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On the line graphs associated to the unit graphs of rings

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Abstract Let R be a commutative ring with unity. Ashrafi et al. [3] introduced the unit graph $G(R)$ of a ring R whose vertices are the elements of R and two distinct vertices x and y are adjacent if and only if $x+y$ is a unit of R. In this article, we investigate some graph-theoretic properties of the line graph $L(G(R))$ associated to $G(R)$. We give some characterization results regarding completeness, bipartiteness, traversability, diameter, girth, and chromatic number of $L(G(R))$.

1 Introduction

Recently, Grimaldi [9] introduced the unit graph $G(\mathbb{Z}_n)$ of ring $\mathbb{Z}_n$, where he took the vertices of $G(\mathbb{Z}_n)$ are the elements of $\mathbb{Z}_n$ and two distinct vertices x and y are adjacent if and only if $x+y$ is a unit of $\mathbb{Z}_n$. After that introduction, Ashrafi et al. [3] generalized the unit graph $G(\mathbb{Z}_n)$ to $G(R)$ for an arbitrary ring R and obtained various characterization results for finite commutative rings regarding connectedness, chromatic index, diameter, girth, and planarity of $G(R)$. Some more discussions of unit graphs of rings can be found in [3], [9], [13], [17], [18] and etc.

Let G be a graph, then one can associate its line graph, denoted by $L(G)$, such that each vertex of $L(G)$ represents an edge of G, and any two distinct vertices in $L(G)$ are adjacent if and only if their corresponding edges in G share a common vertex. Due to Whitney (1932) [19] and Krausz (1934) [12], the line graph became an active area. Whitney proved that the structure of any connected graph can be recovered from its line graph i.e., there is a one-to-one correspondence between the class of connected graphs and the class of connected line graphs. Later, the term line graph comes from a paper from Harary and Norman (1960) [11]. Some more discussions on line graphs associated with some rings and modules can be found in [5], [7], [8], [16], [20] and etc. In this article, we would like to keep an eye on the properties of unit line graph $L(G(R))$ and seek any relation between $G(R)$ and $L(G(R))$.

Now, we recall some needed notions in graph theory. Let $G=(V(G),E(G))$ be a graph with the set of vertices and the set of edges. The degree of the vertex $v\in V$, denoted by $deg(v)$ is the number of vertices adjacent to the vertex v. A path in a graph G is an alternating sequence of vertices and edges of G. A graph G is said to be connected if there is a path between any two distinct vertices of G (for any distinct $x,y\in V(G)$, we write $x\sim y$ if x and y are adjacent; otherwise $x\nsim y$). For a graph G, the distance between two vertices x and y, denoted by $d(x,y)$ is defined as the length of the shortest path from x to y, and $d(x,y)=\infty$ if no such path exists. The diameter of G is defined as $diam(G)=sup\{d(x,y)\mid x,\ y\ are\ vertices\ of\ G\}$. G is called a complete graph if each pair of vertices is connected by an edge. A complete graph on n vertices is denoted by K_n. A cycle graph is a graph that consist of a single cycle. We denote the cycle graph with n vertices by C_n. The length of the shortest cycle in a graph G is called the girth of G and is denoted by $gr(G)$. If G has no cycle, then $gr(G)=\infty$. A connected acyclic graph is called a tree. A circuit in a graph G is a closed trail of length three or more. A circuit C in a graph G is called an Eulerian circuit if C contains every edge of G. A connected graph G is said to be Eulerian if it contains an Eulerian circuit. A graph G is said to be Hamiltonian if it has a circuit which contains all the vertices of G. The chromatic number $\chi(G)$ of a graph G is defined as the minimum number of colors which can be assigned to the vertices of G in such a way that every two adjacent vertices have different colors. The chromatic index $\chi^{/}(G)$ is defined as the minimum number of colors which can be assigned to the edges of graph G in such a way that every two adjacent edges have a different colors. Other undefined terminology related to graph theory can be obtained in [6] and [10].

Throughout this article, R is a ring with unity. We denote cardinality, characteristic, Jacobson radical, and set of units of R by $|R|$, $char(R)$, $J(R)$, and $U(R)$ respectively. A field is denoted by $\mathbb{F}$, and the ring of integer modulo n is denoted by $\mathbb{Z}_n$. For any undefined terminology of ring theory we refer to [4].

The organisation of this article is as follows. In Sec. 2, we study some basic properties of unit line graph $L(G(R))$ associated with R, and finally we give traversability condition of $L(G(R))$ under certain conditions. In Sec. 3, we determine diameter and girth of $L(G(R))$. We prove that $diam(L(G(R))) \in \{0, 2, 3, \infty\}$ if $char(R) = 2$, and $diam(L(G(R))) \in \{0, 1, 2, 3, \infty\}$ for a finite ring R. Finally, we prove that $gr(L(G(R))) \in \{3, 4, 6, \infty\}$, for finite ring R, $gr(L(G(\mathbb{M}_n(R)))) = 3$, for all $n \geq 2$, and we compute the chromatic number of $L(G(R))$.

2 Some Basic Properties of $L(G(R))$

For simplicity of notation, we use $G(R)$ for the unit graph and $L(G(R))$ for its line graph of the ring R with unity. For $x, y \in R$ one has $x + y \in U(R)$, then we have a vertex in the graph $L(G(R))$ and we denote that vertex by $[x, y]$. $L(G(R))$ is connected if and only if $G(R)$ is connected. In this section, we discuss some basic properties of $L(G(R))$.

Theorem 2.1. *Let R be a commutative ring with unity. Then $L(G(R))$ is a complete graph if and only if either $R \cong \mathbb{Z}_2$ or $\mathbb{Z}_3$.*

Proof. Let $L(G(R))$ is a complete graph. Then $G(R)$ is either a complete graph K_3 or a tree. Since there does not exists K_3 in $G(R)$. Therefore, $G(R)$ is a tree. Since, all the units of R is adjacent to zero in $G(R)$. So, $R \cong \mathbb{Z}_2$ or $\mathbb{Z}_3$.
Conversely, let $R \cong \mathbb{Z}_2$ or $\mathbb{Z}_3$. Then R have one or two units and it is easy to see that $L(G(R))$ is K_1 or K_2. Which completes the proof of the Theorem. □

Ashrafi et al. [[3], Theorem 3.2] characterized the unit graphs of rings that have a cycle of length 4 or 6. We know that a line graph $L(G)$ is a cycle graph if and only if G is a cycle graph. Therefore, following theorem is the consequence of [[3], Theorem 3.2].

Theorem 2.2. *Let R be a commutative ring with unity. Then $L(G(R))$ is a cycle graph if and only if R is isomorphic to one of the following rings:*

(i) $\mathbb{Z}_4$;

(ii) $\mathbb{Z}_6$;

(iii) $\left\{ \begin{bmatrix} a & b \\ 0 & a \end{bmatrix} \mid a,\ b \in \mathbb{Z}_2 \right\}$.

Remark 2.3. G is a bipartite graph if and only if G has no odd cycle [König (1936)], and in the case of line graph also it is satisfied. Therefore, $L(G(R))$ is a bipartite graph if and only if R is isomorphic to any one of the rings mentioned in the above Theorem 2.2 and $\mathbb{Z}_3$.

Remark 2.4. From the Theorems 2.1 and 2.2, it is clear that $L(G(R))$ is complete bipartite graph if and only if $R \cong \mathbb{Z}_3,\ \mathbb{Z}_4$ or $\left\{ \begin{bmatrix} a & b \\ 0 & a \end{bmatrix} \mid a,\ b \in \mathbb{Z}_2 \right\}$.

Theorem 2.5 ([3], Proposition 2.4). *Let R be a finite ring. Then the following statements hold for the unit graph of R:*

(i) If $2 \notin U(R)$, then the unit graph $G(R)$ is a $|U(R)|$-regular graph;

(ii) If $2 \in U(R)$, then for every $x \in U(R)$ we have $deg(x) = |U(R)| - 1$ and for every $x \in R \setminus U(R)$ we have $deg(x) = |U(R)|$.

In the following, we compute the degree of $L(G(R))$ using the above Theorem 2.5, and the equality $deg([x,y]) = deg(x) + deg(y) - 2$ for any $[x,y] \in V(L(G))$.

Theorem 2.6. *Let R be a finite ring. Then the following statements hold for $L(G(R))$:*

(i) If $2 \notin U(R)$, then $L(G(R))$ is a $2|U(R)|-2$-regular graph;

(ii) If $2 \in U(R)$, and $x \sim y$ in $G(R)$ then we have:

$$deg([x,y]) = \begin{cases} 2|U(R)|-4 & if\ x,\ y \in U(R) \\ 2|U(R)|-2 & if\ x,\ y \in R \setminus U(R) \\ 2|U(R)|-3 & if x \in U(R)\ and\ y \in R \setminus U(R) \end{cases}$$

Proof. (i) Let $2 \notin U(R)$ and $x, y \in R$. Then we have $deg(x) = deg(y) = |U(R)|$ in $G(R)$ by Theorem 2.5. Now let us assume that $x \sim y$ in $G(R)$, then $[x,y]$ is a vertex of $L(G(R))$. Then, for $[x,y] \in V(L(G(R)))$, we have $deg[x,y] = deg(x)-1+deg(y)-1 \Rightarrow deg[x,y] = |U(R)|-1+|U(R)|-1 = 2|U(R)|-2$.

(ii) Let $2 \in U(R)$ and $x \sim y$ in $G(R)$. Then $[x,y]$ is a vertex in $L(G(R))$. If $x, y \in U(R)$, then by Theorem 2.5, we have $deg(x) = deg(y) = |U(R)|-1$ in $G(R)$. Now, for the unit line graph $L(G(R))$, $deg[x,y] = deg(x)-1+deg(y)-1 \Rightarrow deg[x,y] = 2|U(R)|-4$. If $x, y \in R \setminus U(R)$, then by Theorem 2.5, we have $deg(x) = deg(y) = |U(R)|$ in $G(R)$. Now, for the unit line graph $L(G(R))$, $deg[x,y] = deg(x)-1+deg(y)-1 \Rightarrow deg[x,y] = 2|U(R)|-2$. If $x \in U(R)$ and $y \in R \setminus U(R)$, then for the unit line graph $L(G(R))$, $deg[x,y] = deg(x)-1+deg(y)-1 \Rightarrow deg[x,y] = 2|U(R)|-3$.

□

The following Theorem gives a criterion for the line graph $L(G(R))$ to be Eulerian and it shows that $L(G(R))$ is Eulerian does not necessarily imply that $G(R)$ is Eulerian.

Theorem 2.7. *Let R be a finite ring with unity and $|R| \geq 4$. If $2 \notin U(R)$, then $L(G(R))$ is Eulerian.*

Proof. Let R be a finite ring with unity and $2 \notin U(R)$. Then by part (i) of Theorem 2.6, unit line graph $L(G(R))$ is $2|U(R)|-2$-regular graph, and so $L(G(R))$ is Eulerian. □

Example 2.8. (i) Unit line graph of the rings $\mathbb{Z}_{2n}$ $(n > 1 \in \mathbb{N})$ is Eulerian.

(ii) Let $\mathbb{F}$ be a field with $char(\mathbb{F}) = 2$ and $|\mathbb{F}| \geq 4$. Then $G(R)$ is a complete graph by [Ashrafi et al. [3], Theorem 3.4]. It is clear that, degree of every vertex is odd in $G(R)$. So for $[x,y] \in V(L(G(R)))$, we have $deg([x,y]) = deg(x) + deg(y) - 2$. Which shows that $L(G(\mathbb{F}))$ is Eulerian.

(iii) Let $R = \mathbb{Z}_2 \times \mathbb{F}_1 \times ... \times \mathbb{F}_n$, where $\mathbb{F}_i$ $(1 \leq i \leq n)$ are fields with $|\mathbb{F}_i| \geq 3$. Then $L(G(R))$ is Eulerian.

In the following, we prove the Hamiltonian properties of $L(G(R))$.

Theorem 2.9. *Let R be a finite commutative ring with unity such that $|R| \geq 4$. If $G(R)$ is connected, then $L(G(R))$ is Hamiltonian.*

Proof. Let R be a finite commutative ring with unity such that $|R| \geq 4$, and let $G(R)$ is connected. Now, in view of Theorem 2.1 of [13] it is clear that $G(R)$ is Hamiltonian. Therefore, in view of Theorem 8.8 of [10] $L(G(R))$ is Hamiltonian. □

3 Diameter, Girth & Chromatic Number of $L(G(R))$

In this section, we determine diameter, girth and chromatic number of $L(G(R))$. Ashrafi et al. [3] proved that $diam(G(R)) \in \{1, 2, 3, \infty\}$. Note that for any ring R, $diam(G(R)) - 1 \leq diam(L(G(R))) \leq diam(G(R)) + 1$, and in the following two Theorems 3.1, 3.2 Ramane et al. [14], [15] proved that $diam(G) = diam(L(G))$.

Theorem 3.1. *[15] If $diam(G) \leq 2$ and if none of the three graphs F_1, F_2, and F_3 depicted in Fig. 1 are induced subgraphs of G, then $diam(L(G)) \leq 2$.*

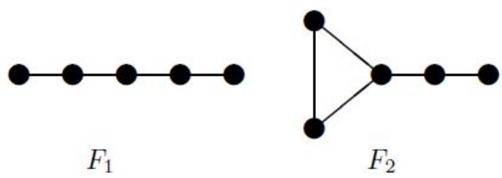

Figure 1. The graphs mentioned in Theorem 3.1

Theorem 3.2. *[14] Let $k \geq 2$. For a connected graph G, $diam(L(G)) \leq k$ if and only if none of the three graphs F_1^k, F_2^k, and F_3^k depicted in Fig. 2 are an induced subgraph of G.*

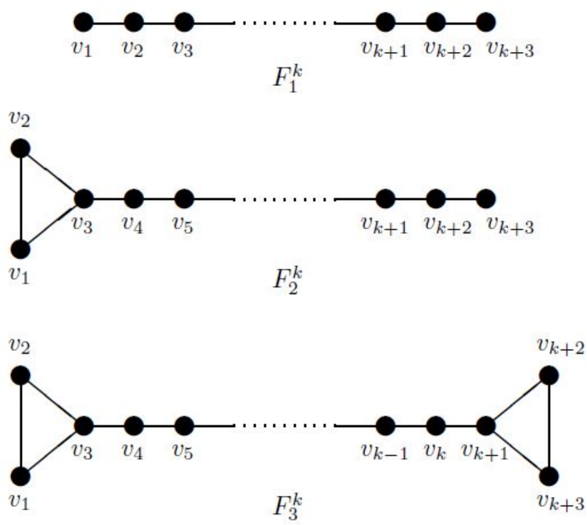

Figure 2. The graphs mentioned in Theorem 3.1 and 3.2

Theorem 3.3. *Let R be a ring with $char(R) = 2$. Then $diam(L(G(R))) \in \{0, 2, 3, \infty\}$.*

Proof. Let R be a ring with $char(R) = 2$. Now the following three cases complete the proof.
Case 1. Suppose that R is a division ring, then $G(R)$ is a complete graph. If $L(G(R))$ is a complete graph, then by Theorem 2.1 $diam(L(G(R))) = 0$. Now, let $L(G(R))$ is not a complete graph and $x, y, u, v \in R$ such that $[x, y], [u, v] \in V(L(G(R)))$. We may assume that $[x, y]$ and $[u, v]$ are not adjacent in $L(G(R))$, then $[x, y]$ —$[y, u]$ —$[u, v]$ is a path of length 2.
Case 2. Suppose that R is not a division ring, then $G(R)$ is not a complete graph, and $diam(G(R)) =$ 2 or 3 by Ashrafi et al. [[3], Lemma 5.5]. Thus $diam(L(G(R)))$ also 2 or 3 by Theorems 3.1 and 3.2, since $G(R)$ have no induced subgraph of Fig. 1 or Fig. 2. Let $x, y, u, v \in R$ such that $[x, y], [u, v] \in V(L(G(R)))$. If $y + u \in U(R)$, then $[x, y]$ —$[y, u]$ —$[u, v]$ is a path of length 2; otherwise $[x, y]$ —$[y, s]$ —$[s, u]$ —$[u, v]$ is a path of length 3.
Case 3. Suppose that $R \cong \mathbb{Z}_2 \times \mathbb{Z}_2$, then $L(G(R))$ is totally disconnected, since $G(R)$ is a disconnected union of K_2. Therefore, $diam(L(G(R))) = \infty$. □

Theorem 3.4. *Let R be a finite commutative ring. Then we have $diam(L(G(R))) \in \{0, 1, 2, 3, \infty\}$.*

Proof. If $R \cong \mathbb{Z}_2$ or Z_3, then by Theorem 2.1, $L(G(R))$ is K_1 or K_2 and so $diam(L(G(R))) = 0$ or 1. If R is any finite ring, then $G(R)$ contains any one induced subgraph of Fig. 1 or 2.

Thus, $diam(G(R)) = diam(L(G(R)))$. Assume that $x, y, u, v \in R$ such that $[x, y], [u, v] \in V(L(G(R)))$. If $y + u \in U(R)$, then there exists a path $[x, y]$ —$[y, u]$ —$[u, v]$ of length 2 in $L(G(R))$. On the other hand, if $y + u \notin U(R)$, then for some $t \in R$, we have $y + t, t + u \in U(R)$, so $[x, y]$ —$[y, t]$ —$[t, u]$ —$[u, v]$ is a path of length 3. If $G(R)$ is disconnected, then $diam(L(G(R))) = \infty$. Thus, we conclude that $diam(L(G(R))) = \{0, 1, 2, 3, \infty\}$. □

Remark 3.5. Note that $diam(L(G(\mathbb{Z}_2))) = 0$, $diam(L(G(\mathbb{Z}_3))) = 1$, $diam(L(G(\mathbb{Z}_4))) = 2$, $diam(L(G(\mathbb{Z}_6))) = 3$, and $diam(L(G(\mathbb{Z}_2 \times \mathbb{Z}_2))) = \infty$.

Ashrafi et al. [3] proved that $gr(G(R)) \in \{3, 4, 6, \infty\}$. In the following, we have also found that $gr(L(G(R))) \in \{3, 4, 6, \infty\}$.

Theorem 3.6. *Let R be a ring. Then the following statements hold for unit line graphs $L(G(R))$:*

(i) If $|U(R)| = 1$, then $gr(L(G(R))) = \infty$;

(ii) If $|U(R)| = 2$, then $gr(L(G(R))) \in \{4, 6, \infty\}$;

(iii) If $|U(R)| \geq 3$, then $gr(L(G(R))) = 3$.

Proof. (i) If $|U(R)| = 1$, then $L(G(R))$ does not contain a cycle, so $gr(L(G(R))) = \infty$.

(ii) Suppose that $U(R) = \{1, x\}$ and $L(G(R))$ contains a cycle. Since $1 + x \notin U(R)$, then there exists $y \in U(R)$ such that $1 + y, x + y \in U(R)$. Therefore, there exists a cycle $[0, 1] \longrightarrow [1, y] \longrightarrow [y, x] \longrightarrow [x, 0] \longrightarrow [0, 1]$ of length 4 in $L(G(R))$. If $1 + y, x + y \notin U(R)$, then there exist $s, t \in R$ such that $1 + s, x + t, s + y, t + y \in U(R)$. Therefore, there exists a cycle $[0, 1] \longrightarrow [1, s] \longrightarrow [s, y] \longrightarrow [y, t] \longrightarrow [t, x] \longrightarrow [x, 0] \longrightarrow [0, 1]$ of length 4 in $L(G(R))$. If $L(G(R))$ does not contain a cycle, then $gr(L(G(R))) = \infty$. Thus $gr(L(G(R))) \in \{4, 6, \infty\}$.

(iii) Suppose that $|U(R)| \geq 3$, then there exists 3 or 4 cycles in $G(R)$. If $G(R)$ contain length of 3 cycle, then obviously in $L(G(R))$ also have a cycle of length 3. Now, we assume that $gr(G(R)) = 4$. Since $|U(R)| = 3$, $G(R)$ is a complete bipartite graph or bipartite graph having $deg(x) \geq 3$, for all $x \in R$. Therefore, for $x, y, z \in U(R)$ we have a cycle $[0, x] \longrightarrow [0, y] \longrightarrow [0, z] \longrightarrow [0, x]$ of length 3 in $L(G(R))$. Thus $gr(L(G(R))) = 3$.

□

Remark 3.7. (i) Let $R = \mathbb{Z}_2$. Then $|U(R)| = 1$ and so $gr(L(G(R))) = \infty$.

(ii) Let $R = \mathbb{Z}_3$. Then $|U(R)| = 2$ and so $gr(L(G(R))) = \infty$. Let $R = \mathbb{Z}_4$. Then $|U(R)| = 2$ and so $gr(L(G(R))) = 4$. Let $R = \mathbb{Z}_6$. Then $|U(R)| = 2$ and so $gr(L(G(R))) = 6$.

(iii) Let $R = \mathbb{Z}_5$. Then $|U(R)| = 4$ and so $gr(L(G(R))) = 3$.

Theorem 3.8. *Let R be a ring with unity. Then the following statements hold:*

(i) If $J(R) \neq 0$ or R contains nonzero nilpotent elements, then $gr(L(G(R))) \in \{3, 4\}$;

(ii) If there exist $x, y \in U(R)$ such that $x \neq y$ and $x + y \in U(R)$, then $gr(L(G(R))) = 3$.

Proof. (i) Let $J(R) \neq 0$ and $j \neq 0 \in J(R)$. Then $[0, 1] \longrightarrow [1, j] \longrightarrow [j, j + 1] \longrightarrow [j + 1, 0] \longrightarrow [0, 1]$ form a length of 4-cycle in $L(G(R))$. Thus $gr(L(G(R))) \leq 4$. Again, let $x \neq 0 \in R$ such that $x^2 = 0$. Then $[0, 1] \longrightarrow [1, x] \longrightarrow [x, 1-x] \longrightarrow [1-x, 0] \longrightarrow [0, 1]$ form a length of 4-cycle in $L(G(R))$, and so $gr(L(G(R))) \leq 4$.

(ii) Let $x, y \in U(R)$ such that $x \neq y$ and $x + y \in U(R)$. Then $[0, x] \longrightarrow [x, y] \longrightarrow [y, 0] \longrightarrow [0, x]$ form a triangle in $L(G(R))$. Thus $gr(L(G(R))) = 3$.

□

Theorem 3.9. *Let R be a division ring with $|R| \geq 4$. Then $gr(L(G(R))) = 3$.*

Proof. Let R be a division ring with $|R| \geq 4$. Then there exist two distinct nonzero elements x, y in R such that $x + y \neq 0$. Therefore, by Theorem 3.8 we have $[0, x] \longrightarrow [x, y] \longrightarrow [y, 0] \longrightarrow [0, x]$ in $L(G(R))$, which yields $gr(L(G(R))) = 3$. □

Theorem 3.10. *Let R be a ring. Then $gr(L(G(\mathbb{M}_n(R)))) = 3$, for all $n \geq 2$.*

Proof. Su and Zhou [[18], Lemma 2.4] proved the existence of units in $\mathbb{M}_n(R)$. Say u and v are units such that $u + v \in U(\mathbb{M}_n(R))$. Then by Theorem 3.8, $[0, u]$, $[u, v]$, and $[v, 0]$ form a cycle in $L(G(\mathbb{M}_n(R)))$. Hence, the result hold. □

We end up this article by discussing the chromatic number of unit line graph $L(G(R))$ of a finite ring R. Since, the chromatic index of a graph leads to the chromatic number of its line graph. Therefore, by [[3], Theorem 5.2] we have the following result for the unit line graph of a finite ring R.

Theorem 3.11. *Let R be a finite ring. Then $\chi(L(G(R))) = |U(R)|$.*

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