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THE FISCHER-CLIFORD MATRICES AND CHARACTER TABLE OF THE SPLIT EXTENSION $2^7 : GL(4,2)$

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Abstract The purpose of this paper is to construct the Fischer-Clifford matrices and the character tables for the group $\overline{G} = 2^7 : GL(4,2)$.

1 Introduction

The theory of Clifford-Fischer matrices, which is based on Clifford Theory (see [3]) was developed by B. Fischer (see [4]). Let $\overline{G} = 2^7 : GL(4,2)$ be the split extension of $N = 2^7$ by G=GL(4,2) where N is the vector space of dimension 7 over GF(2) on which G acts naturally. The aim of this paper is to construct the character table of $\overline{G}$ by using the technique of Fischer-Clifford matrix $M(g)$ for each class representative g of G and the character tables of the inertia factor groups H_i of the inertia groups $\overline{H_i} = 2^7 : H_i$. we use the properties of the Fischer-Clifford matrices discussed in ([1], [2], [9], [10], [11]) to compute entries of these matrices. The Fischer-Clifford matrix $M(g)$ will be portioned row-wise into blocks, where each block corresponds to an inertia group $\overline{H_i}$. Now using the columns of character table of the inertia factor H_i of $\overline{H_i}$ which correspond to classes of H_i which fuse to the class [g] in G and multiply these columns by the rows of the Fischer-Clifford matrix $M(g)$ that correspond to $\overline{H_i}$. Thus, we construct the portion of the character table of $\overline{G}$ which is in the block corresponding to $\overline{H_i}$ for the classes of $\overline{G}$ that come from the coset Ng. For more information about this technique see ([1], [2], [9], [10], [11]). The character table of $\overline{G}$ will be divided row-wise into blocks where each block corresponding to an inertia group $\overline{H_i} = N : H_i$. The computations have been carried out with the aid of computer algebra system MAGMA [7] and GAP [6] and We will follow the notion of Atlas [12].

2 Theory of Fischer-Clifford Matrices

Let $\overline{G} = N : G$ be a split extension of N by G. Then for $\theta \in Irr(N)$, we define $\overline{H} = \left\{x \in \overline{G} : \theta^x = \theta\right\} = I_{\overline{G}}(\theta)$ and $H = \{x \in G : \theta^g = \theta\} = I_G(\theta)$ where $I_{\overline{G}}(\theta)$ is the stabilizer of in the action of $\overline{G}$ on Irr(N), we have that $I_{\overline{G}}(\theta)$ is a subgroup of $\overline{G}$ and N is normal in $I_{\overline{G}}(\theta)$. Also $\left[\overline{G} : I_{\overline{G}}(\theta)\right]$ is the size of the orbit containing . Then it can be shown that $\overline{H} = N : H$, where $\overline{H}$ is the inertia group of in $\overline{G}$. The inertia factor $\overline{H}/N \cong H$ can be regarded as the inertia group of in the factor group $\overline{G}/N \cong G$. Define θ^g by $\theta^g(n) = \theta\left(gng^{-1}\right)$ for $g \in \overline{G}$, $n \in N$, $\theta^g \in Irr(N)$. We say that is extendible to $\overline{H}$ if there exists $\varphi \in Irr\left(\overline{H}\right)$ such that $\varphi\downarrow_N = \theta$. If is extendible to $\overline{H}$, then by Gallagher [5], we have

$$\left\{\alpha : \alpha \in Irr\left(\overline{H}\right), < \alpha\downarrow_N, \theta > \neq 0\right\} = \left\{\beta\varphi : \beta \in Irr\left(\overline{H}/N\right)\right\}.$$

Let $\overline{G}$ has the property that every irreducible character of N can be extended to its inertia group. Now let $\theta_1 = 1_N, \theta_2, \ldots, \theta_t$ be representatives of the orbits of $\overline{G}$ on Irr(N), $\overline{H_i} = I_{\overline{G}}(\varphi_i)$, $1 \leq i \leq t$, $\varphi_i \in Irr\left(\overline{H_i}\right)$ be an extension of θ_i to $\overline{H_i}$ and $\beta \in Irr\left(\overline{H_i}\right)$ such that $N \subseteq Ker(\beta)$. Then it can be shown that

$$Irr\left(\overline{G}\right) = \bigcup_{i=1}^{t}\left\{(\beta\varphi_i)^{\overline{G}} : \beta \in Irr\left(\overline{H_i}\right),\ N \subseteq Ker(\beta)\right\} = \bigcup_{i=1}^{t}\left\{(\beta\varphi_i)^{\overline{G}} : \beta \in \left(\overline{H_i}/N\right)\right\}$$

. Hence the irreducible characters of $\overline{G}$ will be divided into blocks, where each block corresponds to an inertia group $\overline{H_i}$. Let $\overline{H_i}$ be the inertia factor group and φ_i be an extension of θ_i to $\overline{H_i}$. Take $\theta_1 = 1_N$ as the identity character of N, then $\overline{H_1} = \overline{G}$ and $H_1 \cong G$. Let $X(g) = \{x_1, x_2, \ldots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\overline{G}$ from the coset $N\overline{g}$ whose images under the natural homomorphism $\overline{G} \to G$ are in [g] and we take $x_1 = \overline{g}$. We define

$$R(g) = \{(i, y_k) : 1 \leq i \leq t,\ H_i \cap [g] \neq 0, 1 \leq k \leq r\}$$

and we note that y_k runs over representatives of the conjugacy classes of elements of H_i which fuse into [g] in G. Then we define the Fischer-Clifford matrix M(g) by $M(g) = \left(a^j_{(i,y_k)}\right)$, where $a^j_{(i,y_k)} = \sum_l^{t} \frac{|C_{\overline{G}}(x_j)|}{|C(y_{lk})|}\varphi_i(y_{lk})$ with columns indexed by X(g) and rows indexed by R(g) and where $\sum_l^{t}$ is the summation over all l for which $y_{lk} \sim x_j$ in $\overline{G}$. Then the partial character table of $\overline{G}$ on the classes $\{x_1, x_2, \ldots, x_{c(g)}\}$ is given by $\begin{bmatrix} C_1(g)\, M_1(g) \\ C_2(g)\, M_2(g) \\ \vdots \\ C_t(g)\, M_t(g) \end{bmatrix}$ where the Fischer-Clifford matrix $M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix}$ is divided into blocks $M(g)$ with each block corresponding to an inertia group $\overline{H_i}$ and $C_i(g)$ is the partial character table of H_i consisting of the columns corresponding to the classes that fuse into [g] in G. We can also observe that the number of irreducible characters of $\overline{G}$ is the sum of the number of irreducible characters of the inertia factors $H_i^{\prime}s$. The group $\overline{G} = 2^7 : GL(4,2)$ is a split extension with 2^7 abelian and therefore by Mackey's theorem (see [1] - Theorem 4.1.12) , we have each irreducible character of 2^7 can be extended to its inertia group in $\overline{G}$. Hence by the above theoretical outline we can fully determine the character table of $\overline{G} = 2^7 : GL(4,2)$.

3 The conjugacy classes of 2^7 : $GL(4,2)$

In this section, we will use the method of coset analysis to determine the conjugacy classes of $\overline{G} = 2^7 : GL(4,2)$. We refer the reader to ([1], [2], [9], [10], [11]) for full details and background material regarding the method of coset analysis. Most of the information, which involved the conjugacy classes and permution characters, were obtained by using direct computations in GAP [6] and MAGMA [7].

By MAGMA [7], We can generate the direct product group $G = GL(4,2)$ by the following 7×7 matrices:

$$a = \begin{bmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0 \\ 0&0&1&1&0&0&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&1&1&0 \\ 0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1 \end{bmatrix},\quad b = \begin{bmatrix} 1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0 \\ 0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1 \\ 0&0&1&0&0&0&0 \\ 0&0&0&1&0&0&0 \end{bmatrix}$$

We construct the conjugacy class representative of G in terms of 7×7 matrices over $GF(2)$ by GAP [6]. Thus G has 14 conjugacy classes and under the action of G on 2^7 , we obtain 6 orbits of lengths 1, 35, 28,1,35 and 28. The point stabilizers for orbits of lengths 1, 35, 28, 1, 35 and 28 are the subgroups = $GL(4,2)$, $2^2 : (A_4 \times A_4)$, S_6, A_8, $2^2 : (A_4 \times A_4)$ and S_6 with indices 1, 35, 28,1,35 and 28 respectively in G.

Let $\mathcal{X}(G|2^7)$ be the permutation character of G on 2^7. The values of $\mathcal{X}(G|2^7)$ on different classes of G determine the number of k of fixed points of each conjugacy class of G in 2^7. These values of the $k's$ will help us to calculate the conjugacy classes of $\overline{G} = 2^7 : GL(4,2)$ which are listed in Table 1. Consequentially, having obtained the values of the $k's$ for the various classes of G, we then need to calculate the values of $f_i's$ corresponding to the various $'s$, where $f_i's$ are the number of orbits $Q_i's$ for $1 \leq i \leq k$, that fuse together under the action of $C_G(g)$ to form one orbit Δ_j. For this purpose, we used Programme A [1]. For a class representative $dg \in \overline{G}$, where $d \in 2^7$, $g \in G$ and $o(g) = m$, by Theorem 3.3.10 in [10] we have

$$o(dg) = \begin{cases} m & if \quad w = 1_N \\ 2m & otherwise \end{cases}$$

To calculate the orders of the class representative $dg \in \overline{G}$, we used Programme B [1]. If $o(g) = m$ and $w = 1_N$ then $o(dg) = 2m$ otherwise if $\neq 1_N$, then $o(dg) = 2m$. To each class of $\overline{G}$, we have attached some weight m_{ij} which will be used later in computing the Fischer matrices of the extension. These weights are computed by the formula

$$m_{ij} = [N_{\overline{G}}(N\overline{g_i}) : C_{\overline{G}}(g_{ij})] = |N| \frac{|C_G(g_i)|}{|C_{\overline{G}}(g_{ij})|}$$

Thus, we obtained 82 conjugacy classes for the group $\overline{G} = 2^7 : GL(4,2)$ and we list these result about the conjugacy classes of $\overline{G}$ in Table 1.
Table1: Conjugacy Classes of $2^7 : GL(4,2)$

$g_i \in G$	k_i	f_i	$m_{i,j}$	d_i	w	$[x]_{\overline{G}}$	$\lvert[x]_{\overline{G}}\rvert$	$\lvert C_{\overline{G}}(x)\rvert$
1A	128	1	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	1a	1	2580480
		35	35	(0,0,0,0,0,0,1)	(0,0,0,0,0,0,1)	2a	35	73728
		28	28	(0,0,0,1,1,0,0)	(0,0,0,1,1,0,0)	2b	28	92160
		1	1	(1,0,0,0,0,0,0)	(1,0,0,0,0,0,0)	2c	1	2580480
		35	35	(1,0,0,0,0,0,1)	(1,0,0,0,0,0,1)	2d	35	73728
		28	28	(1,0,0,1,1,0,0)	(1,0,0,1,1,0,0)	2e	28	92160
2A	32	1	4	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	2f	840	3072
		3	12	(0,0,0,0,0,1,0)	(0,0,0,1,0,0,0)	4a	2520	1024
		3	12	(0,0,0,0,0,1,1)	(0,0,0,0,0,0,0)	2g	2520	1024
		8	32	(0,0,0,0,1,0,0)	(0,1,1,1,0,1,0)	4b	6720	384
		1	4	(0,0,1,0,0,1,1)	(0,0,0,1,0,0,0)	4c	840	3072
		1	4	(1,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	2h	840	3072
		3	12	(1,0,0,0,0,0,1)	(0,0,0,1,0,0,0)	4d	2520	1024
		3	12	(1,0,0,0,0,1,1)	(0,0,0,0,0,0,0)	2i	2520	1024
		8	32	(1,0,0,0,1,0,0)	(0,1,1,1,0,1,0)	4e	6720	384
		1	4	(1,0,1,0,0,1,1)	(0,0,0,1,0,0,0)	4f	840	3072
3A	32	1	4	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	3a	448	5760
		10	40	(0,0,0,0,0,0,1)	(0,0,1,0,0,1,1)	6a	4480	576
		5	20	(0,0,0,0,1,0,0)	(0,0,0,0,1,0,0)	6b	2240	1152
		1	4	(1,0,0,0,0,0,0)	(1,0,0,0,0,0,0)	6c	448	5760
		10	40	(1,0,0,0,0,0,1)	(1,0,1,0,0,1,1)	6d	4480	576
		5	20	(1,0,0,0,1,0,0)	(1,0,0,0,1,0,0)	6e	2240	1152
6A	8	1	16	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	6f	26880	96
		1	16	(0,0,0,0,0,0,1)	(0,0,0,1,0,0,0)	12a	26880	96
		2	32	(0,0,0,0,1,0,0)	(0,1,1,1,0,1,0)	12b	53760	48
		1	16	(1,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	6g	26880	96
		1	16	(1,0,0,0,0,0,1)	(0,0,0,1,0,0,0)	12c	26880	96
		2	32	(0,0,0,0,1,0,0)	(0,1,1,1,0,1,0)	12d	53760	48

3B	8	1	16	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	3b	17920	144
		1	16	(0,0,0,0,0,0,1)	(0,0,1,1,1,1,1)	6h	17920	144
		1	16	(0,0,0,0,0,1,0)	(0,1,1,0,0,1,0)	6i	17920	144
		1	16	(0,0,0,0,0,1,1)	(0,1,1,0,0,1,0)	6j	17920	144
		1	16	(1,0,0,0,0,0,0)	(1,0,0,0,0,0,0)	6k	17920	144
		1	16	(1,0,0,0,0,0,1)	(1,0,1,1,1,1,1)	6l	17920	144
		1	16	(1,0,0,0,0,1,0)	(1,1,0,1,1,0,1)	6m	17920	144
		1	16	(1,0,0,0,0,1,1)	(1,1,1,0,0,1,0)	6n	17920	144
7A	2	1	64	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	7a	184320	14
		1	64	(1,0,0,0,0,0,0)	(1,0,0,0,0,0,0)	14a	184320	14
7B	2	1	64	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	7b	184320	14
		1	64	(1,0,0,0,0,0,0)	(1,0,0,0,0,0,0)	14b	184320	14
2B	32	1	4	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	2j	420	6144
		12	48	(0,0,0,0,0,0,1)	(0,0,1,0,0,0,0)	4g	5040	512
		1	4	(0,0,0,0,1,0,0)	(0,0,0,0,0,0,0)	2k	420	6144
		1	4	(0,0,0,1,0,0,0)	(0,0,0,0,0,0,0)	2l	420	6144
		1	4	(0,0,0,1,1,0,0)	(0,0,0,0,0,0,0)	2m	420	6144
		1	4	(1,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	2n	420	6144
		12	48	(1,0,0,0,0,0,1)	(0,0,1,0,0,0,0)	4h	5040	512
		1	4	(1,0,0,0,1,0,0)	(0,0,0,0,0,0,0)	2o	420	6144
		1	4	(1,0,0,1,0,0,0)	(0,0,0,0,0,0,0)	2p	420	6144
		1	4	(0,0,0,1,1,0,0)	(0,0,0,0,0,0,0)	2q	420	6144
6B	8	1	16	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	6o	53760	48
		1	16	(0,0,0,0,0,0,1)	(0,0,0,0,0,0,0)	6p	53760	48
		1	16	(0,0,0,0,0,1,0)	(0,0,0,0,0,0,0)	6q	53760	48
		1	16	(0,0,0,0,0,1,1)	(0,0,0,0,0,0,0)	6r	53760	48
		1	16	(1,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	6s	53760	48
		1	16	(1,0,0,0,0,0,1)	(0,0,0,0,0,0,0)	6t	53760	48
		1	16	(1,0,0,0,0,1,0)	(0,0,0,0,0,0,0)	6u	53760	48
		1	16	(1,0,0,0,0,1,1)	(0,0,0,0,0,0,0)	6v	53760	48
5A	8	1	16	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	5a	21504	120
		3	48	(0,0,0,0,0,0,0)	(0,0,1,1,1,0,0)	10a	64512	40
		1	16	(0,0,0,0,0,0,0)	(1,0,0,0,0,0,0)	10b	21504	120
		3	48	(1,0,0,0,0,0,1)	(1,0,1,1,1,0,0)	10c	64512	40
15A	2	1	64	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	15a	86016	30
		1	64	(1,0,0,0,0,0,0)	(1,0,0,0,0,0,0)	30a	86016	30
15B	2	1	64	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	15b	86016	30
		1	64	(1,0,0,0,0,0,0)	(1,0,0,0,0,0,0)	30b	86016	30
4A	8	1	16	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	4q	20160	128
		1	16	(0,0,0,1,0,0,0)	(0,0,0,0,0,0,0)	4r	20160	128
		1	16	(0,0,1,0,0,0,0)	(0,0,0,0,0,0,0)	4s	20160	128
		1	16	(0,0,1,1,0,0,0)	(0,0,0,0,0,0,0)	4t	20160	128
		1	16	(1,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	4u	20160	128
		1	16	(1,0,0,1,0,0,0)	(0,0,0,0,0,0,0)	4v	20160	128
		1	16	(1,0,1,0,0,0,0)	(0,0,0,0,0,0,0)	4w	20160	128
		1	16	(1,0,1,1,0,0,0)	(0,0,0,0,0,0,0)	4x	20160	128

4B	8	1	16	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	4y	40320	64
		1	16	(0,0,0,0,0,0,1)	(0,1,1,0,0,0,0)	8a	40320	64
		1	16	(0,0,0,0,1,0,0)	(0,0,0,0,0,0,0)	4z	40320	64
		1	16	(0,0,0,0,1,0,1)	(0,1,1,0,0,0,0)	8b	40320	64
		1	16	(1,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	4aa	40320	64
		1	16	(1,0,0,0,0,0,1)	(0,1,1,0,0,0,0)	8c	40320	64
		1	16	(1,0,0,0,1,0,0)	(0,0,0,0,0,0,0)	4ab	40320	64
		1	16	(1,0,0,0,1,0,1)	(0,1,1,0,0,0,0)	8d	40320	64

Thus, the group $\overline{G} = 2^7 : GL(4,2)$ has 82 conjugacy classes.

4 The Inertia Factor Group of $\overline{G} = 2^7 : GL(4,2)$

The action of G on N produce 6 orbits of lengths 1, 35, 28,1,35 and 28 hence by Brauer's theorem (Theorem 5.1.4 in [10]) G acts on $Irr(N)$ with the same number of orbits. The lengths of the these orbits will be $1,\ r_1, r_2, r_3, r_4, r_5$ where $1 +\ r_1 + r_2 + r_3 + r_4 + r_5 = 128$, with corresponding point stabilizers H_1, H_2, H_3, H_4, H_5 and H_6 as subgroups of G such that $[G : H_1] = 1$, $[G : H_2] = 35$, $[G : H_3] = 28$, $[G : H_4] = 1$, $[G : H_5] = 35$ and $[G : H_6] = 28$. Considering the indices of these subgroups in G and investigating the maximal and submaximal subgroups of G, we have the Group G acting on Irr(2^7) produce 6 inertia factor groups:

(i) $H_1 = G = GL(4,2)$ of index equal 1.

(ii) $H_2 = 2^2 : (A_4 \times A_4) = H_5$ of index equal 35.

(iii) $H_3 = S_6 = H_6$ of index equal to 28.

(iv) $H_4 = A_8$ of index equal to 1.

Using GAP [6], we can generate the group H_2 in terms of 7×7 matrices over GF(2) by the following matrices:

$$\begin{bmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0 \\ 0&0&1&1&0&0&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&1&1&0 \\ 0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0&0&0&0&0 \\ 0&1&1&0&0&0&0 \\ 0&0&1&0&0&0&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&1 \\ 0&0&0&0&0&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0 \\ 0&0&1&0&1&0&0 \\ 0&0&0&1&0&1&0 \\ 0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1 \end{bmatrix},$$

$$\begin{bmatrix} 1&0&0&0&0&0&0 \\ 0&1&1&1&0&0&0 \\ 0&0&1&0&0&0&0 \\ 0&0&0&1&0&0&0 \\ 0&0&1&0&1&0&1 \\ 0&0&0&1&0&1&1 \\ 0&0&0&0&0&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&0&1&1&0 \\ 0&0&0&1&0&1&1 \\ 0&0&1&1&1&1&0 \\ 0&0&0&1&0&0&1 \\ 0&0&1&1&0&0&0 \\ 0&0&0&0&0&0&1 \end{bmatrix}$$

By using these generators in GAP [6], we obtained the conjugacy classes and the character tables for this group.
We can complete the fusion maps by using matrix conjugation into the group G . This fusion map is listed in table 2.
Table 2: The Fusion of the group H_2 into the group G

$[g]_{H_2} \Longrightarrow [g]_G$	$[g]_{H_2} \Longrightarrow [g]_G$	$[g]_{H_2} \Longrightarrow [g]_G$
1a $\Longrightarrow$ 1a	2a $\Longrightarrow$ 2b	2b $\Longrightarrow$ 2a
3a $\Longrightarrow$ 3b	2c $\Longrightarrow$ 2b	4a $\Longrightarrow$ 4a
3b $\Longrightarrow$ 3b	6a $\Longrightarrow$ 6a	3c $\Longrightarrow$ 3a
6b $\Longrightarrow$ 6b	2d $\Longrightarrow$ 2b	4b $\Longrightarrow$ 4a
6c $\Longrightarrow$ 6b	2e $\Longrightarrow$ 2a	4c $\Longrightarrow$ 4a
4d $\Longrightarrow$ 4b		

Also, by using GAP [6], we can generate the group H_3 in terms of 7×7 matrices over GF(2) by the follows matrices:

$$\begin{bmatrix} 1&0&0&0&0&0&0\\ 0&1&1&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&1\\ 0&0&0&0&0&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0\\ 0&1&0&0&0&0&0 \end{bmatrix},$$

$$\begin{bmatrix} 1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1\\ 0&0&1&1&1&1&0\\ 0&0&0&0&1&1&0\\ 0&0&0&1&0&1&0\\ 0&0&0&0&0&1&0\\ 0&1&0&0&0&0&0 \end{bmatrix}, \begin{bmatrix} 1&0&0&0&0&0&0\\ 0&1&1&1&1&0&1\\ 0&0&0&0&0&1&1\\ 0&1&1&1&0&0&0\\ 0&1&1&0&1&0&0\\ 0&1&0&1&1&0&1\\ 0&1&1&0&0&1&1 \end{bmatrix}$$

By using these generators in GAP [6], we obtained the conjugacy classes and the character tables for this group.
We can complete the fusion maps by using matrix conjugation in the group G . This fusion map is listed in table 3.
Table 3: The Fusion of the group H_3 into the group G

$[g]_{H_3} \Longrightarrow [g]_G$	$[g]_{H_3} \Longrightarrow [g]_G$	$[g]_{H_3} \Longrightarrow [g]_G$
1a $\Longrightarrow$ 1a	2a $\Longrightarrow$ 2a	2b $\Longrightarrow$ 2a
2c $\Longrightarrow$ 2b	3a $\Longrightarrow$ 3a	6a $\Longrightarrow$ 6a
3b $\Longrightarrow$ 3b	4a $\Longrightarrow$ 4b	4b $\Longrightarrow$ 4b
5a $\Longrightarrow$ 5a	6b $\Longrightarrow$ 6b	

Also, by using GAP [6], we can generate the group H_4 in terms of 7×7 matrices over GF(2) by the follows matrices:

$$\begin{bmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&1&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&1&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0\\ 0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0 \end{bmatrix}$$

By using these generators in GAP [6], we obtained the conjugacy classes and the character tables for this group.
We can complete the fusion maps by using matrix conjugation in the group . This fusion map is listed in table 4.
Table 4: The Fusion of the group H_4 into the group G

$[g]_{H_4} \Longrightarrow [g]_G$	$[g]_{H_4} \Longrightarrow [g]_G$	$[g]_{H_4} \Longrightarrow [g]_G$
1a $\Longrightarrow$ 1a	2a $\Longrightarrow$ 2b	4a $\Longrightarrow$ 4a
3a $\Longrightarrow$ 3b	6a $\Longrightarrow$ 6b	2b $\Longrightarrow$ 2a
4b $\Longrightarrow$ 4b	7a $\Longrightarrow$ 7b	7b $\Longrightarrow$ 7a
3b $\Longrightarrow$ 3a	5a $\Longrightarrow$ 5a	15a $\Longrightarrow$ 15b
15b $\Longrightarrow$ 15a	6b $\Longrightarrow$ 6a	

5 The Fischer-Clifford Matrices of $\overline{G} = 2^7 : GL(4,2)$

For each conjugacy class [g] of G with representative $g \in G$, we construct the corresponding Fischer-Clifford matrix $M(g)$ for the group $\overline{G} = 2^7 : GL(4,2)$. We use the properties of the Fischer-Clifford matrices (see [1], [2], [9], [10], [11]) together with fusions of $H_i,\ i = 1\ to\ 6$ into G to compute the entries of these matrices and to construct an algebraic system of linear and non-linear equations with the help of Maxima [8]), we can solve these system of equations and compute all the Fischer matrices of $\overline{G}$. The Fischer-Clifford matrix will be partitioned row-wise into blocks, where each block corresponding to an inertia group $\overline{H_i}$.
We list the Fischer-Clifford matrices of $\overline{G}$ as follows:

1A	1a	2a	2b	2c	2d	2e
$\lvert C_{\overline{G}}(x)\rvert$ $\lvert C_{H_k}\rvert$	2580480	73728	92160	2580480	73728	92160
20160	1	1	1	1	1	1
576	35	3	-5	35	3	-5
720	28	-4	4	28	-4	4
20160	1	1	1	-1	-1	-1
576	35	3	-5	-35	-3	5
720	28	-4	4	-28	4	-4
m_{ij}	1	35	28	1	35	28

2A	2f	4a	2g	4b	4c	2h	4d	2i	4e	4f
$\lvert C_{\overline{G}}(x)\rvert$ $\lvert C_{H_k}\rvert$	3072	1024	1024	384	3072	3072	1024	1024	384	3072
96	1	1	1	1	1	1	1	1	1	1
96	1	1	1	1	-1	1	-1	-1	-1	-1
16	6	-2	2	0	6	6	-2	-2	0	-6
48	2	2	-2	0	2	-2	2	-2	0	-2
16	6	-2	-2	0	-6	6	2	-2	0	6
96	1	1	1	-1	-1	1	-1	-1	1	-1
96	1	1	1	-1	1	1	1	1	-1	1
16	6	-2	2	0	-6	-6	-2	2	0	-6
48	2	2	-2	0	-2	-2	-2	2	0	2
16	6	-2	-2	0	6	-6	2	2	0	6
m_{ij}	4	12	12	32	4	4	12	12	32	4

3A	3a	6a	6b	6c	6d	6e
$\frac{\lvert C_{\overline{G}}(x)\rvert}{\lvert C_{H_k}\rvert}$	5760	576	1152	5760	576	1152
180	1	1	1	1	1	1
36	5	1	-3	5	1	-3
18	10	-2	2	10	-2	2
180	1	-1	1	-1	1	-1
36	5	-1	-3	-5	-1	3
18	10	2	2	-10	-2	-2
m_{ij}	4	40	20	4	40	20

6A	3a
$\frac{\lvert C_{\overline{G}}(x)\rvert}{\lvert C_{H_k}\rvert}$	96
12	1
12	1
6	2
12	1
12	1
6	2
m_{ij}	16

3B	3b	6h	6i	6j	6k	6l	6m	6n
$\frac{\lvert C_{\overline{G}}(x)\rvert}{\lvert C_{H_k}\rvert}$	144	144	144	144	144	144	144	144
18	1	1	1	1	1	1	1	1
18	1	-1	1	-1	1	-1	1	-1
18	1	1	-1	-1	1	1	-1	-1
18	1	-1	-1	1	1	-1	-1	1
18	1	1	1	1	-1	-1	-1	-1
18	1	-1	1	-1	-1	1	-1	1
18	1	1	-1	-1	-1	-1	1	1
18	1	-1	-1	1	-1	1	1	-1
m_{ij}	16	16	16	16	16	16	16	16

7A	7a	14a
$\frac{\lvert C_{\overline{G}}(x)\rvert}{\lvert C_{H_k}\rvert}$	14	14
7	1	1
7	1	-1
m_{ij}	64	64

7B	7b	14b
$\frac{\lvert C_{\overline{G}}(x)\rvert}{\lvert C_{H_k}\rvert}$	14	14
7	1	1
7	1	-1
m_{ij}	64	64

2B	2j	4g	2k	2l	2m	2n	4h	2o	2p	2q
$\frac{\lvert C_{\overline{G}}(x)\rvert}{\lvert C_{H_k}\rvert}$	6144	512	6144	6144	6144	6144	512	6144	6144	6144
192	1	1	1	1	1	1	1	1	1	1
64	3	1	3	3	3	3	-1	3	3	3
48	4	0	-4	4	-4	-4	0	-4	4	4
48	4	0	4	4	4	-4	0	-4	-4	-4
48	4	0	-4	4	-4	4	0	4	4	-4
192	1	-1	1	-1	-1	1	1	-1	1	-1
64	3	1	3	-3	-3	3	-1	-3	3	-3
48	4	0	-4	-4	4	-4	0	4	4	-4
48	4	0	4	-4	-4	-4	0	4	-4	4
48		0	-4	-4	4	4	0	-4	-4	4
m_{ij}	4	48	4	4	4	4	48	4	4	4

6B	6o	6p	6q	6r	6s	6t	6u	6v
$\frac{\lvert C_{\overline{G}}(x)\rvert}{\lvert C_{H_k}\rvert}$	48	48	48	48	48	48	48	48
6	1	1	1	1	1	1	1	1
6	1	-1	1	-1	1	-1	1	-1
6	1	1	-1	-1	1	1	-1	-1
6	1	-1	-1	1	1	-1	-1	1
6	1	1	1	1	-1	-1	-1	-1
6	1	-1	1	-1	-1	1	-1	1
6	1	1	-1	-1	-1	-1	1	1
6	1	-1	-1	1	-1	1	1	-1
m_{ij}	16							

5A	5a	10a	10b	10c
$\frac{\lvert C_{\overline{G}}(x)\rvert}{\lvert C_{H_k}\rvert}$	120	40	120	40
15	1	1	1	1
5	3	-1	-3	1
15	1	1	-1	-1
5	3	1	3	-1
m_{ij}	16	48	16	48

15A	15a
$\frac{\lvert C_{\overline{G}}(x)\rvert}{\lvert C_{H_k}\rvert}$	30
15	1
15	1
m_{ij}	64

4A	4q	4r	4s	4t
$\frac{\lvert C_{\overline{G}}(x)\rvert}{\lvert C_{H_k}\rvert}$	128	128	128	128
16	1	1	1	1
16	1	-1	1	-1
16	1	1	-1	-1
16	1	-1	-1	1
16	1	1	1	1
16	1	-1	1	-1
16	1	1	-1	-1
16	1	-1	-1	1
m_{ij}	16	16	16	16

15B	15b	30b
$\frac{\lvert C_{\overline{G}}(x)\rvert}{\lvert C_{H_k}\rvert}$	30	30
15	1	1
15	1	-1
m_{ij}	64	64

4B	4y	8a	4z	8b	4aa	8c	4ab	8d
$\frac{\lvert C_{\overline{G}}(x)\rvert}{\lvert C_{H_k}\rvert}$	64	64	64	64	64	64	64	64
8	1	1	1	1	1	1	1	1
8	1	-1	1	-1	1	-1	1	-1
8	1	1	-1	-1	1	1	-1	-1
8	1	-1	-1	1	1	-1	-1	1
8	1	1	1	1	-1	-1	-1	-1
8	1	-1	1	-1	-1	1	-1	1
8	1	1	-1	-1	-1	-1	1	1
8	1	-1	-1	1	-1	1	1	-1
m_{ij}	16	16	16	16	16	16	16	16

6 The Character Table of $G = 2^7 : GL(4, 2)$

Now, we have:

(i) The conjugacy classes of $\overline{G} = 2^7 : GL(4, 2)$ (Table 1);

(ii) The character tables of all the inertia factors (by using GAP[6]);

(iii) The fusions of conjugacy classes of the inertia factors into classes of G (Tables 2, 3, 4) ;

(iv) The Fischer matrices of classes of $\overline{G} = 2^7 : GL(4, 2)$ (Section V);

Thus, it is easy to construct the character table of $\overline{G} = 2^7 : GL(4, 2)$ by using the columns of the character tables of the inertia factors H_i which correspond to the classes of H_i fusing to the class $[g]$ in G and multiplying these columns by the rows of the Fischer-Clifford matrix $M(g)$ which correspond to $\overline{H_i} = 2^7 : H_i$, we get parts of columns of the character table of $\overline{G}$ which is in the block corresponding to $\overline{H_i}$ for the classes of $\overline{G}$ coming from the coset Ng.
The character table of $\overline{G}$ will be partitioned row-wise into blocks B_i , where each block corresponds to an inertia group $\overline{H_i} = 2^7 : H_i$. Thus $Irr\left(\overline{G}\right) = \bigcup_{i=1}^{6} B_i$, where $B_1 = \{\mathcal{X}_j : 1 \leq j \leq 14\}$, $B_2 = \{\mathcal{X}_j : 15 \leq j \leq 30\}$, $B_3 = \{\mathcal{X}_j : 31 \leq j \leq 41\}$, $B_4 = \{\mathcal{X}_j : 42 \leq j \leq 55\}$, $B_5 = \{\mathcal{X}_j : 56 \leq j \leq 71\}$ and $B_6 = \{\mathcal{X}_j : 72 \leq j \leq 82\}$. We list the full character table of $\overline{G} = 2^7 : GL(4, 2)$ as follows:

Class	1a	2a	2b	2c	2d	2e	3a	6a	6
Order	1	2	2	2	2	2	3	6	6
Size	1	35	8	1	35	28	448	4480	2

X1	1	1	1	1	1	1	1	1	1
X2	7	7	7	7	7	7	4	4	4
X3	14	14	14	14	14	14	-1	-1	-1
X4	20	20	20	20	20	20	5	5	5
X5	21	21	21	21	21	21	6	6	6
X6	21	21	21	21	21	21	-3	-3	-3
X7	21	21	21	21	21	21	-3	-3	-3
X8	28	28	28	28	28	28	1	1	1
X9	35	35	35	35	35	35	5	5	5
X10	45	45	45	45	45	45	0	0	0
X11	45	45	45	45	45	45	0	0	0
X12	56	56	56	56	56	56	-4	-4	-4
X13	64	64	64	64	64	64	4	4	4
X14	70	70	70	70	70	70	-5	-5	-5
X15	35	3	-5	35	3	-5	5	1	-3
X16	35	3	-5	35	3	-5	5	1	-3
X17	35	3	-5	35	3	-5	5	1	-3
X18	35	3	-5	35	3	-5	5	1	-3
X19	70	6	-10	70	6	-10	-5	-1	3
X20	70	6	-10	70	6	-10	-5	-1	3
X21	70	6	-10	70	6	-10	-5	-1	3
X22	70	6	-10	70	6	-10	-5	-1	3
X23	140	12	-20	140	12	-20	5	1	-3
X24	210	18	-30	210	18	-30	15	3	-9
X25	210	18	-30	210	18	-30	15	3	-9
X26	315	27	-45	315	27	-45	0	0	0
X27	315	27	-45	315	27	-45	0	0	0
X28	315	27	-45	315	27	-45	0	0	0
X29	315	27	-45	315	27	-45	0	0	0
X30	420	36	-60	420	36	-60	-15	-3	9
X31	28	-4	4	28	-4	4	10	-2	2
X32	28	-4	4	28	-4	4	10	-2	2
X33	140	-20	20	140	-20	20	20	-4	4
X34	140	-20	20	140	-20	20	20	-4	4
X35	140	-20	20	140	-20	20	-10	2	-2
X36	140	-20	20	140	-20	20	-10	2	-2
X37	252	-36	36	252	-36	36	0	0	0
X38	252	-36	36	252	-36	36	0	0	0
X39	280	-40	40	280	-40	40	10	-2	2
X40	280	-40	40	280	-40	40	10	-2	2
X41	448	-64	64	448	-64	64	-20	4	-4
X42	1	1	1	-1	-1	-1	1	-1	1
X43	7	7	7	-7	-7	-7	4	-4	4
X44	14	14	14	-14	-14	-14	-1	1	-1
X45	20	20	20	-20	-20	-20	5	-5	5
X46	21	21	21	-21	-21	-21	6	-6	6
X47	21	21	21	-21	-21	-21	-3	3	-3
X48	21	21	21	-21	-21	-21	-3	3	-3
X49	28	28	28	-28	-28	-28	1	-1	1
X50	35	35	35	-35	-35	-35	5	-5	5
X51	45	45	45	-45	-45	-45	0	0	0
X52	45	45	45	-45	-45	-45	0	0	0
X53	56	56	56	-56	-56	-56	-4	4	-4
X54	64	64	64	-64	-64	-64	4	-4	4
X55	70	70	70	-70	-70	-70	-5	5	-5
X56	35	3	-5	-35	-3	5	5	-1	-3
X57	35	3	-5	-35	-3	5	5	-1	-3
X58	35	3	-5	-35	-3	5	5	-1	-3
X59	35	3	-5	-35	-3	5	5	-1	-3
X60	70	6	-10	-70	-6	10	-5	1	3
X61	70	6	-10	-70	-6	10	-5	1	3
X62	70	6	-10	-70	-6	10	-5	1	3
X63	70	6	-10	-70	-6	10	-5	1	3
X64	140	12	-20	-140	-12	20	5	-1	-3
X65	210	18	-30	-210	-18	30	15	-3	-9
X66	210	18	-30	-210	-18	30	15	-3	-9
X67	315	27	-45	-315	-27	45	0	0	0
X68	315	27	-45	-315	-27	45	0	0	0
X69	315	27	-45	-315	-27	45	0	0	0
X70	315	27	-45	-315	-27	45	0	0	0
X71	420	36	-60	-420	-36	60	-15	3	9
X72	28	-4	4	-28	4	-4	10	2	2
X73	28	-4	4	-28	4	-4	10	2	2
X74	140	-20	20	-140	20	-20	20	4	4
X75	140	-20	20	-140	20	-20	20	4	4
X76	140	-20	20	-140	20	-20	-10	-2	-2
X77	140	-20	20	-140	20	-20	-10	-2	-2
X78	252	-36	36	-252	36	-36	0	0	0
X79	252	-36	36	-252	36	-36	0	0	0
X80	280	-40	40	-280	40	-40	10	2	2
X81	280	-40	40	-280	40	-40	10	2	2
X82	448	-64	64	-448	64	-64	-20	-4	-4

Class	2f	4a	2g	4b	4c	2h	4d	2i
Order	2	4	2	4	4	2	4	2
Size	840	2520	2520	6720	840	840	2520	2520

X1	1	1	1	1	1	1	1	1
X2	3	3	3	3	3	3	3	3
X3	2	2	2	2	2	2	2	2
X4	4	4	4	4	4	4	4	4
X5	1	1	1	1	1	1	1	1
X6	1	1	1	1	1	1	1	1
X7	1	1	1	1	1	1	1	1
X8	4	4	4	4	4	4	4	4
X9	-5	-5	-5	-5	-5	-5	-5	-5
X10	-3	-3	-3	-3	-3	-3	-3	-3
X11	-3	-3	-3	-3	-3	-3	-3	-3
X12	0	0	0	0	0	0	0	0
X13	0	0	0	0	0	0	0	0
X14	2	2	2	2	2	2	2	2
X15	7	-1	3	1	5	7	-3	-3
X16	7	-1	3	1	5	7	-3	-3
X17	-5	3	-1	1	-7	-5	1	1
X18	-5	3	-1	1	-7	-5	1	1
X19	2	2	2	2	-2	2	-2	-2
X20	2	2	2	2	-2	2	-2	-2
X21	2	2	2	2	-2	2	-2	-2
X22	2	2	2	2	-2	2	-2	-2
X23	4	4	4	4	-4	4	-4	-4
X24	-10	6	-2	2	-14	-10	2	2
X25	14	-2	6	2	10	14	-6	-6
X26	3	-5	-1	-3	9	3	1	1
X27	-9	-1	-5	-3	-3	-9	5	5
X28	-9	-1	-5	-3	-3	-9	5	5
X29	3	-5	-1	-3	9	3	1	1
X30	4	4	4	4	-4	4	-4	-4
X31	8	0	-4	0	-4	4	4	-4
X32	4	-4	0	0	-8	8	0	0
X33	0	-8	4	0	-12	12	-4	4
X34	12	4	-8	0	0	0	8	-8
X35	4	-4	0	0	-8	8	0	0
X36	8	0	-4	0	-4	4	4	-4
X37	0	-8	4	0	-12	12	-4	4
X38	12	4	-8	0	0	0	8	-8
X39	-16	0	8	0	8	-8	-8	8
X40	-8	8	0	0	16	-16	0	0
X41	0	0	0	0	0	0	0	0
X42	1	1	1	-1	-1	1	-1	-1
X43	3	3	3	-3	-3	3	-3	-3
X44	2	2	2	-2	-2	2	-2	-2
X45	4	4	4	-4	-4	4	-4	-4
X46	1	1	1	-1	-1	1	-1	-1
X47	1	1	1	-1	-1	1	-1	-1
X48	1	1	1	-1	-1	1	-1	-1
X49	4	4	4	-4	-4	4	-4	-4
X50	-5	-5	-5	5	5	-5	5	5
X51	-3	-3	-3	3	3	-3	3	3
X52	-3	-3	-3	3	3	-3	3	3
X53	0	0	0	0	0	0	0	0
X54	0	0	0	0	0	0	0	0
X55	2	2	2	-2	-2	2	-2	-2
X56	7	-1	3	-1	-5	-5	-1	3
X57	7	-1	3	-1	-5	-5	-1	3
X58	-5	3	-1	-1	7	7	3	-1
	-5	3	-1	-1	7	7	3	-1
X59	2	2	2	-2	2	2	2	2
X60	2	2	2	-2	2	2	2	2
X61	2	2	2	-2	2	2	2	2
X62	2	2	2	-2	2	2	2	2
X63	4	4	4	-4	4	4	4	4
X64	-10	6	-2	-2	14	14	6	-2
X65	14	-2	6	-2	-10	-10	-2	6
X66	3	-5	-1	3	-9	-9	-5	-1
X67	-9	-1	-5	3	3	3	-1	-5
X68	-9	-1	-5	3	3	3	-1	-5
X69	3	-5	-1	3	-9	-9	-5	-1
X70	4	4	4	-4	4	4	4	4
X71	8	0	-4	0	4	-8	0	4
X72	4	-4	0	0	8	-4	4	0
X73	0	-8	4	0	12	0	8	-4
X74	12	4	-8	0	0	-12	-4	8
X75	4	-4	0	0	8	-4	4	0
X76	8	0	-4	0	4	-8	0	4
X77	0	-8	4	0	12	0	8	-4
X78	12	4	-8	0	0	-12	-4	8
X79	-16	0	8	0	-8	16	0	-8
X80	-8	8	0	0	-16	8	-8	0
X81	0	0	0	0	0	0	0	0
X82								

Class	6f	12a	12b	6g	12c	12d	7a	14a
Order	6	12	12	6	12	12	7	14
Size	26880	26880	53760	26880	26880	53760	184320	184

X1	1	1	1	1	1	1	1	1
X2	0	0	0	0	0	0	0	0
X3	-1	-1	-1	-1	-1	-1	0	0
X4	1	1	1	1	1	1	-1	-1
X5	-2	-2	-2	-2	-2	-2	0	0
X6	1	1	1	1	1	1	0	0
X7	1	1	1	1	1	1	0	0
X8	1	1	1	1	1	1	0	0
X9	1	1	1	1	1	1	0	0
X10	0	0	0	0	0	0	A	A
X11	0	0	0	0	0	0	/A	/A
X12	0	0	0	0	0	0	0	0
X13	0	0	0	0	0	0	1	1
X14	-1	-1	-1	-1	-1	-1	0	0
X15	1	1	-1	1	1	-1	0	0
X16	1	1	-1	1	1	-1	0	0
X17	1	1	-1	1	1	-1	0	0
X18	1	1	-1	1	1	-1	0	0
X19	-1	-1	1	-1	-1	1	0	0
X20	-1	-1	1	-1	-1	1	0	0
X21	-1	-1	1	-1	-1	1	0	0
X22	-1	-1	1	-1	-1	1	0	0
X23	1	1	-1	1	1	-1	0	0
X24	-1	-1	1	-1	-1	1	0	0
X25	-1	-1	1	-1	-1	1	0	0
X26	0	0	0	0	0	0	0	0
X27	0	0	0	0	0	0	0	0
X28	0	0	0	0	0	0	0	0
X29	0	0	0	0	0	0	0	0
X30	1	1	-1	1	1	-1	0	0
X31	2	-2	0	2	-2	0	0	0
X32	-2	2	0	-2	2	0	0	0
X33	0	0	0	0	0	0	0	0
X34	0	0	0	0	0	0	0	0
X35	-2	2	0	-2	2	0	0	0
X36	2	-2	0	2	-2	0	0	0
X37	0	0	0	0	0	0	0	0
X38	0	0	0	0	0	0	0	0
X39	2	-2	0	2	-2	0	0	0
X40	-2	2	0	-2	2	0	0	0
X41	0	0	0	0	0	0	0	0
X42	1	-1	-1	-1	1	1	1	-1
X43	0	0	0	0	0	0	0	0
X44	-1	1	1	1	-1	-1	0	0
X45	1	-1	-1	-1	1	1	-1	1
X46	-2	2	2	2	-2	-2	0	0
X47	1	-1	-1	-1	1	1	0	0
X48	1	-1	-1	-1	1	1	0	0
X49	1	-1	-1	-1	1	1	0	0
X50	1	-1	-1	-1	1	1	0	0
X51	0	0	0	0	0	0	/A	-/A
X52	0	0	0	0	0	0	A	-A
X53	0	0	0	0	0	0	0	0
X54	0	0	0	0	0	0	1	-1
X55	-1	1	1	1	-1	-1	0	0
X56	1	-1	1	-1	1	-1	0	0
X57	1	-1	1	-1	1	-1	0	0
X58	1	-1	1	-1	1	-1	0	0
X59	1	-1	1	-1	1	-1	0	0
X60	-1	1	-1	1	-1	1	0	0
X61	-1	1	-1	1	-1	1	0	0
X62	-1	1	-1	1	-1	1	0	0
X63	-1	1	-1	1	-1	1	0	0
X64	1	-1	1	-1	1	-1	0	0
X65	-1	1	-1	1	-1	1	0	0
X66	-1	1	-1	1	-1	1	0	0
X67	0	0	0	0	0	0	0	0
X68	0	0	0	0	0	0	0	0
X69	0	0	0	0	0	0	0	0
X70	0	0	0	0	0	0	0	0
X71	1	-1	1	-1	1	-1	0	0
X72	2	2	0	-2	-2	0	0	0
X73	-2	-2	0	2	2	0	0	0
X74	0	0	0	0	0	0	0	0
X75	0	0	0	0	0	0	0	0
X76	-2	-2	0	2	2	0	0	0
X77	2	2	0	-2	-2	0	0	0
X78	0	0	0	0	0	0	0	0
X79	0	0	0	0	0	0	0	0
X80	2	2	0	-2	-2	0	0	0
X81	-2	-2	0	2	2	0	0	0
X82	0	0	0	0	0	0	0	0

Class	3b	6h	6i	6j	6k	6l	6m	6n
Order	3	6	6	6	6	6	6	6
Size	17920	17920	17920	17920	17920	17920	17920	179

X1	1	1	1	1	1	1	1	1
X2	1	1	1	1	1	1	1	1
X3	2	2	2	2	2	2	2	2
X4	-1	-1	-1	-1	-1	-1	-1	-1
X5	0	0	0	0	0	0	0	0
X6	0	0	0	0	0	0	0	0
X7	0	0	0	0	0	0	0	0
X8	1	1	1	1	1	1	1	1
X9	2	2	2	2	2	2	2	2
X10	0	0	0	0	0	0	0	0
X11	0	0	0	0	0	0	0	0
X12	-1	-1	-1	-1	-1	-1	-1	-1
X13	-2	-2	-2	-2	-2	-2	-2	-2
X14	1	1	1	1	1	1	1	1
X15	2	0	0	-2	2	0	0	-2
X16	2	0	0	-2	2	0	0	-2
X17	2	0	0	-2	2	0	0	-2
X18	2	0	0	-2	2	0	0	-2
X19	1	-3	3	-1	1	-3	3	-1
X20	1	-3	3	-1	1	-3	3	-1
X21	1	3	-3	-1	1	3	-3	-1
X22	1	3	-3	-1	1	3	-3	-1
X23	-4	0	0	4	-4	0	0	4
X24	0	0	0	0	0	0	0	0
X25	0	0	0	0	0	0	0	0
X26	0	0	0	0	0	0	0	0
X27	0	0	0	0	0	0	0	0
X28	0	0	0	0	0	0	0	0
X29	0	0	0	0	0	0	0	0
X30	0	0	0	0	0	0	0	0
X31	1	-1	-1	1	1	-1	-1	1
X32	1	-1	-1	1	1	-1	-1	1
X33	-1	1	1	-1	-1	1	1	-1
X34	-1	1	1	-1	-1	1	1	-1
X35	2	-2	-2	2	2	-2	-2	2
X36	2	-2	-2	2	2	-2	-2	2
X37	0	0	0	0	0	0	0	0
X38	0	0	0	0	0	0	0	0
X39	1	-1	-1	1	1	-1	-1	1
X40	1	-1	-1	1	1	-1	-1	1
X41	-2	2	2	-2	-2	2	2	-2
X42	1	1	1	1	-1	-1	-1	-1
X43	1	1	1	1	-1	-1	-1	-1
X44	2	2	2	2	-2	-2	-2	-2
X45	-1	-1	-1	-1	1	1	1	1
X46	0	0	0	0	0	0	0	0
X47	0	0	0	0	0	0	0	0
X48	0	0	0	0	0	0	0	0
X49	1	1	1	1	-1	-1	-1	-1
X50	2	2	2	2	-2	-2	-2	-2
X51	0	0	0	0	0	0	0	0
X52	0	0	0	0	0	0	0	0
X53	1	1	1	1	-1	-1	-1	-1
X54	2	2	2	2	-2	-2	-2	-2
X55	1	1	1	1	-1	-1	-1	-1
X56	2	0	0	-2	-2	0	0	2
X57	2	0	0	-2	-2	0	0	2
X58	2	0	0	-2	-2	0	0	2
X59	2	0	0	-2	-2	0	0	2
X60	1	-3	3	-1	-1	3	-3	1
X61	1	-3	3	-1	-1	3	-3	1
X62	1	3	-3	-1	-1	-3	3	1
X63	1	3	-3	-1	-1	-3	3	1
X64	-4	0	0	4	4	0	0	-4
X65	0	0	0	0	0	0	0	0
X66	0	0	0	0	0	0	0	0
X67	0	0	0	0	0	0	0	0
X68	0	0	0	0	0	0	0	0
X69	0	0	0	0	0	0	0	0
X70	0	0	0	0	0	0	0	0
X71	0	0	0	0	0	0	0	0
X72	1	-1	-1	1	-1	1	1	-1
X73	1	-1	-1	1	-1	1	1	-1
X74	-1	1	1	-1	1	-1	-1	1
X75	-1	1	1	-1	1	-1	-1	1
X76	2	-2	-2	2	-2	2	2	-2
X77	2	-2	-2	2	-2	2	2	-2
X78	0	0	0	0	0	0	0	0
X79	0	0	0	0	0	0	0	0
X80	1	-1	-1	1	-1	1	1	-1
X81	1	-1	-1	1	-1	1	1	-1
X82	-2	2	2	-2	2	-2	-2	2

Class	2j	4g	2k	2l	2m	2n	4h	2o	[illegible]
Order	2	4	2	2	2	2	4	2	[illegible]
Size	420	5040	420	420	420	420	5040	420	[illegible]

X1	1	1	1	1	1	1	1	1	1
X2	-1	-1	-1	-1	-1	-1	-1	-1	-
X3	6	6	6	6	6	6	6	6	6
X4	4	4	4	4	4	4	4	4	4
X5	-3	-3	-3	-3	-3	-3	-3	-3	-
X6	-3	-3	-3	-3	-3	-3	-3	-3	-
X7	-3	-3	-3	-3	-3	-3	-3	-3	-
X8	-4	-4	-4	-4	-4	-4	-4	-4	-
X9	3	3	3	3	3	3	3	3	3
X10	-3	-3	-3	-3	-3	-3	-3	-3	-
X11	-3	-3	-3	-3	-3	-3	-3	-3	-
X12	8	8	8	8	8	8	8	8	8
X13	0	0	0	0	0	0	0	0	0
X14	-2	-2	-2	-2	-2	-2	-2	-2	-
X15	11	1	3	11	3	-5	-1	-5	3
X16	-5	1	3	-5	3	11	-1	11	3
X17	3	1	11	3	11	3	-1	3	-
X18	3	1	-5	3	-5	3	-1	3	1
X19	14	2	-2	14	-2	-2	-2	-2	1
X20	-2	2	14	-2	14	14	-2	14	-
X21	-2	2	-2	-2	-2	14	-2	14	1
X22	14	2	14	14	14	-2	-2	-2	-
X23	12	4	12	12	12	12	-4	12	1
X24	-6	-2	-6	-6	-6	-6	2	-6	-
X25	-6	-2	-6	-6	-6	-6	2	-6	-
X26	-21	1	3	-21	3	27	-1	27	3
X27	3	1	27	3	27	3	-1	3	-
X28	3	1	-21	3	-21	3	-1	3	2
X29	27	1	3	27	3	-21	-1	-21	3
X30	-12	-4	-12	-12	-12	-12	4	-12	-
X31	4	0	-4	4	-4	4	0	4	4
X32	-4	0	4	-4	4	-4	0	-4	-
X33	4	0	-4	4	-4	4	0	4	4
X34	-4	0	4	-4	4	-4	0	-4	-
X35	12	0	-12	12	-12	12	0	12	1
X36	-12	0	12	-12	12	-12	0	-12	-
X37	-12	0	12	-12	12	-12	0	-12	-
X38	12	0	-12	12	-12	12	0	12	1
X39	8	0	-8	8	-8	8	0	8	8
X40	-8	0	8	-8	8	-8	0	-8	-
X41	0	0	0	0	0	0	0	0	0
X42	1	-1	1	-1	-1	1	1	-1	1
X43	-1	1	-1	1	1	-1	-1	1	-
X44	6	-6	6	-6	-6	6	6	-6	6
X45	4	-4	4	-4	-4	4	4	-4	4
X46	-3	3	-3	3	3	-3	-3	3	-
X47	-3	3	-3	3	3	-3	-3	3	-
X48	-3	3	-3	3	3	-3	-3	3	-
X49	-4	4	-4	4	4	-4	-4	4	-
X50	3	-3	3	-3	-3	3	3	-3	3
X51	-3	3	-3	3	3	-3	-3	3	-
X52	-3	3	-3	3	3	-3	-3	3	-
X53	8	-8	8	-8	-8	8	8	-8	8
X54	0	0	0	0	0	0	0	0	0
X55	-2	2	-2	2	2	-2	-2	2	-
X56	11	1	3	-11	-3	-5	-1	5	3
X57	-5	1	3	5	-3	11	-1	-11	3
X58	3	1	11	-3	-11	3	-1	-3	-
X59	3	1	-5	-3	5	3	-1	-3	1
X60	14	2	-2	-14	2	-2	-2	2	1
X61	-2	2	14	2	-14	14	-2	-14	-
X62	-2	2	-2	2	2	14	-2	-14	1
X63	14	2	14	-14	-14	-2	-2	2	-
X64	12	4	12	-12	-12	12	-4	-12	1
X65	-6	-2	-6	6	6	-6	2	6	-
X66	-6	-2	-6	6	6	-6	2	6	-
X67	-21	1	3	21	-3	27	-1	-27	3
X68	3	1	27	-3	-27	3	-1	-3	-
X69	3	1	-21	-3	21	3	-1	-3	2
X70	27	1	3	-27	-3	-21	-1	21	3
X71	-12	-4	-12	12	12	-12	4	12	-
X72	4	0	-4	-4	4	4	0	-4	-
X73	-4	0	4	4	-4	-4	0	4	4
X74	4	0	-4	-4	4	4	0	-4	-
X75	-4	0	4	4	-4	-4	0	4	4
X76	12	0	-12	-12	12	12	0	-12	-
X77	-12	0	12	12	-12	-12	0	12	1
X78	-12	0	12	12	-12	-12	0	12	1
X79	12	0	-12	-12	12	12	0	-12	-
X80	8	0	-8	-8	8	8	0	-8	-
X81	-8	0	8	8	-8	-8	0	8	8
X82	0	0	0	0	0	0	0	0	0

Class	6o	6p	6q	6r	6s	6t	6u
Order	6	6	6	6	6	6	6
Size	53760	53760	53760	53760	53760	53760	53760

X1	1	1	1	1	1	1	1
X2	-1	-1	-1	-1	-1	-1	-1
X3	0	0	0	0	0	0	0
X4	1	1	1	1	1	1	1
X5	0	0	0	0	0	0	0
X6	0	0	0	0	0	0	0
X7	0	0	0	0	0	0	0
X8	-1	-1	-1	-1	-1	-1	-1
X9	0	0	0	0	0	0	0
X10	0	0	0	0	0	0	0
X11	0	0	0	0	0	0	0
X12	-1	-1	-1	-1	-1	-1	-1
X13	0	0	0	0	0	0	0
X14	1	1	1	1	1	1	1
X15	2	0	0	-2	2	0	0
X16	-2	0	0	2	-2	0	0
X17	0	2	-2	0	0	2	-2
X18	0	-2	2	0	0	-2	2
X19	-1	1	-1	1	-1	1	-1
X20	1	-1	1	-1	1	-1	1
X21	1	1	-1	-1	1	1	-1
X22	-1	-1	1	1	-1	-1	1
X23	0	0	0	0	0	0	0
X24	0	0	0	0	0	0	0
X25	0	0	0	0	0	0	0
X26	0	0	0	0	0	0	0
X27	0	0	0	0	0	0	0
X28	0	0	0	0	0	0	0
X29	0	0	0	0	0	0	0
X30	0	0	0	0	0	0	0
X31	1	-1	-1	1	1	-1	-1
X32	-1	1	1	-1	-1	1	1
X33	1	-1	-1	1	1	-1	-1
X34	-1	1	1	-1	-1	1	1
X35	0	0	0	0	0	0	0
X36	0	0	0	0	0	0	0
X37	0	0	0	0	0	0	0
X38	0	0	0	0	0	0	0
X39	-1	1	1	-1	-1	1	1
X40	1	-1	-1	1	1	-1	-1
X41	0	0	0	0	0	0	0
X42	1	1	1	1	-1	-1	-1
X43	-1	-1	-1	-1	1	1	1
X44	0	0	0	0	0	0	0
X45	1	1	1	1	-1	-1	-1
X46	0	0	0	0	0	0	0
X47	0	0	0	0	0	0	0
X48	0	0	0	0	0	0	0
X49	-1	-1	-1	-1	1	1	1
X50	0	0	0	0	0	0	0
X51	0	0	0	0	0	0	0
X52	0	0	0	0	0	0	0
X53	-1	-1	-1	-1	1	1	1
X54	0	0	0	0	0	0	0
X55	1	1	1	1	-1	-1	-1
X56	2	0	0	-2	-2	0	0
X57	-2	0	0	2	2	0	0
X58	0	2	-2	0	0	-2	2
X59	0	-2	2	0	0	2	-2
X60	-1	1	-1	1	1	-1	1
X61	1	-1	1	-1	-1	1	-1
X62	1	1	-1	-1	-1	-1	1
X63	-1	-1	1	1	1	1	-1
X64	0	0	0	0	0	0	0
X65	0	0	0	0	0	0	0
X66	0	0	0	0	0	0	0
X67	0	0	0	0	0	0	0
X68	0	0	0	0	0	0	0
X69	0	0	0	0	0	0	0
X70	0	0	0	0	0	0	0
X71	0	0	0	0	0	0	0
X72	1	-1	-1	1	-1	1	1
X73	-1	1	1	-1	1	-1	-1
X74	1	-1	-1	1	-1	1	1
X75	-1	1	1	-1	1	-1	-1
X76	0	0	0	0	0	0	0
X77	0	0	0	0	0	0	0
X78	0	0	0	0	0	0	0
X79	0	0	0	0	0	0	0
X80	-1	1	1	-1	1	-1	-1
X81	1	-1	-1	1	-1	1	1
X82	0	0	0	0	0	0	0

Class	4q	4r	4s	4t	4u	4v	4w	4x
Order	4	4	4	4	4	4	4	4
Size	20160	20160	20160	20160	20160	20160	20160	2016

X1	1	1	1	1	1	1	1	1
X2	-1	-1	-1	-1	-1	-1	-1	-1
X3	2	2	2	2	2	2	2	2
X4	0	0	0	0	0	0	0	0
X5	1	1	1	1	1	1	1	1
X6	1	1	1	1	1	1	1	1
X7	1	1	1	1	1	1	1	1
X8	0	0	0	0	0	0	0	0
X9	-1	-1	-1	-1	-1	-1	-1	-1
X10	1	1	1	1	1	1	1	1
X11	1	1	1	1	1	1	1	1
X12	0	0	0	0	0	0	0	0
X13	0	0	0	0	0	0	0	0
X14	-2	-2	-2	-2	-2	-2	-2	-2
X15	3	-1	-1	-1	3	-1	-1	-1
X16	-1	-1	-1	3	-1	-1	-1	3
X17	-1	3	-1	-1	-1	3	-1	-1
X18	-1	-1	3	-1	-1	-1	3	-1
X19	2	-2	2	-2	2	-2	2	-2
X20	-2	2	-2	2	-2	2	-2	2
X21	-2	-2	2	2	-2	-2	2	2
X22	2	2	-2	-2	2	2	-2	-2
X23	0	0	0	0	0	0	0	0
X24	2	-2	-2	2	2	-2	-2	2
X25	-2	2	2	-2	-2	2	2	-2
X26	3	-1	-1	-1	3	-1	-1	-1
X27	-1	-1	3	-1	-1	-1	3	-1
X28	-1	3	-1	-1	-1	3	-1	-1
X29	-1	-1	-1	3	-1	-1	-1	3
X30	0	0	0	0	0	0	0	0
X31	0	0	0	0	0	0	0	0
X32	0	0	0	0	0	0	0	0
X33	0	0	0	0	0	0	0	0
X34	0	0	0	0	0	0	0	0
X35	0	0	0	0	0	0	0	0
X36	0	0	0	0	0	0	0	0
X37	0	0	0	0	0	0	0	0
X38	0	0	0	0	0	0	0	0
X39	0	0	0	0	0	0	0	0
X40	0	0	0	0	0	0	0	0
X41	0	0	0	0	0	0	0	0
X42	1	1	1	1	-1	-1	-1	-1
X43	-1	-1	-1	-1	1	1	1	1
X44	2	2	2	2	-2	-2	-2	-2
X45	0	0	0	0	0	0	0	0
X46	1	1	1	1	-1	-1	-1	-1
X47	1	1	1	1	-1	-1	-1	-1
X48	1	1	1	1	-1	-1	-1	-1
X49	0	0	0	0	0	0	0	0
X50	-1	-1	-1	-1	1	1	1	1
X51	1	1	1	1	-1	-1	-1	-1
X52	1	1	1	1	-1	-1	-1	-1
X53	0	0	0	0	0	0	0	0
X54	0	0	0	0	0	0	0	0
X55	-2	-2	-2	-2	2	2	2	2
X56	3	-1	-1	-1	-3	1	1	1
X57	-1	-1	-1	3	1	1	1	-3
X58	-1	3	-1	-1	1	-3	1	1
X59	-1	-1	3	-1	1	1	-3	1
X60	2	-2	2	-2	-2	2	-2	2
X61	-2	2	-2	2	2	-2	2	-2
X62	-2	-2	2	2	2	2	-2	-2
X63	2	2	-2	-2	-2	-2	2	2
X64	0	0	0	0	0	0	0	0
X65	2	-2	-2	2	-2	2	2	-2
X66	-2	2	2	-2	2	-2	-2	2
X67	3	-1	-1	-1	-3	1	1	1
X68	-1	-1	3	-1	1	1	-3	1
X69	-1	3	-1	-1	1	-3	1	1
X70	-1	-1	-1	3	1	1	1	-3
X71	0	0	0	0	0	0	0	0
X72	0	0	0	0	0	0	0	0
X73	0	0	0	0	0	0	0	0
X74	0	0	0	0	0	0	0	0
X75	0	0	0	0	0	0	0	0
X76	0	0	0	0	0	0	0	0
X77	0	0	0	0	0	0	0	0
X78	0	0	0	0	0	0	0	0
X79	0	0	0	0	0	0	0	0
X80	0	0	0	0	0	0	0	0
X81	0	0	0	0	0	0	0	0
X82	0	0	0	0	0	0	0	0

Class	4y	8a	4z	8b	4aa	8c	4ab	8d
Order	4	8	4	8	4	8	4	8
Size	40320	40320	40320	40320	40320	40320	40320	4032

X1	1	1	1	1	1	1	1	1
X2	1	1	1	1	1	1	1	1
X3	0	0	0	0	0	0	0	0
X4	0	0	0	0	0	0	0	0
X5	-1	-1	-1	-1	-1	-1	-1	-1
X6	-1	-1	-1	-1	-1	-1	-1	-1
X7	-1	-1	-1	-1	-1	-1	-1	-1
X8	0	0	0	0	0	0	0	0
X9	-1	-1	-1	-1	-1	-1	-1	-1
X10	1	1	1	1	1	1	1	1
X11	1	1	1	1	1	1	1	1
X12	0	0	0	0	0	0	0	0
X13	0	0	0	0	0	0	0	0
X14	0	0	0	0	0	0	0	0
X15	1	-1	1	-1	1	-1	1	-1
X16	1	-1	1	-1	1	-1	1	-1
X17	-1	1	-1	1	-1	1	-1	1
X18	-1	1	-1	1	-1	1	-1	1
X19	0	0	0	0	0	0	0	0
X20	0	0	0	0	0	0	0	0
X21	0	0	0	0	0	0	0	0
X22	0	0	0	0	0	0	0	0
X23	0	0	0	0	0	0	0	0
X24	0	0	0	0	0	0	0	0
X25	0	0	0	0	0	0	0	0
X26	-1	1	-1	1	-1	1	-1	1
X27	1	-1	1	-1	1	-1	1	-1
X28	1	-1	1	-1	1	-1	1	-1
X29	-1	1	-1	1	-1	1	-1	1
X30	0	0	0	0	0	0	0	0
X31	2	0	-2	0	2	0	-2	0
X32	0	-2	0	2	0	-2	0	2
X33	-2	0	2	0	-2	0	2	0
X34	0	2	0	-2	0	2	0	-2
X35	0	2	0	-2	0	2	0	-2
X36	-2	0	2	0	-2	0	2	0
X37	2	0	-2	0	2	0	-2	0
X38	0	-2	0	2	0	-2	0	2
X39	0	0	0	0	0	0	0	0
X40	0	0	0	0	0	0	0	0
X41	0	0	0	0	0	0	0	0
X42	1	1	1	1	-1	-1	-1	-1
X43	1	1	1	1	-1	-1	-1	-1
X44	0	0	0	0	0	0	0	0
X45	0	0	0	0	0	0	0	0
X46	-1	-1	-1	-1	1	1	1	1
X47	-1	-1	-1	-1	1	1	1	1
X48	-1	-1	-1	-1	1	1	1	1
X49	0	0	0	0	0	0	0	0
X50	-1	-1	-1	-1	1	1	1	1
X51	1	1	1	1	-1	-1	-1	-1
X52	1	1	1	1	-1	-1	-1	-1
X53	0	0	0	0	0	0	0	0
X54	0	0	0	0	0	0	0	0
X55	0	0	0	0	0	0	0	0
X56	1	-1	1	-1	-1	1	-1	1
X57	1	-1	1	-1	-1	1	-1	1
X58	-1	1	-1	1	1	-1	1	-1
X59	-1	1	-1	1	1	-1	1	-1
X60	0	0	0	0	0	0	0	0
X61	0	0	0	0	0	0	0	0
X62	0	0	0	0	0	0	0	0
X63	0	0	0	0	0	0	0	0
X64	0	0	0	0	0	0	0	0
X65	0	0	0	0	0	0	0	0
X66	0	0	0	0	0	0	0	0
X67	-1	1	-1	1	1	-1	1	-1
X68	1	-1	1	-1	-1	1	-1	1
X69	1	-1	1	-1	-1	1	-1	1
X70	-1	1	-1	1	1	-1	1	-1
X71	0	0	0	0	0	0	0	0
X72	2	0	-2	0	-2	0	2	0
X73	0	-2	0	2	0	2	0	-2
X74	-2	0	2	0	2	0	-2	0
X75	0	2	0	-2	0	-2	0	2
X76	0	2	0	-2	0	-2	0	2
X77	-2	0	2	0	2	0	-2	0
X78	2	0	-2	0	-2	0	2	0
X79	0	-2	0	2	0	2	0	-2
X80	0	0	0	0	0	0	0	0
X81	0	0	0	0	0	0	0	0
X82	0	0	0	0	0	0	0	0

Class	15b	30b
Order	15	30
Size	86016	86016

X1	1	1
X2	-1	-1
X3	-1	-1
X4	0	0
X5	1	1
X6	/B	/B
X7	B	B
X8	1	1
X9	0	0
X10	0	0
X11	0	0
X12	1	1
X13	-1	-1
X14	0	0
X15	0	0
X16	0	0
X17	0	0
X18	0	0
X19	0	0
X20	0	0
X21	0	0
X22	0	0
X23	0	0
X24	0	0
X25	0	0
X26	0	0
X27	0	0
X28	0	0
X29	0	0
X30	0	0
X31	0	0
X32	0	0
X33	0	0
X34	0	0
X35	0	0
X36	0	0
X37	0	0
X38	0	0
X39	0	0
X40	0	0
X41	0	0
X42	1	-1
X43	-1	1
X44	-1	1
X45	0	0
X46	1	-1
X47	/B	-/B
X48	B	-B
X49	1	-1
X50	0	0
X51	0	0
X52	0	0
X53	1	-1
X54	1	-1
X55	0	0
X56	0	0
X57	0	0
X58	0	0
X59	0	0
X60	0	0
X61	0	0
X62	0	0
X63	0	0
X64	0	0
X65	0	0
X66	0	0
X67	0	0
X68	0	0
X69	0	0
X70	0	0
X71	0	0
X72	0	0
X73	0	0
X74	0	0
X75	0	0
X76	0	0
X77	0	0
X78	0	0
X79	0	0
X80	0	0
X81	0	0
X82	0	0

Explanation of Character Value Symbols:

$$A = \frac{-1-\sqrt{-7}}{2}; \quad B = \frac{-1-\sqrt{-15}}{2}$$

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