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# SOME PROPERTIES OF PYTHAGOREAN FUZZY IDEALS OF Γ-NEAR-RINGS

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**Abstract** In this article, we introduce the concept of pythagorean fuzzy sets and discuss its relationship with instuitionistic fuzzy sets. We define some basic operations on pythagorean fuzzy sets. The key objective of this paper is to investigate some algebraic structures of pythagorean fuzzy ideal of Γ-near-rings. We define the concept of upper and lower cut of pythagorean fuzzy sets. Finally, we investigate some important results of pythagorean fuzzy ideal of Γ-near-rings using the concept of homomorphism of pythagorean fuzzy sets.

## 1 Introduction

Zadeh [31] introduced the idea of fuzzy set which has a membership function, $\mu$ that assigns to each element of the universe of discourse, a number from the unit interval $[0, 1]$ to indicate the degree of belongingness to the set under consideration. In a fuzzy set, a membership function is defined to describe the degree of membership of an element to a class. The membership value ranges from 0 to 1, where 0 shows that the element does not belong to a class, 1 means belongs, and other values indicate the degree of membership to a class.

Atanassov [6, 7] critically studied these shortcomings and proposed a concept called intuitionistic fuzzy sets (IFSs). The construct (that is, IFSs) incorporates both membership function, $\mu$ and nonmembership function, $\nu$ with hesitation margin, $\pi$ (that is, neither membership nor nonmembership functions), such that $\mu+\nu \leq 1$ and $\mu+\nu+\pi = 1$. The notion of IFSs provides a flexible framework to elaborate uncertainty and vagueness. The basic concept of IFSs and its practical application can be found in [3, 4].

There are situations, where $\mu+\nu \geq 1$ unlike the cases capture in IFSs. This limitation in IFSs naturally led to a construct a new concept, called Pythagorean fuzzy sets (PFSs). Pythagorean fuzzy set (PFS) proposed in [12, 13] is a new tool to deal with vagueness considering the membership grade, $\mu$ and nonmembership grade, $\nu$ satisfying the conditions $\mu+\nu \leq 1$ or $\mu+\nu \geq 1$, and also, it follows that $\mu^2+\nu^2+\pi^2 = 1$, where $\pi$ is the Pythagorean fuzzy set index. The construct of PFSs can be used to characterize uncertain information more sufficiently and accurately than IFSs. Garg [12] presented an improved score function for the ranking order of interval-valued Pythagorean fuzzy sets (IVPFSs). Based on it, a Pythagorean fuzzy technique for order of preference by similarity to ideal solution (TOPSIS) method by taking the preferences of the experts in the form of interval-valued Pythagorean fuzzy decision matrices was discussed. Pythagorean fuzzy set has attracted great attentions of many researchers, and subsequently, the concept has been applied to many application areas such as decision-making, aggregation operators and information measures. Other explorations of the theory of PFSs can be found in [5, 10, 17, 15, 24, 28, 30].

The concept of Γ-rings are introduced by Nobusawa in 1964. After that, Barnes [8], Luh [20], and Kyuno [18] studied the structure of Γ- rings and obtained various properties of ring theory. Sen et.al., [26] present the concept of Γ-semigroups. Then the notion of Γ-semirings introduced by Rao [25]. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set.

In [27], Vougiouklis introduced the notion of a Γ- semihypergroup as a generalization of a semihypergroup. Hedayati et. al., [16] studied the notion of a Γ-semihyperring as a generalization of semiring, a generalization of a semihyperring, and a generalization of a Γ-semiring. There is a considerable amount of work on the connections between fuzzy sets and hyperstructures. In [19], Leoreanu-Fotea et.al., introduced the notion of fuzzy subhypergroups as a generalization of fuzzy subgroups, and this topic was continued by himself and others. Davvaz et al. [9] considered the intuitionistic fuzzification of the concept of algebraic hyperstructurs and investigated some properties of such hyperstructures.

In this paper, we introduce and study the concept of Pythagorean fuzzy ideal in Γ-near-rings. The rest of the paper organized as follows. In Section 2, the preliminaries and some definitions are given and present some algebraic structures of Pythagorean fuzzy sets. In Section 3, we studied the definition of Pythagorean fuzzy ideal of Γ-near-rings and discussed some important results on these ideals. Finally, the paper is concluded including the future research in Section 4.

## 2 Preliminaries and Definitions

To assemble this work self sufficient, we briefly introduce a few definitions engaged in the remaining work.

**Definition 2.1.** [1] A near ring is a non-empty set $R$ with two binary operation $+$ and $\cdot$ satisfying the following axioms:
(i) $(R, +)$ is a group,
(ii) $(R, \cdot)$ is a semigroup,
(iii) $x \cdot (y + z) = x \cdot y + x \cdot z$ for all $x$, $y$, $z \in R$.
Precisely speaking, it is a left near-ring because it satisfied left distributive law. We will use the word "near ring " instead of "left near ring".

**Definition 2.2.** [8] Let $(S, +)$ be a group (not necessarily Abelian) and $\Gamma$ be a non-empty set. Then $S$ is said to be a Γ-near-ring if there exists a mapping $S \times \Gamma \times StoS$ (the image of $(a, \alpha, b)$ is denoted by $a\alpha b$), satisfying the following conditions:
(i) $(a + b)\alpha c = a\alpha c + b\alpha c$; and
(ii) $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, \in S$ and $\alpha$, $\beta \in \Gamma$.
$S$ is said to be a zero-symmetric Γ near-ring if $a\alpha 0 = 0$ for all $a \in S$ and $\alpha \in \Gamma$, where 0 is the additive identity in $S$.

**Definition 2.3.** [8] Let $S$ be a Γ-near ring. Then the non-empty subset $I$ of $(S, +)$ is called
(i) a left ideal if $a\alpha(b + i) - a\alpha b \in I$ for all $a, b \in S$, $\alpha \in \Gamma$ and $i \in I$;
(ii) a right ideal if $i\alpha a \in I$ for all $a, b \in S$, $\alpha \in \Gamma$ and $i \in I$; and
(iii) an ideal if is both a left and right ideal.

**Definition 2.4.** [31]**(Fuzzy set (FS))** A fuzzy set $A$ in a universal set $X$ is defined as $A = \Big\{\langle x, \mu_A(x)\rangle | x \in X\Big\}$ where $\mu_A : X \to [0, 1]$ is a mapping called the membership function of the fuzzy set $A$.

**Definition 2.5.** [6]**(Instuitionstic Fuzzy Set (FS))** Let $X$ be a fixed set. An intuitionistic fuzzy set (IFS) $I$ in $X$ is an expression having the form

$$I = \{\langle x, \mu_I(x), \nu_I(x)\rangle : x \in X\},$$

where the functions $\mu_I(x)$ and $\nu_I(x)$ are the degree of membership and the degree of non-membership of the element $x \in X$ respectively. Also $\mu_I : X \to [0, 1]$, $\nu_I : X \to [0, 1]$ and $0 \leq \mu_I(x) + \nu_I(x) \leq 1$, for all $x \in X$.

The degree of indeterminacy $\pi_I(x) = 1 - \mu_I(x) - \nu_I(x)$.

In practice, may be for some reason, the condition $0 \leq \mu(x) + \nu(x) \leq 1$, is not true. For instance, $0.6 + 0.7 = 1.3 > 1$ but $0.6^2 + 0.7^2 < 1$, or $0.7 + 0.7 = 1.4 > 1$ but $0.7^2 + 0.7^2 < 1$. To overcome this situation, in 2013 Yager[29] introduced the concept of the Pythagorean fuzzy set.

**Definition 2.6.** [29]**(Pythagorean Fuzzy set (PFS))** A pythagorean fuzzy set $P$ in a finite universe oF discourse $X$ is given by

$$P = \{\langle x, \mu_P(x), \nu_P(x)\rangle | x \in X\},$$

where $\mu_P(x) : X \to [0,1]$ denotes the degree of membership and $\nu_P(x) : X \to [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set $A$ respectively with the condition that $0 \leq (\mu_P(x))^2 + (\nu_P(x))^2 \leq 1$.

The degree of indeterminacy $\pi_P(x) = \sqrt{1 - (\mu_P(x))^2 - (\nu_P(x))^2}$.

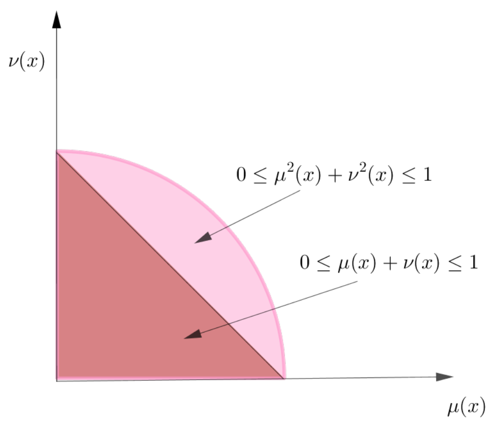


**Figure 1.** Intuitionistic fuzzy set vs Pythagorean fuzzy set

## 2.1 Some Operations on Pythagorean Fuzzy Numbers

Here we discussed some operations on Pythagorean fuzzy numbers and Pythagorean fuzzy sets those are used in the rest of the paper.

Given three Pythagorean fuzzy numbers (PFNs) $\alpha = \langle \mu, \nu \rangle$, $\alpha_1 = \langle \mu_1, \nu_1 \rangle$ and $\alpha_2 = \langle \mu_2, \nu_2 \rangle$. The basic operations can be defined as follows:

(i) $\bar{\alpha} = \langle \nu, \mu \rangle$

(ii) $\alpha_1 \vee \alpha_2 = \langle \max\{\mu_1, \mu_2\}, \min\{\nu_1, \nu_2\} \rangle$

(iii) $\alpha_1 \wedge \alpha_2 = \langle \min\{\mu_1, \mu_2\}, \max\{\nu_1, \nu_2\} \rangle$

(iv) $\alpha_1 \oplus \alpha_2 = \langle \sqrt{\mu_1^2 + \mu_2^2 - \mu_1^2\mu_2^2}, \nu_1\nu_2 \rangle$

(v) $\alpha_1 \otimes \alpha_2 = \langle \mu_1\mu_2, \sqrt{\nu_1^2 + \nu_2^2 - \nu_1^2\nu_2^2} \rangle$

(vi) $\lambda \cdot \alpha = \langle \sqrt{1 - (1 - \mu^2)^\lambda}, \nu^\lambda \rangle, \lambda > 0.$

(vii) $\alpha^\lambda = \langle \mu^\lambda, \sqrt{1 - (1 - \nu^2)^\lambda} \rangle, \lambda > 0.$

**Definition 2.7.** [28] Let $\alpha_1 = \langle \mu_1, \nu_1 \rangle$ and $\alpha_2 = \langle \mu_2, \nu_2 \rangle$ be two PFNs; $S(\alpha_1) = \mu_1^2 - \nu_1^2$ and $S(\alpha_2) = \mu_2^2 - \nu_2^2$ be their score functions; $H(\alpha_1) = \mu_1^2 + \nu_1^2$ and $H(\alpha_2) = \mu_2^2 + \nu_2^2$ be the accuracy degrees of $\alpha_1$ and $\alpha_2$, then Yager and Abbasov [28] defined the following:

1) If $S(\alpha_1) < S(\alpha_2)$ then $\alpha_1$ is smaller than $\alpha_2$, that is $\alpha_1 < \alpha_2$;
2) If $S(\alpha_1) > S(\alpha_2)$ then $\alpha_1 > \alpha_2$,
3) If $S(\alpha_1) = S(\alpha_2)$ then
   if $H(\alpha_1) < H(\alpha_2)$ then $\alpha_1 < \alpha_2$
   if $H(\alpha_1) > H(\alpha_2)$ then $\alpha_1 > \alpha_2$
   if $H(\alpha_1) = H(\alpha_2)$ then $\alpha_1$ and $\alpha_2$ represent the same information, that is $\alpha_1 = \alpha_2$.

## 3 Pythagorean Fuzzy Ideals in Γ-Near-Rings

In this section, we propose the concept of Pythagorean fuzzy ideal of Γ-near-rings. Also, we introduced $t$-level cut sets of Pythagorean fuzzy sets and prove some results on Pythagorean fuzzy ideal of Γ-near-rings.

**Definition 3.1.** Let $S$ be a Γ-near-ring.
A Pythagorean fuzzy set $P = (\mu_P, \nu_P)$ in $S$ is called a Pythagorean fuzzy left (resp. right) ideal of a Γ-near-ring if
(i) $\mu_P(a - b) \geq \{\mu_P(a) \wedge \mu_P(b)\}$,
(ii) $\mu_P(b + a - b) \geq \mu_P(a)$,
(iii) $\mu_P(c\alpha(a + d) - c\alpha d) \geq \mu_P(a)$, (resp. $\mu_P(a\alpha c) \geq \mu_P(a)$)
(iv) $\nu_P(a - b) \leq \{\nu_P(a) \wedge \nu_P(b)\}$,
(v) $\nu_P(b + a - b) \leq \nu_P(a)$,
(vi) $\nu_P(c\alpha(a + d) - c\alpha d) \geq \nu_P(a)$, (resp. $\nu_P(a\alpha c) \leq \nu_P(a)$ )
for all $a$, $b$, $c$, $d \in P$ and $\alpha \in \Gamma$.

**Theorem 3.2.** *Let $P = (\mu_P, \nu_P)$ be a pythagorean fuzzy set. Then $P$ will be a pythagorean fuzzy ideal of a Γ-near-ring $S$ if and only if $\widetilde{P} = (\mu_{\widetilde{P}}, \nu_{\widetilde{P}})$, where*

$$\mu_{\widetilde{P}} = \begin{cases} 1, & x \in P \\ 0, & \textit{otherwise} \end{cases} \qquad \nu_{\widetilde{P}} = \begin{cases} 0, & x \in P \\ 1, & \textit{otherwise} \end{cases}$$

*is a pythagorean fuzzy left (resp. right ) ideal of Γ- near-ring $S$.*

*Proof.* Let $P$ be a left (resp. right ) ideal of $S$.
Let $x_1$, $x_2$, $x_3$, $x_4 \in S$ and $\alpha \in \Gamma$.
If $x_1, x_2 \in P$, then $x_1 - x_2 \in P$, $x_2 + x_1 - x_2 \in P$ and $(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \in P$. Therefore

$$\mu_{\widetilde{P}}(x_1 - x_2) = 1 \geq \{\mu_{\widetilde{P}}(x_1) \wedge \mu_{\widetilde{P}}(x_2)\}, \mu_{\widetilde{P}}(x_2 + x_1 - x_2) = 1 \geq \mu_{\widetilde{P}}(x_1) \text{ and}$$
$$\mu_{\widetilde{P}}(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) = 1 = \mu_{\widetilde{P}}(x_1)(\text{ resp. } \mu_{\widetilde{P}}(x_1\alpha x_3) = \mu_{\widetilde{P}}(x_1) = 1)$$

$$\nu_{\widetilde{P}}(x_1 - x_2) = 0 \leq \{\nu_{\widetilde{P}}(x_1) \vee \nu_{\widetilde{P}}(x_2)\}, \nu_{\widetilde{P}}(x_2 + x_1 - x_2) = 0 \leq \nu_{\widetilde{P}}(x_1) \text{ and}$$
$$\nu_{\widetilde{P}}(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) = 0 = \nu_{\widetilde{P}}(x_1)(\text{ resp. } \nu_{\widetilde{P}}(x_1\alpha x_3) = \nu_{\widetilde{P}}(x_1) = 0).$$

If $x_1 \notin P$ or, $x_2 \notin P$ then $\mu_{\widetilde{P}}(x_1) = 0$ or, $\mu_{\widetilde{P}}(x_2) = 0$.
Thus, we have

$$\mu_{\widetilde{P}}(x_1 - x_2) \geq \{\mu_{\widetilde{P}}(x_1) \wedge \mu_{\widetilde{P}}(x_2)\}, \mu_{\widetilde{P}}(x_2 + x_1 - x_2) \geq \mu_{\widetilde{P}}(x_1) \text{ and}$$
$$\mu_{\widetilde{P}}(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \geq \mu_{\widetilde{P}}(x_1)(\text{ resp. } \mu_{\widetilde{P}}(x_1\alpha x_3) \geq \mu_{\widetilde{P}}(x_1))$$

$$\nu_{\widetilde{P}}(x_1 - x_2) \leq \{\nu_{\widetilde{P}}(x_1) \vee \nu_{\widetilde{P}}(x_2)\}, \nu_{\widetilde{P}}(x_2 + x_1 - x_2) \leq \nu_{\widetilde{P}}(x_1) \text{ and}$$
$$\nu_{\widetilde{P}}(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \geq \nu_{\widetilde{P}}(x_1)(\text{ resp. } \nu_{\widetilde{P}}(x_1\alpha x_3) \leq \nu_{\widetilde{P}}(x_1))$$

Hence, $\widetilde{P}$ is a pythagorean fuzzy left (resp. right ) ideal of near-ring $S$.
Conversely, let $\widetilde{P}$ is a pythagorean fuzzy left (resp. right ) ideal of near-ring $S$.
Let $x_1, x_2 \in S$ and $\alpha \in \Gamma$.
If $x_1, x_2, x_3, x_4 \in P$, then

$$\mu_{\widetilde{P}}(x_1 - x_2) \geq \{\mu_{\widetilde{P}}(x_1) \wedge \mu_{\widetilde{P}}(x_2)\} = 1$$
$$\nu_{\widetilde{P}}(x_1 - x_2) \leq \{\nu_{\widetilde{P}}(x_1) \wedge \nu_{\widetilde{P}}(x_2)\} = 0$$

So, $x_1 - x_2 \in P$. Again,

$$\mu_{\widetilde{P}}(x_2 + x_1 - x_2) \geq \{\mu_{\widetilde{P}}(x_1) = 1$$
$$\nu_{\widetilde{P}}(x_2 + x_1 - x_2) \leq \{\nu_{\widetilde{P}}(x_1) = 0$$

therefore, $(x_2 + x_1 - x_2) \in P$.
So, $x_1 - x_2 \in P$. Again,

$$\mu_{\widetilde{P}}(x_3\alpha(x_1 + x_3) - x_3\alpha x_4) \geq \{\mu_{\widetilde{P}}(x_1) = 1(resp.\mu_{\widetilde{P}}(x_1\alpha x_3) = \mu_{\widetilde{P}}(x_1) = 1)$$
$$\nu_{\widetilde{P}}(x_3\alpha(x_1 + x_3) - x_3\alpha x_4) \leq \{\nu_{\widetilde{P}}(x_1) = 0(resp.\nu_{\widetilde{P}}(x_1\alpha x_3) = \nu_{\widetilde{P}}(x_1) = 0)$$

So, $(x_3\alpha(x_1 + x_3) - x_3\alpha x_4) \in P$.
Hence, $P$ is a pythagorean fuzzy left (resp. right ) ideal of near-ring $S$. □

**Theorem 3.3.** *Let $P = (\mu_P, \nu_P)$ be a pythagorean fuzzy ideal of $\Gamma$-near-ring $S$ and $t \in [0, 1]$, then*
(i) *$U(\mu_P; t)$ is a pythagorean fuzzy ideal of $\Gamma$-near-ring $S$*
(ii) *$L(\nu_P; t)$ is a pythagorean fuzzy ideal of $\Gamma$-near-ring $S$.*

*Proof.* (i) Let $x_1, x_2 \in U(\mu_P; t)$.
Then, $\mu_P(x_1 - x_2) \geq \{\mu_P(x_1) \wedge \mu_P(x_2)\} \geq t$.
Hence, $x_1 - x_2 \in U(\mu_P; t)$.
Again, $\mu_P(x_2 + x_1 - x_2) \geq \mu_P(x_1) \geq t$.
Therefore, $(x_2 + x_1 - x_2) \in U(\mu_P; t)$.
Let $x_1 \in S$, $\alpha \in \Gamma$ and $x_3, x_4 \in U(\mu_P; t)$, then
$\mu_P(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \geq \mu_P(x_1) \geq t$ and so $(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \in U(\mu_P; t)$.
Hence, $U(\mu_P; t)$ is a pythagorean fuzzy ideal of $\Gamma$-near-ring $S$.
(ii) Let $x_1, x_2 \in L(\nu_P; t)$.
Then, $\nu_P(x_1 - x_2) \leq \{\nu_P(x_1) \vee \nu_P(x_2)\} \leq t$.
Hence, $x_1 - x_2 \in L(\nu_P; t)$.
Again, $\nu_P(x_2 + x_1 - x_2) \leq \nu_P(x_1) \leq t$.
Therefore, $(x_2 + x_1 - x_2) \in L(\mu_P; t)$.
Let $x_1 \in S$, $\alpha \in \Gamma$ and $x_3, x_4 \in L(\nu_P; t)$, then
$\nu_P(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \leq \nu_P(x_1) \leq t$ and so $(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \in l(\nu_P; t)$.
Hence, $L(\nu_P; t)$ is a pythagorean fuzzy ideal of $\Gamma$-near-ring $S$. □

**Theorem 3.4.** *Let $I$ be a ideal of a $\Gamma$-near-ring $S$. If the pythagorean fuzzy set $Q = (\mu_Q, \nu_Q)$ defined by*

$$\mu_Q(x) = \begin{cases} p_1 & x \in I \\ p_2 & otherwise \end{cases} ; \nu_Q(x) = \begin{cases} q_1 & x \in Q \\ q_2 & otherwise \end{cases}$$

*for all $x \in S$ and $\alpha \in \Gamma$, where $0 \leq p_1 < p_2$ and $0 \leq q_1 < q_2$, then $Q$ is a pythagorean fuzzy left (resp. right ) ideal of a $\Gamma$-near-ring $S$.*

*Proof.* Let $x_1, x_2 \in S$ and $\gamma \in \Gamma$.
If at least one of $x_1$ and $x_2$ does not belong to $I$, then

$$\mu_P(x_1 - x_2) \geq p_2 = \{\mu_P(x_1) \wedge \mu_P(x_2)\}$$
$$\nu_P(x_1 - x_2) \leq q_2 = \{\nu_P(x_1) \vee \nu_P(x_2)\}$$

If $x_1, x_2 \in I$, then $x_1 - x_2 \in I$ and so,

$$\mu_P(x_1 - x_2) = p_1 = \{\mu_P(x_1) \wedge \mu_P(x_2)\}$$
$$\nu_P(x_1 - x_2) = q_2 = \{\nu_P(x_1) \vee \nu_P(x_2)\}$$

If $x_1, x_2 \in I$, then $(x_2 + x_1 - x_2) \in I$ and so

$$\mu_P(x_2 + x_1 - x_2) = p_1 = \mu_P(x_1) \text{ and } \nu_P(x_2 + x_1 - x_2) = q_2 = \nu_P(x_1)$$

If $x_3, x_4 \in I$, $x_1 \in S$ and $\alpha \in \Gamma$, then $(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \in I$,

$$\mu_P(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) = p_1 = \mu_P(x_1)(\text{resp. } \mu_P(x_1\alpha x_3)) = p_1 = \mu_P(x_1) \text{ and}$$
$$\nu_P(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) = q_1 = \nu_P(x_1)(\text{resp. } \nu_P(x_1\alpha x_3)) = q_1 = \nu_P(x_1)$$

If $x_2 \notin I$, then

$$\mu_P(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) = p_2 = \mu_P(x_1), (\text{resp. } \mu_P(x_1\alpha x_3)) = p_1 = \mu_P(x_1) \text{ and}$$
$$\nu_P(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) = q_2 = \nu_P(x_1)(\text{resp. } \nu_P(x_1\alpha x_3)) = q_1 = \nu_P(x_1)$$

Therefore $P$ is a pythagorean fuzzy left ( resp. right ) ideal of Γ-near-ring $S$. □

**Definition 3.5.** Let $f : S_1 \to S_2$ be a function, where $S_1$ and $S_2$ are Γ-near-rings. The function $f$ is said to be Γ-homomorphism if

$$f(x + y) = f(x) + f(y) \text{ and } f(x\alpha y) = f(x)\alpha f(y),$$

for all $x, y \in S_1$ and $\alpha \in \Gamma$.

**Definition 3.6.** Let $f : S_1 \to S_2$ be a function, where $S_1$ and $S_2$ are Γ-near-rings and $P$ be a pythagorean fuzzy set of $S_2$. Then $f^{-1}(P)$ is the pre-image of $P$ under $f$, if

$$\mu_P^{-1}(x) = \mu_P(f(x)) \text{ and } \nu_P^{-1}(x) = \nu_P(f(x))$$

for all $x \in S_1$.

**Definition 3.7.** Let $f : S_1 \to S_2$ be a function, where $S_1$ and $S_2$ are Γ-near-rings. If $P = (\mu_P, \nu_P)$ and $Q = (\mu_Q, \nu_Q)$ be two pythagorean fuzzy subsets in $S_1$ and $S_2$ respectively, then
(i) the image of $P$ under $f$ is the pythagorean fuzzy set $f(P) = (\mu_{f(P)}, \nu_{f(P)})$ defined by

$$\mu_{f(P)} = \begin{cases} \bigvee_{x \in f^{-1}(y)} \mu_P(x) & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases},$$

$$\nu_{f(P)} = \begin{cases} \bigwedge_{x \in f^{-1}(y)} \nu_P(x) & \text{if } f^{-1}(y) \neq \phi \\ 1 & \text{otherwise} \end{cases}$$

for all $y \in Q$.
(ii) the pre-image of $Q$ under $f$ is the pythagorean fuzzy set $f^{-1}(Q) = (\mu_{f^{-1}(Q)}, \nu_{f^{-1}(Q)})$ defined by

$$\mu_{f^{-1}(Q)} = \begin{cases} \bigvee_{y \in f^{-1}(x)} \mu_Q(y) & \text{if } f^{-1}(x) \neq \phi \\ 0 & \text{otherwise} \end{cases},$$

$$\nu_{f^{-1}(Q)} = \begin{cases} \bigwedge_{y \in f^{-1}(x)} \nu_Q(y) & \text{if } f^{-1}(x) \neq \phi \\ 1 & \text{otherwise} \end{cases}$$

for all $y \in Q$.

**Theorem 3.8.** *Let $S_1$ and $S_2$ be two $\Gamma$- near-rings and $f : S_1 \to S_2$ be a $\Gamma$-epimorphism and let $Q = (\mu_Q, \nu_Q)$ be a pythagorean fuzzy set of $S_2$. If $f^{-1}(Q) = (\mu_f^{-1}(Q), \nu_f^{-1}(Q))$ is a pythagorean fuzzy left (resp. right ) ideal of $S_1$, then $Q$ is a pythagorean fuzzy left (resp. right ) ideal of $\Gamma$-near-ring $S_2$.*

*Proof.* Let $x_1, x_2, x_3, x_4 \in S_2$ and $\alpha \in \Gamma$, then there exists $a_1, a_2, a_3, a_4 \in S_1$ such that $f(a_1) = x_1$, $f(a_2) = x_2$, $f(a_3) = x_3$, $f(a_4) = x_4$.
Therefore,

$$\begin{aligned}
\mu_Q(x_1 - x_2) &= \mu_Q(f(a_1) - f(a_2)) = \mu_Q(f(a_1 - a_2)) \\
&= \mu_{f^{-1}}Q(a_1 - a_2) \geq \{\mu_{f^{-1}(Q)}(a_1) \wedge \mu_{f^{-1}(Q)}(a_2)\} \\
&= \{\mu_Q(f(a_1)) \wedge \mu_Q(f(a_2))\} \\
&= \{\mu_Q(x_1) \wedge \mu_Q(x_2)\}
\end{aligned}$$

$$\begin{aligned}
\nu_Q(x_1 - x_2) &= \nu_Q(f(a_1) - f(a_2)) = \nu_Q(f(a_1 - a_2)) \\
&= \nu_{f^{-1}}Q(a_1 - a_2) \leq \{\nu_{f^{-1}(Q)}(a_1) \wedge \nu_{f^{-1}(Q)}(a_2)\} \\
&= \{\nu_Q(f(a_1)) \vee \nu_Q(f(a_2))\} \\
&= \{\nu_Q(x_1) \wedge \nu_Q(x_2)\}
\end{aligned}$$

$$\begin{aligned}
\mu_Q(x_2 + x_1 - x_2) &= \mu_Q(f(a_2) + f(a_1) - f(a_2)) = \mu_Q(f(a_2 + a_1 - a_2)) \\
&= \mu_{f^{-1}}(Q)(a_2 + a_1 - a_2) \geq \{\mu_{f^{-1}(Q)}(a_1) \\
&= \mu_Q(f(a_1)) \\
&= \mu_Q(x_1)
\end{aligned}$$

$$\begin{aligned}
\nu_Q(x_2 + x_1 - x_2) &= \nu_Q(f(a_2) + f(a_1) - f(a_2)) = \nu_Q(f(a_2 + a_1 - a_2)) \\
&= \nu_{f^{-1}}(Q)(a_2 + a_1 - a_2) \leq \{\nu_{f^{-1}(Q)}(a_1) \\
&= \nu_Q(f(a_1)) \\
&= \nu_Q(x_1)
\end{aligned}$$

Also,

$$\begin{aligned}
\mu_Q(x_3\alpha(x_1 + x_3) - x_3\alpha x_4) &= \mu_Q(f(a_3)\alpha(f(a_1) + f(a_4) - f(a_3)\alpha f(a_4))) \\
&= \mu_Q(f(a_3\alpha(a_1 + a_4) - a_3\alpha a_4)) \\
&= \mu_{f^{-1}(Q)}(a_3\alpha(a_1 + a_4) - a_3\alpha a_4) \\
&\geq \mu_{f^{-1}(Q)}(a_1) \\
&= \mu_Q(f(a_1)) = \mu_Q(x_1)
\end{aligned}$$

$$\begin{aligned}
\nu_Q(x_3\alpha(x_1 + x_3) - x_3\alpha x_4) &= \nu_Q(f(a_3)\alpha(f(a_1) + f(a_4) - f(a_3)\alpha f(a_4))) \\
&= \nu_Q(f(a_3\alpha(a_1 + a_4) - a_3\alpha a_4)) \\
&= \nu_{f^{-1}(Q)}(a_3\alpha(a_1 + a_4) - a_3\alpha a_4) \\
&\leq \nu_{f^{-1}(Q)}(a_1) \\
&= \nu_Q(f(a_1)) = \nu_Q(x_1)
\end{aligned}$$

Similarly, $\mu_Q(x_1\alpha x_3) \geq \mu_Q(x_1)$ and $\nu_Q(x_1\alpha x_3) \leq \nu_Q(x_1)$.
Hence, $Q$ is a pythagorean fuzzy left (resp. right ) ideal in $S_2$. □

**Theorem 3.9.** *A pythagorean fuzzy set $P = (\mu_P, \nu_P)$ of $\Gamma$-near-ring $S$, is a pythagorean fuzzy left (resp. right ) ideal if and only if $P_{<t,s>} = \{x \in S : \mu_P(x) \geq t, \nu_P(x) \leq s\}$ is a left (resp. right ) ideal of $S$.*

*Proof.* Let $P = (\mu_P, \nu_P)$ be a pythagorean fuzzy left ( resp. right ) ideal of Γ-near-ring $S$ and $\mu_P(0) \geq t$, $\nu_P(0) \leq s$.
Let $x_1$, $x_2$, $x_3$, $x_4 \in P_{<t,s>}$ and $\alpha \in \Gamma$.
Then, $\mu_P(x_1) \geq t$, $\nu_P(x_1) \leq t$ and $\mu_P(x_2) \geq t$, $\nu_P(x_2) \leq t$.
Hence,

$$\mu_P(x_1 - x_2) \geq \{\mu_P(x_1) \wedge \mu_P(x_2)\} \geq t$$
$$\nu_P(x_1 - x_2) \leq \{\nu_P(x_1) \vee \nu_P(x_2)\} \leq s.$$

$$\mu_P(x_2 + x_1 - x_2) \geq \mu_P(x_1) \geq t$$
$$\nu_P(x_2 + x_1 - x_2) \leq \nu_P(x_1) \leq s$$

$$\mu_P(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \geq \mu_P(x_1) \geq t$$
$$\text{and} \quad \nu_P(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \leq \nu_P(x_1) \leq s$$
$$(\text{ resp. } \mu_P(x_1\alpha x_3) \geq \mu_P(x_1) \geq t) \text{ and } (\nu_P(x_1\alpha x_3) \leq \nu_P(x_1) \leq s)$$

Therefore, $(x_1 - x_2) \in P_{<t,s>}$, $(x_2 + x_1 - x_2) \in P_{<t,s>}$ and $(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \in P_{<t,s>}$ for all $x_1$, $x_2 \in P_{<t,s>}$ and $\alpha \in \Gamma$.

Hence, $P_{<t,s>}$ is a pythagorean fuzzy left (resp. right ) ideal of Γ-near-ring $S$.
Conversely, let $P_{<t,s>}$ is a pythagorean fuzzy left (resp. right ) ideal of Γ-near-ring $S$ for $\mu_P(0) \geq t$ and $\nu_P(0) \leq s$.
Let $x_1$, $x_2 \in S$ such that $\mu_P(x_1) = t_1$, $\nu_P(x_1) = s_1$ and $\mu_P(x_2) = t_2$, $\nu_P(x_2) = s_2$.
Then $x_1 \in P_{<t,s>}$ and $x_2 \in P_{<t,s>}$.
We now consider that $t_2 \leq t_1$ and $s_2 \geq s_1$.
It follows that $P_{<t_2,s_2>} \subseteq P_{<t_1,s_1>}$ so that $x_1, x_2 \in P_{<t_1,s_1>}$.
Since, $P_{<t_1,s_1>}$ is an pythagorean fuzzy ideal of Γ-near-ring $S$, so we have $(x_1 - x_2) \in P_{<t,s>}$, $(x_2 + x_1 - x_2) \in P_{<t,s>}$ and $(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \in P_{<t,s>}$ for all $\alpha \in \Gamma$.
Now,

$$\mu_P(x_1 - x_2) \geq t_1 \geq t_2 = \{\mu_P(x_1) \wedge \mu_P(x_2)\}$$
$$\nu_P(x_1 - x_2) \leq s_1 \leq s_2 = \{\nu_P(x_1) \vee \nu_P(x_2)\}$$
$$\mu_P(x_2 + x_1 - x_2) \geq t_1 \geq t_2 =\geq \mu_P(x_1)$$
$$\nu_P(x_2 + x_1 - x_2) \leq s_1 \leq s_2 = \nu_P(x_1)$$
$$\mu_P(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \geq t_1 \geq t_2 = \mu_P(x_1)$$
$$\text{and} \quad \nu_P(x_3\alpha(x_1 + x_4) - x_3\alpha x_4) \leq s_1 \leq s_2 = \nu_P(x_1)$$

Hence, $P = (\mu_P, \nu_P)$ is a pythagorean fuzzy ideal of Γ-near-rings $S$. □

**Theorem 3.10.** *Let $P = (\mu_P, \nu_P)$ is a pythagorean fuzzy left (resp. right) ideal of a* Γ- *near-ring $S$ then the set $Q = (\mu_Q, \nu_Q)$, where*

$$\mu_Q = \{x \in S : \mu_P(x) = \mu_P(0)\} \text{ and } \nu_Q = \{x \in S : \nu_P(x) = \nu_P(0)\}$$

*is a pythagorean fuzzy left (resp. right ) ideal of $S$.*

*Proof.* Let $x_1, x_2, x_3, x_4 \in \mu_Q$ and $\alpha \in \Gamma$.
Then $\mu_Q(x_1) = \mu_Q(0)$, $\mu_Q(x_2) = \mu_Q(0)$.
Since, $P$ is a pythagorean fuzzy left (resp. right ) ideal of Γ-near-ring $S$, we get

$$\mu_P(x_1 - x_2) \geq \{\mu_P(x_1) \wedge \mu_P(x_2)\} = \mu_P(0).$$

But, $\mu_P(0) \geq \mu_P(x_2 + x_1 - x_2)$.
So, $x_1 - x_2 \in \mu_Q$.
Again, $\mu_P(x_3\alpha(x_1 + x_3) - x_3\alpha x_4) \in \mu_Q$, ( resp. $\mu_P(x_1\alpha x_3) \geq \mu_P(x_1) = \mu_(0))$.

Hence, $(x_3\alpha(x_1+x_3)-x_3\alpha x_4) \in \mu_Q$.
Therefore, $\mu_Q$ is a pythagorean fuzzy left ( resp. right ) ideal of $\gamma$-near-ring $S$.
Similarly, let $x_1, x_2, x_3, x_4 \in \nu_Q$ and $\alpha \in \Gamma$.
Then $\nu_Q(x_1) = \nu_Q(0), \nu_Q(x_2) = \nu_Q(0)$.
Since, $P$ is a pythagorean fuzzy left (resp. right ) ideal of $\Gamma$-near-ring $S$, we get

$$\nu_P(x_1 - x_2) \leq \{\nu_P(x_1) \vee \nu_P(x_2)\} = \nu_P(0).$$

But, $\nu_P(0) \leq \nu_P(x_2 + x_1 - x_2)$.
So, $x_1 - x_2 \in \nu_Q$.
Again, $\nu_P(x_3\alpha(x_1+x_3) - x_3\alpha x_4) \in \nu_Q$, ( resp. $\nu_P(x_1\alpha x_3) \leq \nu_P(x_1) = \nu_(0))$.
Hence, $(x_3\alpha(x_1+x_3)-x_3\alpha x_4) \in \nu_Q$.
Therefore, $\nu_Q$ is a pythagorean fuzzy left ( resp. right ) ideal of $\Gamma$-near-ring $S$. □

**Definition 3.11.** Let $P = (\mu_P, \nu_P)$ and $Q = (\mu_Q, \nu_Q)$ be two pythagorean fuzzy subsets of a $\Gamma$-near-ring $S$. Then the product $A\Gamma B$ is defined by

$$\mu_{A\Gamma B} = \begin{cases} \bigvee\limits_{x=(u\gamma(v+w)-u\gamma w)} (\mu_P(u) \wedge \mu_Q(v)) & \text{if } x = (u\gamma(v+w) - u\gamma w), u, v, w \in S, \gamma \in \Gamma \\ 0 & \text{otherwise} \end{cases}$$

$$\nu_{A\Gamma B} = \begin{cases} \bigwedge\limits_{x=(u\gamma(v+w)-u\gamma w)} (\mu_P(u) \vee \mu_Q(v)) & \text{if } x = (u\gamma(v+w) - u\gamma w), u, v, w \in S, \gamma \in \Gamma \\ 1 & \text{otherwise} \end{cases}$$

**Theorem 3.12.** *Let $P = (\mu_P, \nu_P)$ and $Q = (\mu_Q, \nu_Q)$ be two pythagorean fuzzy left (resp. right) ideal of $\Gamma$- near-ring $S$, then $P \cap Q$ is a pythagorean fuzzy left (resp. right) ideal of $\Gamma$- near-ring $S$.*

*Proof.* Let $P = (\mu_P, \nu_P)$ and $Q = (\mu_Q, \nu_Q)$ be two pythagorean fuzzy ideal of $\Gamma$- near-ring $S$ and let $x_1$, $x_2$, $x_3$, $x_4 \in S$ and $\alpha \in \Gamma$. Then

$$\begin{aligned} \mu_{P\cap Q}(x_1 - x_2) &= \mu_P(x_1 - x_2) \wedge \mu_Q(x_1 - x_2) \\ &\geq [\mu_P(x_1) \wedge \mu_P(x_2)] \wedge [\mu_Q(x_1) \wedge \mu_Q(x_2)] \\ &= [\mu_P(x_1) \wedge \mu_Q(x_1)] \wedge [\mu_P(x_2) \wedge \mu_Q(x_2)] \\ &= \mu_{P\cap Q}(x_1) \wedge \mu_{P\cap Q}(x_2) \end{aligned}$$

$$\begin{aligned} \nu_{P\cap Q}(x_1 - x_2) &= \nu_P(x_1 - x_2) \vee \nu_Q(x_1 - x_2) \\ &\leq [\nu_P(x_1) \vee \nu_P(x_2)] \vee [\nu_Q(x_1) \vee \nu_Q(x_2)] \\ &= [\nu_P(x_1) \vee \nu_Q(x_1)] \vee [\nu_P(x_2) \vee \nu_Q(x_2)] \\ &= \nu_{P\cap Q}(x_1) \vee \nu_{P\cap Q}(x_2) \end{aligned}$$

$$\begin{aligned} \mu_{P\cap Q}(x_2 + x_1 - x_2) &= \mu_P(x_2 + x_1 - x_2) \wedge \mu_Q(x_2 + x_1 - x_2) \\ &\geq [\mu_P(x_1)] \wedge [\mu_Q(x_1)] \\ &= [\mu_{P\cap Q}(x_1)] \wedge [\mu_{P\cap Q}(x_2)] \end{aligned}$$

$$\begin{aligned} \nu_{P\cap Q}(x_2 + x_1 - x_2) &= \nu_P(x_2 + x_1 - x_2) \vee \nu_Q(x_2 + x_1 - x_2) \\ &\leq [\nu_P(x_1)] \vee [\nu_Q(x_1)] \\ &= [\nu_{P\cap Q}(x_1)] \vee [\nu_{P\cap Q}(x_2)] \end{aligned}$$

Since $P$ and $Q$ are pythagorean fuzzy ideals of $\Gamma$-near-rings $S$, so we have
$\mu_P(x_1\alpha(x_2+x_3) - x_1\alpha x_3) \geq \mu_P(x)$, $\nu_P(x_1\alpha(x_2+x_3) - x_1\alpha x_3) \leq \nu_P(x)$ and

$\mu_Q(x_2\alpha x_1) \geq \mu_Q(x_1), \nu_Q(x_2\alpha x_1) \leq \nu_Q(x_1)$.
Now,

$$\begin{aligned} & \mu_{P\cap Q}(x_1\alpha(x_2+x_3)-x_1\alpha x_3) \\ = \ & \mu_P(x_1\alpha(x_2+x_3)-x_1\alpha x_3) \wedge \mu_B(x_1\alpha(x_2+x_3)-x_1\alpha x_3) \\ \geq \ & \mu_P(x_1) \wedge \mu_Q(x_1) = \mu_{P\cap Q}(x_1)(\text{ resp. } \mu_{P\cap Q}(x_2\alpha x_1) \geq \mu_{P\cap Q}(x_1)) \end{aligned}$$

$$\begin{aligned} & \nu_{P\cap Q}(x_1\alpha(x_2+x_3)-x_1\alpha x_3) \\ = \ & \nu_P(x_1\alpha(x_2+x_3)-x_1\alpha x_3) \vee \nu_B(x_1\alpha(x_2+x_3)-x_1\alpha x_3) \\ \leq \ & \nu_P(x_1) \wedge \nu_Q(x_1) = \nu_{P\cap Q}(x_1)(\text{ resp. } \nu_{P\cap Q}(x_2\alpha x_1) \leq \nu_{P\cap Q}(x_1)) \end{aligned}$$

Hence $P \cap Q$ is a pythagorean fuzzy left (resp. right) ideal of Γ- near-ring $S$. □

## 4 Conclusion

Pythagorean fuzzy set is one of the successful extensions of intuitionistic fuzzy set for handling the uncertainties in the data. Under this environment, in this paper, we introduce the notion of pythagorean fuzzy ideal in Γ-near-rings. Some of its desirable properties have also been investigated. Also, we introduce the concept of lower and upper level sets of pythagorean fuzzy sets and discussed some results on these sets.

In future, we will investigate the decision-making problems based on Interval-valued Pythagorean fuzzy sets. An investigation will be carried out about the Interval-valued Pythagorean fuzzy near-rings and Interval-valued Pythagorean ideals of a near-ring and their algebraic properties.

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