

A FEW COMMENTS ON ACADEMIC PUBLISHING STANDARDS IN RELATION TO THE HORADAM SEQUENCE AND ITS VARIANTS

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Abstract. Time spent surveying work on the so called Horadam sequence $\{w_n\}_{n=0}^{\infty} = \{w_n\}_0^{\infty} = \{w_n(a, b; p, q)\}_0^{\infty}$ —defined by the linear recurrence $w_n = pw_{n-1} - qw_{n-2}$ in its most general form ($n \geq 2$; arbitrary initial values $w_0 = a, w_1 = b$)—has identified considerable variation in the nature of results generated but also, more importantly, in the quality of papers containing them. It urges the following short opinion piece which discusses this observation and touches on some wider issues relating to research.

Quelle est la nature acceptable d'un article de recherche?

1 Introduction

A survey article [6], published in 2013, was the first of its kind and, it seemed, a much needed one on research involving Horadam sequences. An update to, and extension of, that essay has since been disseminated [5], such is the volume of material to be evaluated in tandem with some more recent results that have required reporting. During the gathering of papers for these particular works it occurred to me that the subject in question had suffered somewhat from a tangible lowering of publishing standards through instances whereby collections of *formulas* appeared to have been the main goal—given clear priority over, and happily promoted above, *ideas* central to much of research level mathematics.

This issue surfaced a while ago for me as I was examining works elsewhere on other integer sequences, and sharpened while preparing these two reviews. I will now explain further, and I would imagine more than a few readers will recognise what I am getting at from experience in other fields of work—in other words, the points made may well have an element of generality to them.

1.1 Caveat and Context

The Horadam sequence—so named after its founder Alwyn F. Horadam—is the most general of all second order linear recurrence sequences, and occupies a unique position as fountainhead of that class. To be the originator of a sequence which has been the motivation for, and the subject of, so many research papers over a sustained period of time is noteworthy, and the account [6] was dedicated to Professor Horadam (who was flattered and very appreciative of the gesture) in recognition of precisely this. I stumbled across the sequence as an entity while formalising some results on the Delannoy numbers as it happens. It immediately aroused my curiosity and I began to look back at its history, finding that an impressive body of treatises exists on the sequence and even more so on special case versions whose parameters a, b, p, q are part or fully specified—this co-incided with a nascent interest that I wished to pursue in those Horadam sequences that might reveal periodic attributes (and in any governing conditions).

Depending on values assigned (or not, as the case may be) to the four said characterising variables, the Horadam sequence possesses a vast array of properties which have made it so fascinating to study—to that statement there is no challenge. It is obvious to anyone looking at the relevant literature that many particular instance sequences are connected in some deep and improbable ways, forging the reputation of the general form root sequence and explaining to a degree the longevity of its popularity. All of this has given rise to the two aforementioned expository pieces, since until 2013 no-one had yet documented any such trail of research. Through my attempts to write thematically, I gained a feel for the reasons behind its attraction and was surprised by the sheer variation in article styles and topic matter selected for analysis. It seemed most striking that a simple enough notion had yielded so much, and thus the process of collection, collation and scrutiny of documents began. I must now, though, set down one or two thoughts on what I read—not as an unfounded and sententious proclamation, but as a necessary and honest caveat to the self-evident contribution the Horadam sequence has made to the theory of sequences since Horadam himself published two oft cited papers in 1965 (in the then newly launched *Fibonacci Quarterly* and the established *Duke Mathematical Journal*) which were seminal in laying the groundwork for all subsequent studies [1, 2]. In the Introduction of [1] Horadam wrote of his intention that the article “. . . should be the first of a series investigating aspects of [his sequence].”—little did he know what was to follow, for that series continues to this very day.

“He who day by day finds out where he is deficient, and who month by month never forgets that in which he has become proficient, may truly be called a lover of learning.”

Tzu Hsia (a.k.a. Pu Shang), disciple of Chinese sage Confucius.

2 Issues and Concerns

2.1 Formula Lists, Inter-Related Sequences, and Notation/Nomenclature

If any kind of disapproval were to be levelled at some of the work conducted then it would be directed at the examination of inter-related sequences, which has undoubtedly provided an agreeable and well trodden path over the years. This is an enclave in which it could be argued that individuals have to an extent taken refuge, generating hosts of carefully stacked formulas and extensions thereof (and further extensions) whose complexities sometimes hide as much as (if not more than) they reveal and merely overlay presentations with an artificial, and perhaps deliberate, numinous abstruseness. Whether due to sheer convenience, or a misplaced devotion, or a narrowness of outlook combined with an intransigence in terms of what is considered to constitute a research paper, or some other reason(s), I know not, but it seems to me that certain offerings have the feel of a rather perfunctory attitude to higher mathematical enquiry perhaps at times designed to meet the pressing demands for conspicuity, rather than the quality, of outputs—they are, as a consequence, somewhat dull and dreary, showing a little disregard for subject enhancement and keeping aspects of it compartmentalised and prosaically restrictive. I have it on reliable authority—from a longstanding and respected academic working in this area for five or more decades—that there may well have been periods (particularly in the 1960s and 1970s) when, with little or no critical comment, simply adding to lists of known identities/formulas linking sequence terms and such like was, *ipso facto*, seen as ‘advancement’. Thankfully the practice has gradually decreased of its own accord, largely due to saturation levels (it has not died out, however, to which some recent publications bear testimony). This is not to denigrate all work, of course, for a fair percentage of the articles I have seen are cogent, informative and interesting, but too many for my liking are mechanistic and spiritless, with little or no aesthetic appeal and no great creativity either—bromidic and uninspiring, essentially. I hope I am not stepping out of line, so to speak, in articulating such thoughts, but the realm of publishing in modern day higher education has taken out the purity of research and reduced it to a routine functional part of academe which affects what is deemed to be viable research.

I also feel as if those many specialisations of the basic Horadam sequence, together with a swathe of generalisations (of some) and relations between them, have been casualties of an unfortunate and frustrating lack of consistency in denotation, labelling and terminology, adding

difficulties to any reasonable attempt to understand what results actually mean and what they add to existing knowledge; for someone trying to avoid duplication of, or overlap with, work that has gone before, there are authors who are less helpful than others in this respect and the point at which these things could have been standardised has passed (to address this problem someone—or a resolute and committed team—would have to make a herculean effort to systematically plough through literally hundreds of publications, and this just will not happen).

2.2 What Constitutes Research?

It is indisputable that a linear quantitative increase in publications on and around a topic does not automatically represent a commensurate level of technical progress. I once read the Italian-born American mathematician and philosopher (and renowned expositor) Gian-Carla Rota remind us—in making the point that some pre-supposed ideas of what research should be are, for the most part, at variance with its day-to-day reality—that a significant proportion of mathematical research papers consist of reproving, reformulating, unifying, streamlining, classifying, simplifying, refining, extending, generalising, and so on. There is merit in these, undeniably, but levels of originality and intellectual capital are sometimes difficult to gauge objectively. Other than finding a new or deep theorem/result, or opening up fresh territory for exploration, it is the natural way of mathematicians to embellish existing work in these sorts of ways, and their collective force keeps the discipline expanding and moving forward both iteratively and incrementally for the most part (with occasional insight gained that is profound, or a notable breakthrough of some description made, each quite rare). It will, though, by default spawn some articles that sit towards the lower end of the rankings spectrum, and those that descend into incoherence and confusion, or are just plain poorly written, or devoid of any real depth, or amount to a virtual carbon copy of another with but a modicum of novelty (adding a grain or two of sand to the proverbial pyramid of knowledge), or evidence of a small and one might say overly cultivated mathematical garden, should be seen for what they are and declared as such; I venture to suggest that a number of papers relating to Horadam or Horadam type sequences exhibit these features, which is a shame—some sins are not venial, as it were.

To spend an inordinate length of time on a straitened band of the less demanding of the above types of activities, or to concentrate on them exclusively, is perhaps a sign of weakness in serious purpose, or maybe confirmation of the appealing safety of a professional comfort zone that feeds a craven pusillanimity. Study of the raw components of, and more complex constructs within, mathematics requires a mental capacity which often places its practitioners above their counterparts in the arts, social sciences and humanities. Let not our perception of this *de facto* superiority induce a false sense of security—rather, let us protect and uphold our heritage through the works we fashion, never losing our vibrancy and zeal in fulfilling an exegetical duty to readers through clarity and depth in what we present; it is not easy, and we may falter, but we should always have such aims in mind and show endeavour to this end. To borrow imagery from the influential 20th century French mathematician André Weil (and recast it), the problems, ideas and objects that reveal themselves to us, and in which we immerse ourselves, can be likened to daily food for nourishment of the soul and cerebral sustenance where the dishes we assemble are best prepared with the necessary hygiene of rigour, laced with lavish seasonings of flair and imagination, and dutifully served with formalities observed—this is the world we inhabit, that we cherish, and that we need to safeguard, never allowing junk food to appear in our private kitchens and make its way onto our mathematical menus but instead ever asking “*Que puis-je apporter au dîner?*”

“Be you a scholar of the nobler type, not a scholar of the inferior man’s type.”
Confucius.

3 The Bigger Picture

Notwithstanding all of this, then on a more upbeat note (and taking a broader view) the inception of the generalised Horadam sequence, and the enquiries brought to bear on it since, have created a domain of research which is unquestionably an authentic one as measured by the sheer weight of attention accorded to it. Even the most impartial and dispassionate commentator cannot argue

against the legacy of discourse it has generated (or ignore ongoing developments), against which background the essay [6] and its supplement [5] have been undertaken—I can but hope that with the passage of time they are regarded favourably, produced as they were in order to acknowledge both the man behind the sequence and its genealogy within the field of discrete mathematics.

A tribute to Alwyn Horadam, who passed away in July 2016, was published by this author later in that year [4] (see also [7] by his former student, longtime friend and colleague Anthony G. Shannon, and the more recent obituary penned by daughter Kathy [3]). As a man of reportedly impeccable manners, and one who took care in the way he composed research papers (mathematically, grammatically and structurally), I feel sure Alwyn would endorse any call to maintain high publishing principles throughout the mathematical community—embodying such things as conscientiousness, integrity and distinction as its core constituents—but especially inside that rich arena of linear recurrence sequences initiated by him over fifty years ago, indeed to honour his memory and achievements properly it is incumbent to do so. Polish applied mathematician Stanisław M. Ulam¹ wrote, in his autobiography,

“In many cases, mathematics is an escape from reality. The mathematician finds his own monastic niche and happiness in pursuits that are disconnected from external affairs. Some practice it as if using a drug . . . In their unhappiness over the events of this world, some immerse themselves in a kind of self-sufficiency in mathematics. (Some have engaged in it for this reason alone.) Yet one cannot be sure that this is the sole reason; for others, mathematics is what they can do better than anything else.” [8, p. 120].

The truth of these words, and the inescapable human elements that shape and fine-tune those incentives for our travails, beg the following questions to which each individual will have his or her own answers: *Why am I in the game of mathematical research, and how much do I care about the fruits of my labours in what I present to compatriots in the sphere of academe as a reflection of my personality and work ethic?*

“The man of virtue makes the difficulty to be overcome his first interest;
success only comes later.”

Confucius.

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¹Best known for his work on the World War II hydrogen bomb Manhattan Project at the secret Los Alamos Laboratory in New Mexico, and for fundamental ideas underpinning modern day Monte Carlo methods.