{aligned}$$

then

$$\begin{aligned} K^{\mathbb{G}}(\varepsilon_1, \tilde{\xi}) = & \{k \in \mathbb{N} : G_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}) > 1 - \varepsilon_1 \text{ or} \\ & B_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}) < \varepsilon_1, \\ & Y_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}) < \varepsilon_1\}. \end{aligned}$$

Since $S_\beta - \lim_{n \rightarrow \infty} w_{e_k}^k = w_e$, so $\psi_\beta(K(\varepsilon_1, \tilde{\xi})) = 0$ and $\psi_\beta(K^{\mathbb{G}}(\varepsilon_1, \tilde{\xi})) = 1$. Let $\gamma \in K^{\mathbb{G}}(\varepsilon_1, \tilde{\xi})$, then

$$\begin{aligned} G_N\left(w_{e_\gamma}^\gamma \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) &> 1 - \varepsilon_1 \text{ and} \\ B_N\left(w_{e_\gamma}^\gamma \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) &< \varepsilon_1, \\ Y_N\left(w_{e_\gamma}^\gamma \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) &< \varepsilon_1. \end{aligned} \quad (3.5)$$

Now let

$$\begin{aligned} T(\varepsilon, \tilde{\xi}) &= \{k \in \mathbb{N} : G_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}) \leq 1 - \varepsilon_1 \text{ or} \\ &\quad B_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}) \geq \varepsilon_1, \\ &\quad Y_N(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}) \geq \varepsilon_1\} \end{aligned}$$

then we show that $T(\varepsilon, \tilde{\xi}) \subseteq K(\varepsilon_1, \tilde{\xi})$. Let $m \in T(\varepsilon, \tilde{\xi})$, then

$$\begin{aligned} G_N\left(w_{e_m}^m \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}\right) &\leq 1 - \varepsilon \text{ or} \\ B_N\left(w_{e_m}^m \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}\right) &\geq \varepsilon, \\ Y_N\left(w_{e_m}^m \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}\right) &\geq \varepsilon. \end{aligned} \quad (3.6)$$

If $G_N(w_{e_m}^m \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \leq 1 - \varepsilon$, then $G_N(w_{e_m}^m \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}) \leq 1 - \varepsilon_1$ and so $m \in T(\varepsilon_1, \tilde{\xi})$. As otherwise, if $G_N(w_{e_m}^m \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}) > 1 - \varepsilon_1$, then by (3.4), (3.5) and (3.6) we get

$$\begin{aligned} 1 - \varepsilon &\geq G_N(w_{e_m}^m \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) = G_N\left(w_{e_m}^m \ominus w_e \oplus w_e \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, \right. \\ &\quad \left. v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2} \oplus \frac{\tilde{\xi}}{2}\right) \\ &\geq G_N\left(w_{e_m}^m \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) \circ G_N\left(w_{e_\gamma}^\gamma \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, \right. \\ &\quad \left. v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) \\ &> (1 - \varepsilon_1) \circ (1 - \varepsilon_1) \\ &> (1 - \varepsilon), \end{aligned}$$

which is not possible. Thus, $T(\varepsilon, \tilde{\xi}) \subseteq K(\varepsilon_1, \tilde{\xi})$.

Similarly, if $B_N(w_{e_m}^m \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon$, then $m \in T(\varepsilon_1, \tilde{\xi})$ and if $Y_N(w_{e_m}^m \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) \geq \varepsilon$, then $m \in T(\varepsilon_1, \tilde{\xi})$. So in all the cases $T(\varepsilon, \tilde{\xi}) \subseteq K(\varepsilon_1, \tilde{\xi})$ and therefore $\psi_\beta(T(\varepsilon, \tilde{\xi})) = 0$ as $\psi_\beta(K(\varepsilon_1, \tilde{\xi})) = 0$. This shows that $w = (w_{e_k}^k)$ is Cauchy. $\square$

Definition 3.8. A $NS - n - NLS : \tilde{W}$ is said to be statistical complete of order β with respect to the neutrosophic soft $-n$ -norm (G_N, B_N, Y_N) if every statistically Cauchy sequence of order β is S_β -summable in $\tilde{W}$.

Theorem 3.9. $\widetilde{W}$ is statistical complete if every statistical Cauchy sequence of order β in $\widetilde{W}$ has a S -summable subsequence of order β .

Proof. Let $w = (w_{e_k}^k)$ be statistical Cauchy sequence of order β in $\widetilde{W}$ which has a statistical convergent subsequence $(w_{e_{k(j)}}^{k(j)})$ i.e., $S_\beta - \lim_{j \rightarrow \infty} w_{e_{k(j)}}^{k(j)} = w_e$ for some w_e in $\widetilde{W}$. Since $w = (w_{e_k}^k)$ is statistically Cauchy of order β , so for $\varepsilon > 0$, $\widetilde{W} > \widetilde{0}$ and ε_1 satisfying (4) we have, $\psi_\beta(P) = 0$ where

$$P = \left\{ k \in \mathbb{N} : G_N \left(w_{e_k}^k \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) \leq 1 - \varepsilon_1 \text{ or} \right. \\ \left. B_N \left(w_{e_k}^k \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) \geq \varepsilon_1, \right. \\ \left. Y_N \left(w_{e_k}^k \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) \geq \varepsilon_1 \right\}.$$

Again since $S_\beta - \lim_{j \rightarrow \infty} w_{e_{k(j)}}^{k(j)} = w_e$. So we have $\psi_\beta(D) = 0$, where

$$D = \left\{ k(j) \in \mathbb{N} : G_N \left(w_{e_{k(j)}}^{k(j)} \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) \leq 1 - \varepsilon_1 \text{ or} \right. \\ \left. B_N \left(w_{e_{k(j)}}^{k(j)} \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) \geq \varepsilon_1, \right. \\ \left. Y_N \left(w_{e_{k(j)}}^{k(j)} \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) \geq \varepsilon_1 \right\}.$$

Now define

$$R = \left\{ k \in \mathbb{N} : G_N \left(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) \leq 1 - \varepsilon \text{ or} \right. \\ \left. B_N \left(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) \geq \varepsilon, \right. \\ \left. Y_N \left(w_{e_k}^k \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) \geq \varepsilon \right\}.$$

Now we claim that $P^{\mathbb{C}} \cap D^{\mathbb{C}} \subseteq R^{\mathbb{C}}$. Let $m \in P^{\mathbb{C}} \cap D^{\mathbb{C}}$, then $m \in P^{\mathbb{C}}$ and $m \in D^{\mathbb{C}}$. If $m \in P^{\mathbb{C}}$, then

$$G_N \left(w_{e_m}^m \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) \leq 1 - \varepsilon_1 \text{ and} \\ B_N \left(w_{e_m}^m \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) \geq \varepsilon_1, \quad (3.7) \\ Y_N \left(w_{e_m}^m \ominus w_{e_\gamma}^\gamma, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) \geq \varepsilon_1,$$

and if $m \in D^{\mathbb{C}}$, then $m = k(j_0)$ for $j_0 \in \mathbb{N}$ and

$$G_N \left(w_{e_{k(j_0)}}^{k(j_0)} \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) > 1 - \varepsilon_1 \text{ and} \\ B_N \left(w_{e_{k(j_0)}}^{k(j_0)} \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) < \varepsilon_1, \quad (3.8) \\ Y_N \left(w_{e_{k(j_0)}}^{k(j_0)} \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\widetilde{\xi}}{2} \right) < \varepsilon_1$$

Now

$$\begin{aligned}
 G_N(w_{e_m}^m \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) &= G_N\left(w_{e_m}^m \ominus w_{e_{k(j_0)}}^{k(j_0)} \oplus w_{e_{k(j_0)}}^{k(j_0)} \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \right. \\
 &\quad \left. \frac{\tilde{\xi}}{2} \oplus \frac{\tilde{\xi}}{2}\right) \\
 &\geq G_N\left(w_{e_m}^m \ominus w_{e_{k(j_0)}}^{k(j_0)}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) \circ G_N\left(w_{e_{k(j_0)}}^{k(j_0)} \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \right. \\
 &\quad \left. \frac{\tilde{\xi}}{2}\right) \\
 &> (1 - \varepsilon_1) \circ (1 - \varepsilon_1) \text{ for } \gamma = k(j_0) \\
 &> 1 - \varepsilon,
 \end{aligned}$$

and

$$\begin{aligned}
 B_N(w_{e_m}^m \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) &= B_N\left(w_{e_m}^m \ominus w_{e_{k(j_0)}}^{k(j_0)} \oplus w_{e_{k(j_0)}}^{k(j_0)} \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \right. \\
 &\quad \left. \frac{\tilde{\xi}}{2} \oplus \frac{\tilde{\xi}}{2}\right) \\
 &\leq B_N\left(w_{e_m}^m \ominus w_{e_{k(j_0)}}^{k(j_0)}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) \star B_N\left(w_{e_{k(j_0)}}^{k(j_0)} \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \right. \\
 &\quad \left. \frac{\tilde{\xi}}{2}\right) \\
 &< \varepsilon_1 \star \varepsilon_1 \text{ for } \gamma = k(j_0) \\
 &< \varepsilon,
 \end{aligned}$$

$$\begin{aligned}
 Y_N(w_{e_m}^m \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \tilde{\xi}) &= Y_N\left(w_{e_m}^m \ominus w_{e_{k(j_0)}}^{k(j_0)} \oplus w_{e_{k(j_0)}}^{k(j_0)} \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \right. \\
 &\quad \left. \frac{\tilde{\xi}}{2} \oplus \frac{\tilde{\xi}}{2}\right) \\
 &\leq Y_N\left(w_{e_m}^m \ominus w_{e_{k(j_0)}}^{k(j_0)}, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \frac{\tilde{\xi}}{2}\right) \star Y_N\left(w_{e_{k(j_0)}}^{k(j_0)} \ominus w_e, v_{e_1}^1, v_{e_2}^2, \dots, v_{e_{n-1}}^{n-1}, \right. \\
 &\quad \left. \frac{\tilde{\xi}}{2}\right) \\
 &< \varepsilon_1 \star \varepsilon_1 \text{ for } \gamma = k(j_0) \\
 &< \varepsilon, \quad \text{by (3.7) and (3.8)}
 \end{aligned}$$

which implies that $m \in R^{\mathcal{G}}$, so $P^{\mathcal{G}} \cap D^{\mathcal{G}} \subseteq R^{\mathcal{G}}$ or $R \subseteq P \cup D$. Therefore, $\psi_{\beta}(R) \leq \psi_{\beta}(P \cup D) = 0$. This shows that $w = (w_{e_k}^k)$ is statistically convergent and complete. $\square$

4 Conclusion remarks

Neutrosophic soft norm is a powerful tool to estimate length and differences in many problems including decision making, medical diagnose and data mining which can be parameterized and contain huge uncertainty. In this paper a new method of summability is given in more general setting i.e. neutrosophic soft- n - normed linear spaces. The results presented in this paper will be helpful to resolve many convergence related problems where neutrosophic norm is not sufficient and we look forward towards a more general setting i.e. neutrosophic soft- n - norm due to parameterized family.

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