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WEAK SYMMETRIC MANIFOLDS EQUIPPED WITH SEMI-SYMMETRIC METRIC CONNECTION

B. B. Chaturvedi and Pankaj Pandey

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Abstract. In this paper, examples of weakly symmetric manifold, weakly Ricci-symmetric manifold, conformally flat weakly symmetric manifold and weakly projective symmetric manifold with semi-symmetric metric connection satisfying a special type condition have been constructed. Also, it is shown that these examples satisfy the condition of 4-dimensional Lorentzian space-time.

1 Introduction

The weak symmetric property of manifolds play an important role in the study of manifolds as well as in general relativity theory. The idea of weakly symmetric and weakly Ricci-symmetric manifolds are introduced by L. Tamassy and T. Q. Binh [2, 27]. According to Tamassy and Binh [2, 27], an n-dimensional (pseudo) Riemannian manifold is said to be weakly symmetric if the following condition satisfies:

$$\begin{aligned}(\nabla_X R)(Y,Z,U,V) =& A(X)R(Y,Z,U,V) + B(Y)R(X,Z,U,V)\\ &+ C(Z)R(Y,X,U,V) + D(U)R(Y,Z,X,V)\\ &+ E(V)R(Y,Z,U,X).\end{aligned} \tag{1.1}$$

If the Ricci tensor S of the manifold satisfies

$$(\nabla_X S)(Y,Z) = A(X)S(Y,Z) + B(Y)S(X,Z) + C(Z)S(Y,X), \tag{1.2}$$

then the manifold is called weakly Ricci-symmetric manifold, where A, B, C, D, E are simultaneously non-vanishing 1-forms and X, Y, Z, U, V are vector fields. Further, these ideas are extended by Prvanovic [22], De and Bandyopdhyay [8], De and Mallick [11], Chaturvedi and Pandey [5] and by the several authors(see: [13], [10], [12], [19], [18], [16], [20]). Many authors expanded the study of manifolds by using semi-symmetric metric connection, quarter-symmetric metric connection, and other type of connections (see: [1], [21], [24], [26], [17], [30], [29], [15], [6]). Also, many examples of several type of manifolds are given by several authors (see: [8], [1], [16], [9], [20], [11], [14] [25], [15], [7]).

In this type of study De and Sengupta [13] studied weakly symmetric Riemannian manifolds admitting a special type of semi-symmetric metric connection and obtained some interesting results. Recently, Chaturvedi and Panndey [5] (2015) also introduced some important results in the study of weak symmetric manifolds with special type semi-symmetric metric connection in Kähler manifolds. But in these papers there is lack of examples satisfying such a special type condition. Therefore, we motivate to find the examples in weak symmetric manifolds with special type semi-symmetric metric connection.

In 1995 Prvanovic [22] proved that if the manifold M be a weakly symmetric manifold satisfying (1.1) then $B = C = D = E$. In this paper, we have considered $B = C = D = E = \omega$ (say), therefore the equations (1.1) and (1.2) can be written as

$$\begin{aligned}(\nabla_X R)(Y,Z,U,V) =& A(X)R(Y,Z,U,V) + \omega(Y)R(X,Z,U,V)\\ &+ \omega(Z)R(Y,X,U,V) + \omega(U)R(Y,Z,X,V)\\ &+ \omega(V)R(Y,Z,U,X),\end{aligned} \tag{1.3}$$

and

$$(\nabla_X S)(Y,Z) = A(X)S(Y,Z) + \omega(Y)S(X,Z) + \omega(Z)S(Y,X), \tag{1.4}$$

where A and ω are arbitrary 1-forms defined by $g(X,\alpha) = A(X)$ and $g(X,\rho) = \omega(X)$, α and ρ being associated vector fields.

Now, if ∇ be a Levi-Civita connection then the connection defined by

$$\nabla^*_X Y = \nabla_X Y + \omega(Y)X - g(X,Y)\rho, \tag{1.5}$$

is called a semi-symmetric metric connection if the torsion tensor T of the connection ∇^* satisfies the condition

$$T(X,Y) = \omega(Y)X - \omega(X)Y, \tag{1.6}$$

In 1970 Yano [28] obtained an expression for the curvature tensor R^* of the semi-symmetric metric connection ∇^* is given by

$$\begin{aligned} R^*(X,Y,Z,U) =& R(X,Y,Z,U) - \theta(Y,Z)g(X,U) + \theta(X,Z)g(Y,U) \\ &- \theta(X,U)g(Y,Z) + \theta(Y,U)g(X,Z), \end{aligned} \tag{1.7}$$

where θ is a tensor field of type (0,2) defined by

$$\theta(X,Y) = (\nabla_X \omega)Y - \omega(X)\omega(Y) + \frac{1}{2}\omega(\rho)g(X,Y). \tag{1.8}$$

The semi-symmetric metric connection ∇^* defined by (1.5) is said to be a special type of semi-symmetric metric connection if the torsion tensor T and the curvature tensor R^* with respect to connection ∇^* satisfy

$$(\nabla^*_X T)(Y,Z) = \omega(X)T(Y,Z), \tag{1.9}$$

and

$$R^*(X,Y)Z = 0. \tag{1.10}$$

The paper is organize in four sections. First section is introductory, which contains some basic definitions and useful results. In the second section, we have constructed an example of weakly symmetric manifold with special type of semi-symmetric metric connection which is related to Bessel's function. The purpose of constructing this example is that to verify the existence of weakly symmetric manifold with special type semi-symmetric metric connection. In the next section, examples of weakly symmetric, weakly Ricci-symmetric, conformally flat weakly symmetric and weakly projective-symmetric manifolds with special type semi-symmetric metric connection have been constructed. In the same section, it is also shown that the examples described above are also examples of 4-dimensional Lorentzian space-time with special type semi-symmetric metric connection. The last section contains the discussion and conclusions of the results and then the references are given.

2 Weakly Symmetric Manifold

Example 2.1. Let g be a (pseudo) Riemannian metric in an n-dimensional manifold $\mathbb{R}^n$ defined by

$$ds^2 = \varphi(dx^1)^2 + K_{\alpha\beta}dx^\alpha dx^\beta + 2dx^1 dx^n, \tag{2.1}$$

where $[K_{\alpha\beta}]$ is a non-singular symmetric matrix with entries as constants such that α, β varies from 2 to $(n-1)$ and φ is a function of $x^1, x^2, ..., x^{n-1}$.

The non-zero components of Christoffel symbols, the curvature tensor R and the Ricci tensor S are given by [23]

$$\Gamma^\beta_{11} = -\frac{1}{2}K^{\alpha\beta}\varphi_{\cdot\alpha}, \quad \Gamma^n_{11} = \frac{1}{2}\varphi_{\cdot 1}, \quad \Gamma^n_{1\alpha} = \frac{1}{2}\varphi_{\cdot\alpha}, \tag{2.2}$$

and

$$R_{1\alpha\beta 1} = \frac{1}{2}\varphi_{\cdot\alpha\beta}, \qquad R_{11} = \frac{1}{2}K^{\alpha\beta}\varphi_{\cdot\alpha\beta}, \tag{2.3}$$

where $(.)$ denotes the partial differentiation and $[K^{\alpha\beta}]$ is the inverse matrix of $[K_{\alpha\beta}]$. If we take $K_{\alpha\beta}$ as $\delta_{\alpha\beta}$ $(=\{^{1\ \ if \alpha=\beta}_{0,\ \ oherwise})$ and $\varphi = ie^{-iK_{\alpha\beta}\frac{\pi}{2}}x^\alpha x^\beta f(x^1)$ then φ becomes

$$\varphi = \sum_{\alpha=2}^{n-1}(x^\alpha)^2 f(x^1), \tag{2.4}$$

where f is an arbitrary non-zero function of x^1.
From (2.4), we can easily get

$$\varphi_{\cdot\alpha\alpha} = 2f(x^1), \quad \varphi_{\cdot\alpha\beta} = 0, \alpha \neq \beta. \tag{2.5}$$

Equations (2.3) and (2.5) give the non-zero component of the curvature tensor R_{hijk} and its derivative $R_{hijk.l}$ as

$$R_{1\alpha\alpha 1} = f(x^1) = -R_{1\alpha 1\alpha} = -R_{\alpha 1\alpha 1}. \tag{2.6}$$

and

$$R_{1\alpha\alpha 1.1} = \frac{d}{dx^1}f(x^1). \tag{2.7}$$

Let

$$\omega_h = \{^{\phi(x^1),\quad h=1}_{0\quad otherwise}, \tag{2.8}$$

then from (1.8), we have

$$\theta_{11} = \nabla_1\omega_1 - \omega_1\omega_1 + \frac{1}{2}g_{11}\omega_h\omega^h, \tag{2.9}$$

or

$$\theta_{11} = \frac{\partial\omega_1}{\partial x^1} - \omega_h\Gamma^h_{11} - \omega_1\omega_1 + \frac{1}{2}g_{11}\omega_h\omega^h, \tag{2.10}$$

where $\omega^h = g^{hm}\omega_m$.
Using (2.2) in (2.10), we get

$$\theta_{11} = \frac{d\phi(x^1)}{dx^1} - \phi^2(x^1). \tag{2.11}$$

Putting the values from equations (2.6) and (2.11) in (1.7), we get the non-zero component of the curvature tensor R^*_{hijk} as follows:

$$R^*_{1\alpha\alpha 1} = R_{1\alpha\alpha 1} - \theta_{\alpha\alpha}g_{11} + \theta_{\alpha 1}g_{\alpha 1} - \theta_{11}g_{\alpha\alpha} + \theta_{1\alpha}g_{1\alpha} = R_{1\alpha\alpha 1} - \theta_{11}g_{\alpha\alpha}. \tag{2.12}$$

Using the condition (1.10) in (2.12), we have

$$R_{1\alpha\alpha 1} = \theta_{11}g_{\alpha\alpha}. \tag{2.13}$$

Taking covariant differentiation with respect to x^1 of equation (2.13), we can write

$$R_{1\alpha\alpha 1.1} = g_{\alpha\alpha}\theta_{11.1}. \tag{2.14}$$

If we take

$$f(x^1) = \frac{d\phi(x^1)}{dx^1} - \phi^2(x^1), \tag{2.15}$$

$$a_i = \{^{\frac{1}{2}[\frac{\frac{d}{dx^1}f(x^1)}{f(x^1)} - \phi(x^1)],\quad i=1}_{0\quad otherwise,} \tag{2.16}$$

and $g_{\alpha\alpha} = 1$, for all α then equation (1.3) can be written as

$$R_{1\alpha\alpha 1.1} = a_1R_{1\alpha\alpha 1} + \omega_1R_{1\alpha\alpha 1} + \omega_\alpha R_{1\alpha\alpha 1} + \omega_\alpha R_{1\alpha\alpha 1} + \omega_1 R_{1\alpha\alpha 1}, \tag{2.17}$$

or

$$R_{1\alpha\alpha 1.1} = (a_1 + 2\omega_1)R_{1\alpha\alpha 1}. \tag{2.18}$$

Now from (2.6), (2.8) and (2.16), we have the right hand side of (2.18)

$$(a_1 + 2\omega_1)R_{1\alpha\alpha 1} = \frac{d}{dx^1}f(x^1). \tag{2.19}$$

Also from (2.11) and (2.15), we get the right hand side of (2.14)

$$g_{\alpha\alpha}\theta_{11.1} = \frac{d}{dx^1} f(x^1). \tag{2.20}$$

Hence from equations (2.19) and (2.20), we can say this is an example of weakly symmetric manifold satisfying the condition of special type of semi-symmetric metric connection.

Now, for obtaining the function ϕ, we take a transformation $\phi = -\frac{1}{U}\frac{dU}{dx^1}$ in (2.15) and get

$$U'' + f(x^1)U = 0, \tag{2.21}$$

where $(')$ denote the derivative with respect to x^1.
Again by a transformation $f(x^1) = t^2$ in (2.21), we have

$$\left[\frac{f'(x^1)}{2t}\right]^2 \frac{d^2U}{dt^2} + \left[\frac{f''(x^1)}{2t} - \frac{1}{t}\left(\frac{f'(x^1)}{2t}\right)^2\right]\frac{dU}{dt} + f(x^1)U = 0, \tag{2.22}$$

which is a Bessel's differential equation.
In particular, if we take $f(x^1) = e^{x^1}$ then (2.22) reduces to

$$t^2\frac{d^2U}{dt^2} + t\frac{dU}{dt} + 4t^2U = 0. \tag{2.23}$$

The general solution of (2.23) in terms of Bessel's function is given by

$$U = AJ_0(2t) + BY_0(2t). \tag{2.24}$$

Using (2.24) in $\phi = -\frac{1}{U}\frac{dU}{dx^1}$ and by the help of $t = \sqrt{f(x^1)} = \sqrt{e^{x^1}}$, we get the function ϕ as follows

$$\phi(x^1) = \frac{ae^{\frac{x^1}{2}}J_1(2e^{\frac{x^1}{2}}) + 2e^{\frac{x^1}{2}}Y_1(2e^{\frac{x^1}{2}})}{aJ_0(2e^{\frac{x^1}{2}}) + 2Y_0(2e^{\frac{x^1}{2}})}, \tag{2.25}$$

where J_0 and J_1 are the Bessel's function and Y_0 and Y_1 are the Bessel's Y function.

3 Different Type of Examples Satisfying the Special Condition

In this section, we have constructed the examples of weakly symmetric, weakly Ricci-symmetric, conformally flat weakly symmetric and weakly projective-symmetric manifolds with special type semi-symmetric metric connection.

3.1 Weakly Symmetric Manifold

Example 3.1. Let g be the Riemannian metric in an n-dimensional manifold $\mathbb{R}^n$ defined by (2.1) and if we take $\varphi = K_{\alpha\beta}\, x^\alpha x^\beta f(x^1)$ then φ becomes

$$\varphi = \sum_{\alpha=2}^{n-1}(x^\alpha)^2 f(x^1), \tag{3.1}$$

where f is an arbitrary non-zero function of x^1.
From (3.1), we get

$$\varphi_{\cdot\alpha\alpha} = 2f(x^1), \quad \varphi_{\cdot\alpha\beta} = 0, \alpha \neq \beta. \tag{3.2}$$

Equations (2.3) and (3.2) imply, the non-zero components of the curvature tensor R_{hijk}, its derivative $R_{hijk.l}$, the Ricci tensor R_{ij} and its derivative $R_{ij.l}$ are

$$R_{1\alpha\alpha 1} = f(x^1) = -R_{1\alpha 1\alpha} = -R_{\alpha 1\alpha 1}, \tag{3.3}$$

$$R_{1\alpha\alpha 1.1} = \frac{d}{dx^1} f(x^1), \tag{3.4}$$

$$R_{11} = (n-2)f(x^1), \tag{3.5}$$

and

$$R_{11.1} = (n-2)\frac{d}{dx^1}f(x^1). \tag{3.6}$$

If we take ω_h and θ_{11} as defined in (2.8) and (2.11) respectively then by using (3.3) equation (1.7) gives the non-zero component of the curvature tensor R^*_{hijk} as follows:

$$R^*_{1\alpha\alpha 1} = R_{1\alpha\alpha 1} - \theta_{\alpha\alpha}g_{11} + \theta_{\alpha 1}g_{\alpha 1} - \theta_{11}g_{\alpha\alpha} + \theta_{1\alpha}g_{1\alpha} = R_{1\alpha\alpha 1} - \theta_{11}g_{\alpha\alpha}. \tag{3.7}$$

Using the condition (1.10) in (3.7), we have

$$R_{1\alpha\alpha 1} = \theta_{11}g_{\alpha\alpha}. \tag{3.8}$$

The covariant derivative with respect to x^1 of equation (3.8) implies

$$R_{1\alpha\alpha 1.1} = g_{\alpha\alpha}\theta_{11.1}. \tag{3.9}$$

If we take a_i as in (2.16) and $g_{\alpha\alpha} = 1$ then equation (1.3) can be written as

$$R_{1\alpha\alpha 1.1} = a_1 R_{1\alpha\alpha 1} + \omega_1 R_{1\alpha\alpha 1} + \omega_\alpha R_{1\alpha\alpha 1} + \omega_\alpha R_{1\alpha\alpha 1} + \omega_1 R_{1\alpha\alpha 1}, \tag{3.10}$$

or

$$R_{1\alpha\alpha 1.1} = (a_1 + 2\omega_1)R_{1\alpha\alpha 1}. \tag{3.11}$$

Now from (2.8), (2.16) and (3.3) we have

$$(a_1 + 2\omega_1)R_{1\alpha\alpha 1} = \frac{d}{dx^1}f(x^1). \tag{3.12}$$

Also from (2.11) and (2.15), the right hand side of (3.9) becomes

$$g_{\alpha\alpha}\theta_{11.1} = \frac{d}{dx^1}f(x^1). \tag{3.13}$$

Hence from equations (3.12) and (3.13) we can say that this is an example of weakly symmetric manifold satisfying the condition of special type of semi-symmetric metric connection.

3.2 Conformally Flat Weakly Symmetric Manifold

Example 3.2. It is well known that the conformal curvature tensor C on an n-dimensional manifold M is given by

$$\begin{aligned} C(X,Y,Z,U) =& R(X,Y,Z,U) - \frac{1}{(n-2)}[S(Y,Z)g(X,U) \\ & - S(X,Z)g(Y,U) + S(X,U)g(Y,Z) \\ & - S(Y,U)g(X,Z)] \\ & + \frac{r}{(n-1)(n-2)}[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)]. \end{aligned} \tag{3.14}$$

Now contracting (3.8) with $g^{\alpha\alpha}$, we get

$$R_{11} = (n-2)\theta_{11}. \tag{3.15}$$

Also from (2.1) we easily obtained $g_{ni} = g_{in} = 0$ for $i \neq 1$, which implies $g^{11} = 0$. Hence the scalar curvature tensor r becomes $r = g^{ij}R_{ij} = g^{11}R_{11} = 0$. Therefore equation (3.14) gives the non-zero component of the conformal curvature tensor C_{hijk} as follows

$$C_{1\alpha\alpha 1} = R_{1\alpha\alpha 1} - \frac{1}{(n-2)}g_{\alpha\alpha}R_{11}. \tag{3.16}$$

Using (3.8) and (3.15) in (3.16) we have

$$C_{1\alpha\alpha 1} = 0. \tag{3.17}$$

Hence, equation (3.17) shows that this is an example of conformally flat weakly symmetric manifold satisfying the condition of special type of semi-symmetric metric connection.

3.3 Weakly Ricci-Symmetric Manifold

Example 3.3. Again, equation (1.4) implies

$$R_{11.1} = a_1 R_{11} + \omega_1 R_{11} + \omega_1 R_{11}. \tag{3.18}$$

or

$$R_{11.1} = (a_1 + 2\omega_1) R_{11}. \tag{3.19}$$

Now, by the help of (2.8) and (2.16) the right hand side of (3.19) becomes

$$(a_1 + 2\omega_1) R_{11} = (n-2)\frac{d}{dx^1} f(x^1). \tag{3.20}$$

Also, the covariant derivative with respect to x^1 of (3.15) gives

$$R_{11.1} = (n-2)\theta_{11.1}. \tag{3.21}$$

By using (2.11) and (2.15), right hand side of (3.21) becomes

$$(n-2)\theta_{11.1} = (n-2)\frac{d}{dx^1} f(x^1). \tag{3.22}$$

Hence from equations (3.20) and (3.22) we can say, this is an example of weakly Ricci symmetric manifold satisfying the condition of special type of semi-symmetric metric connection.

3.4 Weakly Projective Symmetric Manifold

Example 3.4. We know that the projective curvature tensor P on an n-dimensional manifold is defined by

$$P(X,Y,Z,U) = R(X,Y,Z,U) - \frac{1}{(n-1)}[S(Y,Z)g(X,U) - S(X,Z)g(Y,U)]. \tag{3.23}$$

By (3.3), (3.5) and (3.23) we have the non-zero component of the projective curvature tensor P_{hijk} and its derivative $P_{hijk.l}$ as follows

$$P_{1\alpha\alpha 1} = R_{1\alpha\alpha 1}, \tag{3.24}$$

and

$$P_{1\alpha\alpha 1.1} = R_{1\alpha\alpha 1.1}. \tag{3.25}$$

Using (3.9) in (3.25), we get

$$P_{1\alpha\alpha 1.1} = g_{\alpha\alpha}\theta_{11.1}. \tag{3.26}$$

Using (3.13) in (3.26), we obtained

$$P_{1\alpha\alpha 1.1} = \frac{d}{dx^1} f(x^1). \tag{3.27}$$

Using (3.12) in (3.27), we get

$$P_{1\alpha\alpha 1.1} = (a_1 + 2\omega_1) R_{1\alpha\alpha 1}. \tag{3.28}$$

From (3.24) and (3.28), we can write

$$P_{1\alpha\alpha 1.1} = (a_1 + 2\omega_1) P_{1\alpha\alpha 1}. \tag{3.29}$$

Using (2.8) and (2.16), we can easily verify that equation (3.29) holds.
Hence, this is an example of weakly projective symmetric manifold.

3.5 4-Dimensional Lorentzian Space-Time

In this section, it is shown that the examples constructed in previous sections are also the examples of 4-dimensional Lorentzian manifold with special type of semi-symmetric metric connection.
If we take the the function $f(x^1)$ with negative values $i.e.$ $f(x^1) = -h(x^1)$ for a positive valued function $h(x^1)$ then from (2.4) we can easily check that the function φ be negative valued. Therefore, let $\varphi = -\xi$ then for the 4-dimensional manifold, the metric (2.1) reduces to

$$ds^2 = -\xi(dx^1)^2 + (dx^2)^2 + (dx^3)^2 + 2dx^1dx^4. \tag{3.30}$$

Clearly, the signature of the metric defined by (3.30) is (-,+,+,+) $i.e.$ the metric (3.30) is a Lorentzian metric.

Hence, if we take 4-dimensional manifold with metric (3.30) then the examples (3.1), (3.2), (3.3) and (3.4) are also the examples of 4-dimensional weakly symmetric space-time, weakly Ricci-symmetric space-time, Conformally flat weakly symmetric space-time and the weakly projective symmetric space-time.

4 Discussion and Conclusion

In the process of the study of weakly symmetric manifold, weakly Ricci-symmetric manifold, weakly projective symmetric manifold, we observe that several differential Geometers investigated these manifolds with suitable examples. Also, many authors constructed examples of these type of manifolds equipped with semi-symmetric metric connection. After these studies a natural question arise, can we construct a metric in (pseudo) Riemannian manifold which satisfy the conditions of weakly symmetric manifold together with conditions of other weak symmetric manifolds like weakly Ricci-symmetric, conformally flat weakly symmetric and weakly projective symmetric manifolds with respect to semi-symmetric metric connection ? In this paper we find that the answer of this question is positive. These type of symmetries are very useful in the study of theory of relativity.

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Author information

B. B. Chaturvedi and Pankaj Pandey,
Department of Pure & Applied Mathematics,
Guru Ghasidas Vishwavidyalaya, Bilaspur (C.G.)-495009, India.
E-mail: brajbhushan25@gmail.com; pankaj.anvarat@gmail.com

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