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INITIAL CHEBYSHEV POLYNOMIAL CLASS OF ANALYTIC FUNCTIONS BASED ON QUASI-SUBORDINATION

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Abstract. In this work, the authors investigated the initial Chebyshev polynomial class of analytic functions based on quasi-subordination. The coefficient estimates including the relevant connection to the Fekete-Szegö inequality of functions belonging to the class $\mathcal{G}_n^(q, \gamma, t)$ were derived. Also, certain results for the associated classes involving subordination and majorization were presented.

1 Introduction

Let Γ denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1.1}$$

which is analytic in the unit disk $E = \{z : |z| < 1\}$ and normalized by $f(0) = f'(0) - 1 = 0$

Recall that S denotes the class of univalent functions. A function is said to be starlike if $Re\left\{\frac{zf'(z)}{f(z)}\right\} > 0$ and denoted by S^*, while a function is said to be convex if $Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 0$.

For two analytic functions f and g such that $f(0) = g(0)$, we say that f is subordinate to g in E and write $f(z) \prec g(z)$, $z \in E$, if there exists a Schwarz function $w(z)$ with $w(0) = 0$ and $|w(z)| \leq |z|$ such that $f(z) = g(w(z))$. Furthermore, if the function g is univalent in E, then we have the following equivalence: $f(z) \prec g(z) \Leftrightarrow f(0) = g(0)$ and $f(E) \subset g(E)$. (See details in [4].

A function f is said to be quasi-subordinate to g in E and written as $f(z) \prec_q g(z)$, $z \in E$, if there exists an analytic function $\varphi(z)$ with $|\varphi| \leq 1$, $(z \in E)$ such that $\frac{f(z)}{\varphi(z)}$ is analytic in E and $\frac{f(z)}{\varphi(z)} \prec g(z)$, $(z \in E)$, that is, there exists a Schwarz function $w(z)$ such that $f(z) = \varphi(z)g(w(z))$, $z \in E$ defined by [6].

It is also seen that if $\varphi(z) \equiv 1$, $(z \in E)$, then the quasi-subordination $\prec_q$ becomes the usual subordination $\prec$, and for the Schwarz function $w(z) = z, (z \in E)$, the quasi-subordination $\prec_q$ becomes the majorization '$\ll$'. That is, $f(z) \prec_q g(z) \Rightarrow f(z) = \varphi(z)g(z) \Rightarrow f(z) \ll g(z)$, $z \in E$. The concept of majorization was revealed by [3].

The Sălăgean[8] differential operator is defined as follows: Let $D^n : \Gamma \longrightarrow \Gamma$, then

$$\begin{aligned} D^0 f(z) &= f(z) \\ D^1 f(z) &= zf'(z) \\ &\vdots \\ D^n f(z) &= z(D^{n-1}f(z))' \end{aligned} \tag{1.2}$$

Applying (1.2) in (1.1), then we have

$$D^n f(z) = z\left(D^{n-1}f(z)\right)' = z + \sum_{k=2}^{\infty} k^n a_k z^k \tag{1.3}$$

Let $\gamma > 0$ (γ is real) such that (1.1) gives

$$f(z)^\gamma = \left(z + \sum_{k=2}^{\infty} a_k z^k\right)^\gamma \tag{1.4}$$

Thus, (1.4) can be rewritten as

$$f(z)^\gamma = z^\gamma(1 + a_2 z + a_3 z^2 + a_4 z^3 + ...)^\gamma. \tag{1.5}$$

then, the binomial expansion of (1.5) yields

$$\begin{aligned} f(z)^\gamma = z^\gamma + \gamma a_2 z^{\gamma+1} + \left(\frac{\gamma(\gamma-1)}{2}a_2^2 + \gamma a_3\right) z^{\gamma+2} + \\ \left(\frac{\gamma(\gamma-1)(\gamma-2)}{6}a_2^3 + \gamma(\gamma-1)a_2 a_3 + \gamma a_4\right) z^{\gamma+3} + ... \end{aligned} \tag{1.6}$$

Finally applying (1.2) in (1.6), we obtain

$$\begin{aligned} D^n f(z)^\gamma = z^\gamma + 2^n \gamma a_2 z^{\gamma+1} + 3^n \left(\frac{\gamma(\gamma-1)}{2}a_2^2 + \gamma a_3\right) z^{\gamma+2} \\ +4^n \left(\frac{\gamma(\gamma-1)(\gamma-2)}{6}a_2^3 + \gamma(\gamma-1)a_2 a_3 + \gamma a_4\right) z^{\gamma+3} + ... \end{aligned} \tag{1.7}$$

Chebyshev polynomial is a normal function used by numerical analyst and it can be categorized into four kind. The first and second kind $T_n(x)$ and $U_n(x)$ are the most related kind of polynomial found in literature and they have numerous application in different fields.
The usual kind of the Chebyshev polynomials are the first and the second kind. In the case of real variable x on $[-1, 1]$, the first and second kind are defined by

$$T_n(x) = cosn\theta$$

and

$$U_n(x) = \frac{sin(n+1)\theta}{sin\theta}$$

where the subscript n denotes the polynomial degree and where $x = cos\theta$ respectively.
Letting $t = cos\alpha, \alpha \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$, one can obtain

$$H(z,t) = \frac{1}{1 - 2tz + z^2} = 1 + \sum_{k=1}^{\infty} \frac{sin(k+1)\alpha}{sin\alpha} z^k = 1 + 2cos\alpha z + (3cos^2\alpha - sin^2\alpha)z^2 + ...(z \in E)$$

So, we write

$$H(z,t) = 1 + U_1(t)z + U_2(t)z^2 + ...(z \in U, t \in (-1, 1))$$

where $U_{n-1} = \frac{sin(k\ arccost)}{\sqrt{1-t^2}}$ for $k \in \mathcal{N}$, are the second kind of the Chebyshev polynomials. More so, we know that

$$U_n(t) = 2tU_{n-1}(t) - U_{n-2}(t) \tag{1.8}$$

and

$$U_1(t) = 2t; U_2(t) = 4t^2 - 1; U_3(t) = 8t^3 - 4t; U_4(t) = 16t^4 - 12t^2 + 1, ... \tag{1.9}$$

The Chebyshev polynomials $T_n(t); t \in [-1, 1]$ of the first kind have generating function of the form

$$\sum_{n=0}^{\infty} T_n(t)z^n = \frac{1 - tz}{1 - 2tz + z^2}. \qquad z \in E \tag{1.10}$$

All the same, there is the following relationship between the Chebyshev polynomial of the first kind $T_n(x)$ and the second kind $U_n(t)$

$$\frac{dT_n(t)}{dt} = nU_{n-1}(t)T_n(t) = U_n(t) - tU_{n-1}(t)2T_n(t) = U_n(t) - U_{n-2}(t).$$

See details in [1].
In this present work, we focus mainly on determining the coefficient estimates including a Fekete-Szegö inequality [2], [5], [7], [9] of functions belonging to the classes $\mathcal{G}_n^(q, \gamma, t)$, $\mathcal{G}_n(\gamma, t)$ and the class involving majorization. In order to obtain our results, the following lemmas and definitions shall be required.

Lemma 1.1. *[10] If a function $p \in P$ is given by*

$$p(z) = 1 + p_1z + p_2z^2 + ... \quad (z \in E), \tag{1.11}$$

then $|p_k| \leq 2$, $(k \in N)$, where P is the family of all functions analytic in E for which $p(0) = 1$ and Re $p(z) > 0$, $(z \in E)$.

Lemma 1.2. *[11] Let the Schwarz function $w(z)$ be given by*

$$w(z) = w_1z + w_2z^2 + w_3z^3 + ..., (z \in E) \tag{1.12}$$

then

$$|w_1| \leq 1, |w_2 - tw_1^2| \leq 1 + (|t| - 1)|w_1|^2 \leq max\{1, |t|\}, \tag{1.13}$$

where $t \in \mathcal{C}$.

Definition 1.3. Let $H(z,t) \in P$ be univalent and $H(z,t)(U)$ symmetrical about the real axis $(H(z,t))'(0) > 0$. For $t(-1,1), \gamma > 0, n \in \mathcal{N}$ and , a function $f \in \Gamma$ is said to be in the class $\mathcal{G}_n^*(q, \gamma, t)$ if

$$\left(\frac{D^{n+1}f(z)^\gamma}{D^nf(z)^\gamma} - 1\right) \prec_q (H(z,t) - 1), \quad z \in E. \tag{1.14}$$

and $\varphi(z)$ which is analytic in E is of the form

$$\varphi(z) = d_0 + d_1z + d_2z^2 + ... \tag{1.15}$$

2 Main Results

Theorem 2.1. *Let $f(z)^\gamma \in \Gamma$ of the form (1.4) belong to the class $\mathcal{G}_n^*(q, \gamma, t)$, then*

$$|a_2| \leq \frac{2t}{|\gamma(2^{n+1} - 2^n)|} \tag{2.1}$$

and for some $\mu \in \mathcal{C}$,

$$|a_3 - ca_2^2| \leq \frac{2t}{|\gamma(3^{n+1} - 3^n)|} \max\left\{1, \left|\frac{4t^2 - 1}{2t} - \frac{t\left[(3^{n+1} - 3^n)(\gamma(\gamma - 1) + 2c) - 2\gamma^2(2^{2n+1} - 2^{2n})\right]}{\gamma^2(2^{n+1} - 2^n)^2}\right|\right\} \tag{2.2}$$

where $\gamma > 0$, (γ is real),$t \in (-1, 1)$ and $n \in \mathcal{N}$. The result is sharp.

Proof. Let $f \in \Gamma$, then for a Schwarz function $\omega(z)$ given by (1.12) and for an analytic function $\varphi(z)$ given by (1.15), we have

$$\left(\frac{D^{n+1}f(z)^\gamma}{D^n f(z)^\gamma} - 1\right) = \varphi(H(\omega(z),t)-1), \quad z \in E. \tag{2.3}$$

In view of (2.3) we obtain

$$\varphi(H(\omega(z),t)-1) = \left(d_0 + d_1 z + d_2 z^2 + ...\right)\left(U_1(t)\omega_1 z + (U_2(t)\omega_1^2 + U_1(t)\omega_2)z^2 + ...\right)$$

$$= d_0 U_1(t)\omega_1 z + \left\{d_0(U_2(t)\omega_1^2 + U_1(t)\omega_2) + d_1 U_1(t)\omega_1\right\} z^2 + ... \tag{2.4}$$

Using the series expansion for

$$\left(\frac{D^{n+1}f(z)^\gamma}{D^n f(z)^\gamma} - 1\right)$$

gives

$$\left(2^{n+1} - 2^n\right)\gamma a_2 z + \left[\left(2^{2n} - 2^{2n+1}\right)\gamma^2 a_2^2 + \left(3^{n+1} - 3^n\right)\left(\frac{\gamma(\gamma-1)}{2}a_2^2 + a_3\gamma\right)\right] z^2 + ... \tag{2.5}$$

From the expansion (2.4) and (2.5) ,on equating the coefficients of z and z^2 in (2.3) find that

$$\left(2^{n+1} - 2^n\right)\gamma a_2 = d_0 U_1(t)\omega_1 \tag{2.6}$$

$$\left(2^{2n} - 2^{2n+1}\right)\gamma^2 a_2^2 + \left(3^{n+1} - 3^n\right)\left(\frac{\gamma(\gamma-1)}{2}a_2^2 + a_3\gamma\right) = d_0(U_2(t)\omega_1^2 + U_1(t)\omega_2) + d_1 U_1(t)\omega_1 \tag{2.7}$$

Now (2.6) gives

$$a_2 = \frac{d_0 U_1(t)\omega_1}{\left(2^{n+1} - 2^n\right)\gamma} \tag{2.8}$$

which in view of (2.7) yields that

$$\gamma\left(3^{n+1} - 3^n\right)a_3 = \frac{\left(2\left(2^{2n+1} - 2^{2n}\right)\gamma - \left(3^{n+1} - 3^n\right)(\gamma - 1)\right)d_0^2 U_1^2(t)\omega_1^2}{2\gamma\left(2^{n+1} - 2^n\right)^2} + d_0(U_2(t)\omega_1^2 + U_1(t)\omega_2) + d_1 U_1(t)\omega_1 \tag{2.9}$$

and therefore

$$a_3 = \frac{U_1(t)}{\gamma\left(3^{n+1} - 3^n\right)} \left[d_1\omega_1 + d_0\left\{\omega_2 + \left(\frac{\left(2\left(2^{2n+1} - 2^{2n}\right)\gamma - \left(3^{n+1} - 3^n\right)(\gamma-1)\right)d_0 U_1(t)}{2\gamma\left(2^{n+1} - 2^n\right)^2}\right) + \frac{U_2(t)}{U_1(t)}\right\}\omega_1^2\right] \tag{2.10}$$

For some $c \in \mathcal{C}$, we obtain from (2.8) and (2.10)

$$a_3 - ca_2^2 = \frac{U_1(t)}{\gamma\left(3^{n+1} - 3^n\right)} \left[d_1\omega_1 + d_0\left(\omega_2 + \frac{U_2(t)}{U_1(t)}\omega_1^2\right) + \left(\frac{\left(2\gamma^2\left(2^{2n+1} - 2^{2n}\right) - \left(3^{n+1} - 3^n\right)(\gamma(\gamma-1) + 2c)\right)d_0^2 U_1^2(t)\omega_1^2}{2\gamma^2\left(2^{n+1} - 2^n\right)^2}\right)\right] \tag{2.11}$$

Since $\varphi(z)$ given by (1.15) is analytic and bounded in U, therefore on using [4],[p.172], we have for some $y(|y| \leq 1)$;

$$|d_0| \leq 1 \qquad and \qquad d_1 = (1 - d_0^2)y \tag{2.12}$$

On putting the value of d_1 from (2.12) into (2.11), we get

$$a_3 - ca_2^2 = \frac{U_1(t)}{\gamma\left(3^{n+1}-3^n\right)}$$

$$\left[y\omega_1 + d_0\left(\omega_2 + \frac{U_2(t)}{U_1(t)}\omega_1^2\right) - \left(\frac{\left(\left(3^{n+1}-3^n\right)\left(\gamma(\gamma-1)+2c\right)-2\gamma^2\left(2^{2n+1}-2^{2n}\right)\right)U_1(t)\omega_1^2}{2\gamma^2\left(2^{n+1}-2^n\right)^2} + y\omega_1\right)d_0^2\right] (2$$

If $d_0 = 0$ in (2.13) then,

$$\left|a_3 - ca_2^2\right| \leq \frac{U_1(t)}{\left|\gamma\left(3^{n+1}-3^n\right)\right|}. \tag{2.14}$$

But if $d_0 \neq 0$, let us then suppose that

$$F(d_0) = y\omega_1 + d_0\left(\omega_2 + \frac{U_2(t)}{U_1(t)}\omega_1^2\right) -$$

$$\left(\frac{\left(\left(3^{n+1}-3^n\right)\left(\gamma(\gamma-1)+2c\right)-2\gamma^2\left(2^{2n+1}-2^{2n}\right)\right)U_1(t)\omega_1^2}{2\gamma^2\left(2^{n+1}-2^n\right)^2} + y\omega_1\right)d_0^2, \tag{2.15}$$

which is a polynomial in d_0 and have analytic in $|d_0| \leq 1$, and maximum of $|F(d_0)|$ is attained at $d_0 = e^{i\theta}(0 \leq \theta < 2\pi)$. We find that $max_{0\leq\theta<2\pi}|F(e^{i\theta})| = |F(1)|$ and

$$\left|a_3 - ca_2^2\right| \leq \frac{U_1(t)}{\gamma\left(3^{n+1}-3^n\right)}$$

$$\left|\omega_2 - \left(\frac{\left(\left(3^{n+1}-3^n\right)\left(\gamma(\gamma-1)+2c\right)-2\gamma^2\left(2^{2n+1}-2^{2n}\right)\right)}{2\gamma^2\left(2^{n+1}-2^n\right)^2}U_1(t) - \frac{U_2(t)}{U_1(t)}\right)\omega_1^2\right|. \tag{2.16}$$

Using lemma 1.2 gives

$$\left|a_3 - ca_2^2\right|$$

$$\leq \frac{U_1(t)}{\left|\gamma\left(3^{n+1}-3^n\right)\right|} max\left\{1, \left|\frac{\left(\left(3^{n+1}-3^n\right)\left(\gamma(\gamma-1)+2c\right)-2\gamma^2\left(2^{2n+1}-2^{2n}\right)\right)}{2\gamma^2\left(2^{n+1}-2^n\right)^2}U_1(t) - \frac{U_2(t)}{U_1(t)}\right|\right\} (2.17)$$

and the above inequality together with (2.14) establishes the result in (2.2).
□

Theorem 2.2. *Let $f(z)^\gamma \in \Gamma$ of the form (1.4) belong to the class $\mathcal{G}_n^*(q,\gamma,t)$, then*

$$|a_2| \leq \frac{2t}{|\gamma(2^{n+1}-2^n)|}$$

and for some $\mu \in \mathcal{C}$,

$$|a_3 - ca_2^2| \leq \frac{2t}{|\gamma(3^{n+1}-3^n)|}\max\left\{1, \left|\frac{4t^2-1}{2t} - \frac{t\left[(3^{n+1}-3^n)(\gamma(\gamma-1)+2c)-2\gamma^2(2^{2n+1}-2^{2n})\right]}{\gamma^2(2^{n+1}-2^n)^2}\right|\right\}$$

where $\gamma > 0$, (γ is real),$t \in (-1,1)$ and $n \in \mathcal{N}$. The result is sharp.

Proof. Let $f \in \mathcal{G}_n^(q,\gamma,t)$. Similar to the proof of Theorem 1. If $\varphi(z) \equiv 1$, then (1.15) evidently implies that $d_0 = 1$ and $d_n = 0, n \in \mathcal{N}$, hence, in view of (2.8) and (2.10) and Lemma 1.2, we obtain the desired result of Theorem 2.2. □
Here, the majorization and the result pertaining to it is contained in the following:

Theorem 2.3. *If a function $f \in \Gamma$ of the form (1.4) satisfies*

$$\frac{D^{n+1}f(z)^\gamma}{D^n f(z)^\gamma} - 1 << (H(z,t)-1), \quad z \in E. \tag{2.18}$$

then

$$|a_2| \leq \frac{2t}{|\gamma(2^{n+1}-2^n)|} \tag{2.19}$$

and for some $\mu \in \mathcal{C}$,

$$|a_3 - ca_2^2| \leq \frac{2t}{|\gamma(3^{n+1}-3^n)|} \max\left\{1, \left|\frac{4t^2-1}{2t} - \frac{t\left[(3^{n+1}-3^n)(\gamma(\gamma-1)+2c) - 2\gamma^2(2^{2n+1}-2^{2n})\right]}{\gamma^2(2^{n+1}-2^n)^2}\right|\right\} \tag{2.20}$$

where $\gamma > 0$, $t \in (-1,1)$,(γ is real) and $n \in \mathcal{N}$. The result is sharp.

Proof. Following the proof of Theorem 2.1, if $\omega(z) \equiv z$ in (1.13), so that $\omega_1 = 1$ and $\omega_n = 0, n = 2,3,4,5,...$ then in view of (2.8) and (2.10), we get

$$|a_2| \leq \frac{U_1(t)}{|\gamma(2^{n+1}-2^n)|} \tag{2.21}$$

and

$$a_3 - ca_2^2 = \frac{U_1(t)}{\gamma\left(3^{n+1}-3^n\right)} \left[d_1 + d_0\frac{U_2(t)}{U_1(t)} - \left(\frac{\left(3^{n+1}-3^n\right)\left(\gamma(\gamma-1)+2c\right) - 2\gamma^2\left(2^{2n+1}-2^{2n}\right)}{2\gamma^2\left(2^{n+1}-2^n\right)^2}\right) d_0^2 U_1(t)\right]. \tag{2.22}$$

On putting the value of d_1 from (2.12) in (2.22), we get

$$a_3 - ca_2^2 = \frac{U_1(t)}{\gamma\left(3^{n+1}-3^n\right)} \left[y + d_0\frac{U_2(t)}{U_1(t)} - \left(\frac{\left[\left(3^{n+1}-3^n\right)\left(\gamma(\gamma-1)+2c\right) - 2\gamma^2\left(2^{2n+1}-2^{2n}\right)\right]U_1(t)}{2\gamma^2\left(2^{n+1}-2^n\right)^2} + y\right) d_0^2\right]. \tag{2.23}$$

If $d_0 = 0$, we get

$$\left|a_3 - ca_2^2\right| \leq \frac{U_1(t)}{\left|\gamma\left(3^{n+1}-3^n\right)\right|}. \tag{2.24}$$

and if $d_0 \neq 0$, let

$$G(d_0) := y + d_0\frac{U_2(t)}{U_1(t)} - \left(\frac{\left[\left(3^{n+1}-3^n\right)\left(\gamma(\gamma-1)+2c\right) - 2\gamma^2\left(2^{2n+1}-2^{2n}\right)\right]U_1(t)}{2\gamma^2\left(2^{n+1}-2^n\right)^2} + y\right) d_0^2 \tag{2.25}$$

which being a polynomial in d_0 is analytic in $|d_0| \leq 1$, and maximum of $|G(d_0)|$ is attained at $d_0 = e^{i\theta}(0 \leq \theta < 2\pi)$. We thus find $max_{0\leq\theta<2\pi}|G(e^{i\theta})| = |G(1)|$ and consequently

$$\left|a_3 - ca_2^2\right| \leq \frac{U_1(t)}{\left|\gamma\left(3^{n+1}-3^n\right)\right|} \left|\frac{\left[\left(3^{n+1}-3^n\right)\left(\gamma(\gamma-1)+2c\right) - 2\gamma^2\left(2^{2n+1}-2^{2n}\right)\right]U_1(t)}{2\gamma^2\left(2^{n+1}-2^n\right)^2} - \frac{U_2(t)}{U_1(t)}\right|. \tag{2.26}$$

This completes the proof. □

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