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C-Bochner curvature tensor on almost $C(\lambda)$ manifolds

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Abstract. In this paper we have studied C-Bochner curvature tensor in almost $C(\lambda)$ manifolds with the conditions $B(\xi, X).S = 0$, $B(\xi, X).R = 0$ and $B(\xi, X).B = 0$, where R, S and B denotes Riemannian curvature tensor, Ricci tensor and C-Bochner curvature tensor respectively. Also, we have studied ξ-C-Bochner flat $C(\lambda)$ manifold.

1 Introduction

An almost contact metric manifold $M^{2n+1}(\phi, \xi, \eta, g)$ is said to be an almost $C(\lambda)$ manifold if the curvature tensor R of the manifold have the form [12]

$$R(X,Y)Z = R(\phi X, \phi Y)Z - \lambda[g(Y,Z)X - g(X,Z)Y - \phi X g(\phi Y, Z) + g(\phi X, Z)\phi Y], \quad (1.1)$$

for any vector fields $X, Y, Z \in TM$ and λ is real number.
S. V. Kharitonova [12] proved that if $\lambda = 0$, $\lambda = 1$ and $\lambda = -1$ then $C(\lambda)$ manifolds becomes cosymplectic, Sasakian, and Kenmotsu manifolds respectively. In 2013, Ali Akber and Avijit Sarkar[1] studied conharmonic and concircular curvature tensors. They proved that the concircular and conharmonic curvature tensors in $C(\lambda)$ manifold vanish if either $\lambda = 0$ or the manifold be a special type of η-Einstein manifold. In 1949, S. Bochner [13] gave the idea of Bochner curvature tensor. D. E. Blair[5] explain the Bochner curvature tensor geometrically in 1975, Matsumoto and Chuman [9] constructed a curvature tensor from the Bochner curvature tensor with the help of Boothby-Wangs fibrations[17] and called it C-Bochner curvature tensor. J. S. Kim, M. M. Tripathi and J.Choi[8] studied C-Bochner curvature tensor of a contact metric manifold in 2005. C-Bochner curvature tensor studied by several authors, viz., [4, 7, 11, 16] by different aproaches.

The C-Bochner curvature tensor is defined by [9]

$$\begin{aligned} B(X,Y)Z = R(X,Y)Z &+ \frac{1}{2(n+2)}\Big\{S(X,Z)Y - S(Y,Z)X \\ &+ g(X,Z)QY - g(Y,Z)QX + S(\phi X,Z)\phi Y \\ &- S(\phi Y,Z)\phi X + g(\phi X,Z)Q\phi Y - g(\phi Y,Z)Q\phi X \\ &+ 2S(\phi X,Y)\phi Z + 2g(\phi X,Y)Q\phi Z - S(X,Z)\eta(Y)\xi \\ &+ S(Y,Z)\eta(X)\xi - \eta(X)\eta(Z)QY + \eta(Y)\eta(Z)QX\Big\} \\ &- \frac{\tau+2n}{2(n+2)}\Big\{g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X \\ &+ 2g(\phi X,Y)\phi Z\Big\} - \frac{\tau-4}{2(n+2)}\{g(X,Z)Y - g(Y,Z)X\} \\ &+ \frac{\tau}{2(n+2)}\Big\{g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi \\ &+ \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X\Big\}, \end{aligned} \tag{1.2}$$

where $\tau = \frac{r+2n}{2(n+2)}$, Q is Ricci operator i.e. $g(QX,Y) = S(X,Y)$ for all X and Y and r is a scalar curvature of the manifold.

2 Preliminaries

A Riemannian manifold (M^{2n+1}, g) of dimension $(2n+1)$ is said to be an almost contact metric manifold [3] if there exist a tensior field ϕ of type (1, 1), a vector field ξ (called the structure vector field) and a 1-form η on M such that

$$\phi^2(X) = -X + \eta(X)\xi, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.2}$$

and

$$\eta(\xi) = 1, \tag{2.3}$$

for any vector fields X, Y on M . In an almost contact metric manifold, we have

$$\phi\xi = 0, \quad \eta o\phi = 0. \tag{2.4}$$

Then such type of manifold is a called contact metric manifold if $d\eta = \Phi$, where $\Phi(X,Y) = g(X,\phi Y)$, is called the fundamental 2-form of $M^{(2n+1)}$.
A contact metric manifold is said to be K-contact manifold if and only if the covariant derivative of ξ satisfies

$$\nabla_X\xi = -\phi X, \tag{2.5}$$

for any vector field X on M.
The almost contact metric structure of M is said to be normal if

$$[\phi,\phi](X,Y) = -2d\eta(X,Y)\xi, \tag{2.6}$$

for any vector fields X and Y, where $[\phi,\phi]$ denotes the Nijenhuis torsion of ϕ.
A normal contact metric manifold is called Sasakian manifold. An almost contact metric manifold is Sasakian if and only if

$$(\nabla_X\phi)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.7}$$

for any vector fields X and Y.
An almost $C(\lambda)$ manifold satisfies the following relations [12]

$$R(X,\,Y)\,\xi = R(\phi X, \phi Y)\xi - \lambda\,\{\eta(Y)X - \eta(X)Y\}\,, \tag{2.8}$$

$$R(\xi,\, X)\, Y = \lambda \left\{ \eta(Y)X - g(X,Y)\xi \right\}, \tag{2.9}$$

$$R(X,\, \xi)\, Z = \lambda \left\{ \eta(Z)X - g(X,Z)\xi \right\}, \tag{2.10}$$

$$R(X,\, \xi)\, \xi = \lambda \left\{ \eta(Y)\xi - X \right\}, \tag{2.11}$$

$$R(\xi,\, X)\, \xi = \lambda \left\{ Y - \eta(Y)\xi \right\}, \tag{2.12}$$

$$S(X,Y) = Ag(X,Y) + B\eta(X)\eta(Y), \tag{2.13}$$

where $A = -\lambda(2n-1)$ and $B = -\lambda$, since $g(QX,Y) = S(X,Y)$, where Q is the Ricci-operator. From straight forward calculation of (2.10) we can write the following

$$QX = AX + B\eta(X)\xi, \tag{2.14}$$

$$S(X,\xi) = (A+B)\eta(X), \tag{2.15}$$

$$S(\xi,\xi) = (A+B), \tag{2.16}$$

and

$$r = -4n^2\lambda. \tag{2.17}$$

Replacing X by ξ and using equations (2.4) and (2.9)-(2.16) in equation (1.2), we have

$$B(\xi,Y)Z = \frac{2(\lambda+1)}{(n+2)}[Y\eta(Z) - g(Y,Z)\xi], \tag{2.18}$$

again replacing Z by ξ and using equations (2.4) and (2.8)-(2.16) in equation (1.2), we get

$$B(X,Y)\xi = R(\phi X,\phi Y)\xi + \frac{2(\lambda+1)}{(n+2)}[\eta(X)\,Y - \eta(Y)\,X]. \tag{2.19}$$

Also from equation (1.2), we get

$$\eta(B(X,Y)Z) = \eta(R(\phi X,\phi Y)Z) + \frac{2(\lambda+1)}{(n+2)}[\eta(X)g(Y,Z) \; - \eta(Y)\,g(X,Z)]. \tag{2.20}$$

This is required C-Bochner curvature tensor in $C(\lambda)$ manifolds.

3 C-Bochner Pseudosymmetric $C(\lambda)$ manifolds

Let (M,g) be a Riemannian manifold and let ∇ be the Levi-Civita connection of (M,g). A Riemannian manifold is called locally symmetric if $\nabla R = 0$, where R is the Riemannian curvature tensor of (M,g). The locally symmetric manifold have been studied by different differetial geometer through different aproaches and they exten it i.e. semisymmetric manifold by Szabo [18], recurrent manifold by Walker [2], conformally recurrent manifold by Adati and Miyazawa [14].
According to Z. I. $Szab^{'}o$[18], if the manifold M satisfies the condition

$$(R(X,Y).R)(U,V)W = 0, \quad X,Y,U,V,W \in \chi(M) \tag{3.1}$$

for all vector fields X and Y then the manifold is called semisymmetric manifold. For a (0, k)-tensor field T on M, $k \geq 1$ and a symmetric (0, 2)-tensor field A on M the (0, k+2)-tensor fields R.T and Q(A, T) are defined by

$$\begin{aligned}(R.T)(X_1,.....X_k;X,Y) = -T(R(X,Y)X_1,X_2,.......X_k)\\ - .... - T(X_1,..........X_{k-1},R(X,Y)X_k),\end{aligned} \tag{3.2}$$

and

$$Q(A,T)(X_1,.....X_k;X,Y) = -T((X \wedge_A Y)X_1, X_2, ...X_k) - .... - T(X_1,..........X_{k-1},(X \wedge_A Y)X_k), \tag{3.3}$$

where $X \wedge_A Y$ is the endomorphism given by

$$(X \wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y. \tag{3.4}$$

According to R. Deszcz [10] a Riemannian manifold is said to be pseudosymmetric if

$$R.R = L_R\, Q(g,R), \tag{3.5}$$

holds on $U_r = \left\{x \in M | R - \frac{r}{n(n-1)} G \neq 0 \;\; at \;\; x\right\}$, where G is (0, 4)-tensor defined by $G(X_1, X_2, X_3, X_4) = g((X_1 \wedge X_2)X_3, X_4)$ and L_R is some smooth function on U_R.
A Riemannian manifold M is said to be C-Bochner pseudosymmetric if

$$R.B = L_B\, Q(g,B), \tag{3.6}$$

holds on the set $U_B = \{x \in M : B \neq 0 \;\; at \;\; x\}$, where L_B is some function on U_B and B is the C-Bochner curvature tensor.
Let M^{2n+1} be C-Bochner pseudosymmetric $C(\lambda)$ manifold then from equation(3.6), we have

$$(R(X,\xi).B)(U,V)W = L_B[((X \wedge_g \xi).B)(U,V)W]. \tag{3.7}$$

Using equations (3.2) and (3.3) in equation (3.7), we get

$$\begin{aligned} &R(X,\xi)B(U,V)W - B(R(X,\xi)U,V)W \\ &- B(U,R(X,\xi)V)W - B(U,V)R(X,\xi)W \\ &= L_B\Big\{(X \wedge_g \xi)B(U,V)W - B((X \wedge_g \xi)U,V)W \\ &- B(U,(X \wedge_g \xi)V)W - B(U,V)(X \wedge_g \xi)W\Big\}. \end{aligned} \tag{3.8}$$

Again using equations (2.9) and (3.4) in (3.8), we infer

$$\begin{aligned} &(\lambda)\Big\{g(X,B(U,V)W)\xi - g(\xi,B(U,V)W)X + \eta(U)B(X,V)W \\ &- g(X,U)B(\xi,V)W + \eta(V)B(U,X)W - g(X,V)B(U,\xi)W \\ &+ \eta(W)B(U,V)X - g(X,W)B(U,V)\xi\Big\} \\ &= L_B\Big\{g(\xi,B(U,V)W)X - g(X,B(U,V)W)\xi - \eta(U)B(X,V)W \\ &+ g(X,U)B(\xi,V)W - \eta(V)B(U,X)W + g(X,V)B(U,\xi)W \\ &- \eta(W)B(U,V)X + g(X,W)B(U,V)\xi\Big\}. \end{aligned} \tag{3.9}$$

The above expression can be written as

$$\begin{aligned} &(L_B + \lambda)\Big\{g(\xi,B(U,V)W)X - g(X,B(U,V)W)\xi - \eta(U)B(X,V)W \\ &+ g(X,U)B(\xi,V)W - \eta(V)B(U,X)W + g(X,V)B(U,\xi)W \\ &- \eta(W)B(U,V)X + g(X,W)B(U,V)\xi\Big\} = 0, \end{aligned} \tag{3.10}$$

which implies that either

$$\begin{aligned} &(a)\; L_B = -\lambda \\ &or \\ &(b)\; \Big\{g(\xi,B(U,V)W)X - g(X,B(U,V)W)\xi - \eta(U)B(X,V)W \\ &+ g(X,U)B(\xi,V)W - \eta(V)B(U,X)W + g(X,V)B(U,\xi)W \\ &- \eta(W)B(U,V)X + g(X,W)B(U,V)\xi\Big\} = 0. \end{aligned} \tag{3.11}$$

Putting $U=\xi$ and using equations (2.3) and (2.18) in equation (3.11(b)), we have

$$B(X,V)W = \frac{2(\lambda+1)}{(n+2)}(g(X,W)V - g(V,W)X), \tag{3.12}$$

contracting X in the above equation, we have

$$\frac{2(\lambda+1)}{(n+2)}2ng(W,V) = 0, \tag{3.13}$$

this implies that

$$\lambda = -1, \tag{3.14}$$

Using equation (3.14) in (2.18) and (3.12), we get

$$B(X,V)W = 0, \quad and \quad B(\xi,Y)Z = 0, \tag{3.15}$$

this means M^{2n+1} is C-Bochner flat manifold.
Therefore with the help of equations (3.11) and (3.15), we conclude that:

Theorem 3.1. *A (2n+1)-dimensional $C(\lambda)$ manifold M^{2n+1} $(n>1)$ will be C-Bochner pseudosymmetric if either M^{2n+1} is C-Bochner flat, for which $\lambda=-1$, or $L_B=-\lambda$.*

Now, since λ is real number and if $C(\lambda)$ manifold is C-Bochner pseudosymmetric then we have $\lambda=-1$, or $L_B=-\lambda$ holds on M^{2n+1} which implies that $L_B=-\lambda$ will be real number in both cases means, we can state the following corollary.

Corollary 3.2. *Every $C(\lambda)$ manifold is C-Bochner pseudosymmetric and have the form $R.B = -\lambda\, Q(g,B)$.*

Corollary 3.3. *Every $C(\lambda)$ manifold is C-Bochner pseudosymmetric and have the form $R.B = Q(g,B)$.*

Now we propose:

Theorem 3.4. *If in a (2n+1)-dimensional $C(\lambda)$ manifold M^{2n+1} $(n>1)$ satisfies $B(\xi,X).S=0$ then either $\lambda=-1$ or $S(X,U)=-2\lambda ng(X,U)$.*

Proof. If in a $C(\lambda)$ manifold satisfies $B(\xi,X).S=0$,
then from equation (3.2), we have

$$S(B(\xi,X)U,\xi) + S(U,B(\xi,X)\xi) = 0, \tag{3.16}$$

From equation (2.15), we have

$$S(B(\xi,X)U,\xi) = -2n\lambda\eta(B(\xi,X)U). \tag{3.17}$$

Now with the help of equations (2.18) and (3.17), we can write

$$S(B(\xi,X)U,\xi) = -2n\lambda\frac{2(\lambda+1)}{(n+2)}(\eta(X)\eta(U) - g(X,U)). \tag{3.18}$$

Again in view of the equation (2.18), we have

$$S(B(\xi,X)\xi,U) = \frac{2(\lambda+1)}{(n+2)}(S(X,U) + 2n\lambda\eta(X)\eta(U)). \tag{3.19}$$

By using expressions (3.18) and (3.19) in (3.16), we infer

$$\frac{2(\lambda+1)}{(n+2)}(S(X,U) + 2n\lambda g(X,U)) = 0, \tag{3.20}$$

which implies that if $B(\xi,X).S=0$ then either $\lambda=-1$ or $S(X,U)=-2n\lambda g(X,U)$. □

As a perticular case of Theorem 3.4 we can state the following corollary:

Corollary 3.5. *A (2n+1)-dimensional $C(\lambda)$ manifold M^{2n+1} $(n > 1)$ $B(\xi, X).S = 0$ is an Einstein manifold.*

Now if we take $B(\xi, U).R = 0$,
then from equation (3.2), we have

$$\begin{aligned} &B(\xi, U)R(X, Y)Z - R(B(\xi, U)X, Y)Z \\ &- R(X, B(\xi, U)Y)Z - R(X, Y)B(\xi, U)Z = 0. \end{aligned} \tag{3.21}$$

In the view of the equation (2.18), we can write

$$\begin{aligned} &\frac{2(\lambda+1)}{(n+2)}\Big\{\eta(R(X,Y)Z)U - g(U, R(X,Y)Z)\xi \\ &- \eta(X)R(U,Y)Z + g(U,X)R(\xi,Y)Z - \eta(Y)R(X,U)Z + g(U,Y)R(X,\xi)Z \\ &- \eta(Z)R(X,Y)U + g(U,Z)R(X,Y)\xi\Big\} = 0. \end{aligned} \tag{3.22}$$

Using $X = \xi$ and equation (2.3) in the above equation, we get

$$\frac{2(\lambda+1)}{(n+2)}\Big\{\lambda(g(U,Z)Y - g(Y,Z)U) - R(U,Y)Z\Big\} = 0, \tag{3.23}$$

which implies that if $B(\xi, X).R = 0$ then either $\lambda = -1$ or $R(U,Y)Z = \lambda(g(U,Z)Y - g(Y,Z)U)$.
Thus, we conclude:

Theorem 3.6. *If in a (2n+1)-dimensional $C(\lambda)$ manifold M^{2n+1} $(n > 1)$ satisfies $B(\xi, X).R = 0$ then either $\lambda = -1$ or $R(U,Y)Z = \lambda(g(U,Z)Y - g(Y,Z)U)$.*

As a particular case of Theorem 3.6 we can state the following corollary:

Corollary 3.7. *A (2n+1)-dimensional $C(\lambda)$ manifold M^{2n+1} $(n > 1)$ $B(\xi, X).R = 0$ is a manifold of constant scalar curvature tensor (-1).*

Now we propose:

Theorem 3.8. *A (2n+1)-dimensional $C(\lambda)$ manifold M^{2n+1} $(n > 1)$ satisfies $B(\xi, X).B = 0$ if $\lambda = -1$.*

Proof. If in a $C(\lambda)$ manifold $B(\xi, X).B = 0$, then from equation (3.2), we have

$$\begin{aligned} &B(\xi, X)B(U, V)W - B(B(\xi, X)U, V)W \\ &- B(U, B(\xi, X)V)W - B(U, V)B(\xi, X)W = 0. \end{aligned} \tag{3.24}$$

In the view of the equation (2.18), we can write

$$\begin{aligned} &\frac{2(\lambda+1)}{(n+2)}\Big\{\eta(B(U,V)W)X - g(X, B(U,V)W)\xi \\ &- \eta(U)B(X,V)W + g(X,U)B(\xi,V)W \\ &- \eta(V)B(U,X)W + g(X,V)B(U,\xi)W \\ &+ g(W,X)B(U,V)\xi - \eta(W)B(U,V)X\Big\} = 0. \end{aligned} \tag{3.25}$$

By using $U = \xi$ in above equation, we get

$$\frac{2(\lambda+1)}{(n+2)}\Big\{(\frac{2(\lambda+1)}{(n+2)}(g(X,W)V) - g(V,W)X) - B(X,V)W\Big\} = 0, \tag{3.26}$$

which implies that either $\lambda = -1$ or

$$B(X,V)W = \frac{2(\lambda+1)}{(n+2)}(g(X,W)V - g(V,W)X), \tag{3.27}$$

contracting V in above equation, we have

$$\frac{2(\lambda+1)}{(n+2)}2ng(X,U) = 0, \tag{3.28}$$

this implies $\lambda = -1$. □

4 ξ-C-Bochner flat $C(\lambda)$ manifold

A contact metric manifold is said to be ξ-conformally flat contact metric manifold if the conformal curvature tensor of the manifold satisfies

$$C(X,Y)\xi = 0, \tag{4.1}$$

for any vector fields X and Y.
This idea was introduced by Zhen, Cabrerizo, M. Fernandez and Fernandez [6] in 1997. In 2012 U.C.De, Ahmet Yildiz, Mine Turan and Bilal E. Acet [15] defined ξ-concircularly flat manifold if the concircular curvature tensor $\tilde{C}(X,Y)\xi = 0$ holds on M.
Now, we define ξ- C-Bochner flat $C(\lambda)$ manifold.

Definition 4.1. The C-Bochner curvature tensor B of type (1, 3) on a Riemannian manifold (M, g) of dimension (2n+1) is called ξ-C-Bochner flat $C(\lambda)$ manifold if the C-Bochner curvature tensor of the manifold satisfies

$$B(X,Y)\xi = 0, \tag{4.2}$$

for any vector fields X and Y.
Putting $Z = \xi$ and using equations (2.4) and (4.2) in equation (1.2), we infer

$$\begin{aligned} R(X,Y)\xi &+ \frac{1}{2(n+2)}\Big[S(X,\xi)Y - S(Y,\xi)X - S(X,\xi)\eta(Y)\xi \\ &+ S(Y,\xi)\eta(X)\xi\Big] + \frac{2}{n+2}[\eta(X)Y - \eta(Y)X] = 0. \end{aligned} \tag{4.3}$$

Using equation (2.15) in equation (4.3), we get

$$R(X,Y)\xi - \frac{(n\lambda-2)}{(n+2)}[\eta(X)Y - \eta(Y)X] = 0 \tag{4.4}$$

From (2.8) equation (4.4) can be written as

$$R(\phi X,\phi Y)\xi + \frac{2(\lambda+1)}{n+2}[\eta(X)Y - \eta(Y)X] = 0 \tag{4.5}$$

putting $Y = \xi$ in above equation we have

$$\frac{2(\lambda+1)}{n+2}(\eta(X)\xi - X) = 0. \tag{4.6}$$

Now taking inner product with a vector field V, we have

$$\frac{2(\lambda+1)}{n+2}(\eta(X)\eta(V) - g(X,V)) = 0. \tag{4.7}$$

Replacing X by QX in above equation, we get

$$\frac{2(\lambda+1)}{n+2}((\eta(QX)\eta(V) - g(QX,V))) = 0, \tag{4.8}$$

since S(X,Y)=g(QX, Y), then from above equation we have

$$\frac{2(\lambda+1)}{n+2}((2n\lambda\eta(X)\eta(V))+S(X,V))=0. \tag{4.9}$$

this implies that either

$$\lambda=-1, \tag{4.10}$$

or

$$S(X,V)=-2n\lambda\eta(X)\eta(V). \tag{4.11}$$

Theorem 4.2. *A (2n+1)-dimensional $C(\lambda)$ manifold M^{2n+1} $(n>1)$ will be ξ-C-Bochner flat $C(\lambda)$ manifold if either $\lambda=-1$ or Ricci tensor S satisfies equation (4.11).*

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