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COLLOCATION METHODS GENERAL APPROACH FOR ORDINARY DIFFERENTIAL EQUATIONS

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Abstract The article discusses generalized method for solving ordinary differential equation using wavelet collocation method. The method is implemented for both the initial valued problems as well as boundary valued problems. We can directly solve the boundary valued problem as against the traditional shooting methods where the boundary valued problem itself is approximated by Initial valued problem and hence we achieve better accuracy with wavelet collocation methods.

1 Introduction

The theory of wavelets is being utilized for the past two decades in various applications of signal processing, fingerprint verification [7], storing fingerprint electronically using wavelet, denoising data , musical tones, etc see [8] and solution of differential equations [5],[3], [4]. Wavelets have several properties which are encouraging their use for numerical solutions of differential equations. The orthogonal, compactly supported wavelet basis exactly approximates polynomial of increasingly higher order. This wavelet basis can provide an accurate and stable representation of differential operations even in region of strong gradients or oscillations. In addition, the orthogonal wavelet basis has the inherent advantage of multi resolution analysis over the traditional methods [4]. The adaptive wavelet collocation method is able to dynamically track the evolution of the solution's irregular features and to allocate higher grid density to the necessary regions. Therefore, the number of collocation points needed is optimized, without damaging the accuracy of the solution [5]. The benefit of Haar wavelet approach is their sparse matrices representation, fast transformation and possibility of implementation of fast algorithms [8].

In this article we brief the collocation method used for solving initial valued problem and boundary valued problem with simple examples to establish a common unified approach of approximating derivative using wavelet function as basis.

In this method higher order derivative is approximated using wavelet function and the lower order derivatives and functions itself are expressed by repeated integration. The orthogonal set of Haar functions is used. This group of square waves has magnitude ± 1 in some interval and zero elsewhere. These zeros make Haar transform faster than other square functions such as Walshs functions. Haar wavelet basis lacks differentiability and hence the integration approach will be used instead of the differentiability for calculation of coefficients. Due to the local property of the powerful Haar wavelet the new method is simpler.

This article has been organized in the following way. Section 2 discusses the prerequisites for understanding Haar wavelets as a basis function and their properties. Section 3 describes how a function can be represented using Haar wavelets as basis function. Section 4 discusses initial value problem formation using haar wavelets. Section 5 describes solution for boundary value problem with four different types of boundary conditions. Section 6 explains a common procedure to handle examples of both initial value and boundary value problem. Section 9 explains normalization procedure for a general solution of IVP and BVP within domain $[A, B]$. Section [11] and 8 discusses examples of IVP and BVP respectively. Section 10 describes transformation of a boundary valued problem using the general transformation discussed in section 9. Section 11 explains along with an example of IVP, how to extend the solution to a specified value with increased resolution , since the basis is valid within $[0, 1]$. The examples are validated using plots utilizing Matlab code.

2 Basis wavelet

Haar wavelets are considered as basis wavelets in the discussion, haar transform has been used as an earliest example for orthonormal wavelet transform with compact support. Haar wavelet transform has been used as an earliest example for orthonormal wavelet transform with compact support. The Haar wavelet transform is the first known wavelet and was proposed in 1909 by Alfred Haar. The Haar wavelet family for,$x \in [0, 1)$ Is defined by,

$$h_i(x) = \begin{cases} 1 & \text{if } x \in \alpha, \beta) \\ -1 & \text{if } x \in [\beta, \gamma) \\ 0 & \text{elsewhere.} \end{cases} \tag{2.1}$$

here,

$$\alpha = \frac{k}{m} \quad \beta = \frac{k + 0.5}{m} \quad \gamma = \frac{k + 1}{m} \tag{2.2}$$

here $m = 2^j, j = 0, 1, , J.$ indicates the levels of wavelet and integer $k = 0, 1, .., (m - 1)$, the shift parameter. Maximum resolution is J, and $i = m + k + 1$. Incase of minimum value $m = 1, k = 0, i = 2$ The maximal value of i is $i = 2M = 2J + 1$ The function

$$h_1(x) = \begin{cases} 1 & x \in [0,\ 1) \\ 0 & elsewhere. \end{cases} \tag{2.3}$$

From equation (2)

$$h_2(x) = \begin{cases} 1 & \text{if } x \in 0, 0.5) \\ -1 & \text{if } x \in [0.5, 1) \\ 0 & \text{elsewhere.} \end{cases} \tag{2.4}$$

which can be graphically vizualized as,

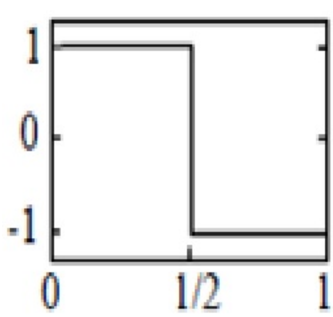

Figure 1. Haar function

The equation 2.3 is also called mother wavelet. In order to perform wavelet transform, Haar wavelet uses translations and dilations of the function, i.e. the transformation uses the following function

$$h(x) = h(2^j - k) \tag{2.5}$$

Translation/Shifting with $h(x) = h(x - k)$
Dilation/Scaling with $h(x) = h(2^j x)$ where this is the basic work for wavelet expansion. With

the dilation and translation process as in Eq.(2.5), one can easily obtain father wavelet, daughter wavelet, granddaughter wavelet etc.

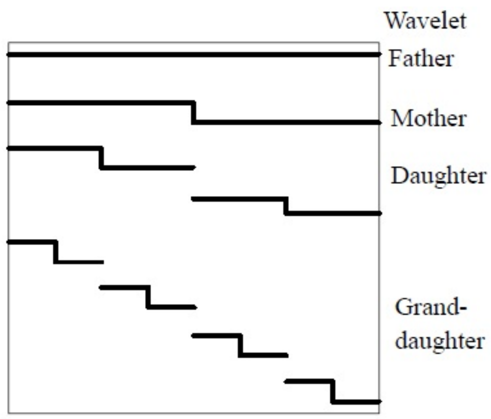

Figure 2. Haar wavelets representation upto second resolution

We can obtain coefficient matrix H of order $2m \times 2m$ as

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

for $m = 2$, The Haar wavelets are orthogonal as, i.e. The operational matrix P which is a $2m$ square matrix is define as in [12] by

$$p_{i,1}(x) = \int_0^x h_i(x')dx' \tag{2.6}$$

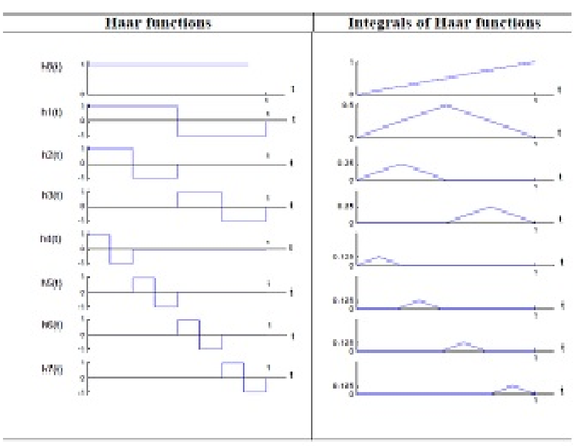

Figure 3. Integratral representation of Haar wavelets

and the recurrence relation is given by

$$p_{i,v+1}(x) = \int_0^x p_{i,v}(x')dx' \quad where\ v = 1,\ 2,\ \ldots \tag{2.7}$$

We will need the integral

$$P(x) = \underbrace{\int_A^x \int_A^x ... \int_A^x h_i(t)dt^u} = \frac{1}{(u-1)!} \int_A^x (x-t)^{u-1} h_i(t)dt \tag{2.8}$$

with $u = 2, 3, ..., n$ and $i = 1, 2, ..., 2m$ The above integrals can be evaluated using equation 2 the first two are given by

$$p_{i,1}(x) = \begin{cases} x - \alpha & \text{for } x \in [\alpha, \beta) \\ \gamma - x & \text{for } x \in [\beta, \gamma) \\ 0 & \text{elsewhere.} \end{cases} \tag{2.9}$$

$$p_{i,2}(x) \quad = \quad \begin{cases} \frac{1}{2}(x-\alpha)^2, & \text{for } x \in [\alpha, \beta) \\ \frac{1}{4m^2} - \frac{1}{2}(\gamma - x)^2, & \text{for } x \in [\beta, \gamma) \\ \frac{1}{4m^2}, & \text{for } x \in [\gamma, 1) \\ 0 & \text{elsewhere.} \end{cases} \tag{2.10}$$

and so on will be utilized in representing the derivative and function values in the further discussion.

3 Function representation

Considering the fact that Haar functions are orthogonal, we may take any function $f(x)$ which is square integrable in the interval $[0, 1)$ as an infinite sum of Haar wavelets,

$$f(x) = \sum_{i=1}^{\infty} a_i h_i(x) \tag{3.1}$$

where a_i are haar coefficients and $h_i(x)$ are haar wavelet functions. $f(x)$ has finite terms and if $f(x)$ is piecewise constant or can be approximated as piecewise constant during each subinterval as,

$$f(x) = \sum_{i=1}^{2M} a_i h_i(x) \tag{3.2}$$

In this scheme the highest order derivative of the function is approximated by haar wavelets and the consecutive lower order derivatives and function itself is obtained by repeated integration as explained in the section IV and V.

4 Initial Value Problem IVP

Consider the general nth order linear differential equation

$$M_1 y^n(x) + M_2 y^{(n-1)}(x) + ... + M_n y(x) = f \tag{4.1}$$

for $x \in [A, B]$ with initial condition

$$y^{(n-1)}(A), \quad y^{(n-2)}(A), ..., y(A) \tag{4.2}$$

and all lower order derivative approximations are obtained by repeated integrals. For example rth order derivative of y is obtained as

$$y^r(x) = \sum_{i=1}^{2M} a_i p_{i,n-r}(x) + \sum_{\sigma=0}^{n-r-1} \frac{1}{\sigma!}(x-A)^{\sigma} y_0^{(r+\sigma)} \tag{4.3}$$

We obtain $y^{(n-1)}(x), y^{(n-2)}(x), ... \quad and \quad y(x)$ The collocation points are obtained by

$$x_p = \frac{(p-\frac{1}{2})}{2M}, \quad p = 1,\ 2,\ ...\ 2M \tag{4.4}$$

The expressions of $y^n(x),\ \ y^{(n-1)}(x),$ and $y(x)$ are substituted in differential equation, discretization is applied along the points given by equation (4.4) resulting in a linear or non linear system of $2M \times 2M$. Solving the system for haar coefficients the approximate solution is achieved.

5 Boundary value problems BVP

Consider a second order boundary valued problem

$$y"(x) = f(x, y, y') \tag{5.1}$$

for $x \in [0, 1]$ For second order ordinary differential equations, there are four different types of boundary conditions possible. They are treated differently as follows

Case 1 $y(0) = R$ and $y(1) = Q$ then integrating equation (5.1) yields
$y'(x) = \sum_{i=1}^{2M} a_i p_{i,1}(x) + y'(0) \quad$ as $p_{i,1}(0) = 0$.
Integrating again and using condition $y(0) = R$ we get

$$y(x) = R + y'(0)x + \sum_{i=1}^{2M} a_i p_{i,2}(x). \tag{5.2}$$

Now utilizing second condition $y(1) = Q$ we obtain $y'(0) = (Q - R) - \sum_{i=1}^{2M} a_i c_{i1}$ with $c_{i1} = \int_0^1 p_{i,1}(x)dx$ further simplifying we get

$$y(x) = R + (Q - R)x + \sum_{i=1}^{2M} a_i(p_{i,2}(x) - xc_{i1}) \tag{5.3}$$

and

$$y'(x) = Q - R + \sum_{i=1}^{2M} a_i(p_{i,1}(x) - c_{i1}) \tag{5.4}$$

Case 2 $y'(0) = R_1$ and $y(1) = Q_1$ Integrating equation (5.1) and using boundary condition, $y'(0) = R_1$ we get

$$y'(x) = R_1 + \sum_{i=1}^{2M} a_i p_{i,1}(x) \tag{5.5}$$

$$y(x) = Q_1 - R_1(1 - x) - \sum_{i=1}^{2M} a_i(c_{i1} - p_{i,2}(x)) \tag{5.6}$$

Case 3 $y(0) = R_2$ and $y'(1) = Q_2$ we get

$$y'(x) = Q_2 - a_1 + \sum_{i=1}^{2M} a_i p_{i,1}(x) \tag{5.7}$$

and

$$y(x) = R_2 + (Q_2 - a_1)x + \sum_{i=1}^{2M} a_i(p_{i,2}(x)) \tag{5.8}$$

Case 4 $y'(0) = R_3$ and $y'(1) = Q_3$ here by integrating and applying the first condition we obtain,

$$\begin{aligned} y'(x) &= R_3 + \sum_{i=1}^{2M} a_i p_{i,1}(x) \quad using\ y'(1) \\ (Q_3 - R_3) &= a_1 \quad as \quad p_{i,1}(1) = 1. \end{aligned} \tag{5.9}$$

so we get

$$y''(x) = (Q_3 - R_3)h_1(x) + \sum_{i=2}^{2M} a_i h_i(x) \tag{5.10}$$

$$y'(x) = R_3 + (Q_3 - R_3)p_{11}(x) + \sum_{i=2}^{2M} a_i p_{i1}(x) \tag{5.11}$$

(5.12)

$$y(x) = y(0) + R_3 x + (Q_3 - R_3)p_{12}(x) + \sum_{i=2}^{2M} a_i p_{i2}(x) \tag{5.13}$$

which is obtained by equation (5.9), by integrating from 0 to x. Similarly we can extend the approach to higher order differential equations with boundary conditions.

6 Procedure

- The highest order derivative is approximated by haar wavelet function.
- The successive lower order derivatives and the function itself is replaced by the expressions obtained by repeated integration obtained in (i)
- The algebraic expression in terms of haar coefficients is represented in matrix form.
- The matrix is solved to obtain the haar coefficients a_i which are then substituted in the expression of solution function.

Separate MATLAB routines are generated for computation of the matrix P and C which appear in the algebraic representation. P represents the matrix formed by $P_{i,k}$ and C represents the matrix formed by required $C'_{i,k}s$, where k depends on the order of the equation handled. Now to get a clear idea of the methods we give examples, one each for second order initial valued problem and second order boundary value problem in the next sections.

7 Example 1

Consider an initial valued ordinary differential equation,

$$y'' + y = \sin(x) + x\cos(x) \tag{7.1}$$

with $x \in [0, 1]$ and $y(0) = 1$ and $y'(0) = 1$ The analytic solution is given by

$$y(x) = \cos x + \frac{5}{4}\sin x + \frac{1}{4}(x^2 \sin x - x\cos x) \tag{7.2}$$

Wavelet formulation is obtained by substituting $y''(x) = \sum_{i=1}^{2M} a_i h_i(x)$ and integrating twice this equation we get

$$y(x) = \sum_{i=1}^{2M} a_i p_{i2}(x) + 1 + x \tag{7.3}$$

The differential equation gets converted to

$$\sum_{i=1}^{2M} a_i(h_i(x) + p_{i,2}(x)) = \sin x + x\cos x - 1 - x \qquad (7.4)$$

Solving the system $A[H + P] = B$ we obtain the wavelet coefficient matrix A, where H is the haar matrix, P is the matrix consisting with rows $P'_{i,k}s$ and B is right hand side vector obtained by considering values of x at collocation points. From the expression $y(x)$ of by substituting the wavelet coefficients, the solution function is generated. For j=3, that is m=16. We obtained the result, compared it with analytical solution using MATLAB program. The plot is given in fig (4).

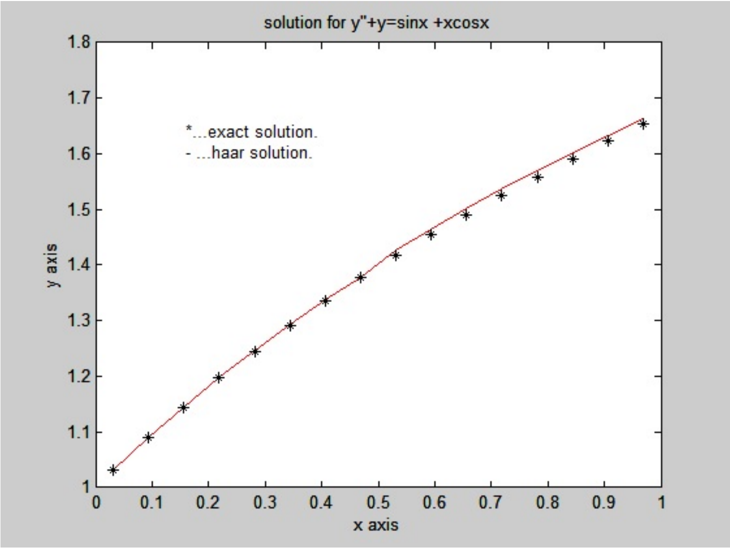

Figure 4. Comparison of haar solution with exact solution for example 1 using $j = 3$

Now fig (5) represents the graph of the haar, analytical and inbuilt function implementation of matlab results for the initial valued problem (Example 1).

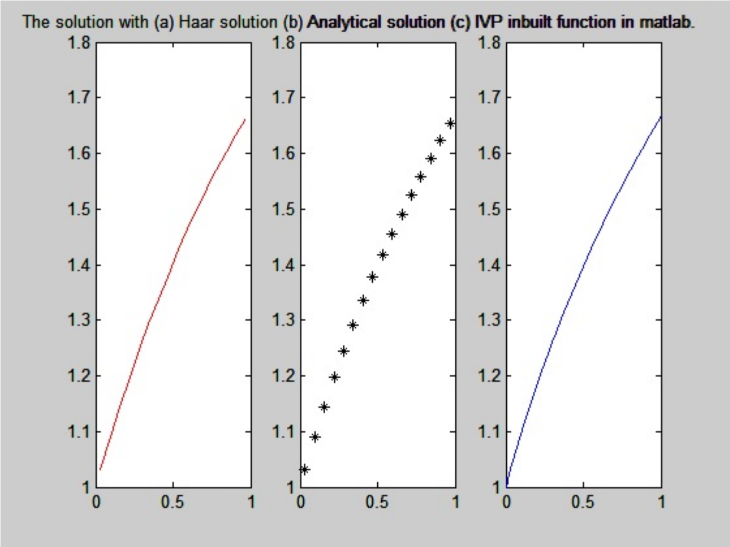

Figure 5. Plot for Haar implementation, analytic and inbuilt solution

8 Example 2

Consider a second order boundary valued problem as $y" = y' + y + \exp x(1-2x)$ with $x \in [0,1]$ with condition $y(0) = 1$ and $y(1) = 3e$ which are the boundary conditions as mentioned in case 1. The analytical solution of this boundary valued problem is $y = \exp x(1+2x)$ Taking the wavelet approximation for second derivative, we obtain

$$\begin{aligned} y'(x) &= 3e - 1 + \sum_{i=1}^{2M} a_i(p_{i,1}(x) - c_{i1}) \quad and \\ y(x) &= 1 + (3e-1)x + \sum_{i=1}^{2M} a_i(p_{i,2}(x) - xc_{i1}) \end{aligned} \tag{8.1}$$

Therefore the boundary valued problem gets transformed as

$$\begin{aligned} &\sum_{i=1}^{2M} a_i(h_i(x) - p_{i,1}(x) + c_{i1}(1+x) - p_{i,2}(x)) = x(3e-1) \\ &+e^x(1-2x) + 3e. \end{aligned} \tag{8.2}$$

For $2m$ collocation points we get $2m$ linear algebraic equations with unknowns $2m$ haar coefficients, which are obtained by solving the system of equations. By substituting the coefficients a_i so obtained in the equation for $y(x)$ we get the required solution. This result is compared with the analytical solution which is exactly similar at $j = 3$, that is $m = 16$, as in fig (9)

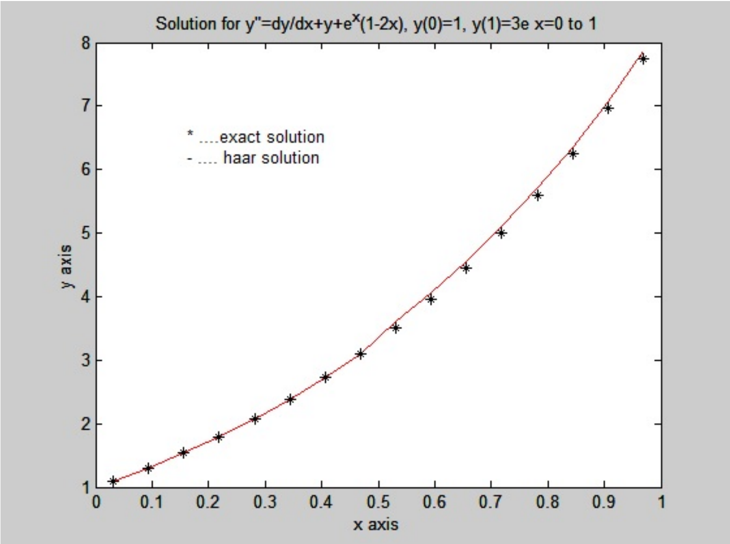

Figure 6. Comparison of haar solution with exact solution for example 2

The graph of the haar, analytical and inbuilt function implementation of matlab results for the boundary valued problem (Example 2) using , $j = 3$, is below.

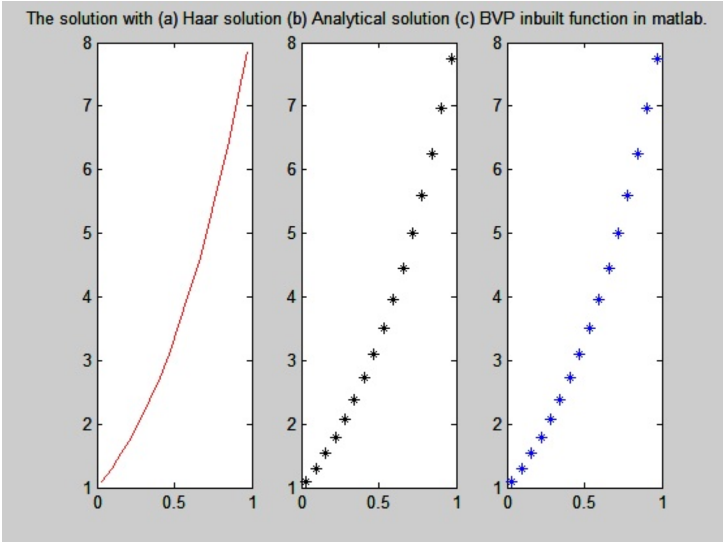

Figure 7. Comparison of haar solution with exact solution for BVP

8.1 Observations

We have presented a unified way for solving the initial valued problem and the boundary valued problem using the wavelet collocation method. As we stated earlier this method is bound to give more accurate solution for boundary valued problem compared to the traditional shooting methods. Also an appropriate higher wavelet resolution may be chosen if the ordinary differential equation is stiff. To improve the accuracy and optimize the computation an appropriate dynamic resolution adaptive scheme could be formulated. In case of nonlinear differential equation with relatively less non linearity, it generates manageable non linear algebraic equations; otherwise it becomes a complicated system. Our method is more amicable for linear differential equations.

9 Normalization procedure for general solution of ivp and bvp within the domain [0, 1].

Since Haar wavelet function is defined in $[0,1]$ the collocation method with haar basis function can be used for obtaining solution in interval [0,1], when we seek solution either for initial value or boundary value ordinary differential equation in domain $[A,B]$, we need to carryout transformation as follows Consider a boundary valued ordinary differential equation as (5.1) $x \in [A,B]$ with conditions specified at any random points A and B as $y(A)$ and $y(B)$ we transform the variable x to x_1 such that x_1 lies in the interval $[0,1]$ with the transformation, $x_1 = \frac{x-A}{B-A}$ which leads to a change in the differential equation and boundary conditions. Derivatives will change with this transformation.

$$\frac{dy}{dx} = \frac{dy}{dx_1}(\frac{dx_1}{dx}), \tag{9.1}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx}) = \frac{d}{dx_1}(\frac{dy}{dx})\frac{dx_1}{dx}$$

Accordingly the boundary conditions are modified and obtained between $[0,1]$, once the above formulation is done, the equation can be solved as specified in an example. Finally the solution is again transformed back to the original variable with specified parametric changes to obtain the results in the domain $[A,B]$. Another possibility is extending the solution for an arbitrary interval $[A,B]$ with initial conditions specified at 0 , we convert the interval first to an interval $[B-A,0]$ and solving it for $[0,B-A]$ which is further transformed into $[0,1]$ as required. This conversion helps in converting the problem to the required domain $[0,1]$ where haar collocation in implemented. Reverse conversion to $[A,B]$ gives the required solution. Here the variable

$x_1 = B - x$ which simply converts say for $n = 1$

$$\frac{dy}{dx} = \frac{d}{dx_1}(\frac{dx_1}{dx}) \tag{9.2}$$

$$\tag{9.3}$$

for $\quad n = 2$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx}) = \frac{d}{dx_1}(\frac{-dy}{dx_1}) \tag{9.4}$$

$$\frac{dx_1}{dx} = \frac{d^2y}{dx_1{}^2}$$

$$\tag{9.5}$$

The above concept is used in example as discussed in this section.

10 Example 3

Consider a simple second order boundary valued problem as

$$y" = 1, \tag{10.1}$$

$$x \in [-2, 2] \tag{10.2}$$

$$y(-2) = y(2) = 4. \tag{10.3}$$

Here a change in variable as mentioned in the above section is done with $x_1 = \frac{x+2}{4}$ the modified equation obtained using the change of variable both for the differential operator and the boundary conditions is $\frac{y"(x)}{16} = 1$ with $y(0) = y(1) = 4$ The analytical solution of this boundary valued problem is $y = 8x^2 - 8x + 4$ the wavelet formulation is obtain as,

$$y"(x) = \sum_{1=1}^{2M} a_i h_i(x) \tag{10.4}$$

$$y'(x) = \sum_{i=1}^{2M} a_i(p_{i,1}(x) - c_{i1})$$

$$y(x) = 4 + \sum_{i=1}^{2M} a_i(p_{i,2}(x) - xc_{i1})$$

Therefore the boundary valued problem gets transformed as $\sum_{1=1}^{2M} a_i h_i(x) = 16$ For $2m$ collocation points we get $2m$ linear algebraic equations with unknowns $2m$ haar coefficients, which are obtained by solving the system of equations. By substituting the coefficients a_i so obtained in the equation for $y(x)$ we get the required solution. This result is compared with the analytical solution which is exactly similar at $j = 4$, that is $m = 32$, as in figure 8.

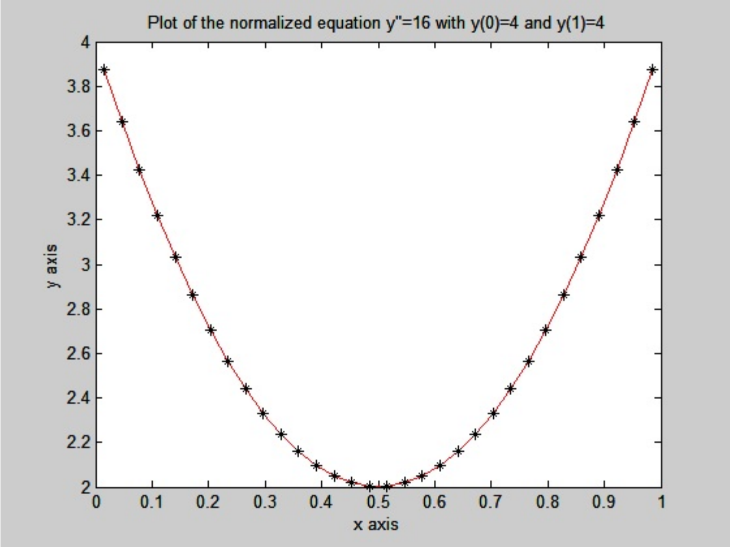

Figure 8. Comparitive study of normalized approche

The comparison of haar solution with exact solution for $x \in [0, 1]$ boundary valued ordinary differential equation , with $j = 4$. Now 9 represents the graph of the haar solution after transforming back to the original domain , with its analytic solution.

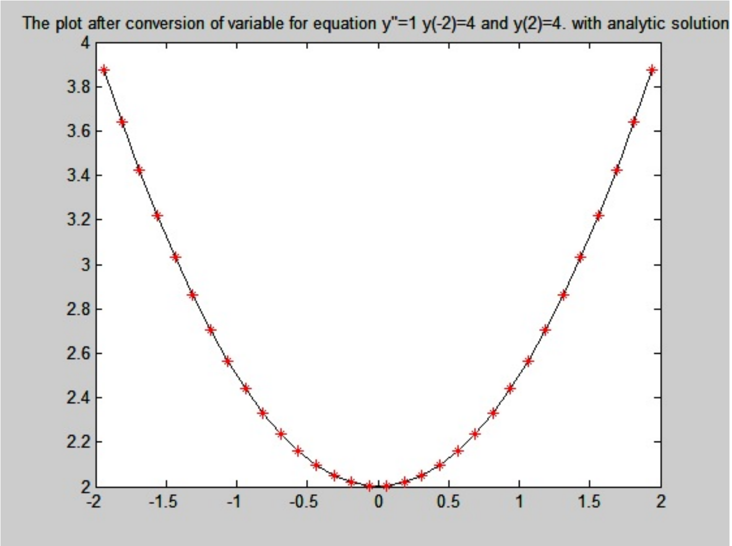

Figure 9. Haar solution after transforming

Extension of region is demonstrated using examples with cases , where the extension in example 1 is done for positive domain and then the example solution is done in Example 2 in negative domain where initial condition is still specified at origin. Normalization discussed in section 9 is utilized.

11 Extension of region of solution using haar wavelets

Consider an equation

$$y" + y = \cos wt \tag{11.1}$$

with $y'(0) = 0$ and $y(0) = 0$ The analytic solution for the equation is

$$y(t) = \frac{2}{1-w^2}\sin(\frac{(1+w)t}{2})\sin(\frac{(1-w)t}{2}) \tag{11.2}$$

- Case 1 The haar solution is done with $w = 0.9$ and the solution is then extended from $(0, 1)$ to $(0, 10)$. As shown in figure 10

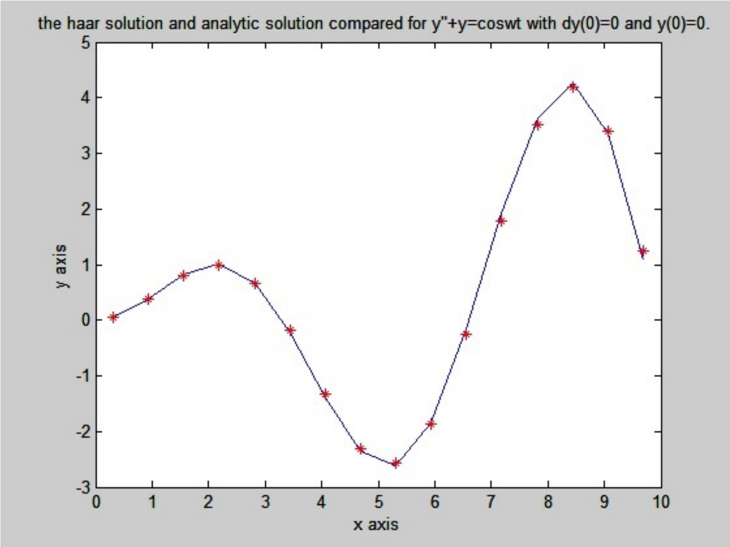

Figure 10. Generalized solution for case 1 in the domain $(0, 10)$

- Case 2 The haar solution for solving the equation in the interval $[-10, 0]$ with conditions specified at 0. The conversions to the differential equation is done as specified in previous section , using the variable. After reconversion of the domain haar solution and its comparison with analytic solution is shown in figure 11.

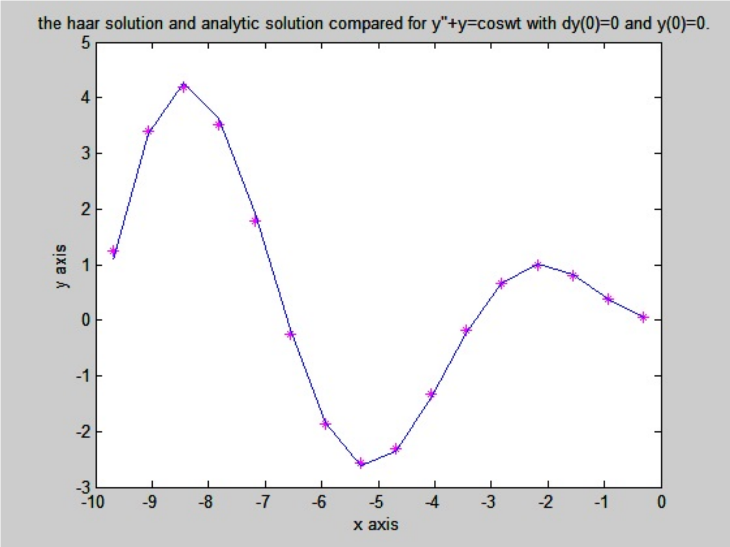

Figure 11. Extended solution case 1 in the domain $(-10, 0)$

12 Conclusion

The article presents a unified procedure for solving ordinary differential equation using wavelet collocation method for both initial value and boundary value problems. These are discussed with examples specifying the details of formulation and validation is done using programs developed by us. Results are shown with plots. It demonstrates methodology for solving ordinary differential equation using wavelets in any given domain even though the basis function is defined in $[0, 1]$. Hence generalizing is done using the collocation approach with change in variables. Graphical details are interpreted using plots depicting higher resolution in fig 11. It is an efficient and more accurate numerical method for solving ordinary differential equation as we solve the system of algebraic equation with less calculation rather than solving the differential equations. The results obtained shows good agreement with analytical solutions.

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