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Some integrals involving Wright generalized Bessel-Maitland function with Jacobi polynomial

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Abstract. The main object of this paper, is to evaluate some interesting integrals involving the product of Wright generalized Bessel-Maitland function and Jacobi polynomial, which are expressed in terms of generalized hypergeometric function. Further, some special cases of our main results are also considered.

1 Introduction

Recently, many authors (see [1], [2], [3], [4], [5], [7]) have been developed many integral formulas involving a variety of special functions. Several integrals involving Bessel functions play an important role in different kind of physical problems. In fact, Bessel functions are associated with a wide rang of problems in divers areas of mathematical physics. Due to great importance of Bessel functions, in this paper, we introduce some integrals involving the product of generalized Bessel function and Jacobi polynomials, which are expressed in term of generalized Bessel-Maitland functions. Furthermore, some interesting special case of our present investigation are also considered.

The Bessel-Maitland function is defined by (see [17]):

$$J_{\nu}^{\mu}(z)=\sum_{m=0}^{\infty}\frac{(-z)^m}{\Gamma(\nu+\mu m+1)},(\mu>0;z\in\mathbb{C}). \tag{1.1}$$

Singh et al. [16] introduced the following generalization of Bessel-Maitland function:

$$J_{\nu,\gamma}^{\mu,q}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}(-z)^n}{n!\Gamma(n\mu+\nu+1)}, \tag{1.2}$$

where $\mu,\nu,\gamma\in\mathbb{C};\Re(\mu)\geq 0,\Re(\nu)\geq -1,\Re(\gamma)\geq 0$, and $q\in(0,1)\bigcup\mathbb{N}$ and $(\gamma)_{qn}=\frac{\Gamma(\gamma+qn)}{\Gamma(\gamma)}$ denotes the generalized Pochhammer symbol (see [14]).

Very recently, Ghayasuddin and Khan [6] introduced a new extension of Bessel-Maitland function as follows:

$$J_{\nu,\gamma,\delta}^{\mu,q,p}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}(-z)^n}{\Gamma(n\mu+\nu+1)(\delta)_{pn}}, \tag{1.3}$$

where $\mu,\nu,\gamma,\delta\in\mathbb{C};\Re(\mu)\geq 0,\Re(\nu)\geq -1,\Re(\gamma)\geq 0,\Re(\delta)\geq 0;p,q>0$, and $q<\Re(\alpha)+p$.

They also investigated some special case of the generalized Bessel-Maitland function (1.3) by assigning some particular values to the parameters $\mu,\nu,\delta,\gamma,p,q$, which are given as follows:

Relation with Mittag-leffler functions:

(i) On replacing ν by $\nu-1$, (1.3) reduces to

$$J_{\nu-1,\gamma,\delta}^{\mu,q,p}(-z)=E_{\nu,\gamma,\delta}^{\mu,q,p}(z), \tag{1.4}$$

where $E_{\nu,\gamma,\delta}^{\mu,q,p}(z)$ is the Mittag-Leffler function defined by Salim and Faraj [14, 15].

(ii) On setting $p=\delta=1$ and replacing ν by $\nu-1$, (1.3) gives

$$J_{\nu-1,\gamma,1}^{\mu,q,1}(-z)=E_{\nu,\gamma}^{\mu,q}(z), \tag{1.5}$$

where $E_{\nu,\gamma}^{\mu;q}(z)$ is the Mittag-Leffler function defined by Shukla and Prajapati [12].

(iii) On setting $p=q=\delta=1$ and replacing ν by $\nu-1$, (1.3) reduces to

$$J_{\nu-1,\gamma,1}^{\mu,1,1}(-z)=E_{\nu,\gamma}^{\mu}(z), \tag{1.6}$$

where $E_{\nu,\gamma}^{\mu}(z)$ is the Mittag-Leffler function defined by Prabhakar [9].

(iv) On setting $p=q=\delta=\gamma=1$ and the replacing ν by $\nu-1$, (1.3) gives

$$J_{\nu-1,1,1}^{\mu,1,1}(z)=E_{\nu,}^{\mu}(z), \tag{1.7}$$

where $E_{\nu}^{\mu}(z)$ is the Mittag-Leffler function defined by Wiman [18].

(v) On setting $p=q=\delta=\gamma=1$ and $\nu=0$, (1.3) reduces to

$$J_{0,1,1}^{\mu,1,1}(-z)=E_{\mu}(z), \tag{1.8}$$

where $E_{\mu}(z)$ is the Mittag-Leffler function defined by Ghosta Mittag-Leffler [8].

2 Integrals involving Wright generalized Bessel-Maitland with Jacobi polynomials

The Jacobi polynomial $P_n^{(\alpha,\beta)}(z)$ is defined by the (see, [10], [13]):

$$P_n^{(\alpha,\beta)}(z)=\frac{(1+\alpha)_n}{n!}{}_2F_1\left[\begin{array}{ccc} -n, & (1+\alpha+\beta+n) & ; \\ & & & \dfrac{1-z}{z} \\ (1+\alpha) & & ; \end{array}\right], \tag{2.1}$$

or equivalently

$$P_n^{(\alpha,\beta)}(z)=\sum_{k=0}^{n}\frac{(1+\alpha)_n(1+\alpha+\beta)_{n+k}}{k!(n-k)!(1+\alpha)_k(1+\alpha+\beta)_n}\left(\frac{z-1}{z}\right)^k. \tag{2.2}$$

From (2.1) and (2.2), we find

$$P_n^{(\alpha,\beta)}(1)=\frac{(1+\alpha)_n}{n!}, \tag{2.3}$$

where $P_n^{(\alpha,\beta)}(z)$ is a polynomial of degree precisely n.

In this section, we establish some interesting integral formulas involving a product of Bessel-Maitland function and Jacobi polynomials as follows:

$$I_1=\int_{-1}^{1}x^{\lambda}(\,1-x)^{\alpha}(1+x)^{\delta}P_n^{(\alpha,\beta)}(x)J_{\nu,\gamma,\rho}^{\mu,q,p}[z(1+x)^h]dx$$

$$=\int_{-1}^{1}x^{\lambda}(\,1-x)^{\alpha}(1+x)^{\delta}P_n^{(\alpha,\beta)}(x)\sum_{k=0}^{\infty}\frac{(\gamma)_{qk}(-z)^k(1+x)^{hk}}{\sqrt{(\mu k+\nu+1)(\rho)_{pk}}}dx. \tag{2.4}$$

Interchanging the order of summation and integration, we can write above expression as

$$=\sum_{k=0}^{\infty}\frac{(\gamma)_{qk}(-z)^k}{\Gamma(\mu k+\nu+1)(\rho)_{pk}}\int_{-1}^{1}x^{\lambda}(\ 1-x)^{\alpha}(1+x)^{\delta+kh}P_n^{(\alpha,\beta)}(x)dx. \tag{2.5}$$

Using the formula given in ([11], p.52):

$$\int_{-1}^{1}x^{\lambda}(\ 1-x)^{\alpha}(1+x)^{\delta}P_n^{(\alpha,\beta)}(x)dx=\frac{(-1)^n2^{\alpha+\delta+1}\Gamma(\delta+1)\Gamma(\alpha+n+1)\Gamma(\delta+\beta+1)}{n!n!\Gamma(\delta+\beta+n+1)\Gamma(\delta+\alpha+n+2)}$$

$$\times{}_3F_2\left[\begin{matrix}-\lambda, & \delta+\beta+1, & \delta+1; & \\ & & & 1\\ \delta+\beta+n+1, & \delta+\alpha+n+2; & & \end{matrix}\right], \tag{2.6}$$

in the above (2.5), we get

$$I_1=\frac{(-1)^n2^{\alpha+\delta+1}\Gamma(\delta+kh+1)\Gamma(\alpha+n+1)\Gamma(\delta+kh+1)}{n!\Gamma(\delta+kh+\beta+n+1)\Gamma(\delta+kh+\alpha+n+2)}J_{\nu,\gamma,\rho}^{\mu,q,p}[z(2)^h]$$

$$\times{}_3F_2\left[\begin{matrix}-\lambda, & \delta+kh+\beta+1, & \delta+1; & \\ & & & 1\\ \delta+kh+\beta+n+1, & \delta+kh+\alpha+n+2; & & \end{matrix}\right]. \tag{2.7}$$

Provided
(i) $\mu,\nu,\gamma,\rho,\in\mathbb{C}$; $\Re(\mu)\geq 0,\Re(\nu)\geq -1,\Re(\gamma)\geq 0,\Re(\rho)\geq 0$; $p,q>0$ and $q<\Re(\alpha)+1$
(ii) $\Re(\lambda)>-1,\alpha>-1$ and $\beta>-1$.

$$I_2=\int_{-1}^{1}(1-x)^{\delta}(\ 1+x)^{\beta}P_n^{(\alpha,\beta)}(x)P_m^{(\rho,\sigma)}(x)J_{\nu,\gamma,\omega}^{\mu,q,p}[z(1-x)^h]dx \tag{2.8}$$

$$=\sum_{k=0}^{\infty}\frac{(\gamma)_{qk}(-z)^k}{\Gamma(\mu k+\nu+1)(\omega)_{pk}}\int_{-1}^{1}(\ 1-x)^{\delta+kh}(1+x)^{\beta}P_n^{(\alpha,\beta)}(x)P_m^{(\rho,\sigma)}(x)dx. \tag{2.9}$$

Using (2.2) in above expression, we get

$$=\sum_{k=0}^{\infty}\frac{(\gamma)_{qk}(-z)^k}{\Gamma(\mu k+\nu+1)(\omega)_{pk}}\frac{(1+\rho)_m}{m!}\sum_{k=0}^{\infty}\frac{(-m)_k(1+\rho+\sigma+m)_k}{(1+\rho)_k2^kk!}$$

$$\times\int_{-1}^{1}(\ 1-x)^{\delta+kh+k}(1+x)^{\beta}P_n^{(\alpha,\beta)}(x). \tag{2.10}$$

Again using (2.2) in (2.10), we get

$$=\sum_{k=0}^{\infty}\frac{(\gamma)_{qk}(-z)^k}{\Gamma(\mu k+\nu+1)(\omega)_{pk}}\frac{\Gamma(1+\rho+m)\Gamma(1+\alpha+n)}{m!n!}\sum_{k=0}^{\infty}\frac{(-m)_k(-n)_k(1+\rho+\sigma+m)_k(1+\alpha+\beta+n)}{\Gamma(1+\rho+k)\Gamma(1+\alpha+k)2^{2k}(k!)^2}$$

$$\times\int_{-1}^{1}(\ 1-x)^{\delta+kh+k}(1+x)^{\beta}dx. \tag{2.11}$$

By using the formula

$$\int_{-1}^{1}(\ 1-x)^{n+\alpha}(1+x)^{n+\beta}dx=2^{2n+\alpha+\beta+1}B(1+\alpha+n,1+\beta+n). \tag{2.12}$$

Equation (2.11) becomes

$$I_2 = \frac{2^{\delta+\beta+1}\Gamma(1+\rho+m)\Gamma(1+\alpha+n)}{m!n!} \sum_{k=0}^{\infty} \frac{(-m)_k(-n)_k(1+\rho+\sigma+m)_k(1+\alpha+\beta+n)_k}{\Gamma(1+\rho+k)\Gamma(1+\alpha+k)2^{2k}(k!)^2}$$

$$\times J_{\nu,\gamma,\omega}^{\mu,q,p}[z(2)^h] B(1+\delta+kh+2k, 1+\beta). \tag{2.13}$$

Provided
(i) $\mu, \nu, \gamma, \omega \in \mathbb{C}$; $\Re(\mu) \geq 0, \Re(\nu) \geq -1, \Re(\gamma) \geq 0, \Re(\omega) \geq 0, p, q > 0$ and $q < \Re(\alpha) + p$.
(ii) $\Re(\beta) > -1, h$ and δ are positive numbers.

$$I_3 = \int_{-1}^{1} (\ 1-x)^{\rho}(1+x)^{\sigma} P_n^{(\alpha,\beta)}(x) J_{\nu,\gamma,\delta}^{\mu,q,p}[z(1-x)^h(1+x)^t]dx$$

$$= \int_{-1}^{1} (\ 1-x)^{\rho}(1+x)^{\sigma} P_n^{(\alpha,\beta)}(x) \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}(-z)^k(1-x)^{kh}(1+x)^{tk}}{\Gamma(\mu k+\nu+1)(\delta)_{pk}} dx$$

$$= \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}(-z)^k}{\Gamma(\mu k+\nu+1)(\delta)_{pk}} \int_{-1}^{1} (\ 1-x)^{\rho+kt}(1+x)^{\sigma+hk} P_n^{(\alpha,\beta)}(x)dx. \tag{2.14}$$

Now, by using (2.2) in (2.14), we get

$$I_3 = \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}(-z)^k}{\Gamma(\mu k+\nu+1)(\delta)_{pk}} \frac{(1+\alpha)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k(1+\alpha+\beta+n)_k}{(1+\alpha)_k 2^k k!}$$

$$\times \int_{-1}^{1} (\ 1-x)^{\rho+kh+k}(1+x)^{\sigma+kt}dx. \tag{2.15}$$

Using(2.12) in (2.15), we get

$$I_3 = \frac{2^{\rho+\sigma+1}(1+\alpha)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k(1+\alpha+\beta+n)_k}{(1+\alpha)_k(k!)}$$

$$\times J_{\nu,\gamma,\delta}^{\mu,q,p}[z(2)^{h+t}] B(1+\rho+kh+k, 1+\sigma+tk). \tag{2.16}$$

Provided
(i) $\mu, \nu, \gamma, \delta \in \mathbb{C}$; $\Re(\mu) \geq 0, \Re(\nu) \geq -1, \Re(\gamma) \geq 0, \Re(\delta) \geq 0, p, q > 0$ and $q < \Re(\alpha) + p$.
(ii) $\Re(\alpha) > -1$ and $\Re(\beta) > -1$.

$$I_4 = \int_{-1}^{1} (\ 1-x)^{\rho}(1+x)^{\sigma} P_n^{(\alpha,\beta)}(x) J_{\nu,\gamma,\delta}^{\mu,q,p}[z(1-x)^h(1+x)^{-t}]dx$$

$$= \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}(-z)^k}{\Gamma(\mu k+\nu+1)(\delta)_{pk}} \int_{-1}^{1} (\ 1-x)^{\rho-kt}(1+x)^{\sigma+hk} P_n^{(\alpha,\beta)}(x)dx. \tag{2.17}$$

Now, by using (2.2) in (2.17), we obtain

$$I_4 = \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}(-z)^k}{\Gamma(\mu k+\nu+1)(\delta)_{pk}} \frac{(1+\alpha)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k(1+\alpha+\beta+n)_k}{(1+\alpha)_k 2^k k!}$$

$$\times \int_{-1}^{1} (\ 1-x)^{\rho+kh+k}(1+x)^{\sigma-kt}dx. \tag{2.18}$$

Using (2.12) in (2.18), we get

$$I_4 = \frac{2^{\rho+\sigma+1}(1+\alpha)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k(1+\alpha+\beta+n)_k}{(1+\alpha)_k(k!)}$$

$$\times J^{\mu,q,p}_{\nu,\gamma,\delta}[z(2)^{h-t}]B(1+\rho+kh+k, 1+\sigma-tk). \tag{2.19}$$

Provided
(i) $\mu, \nu, \gamma, \delta \in \mathbb{C}$; $\Re(\mu) \geq 0, \Re(\nu) \geq -1, \Re(\gamma) \geq 0, \Re(\delta) \geq 0, p, q > 0$ and $q < \Re(\alpha) + p$.
(ii) $\Re(\alpha) > -1$, and $\Re(\beta) > -1$.

$$I_5 = \int_{-1}^{1} (\,1-x)^{\rho}(1+x)^{\sigma} P_n^{(\alpha,\beta)}(x) J^{\mu,q,p}_{\nu,\gamma,\delta}[z(1-x)^{-h}]dx$$

$$= \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}(-z)^k}{\Gamma(\mu k+\nu+1)(\delta)_{pk}} \int_{-1}^{1} (\,1-x)^{\rho}(1+x)^{\sigma-hk} P_n^{(\alpha,\beta)}(x)dx. \tag{2.20}$$

By using (2.2) in (2.20), we get

$$I_5 = \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}(-z)^k}{\Gamma(\mu k+\nu+1)(\delta)_{pk}} \frac{(1+\alpha)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k(1+\alpha+\beta+n)_k}{(1+\alpha)_k 2^k k!}$$

$$\times \int_{-1}^{1} (\,1-x)^{\rho+k}(1+x)^{\sigma-kh}dx. \tag{2.21}$$

Again using (2.12) in (2.21), we get

$$I_5 = \frac{2^{\rho+\sigma+1}(1+\alpha)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k(1+\alpha+\beta+n)_k}{(1+\alpha)_k(k!)}$$

$$\times J^{\mu,q,p}_{\nu,\gamma,\delta}[z(2)^{h+t}]B(1+\rho+kh+k, 1+\sigma-hk). \tag{2.22}$$

Provided
(i) $\mu, \nu, \gamma, \delta \in \mathbb{C}$; $\Re(\mu) \geq 0, \Re(\nu) \geq -1, \Re(\gamma) \geq 0, \Re(\delta) \geq 0, p, q > 0$ and $q < \Re(\alpha) + p$.
(ii) $\Re(\alpha) > -1$, and $\Re(\beta) > -1$.

3 Special Cases

(i). On setting $\alpha = \beta = \rho = \sigma = 0$ and replacing δ by $\lambda - 1$, the integral I_2 transforms into the following integral involving Legendre polynomials (see [10], [13]):

$$I_6 = \int_{-1}^{1} (1-x)^{\lambda-1}(\,1+x)^{\beta} P_n(x) P_m(x) J^{\mu,q,p}_{\nu,\gamma,\omega}[z(1-x)^h]dx$$

$$= 2^{\lambda} \sum_{k=0}^{\infty} \frac{(-m)_k(-n)_k(1+m)_k(1+n)_k}{2^{2k}(k!)^2(k!)^2} J^{\mu,q,p}_{\nu,\gamma,\omega}[z(2)^h]B(\lambda+kh+2k, 1). \tag{3.1}$$

(ii). On setting $\alpha = \beta = 0$ and replacing ρ by $\rho - 1$ and σ by $\sigma - 1$, then I_3 transforms into the following integral involving Legendre polynomial (see [10], [13]):

$$I_7 = \int_{-1}^{1} (\,1-x)^{\rho-1}(1+x)^{\sigma-1} P_n(x) J^{\mu,q,p}_{\nu,\gamma,\delta}[z(1-x)^h(1+x)^t]dx$$

$$= 2^{\rho+\sigma-1} \sum_{k=0}^{\infty} \frac{(-n)_k(1+n)_k}{(k!)^2} J^{\mu,q,p}_{\nu,\gamma,\delta}[z(2)^{h+t}]B(\rho+kh+k, \sigma+tk). \tag{3.2}$$

(iii). On taking $\alpha=\beta=0$ and replacing ρ by $\rho-1$ and σ by $\sigma-1$, then I_4 transforms into following integral involving Legendre polynomial (see [10], [13]):

$$I_8=\int_{-1}^{1}(\,1-x)^{\rho-1}(1+x)^{\sigma-1}P_n(x)J_{\nu,\gamma,\delta}^{\mu,q,p}[z(1-x)^h(1+x)^{-t}]dx$$

$$=2^{\rho+\sigma-1}\sum_{k=0}^{\infty}\frac{(-n)_k(1+n)_k}{(k!)^2}J_{\nu,\gamma,\delta}^{\mu,q,p}[z(2)^{h-t}]B(\rho+kh+k,\sigma-tk). \tag{3.3}$$

(iv). On taking $\alpha=\beta=0$ and replacing ρ by $\rho-1$ and σ by $\sigma-1$, then I_5 transforms into the following integral involving Legendre polynomial (see [10], [13]):

$$I_9=\int_{-1}^{1}(\,1-x)^{\rho-1}(1+x)^{\sigma-1}P_n(x)J_{\nu,\gamma,\delta}^{\mu,q,p}[z(1-x)^{-h}]dx$$

$$=2^{\rho+\sigma-1}\sum_{k=0}^{\infty}\frac{(-n)_k(1+n)_k}{(k!)^2}J_{\nu,\gamma,\delta}^{\mu,q,p}[z(2)^{h+t}]B(\rho+kh+k,\sigma-hk). \tag{3.4}$$

4 Concluding Remark

In this paper, we have evaluated more interesting integrals associated with Wright generalized Bessel-Maitland function and Jacobi polynomial, whose explicit representations are given in terms of generalized Bessel-Maitland function with different arguments, respectively. By means of integrals I_1 to I_5, we have derived some (presumably) new integrals as their special cases. Further, it can be easily seen that the Bessel-Maitland function is a special case of Mittag-Leffler function. Therefore, the results presented in this paper are easily converted interms of the Mittag-Leffler function after some suitable parametric replacement. Also, it is remarked that if we replace the integral operator (1.3) (which is used to establish the main results) by any other operator, then we get a number of new interesting results.

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