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AN ACCOUNT OF SPECIALITY OF PARTIAL WORDS WITH RESPECT TO PERIODICITY

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Abstract A study on the behaviour of DNA strands lead to the introduction of the mathematical term 'partial word' by Berstel and Boasson and extended partial words with an arbitrary number of holes. The concept of a special partial word is crucial for these extensions. In this paper, we consider some cases on non-primitive partial words in which we have fixed the position of the hole and constructed a sequence of (k, l)-special for a fixed k where l varies. The periodicity is generalized for the considered partial words with the help of $gcd(k, l)$ and it can be determined whether a partial word is (k, l)-special or not.

1 Introduction

The origin of combinatorics on words goes back to the 20th century and initiated by the work of Axel Thue on square-free words [2]. Loithaire's [13, 10, 11] works on combinatorics on words was the stimulus for recent works on partial words. A study on the behaviour of DNA strands lead to the introduction of the mathematical term 'partial word' by Berstel and Boasson in [1]. Since then, the term has evolved into a theory due to its many diverse applications. Blanchet-Sadri established the fact that partial words are useful in the determination of suitable encoding for DNA calculations [7]. Relations like containment $(\subset)$ and compatibility $(\uparrow)$ were defined by Blanchet-Sadri to aid in reconstruction of information of a partial word [6]. Blanchet-Sadri and Luhmann have extended many results from words to partial words [3] and they have established that a partial word with a single hole behave similar to a word [4] while partial words with more number of holes act differently. Tomasz Kociumaka et.al., extend Fine and Wilf's result to partial words and followed with extensions of the periodicity lemma which include a variant with three and an arbitrary number of specified periods [9]. On an extension work of a theorem due to Berstel et.al., defined the concepts of special partial words: (k, l) which contains at least two hole and $\{k, l\}$ which contains two consecutive holes speciality. Primitivity of partial words with finite alphabet was studied in [5] and their perioidcity properties were established by A.M.Shur et al. in [14, 12]. All the existing literature related to speciality and primitivity of partial words deal only with a single hole or only a restricted rule of two holes. For certain higher powers, John Machacek [8] exhibits binary partial words containing three powers all of which start at the same position. Motivated by the work on [3], in this paper we have generalized (k, l) special concept to partial words with three holes. In section 2, we first summarize the properties of partial words and (k, l)-speciality, which aid in understanding the constructions and the results. In section 3, the periodicity is generalized for the considered partial words with the help of $gcd(k, l)$ and it can be determined whether a partial word is (k, l)-special or not.

2 Preliminaries

In this section, we analyse some basic notions including: partial word, periodicity, compatible and (k, l)-special.

A finite set of symbols, denoted by Σ is defined as alphabet. Symbols in Σ are referred to as letters. A word is any finite sequence of letters. Let Σ^* denotes the set of words from Σ

and $\Sigma^+ = \Sigma^* \setminus \{\epsilon\}$. A factor r (or subword) of a word x if there exists the word u and v such that $r = uxv$. x is a proper subword of r if $uv \neq \epsilon$. The factor x is prefix of r if $u = \epsilon$. Likewise, x is a suffix of r if $v = \epsilon$. A partial word t of length l over Σ is a partial function $t : \{0, 1, 2 \cdots, y-1\} \to \Sigma$. If $0 \leq j < y$, then $j \in Domain(t)$ (denoted by $D(t)$), otherwise $j \in Hole(t)$ (denoted by $H(t)$ the set of holes). The companion of t, denoted by $t_\diamondsuit$ is the total function $t_\diamondsuit : \{0, 1, 2 \cdots, y-1\} \to \Sigma_\diamondsuit = \Sigma \cup \{\diamondsuit\}$ defined by

$$r_\diamondsuit(j) = \begin{cases} t(j) & \text{if } j \in \text{D(t)} \\ \diamond & \text{if } j \in \text{H(t).} \end{cases}$$

If t is p-periodic, it can be interpreted by writing the letters of t into p-columns with the same symbol in each column (if any). Let x, y be partial words such that $|x| = |y|$. If $D(x)$ are in $D(y)$ and $x(i) = y(i)$ for all $i \in D(x)$ then x is said to be contained in y, indicated by $x \subset y$. If $y = \beta x \gamma$ for strings β and γ then x is a substring of y . If x and y partial words are said to be compatible, denoted by $x \uparrow y$, if a partial word z exists such that $x \subset z$ and $y \subset z$. It is clear that $x \uparrow y$ implies $y \uparrow x$.

Let Γ be a partial word of length $k + l$ where k, l be positive integers satisfying k less than or equal to l. Γ is (k, l)-special if there exists $0 \leq j < gcd(k, l)$ such that sequence $seq_{(k,l)}(j) = \{j_0, j_1, \cdots, j_n, j_{n+1}\}$ contains atleast two holes while $\Gamma(j_0)\Gamma(j_1)\cdots\Gamma(j_{n+1})$ is not 1-periodic. For $\{k, l\}$-special partial word it's the extension of (k, l)-special whereas the conditions are it contains two consective holes or not 1-periodic.

3 Main Results

All the following cases have $\Sigma^+_\diamondsuit = \{p, q, r\}$. Let the partial word v be fixed as $v_\diamondsuit = pqrpq\diamondsuit$ with length 6, where $v_\diamondsuit$ is 3-periodic. For an even integer $l \geq 6$, define $W_\diamondsuit = \{w_\diamondsuit | w_\diamondsuit = (pqr)^{(l/2)}\}$ where $l = 2t \; \forall t = 1, 2, 3, \cdots$. In the cases below, we consider the partial word $v_\diamondsuit$ and $w_\diamondsuit$, for $w_\diamondsuit$ we place the hole in alternate positions as qr, pq and qp whereas the concatenation and reverse of $v_\diamondsuit w_\diamondsuit$ are denoted as Γ and $\widetilde{\Gamma}$.

3.1 Cases

(i) $Case$ $c_{1,1}$

Given $v_\diamondsuit$ defined above, let $v_\diamondsuit \subset (pqr)^3$ and $w_\diamondsuit \subset (pqr)^{(l/2)}$ where l is any even number $(l \geq 6)$ be two partial words. According to the position of hole, let $v_\diamondsuit = a_\diamondsuit$ and $w_\diamondsuit = a_w\diamondsuit b_w\diamondsuit c_w$ where a_x, b_x and c_x designate the partial word x. The product of $\Gamma_{1,1}$ of $v_\diamondsuit w_\diamondsuit = a_v\diamondsuit a_w\diamondsuit b_w\diamondsuit c_w$.

(ii) $Case$ $c_{1,2}$

For the same $v_\diamondsuit, w_\diamondsuit$ mentioned in $c_{1,1}$, define $\widetilde{\Gamma}_{1,2} = rev(a_v)\diamondsuit rev(a_w)\diamondsuit rev(b_w)\diamondsuit rev(c_w)$.

(iii) $Case$ $c_{2,1}$

Given $v_\diamondsuit$ and for all partial words $w_\diamondsuit \in W_\diamondsuit$. Referring to the position of holes, let $v_\diamondsuit = a_v\diamondsuit$ and $w_\diamondsuit = \diamondsuit a_w\diamondsuit b_w$. The product of $\Gamma_{2,1}$ of $v_\diamondsuit w_\diamondsuit = a_v\diamondsuit\diamondsuit a_w\diamondsuit b_w$.

(iv) $Case$ $c_{2,2}$

For the same $v_\diamondsuit, w_\diamondsuit$ mentioned in $c_{2,1}$, define $\widetilde{\Gamma}_{2,2} = rev(a_v)\diamondsuit\diamondsuit rev(a_w)\diamondsuit rev(b_w)$.

(v) $Case$ $c_{3,1}$

Given $v_\diamondsuit$ and for all partial words $w_\diamondsuit \in W_\diamondsuit$. According to the position of holes, let $v_\diamondsuit = a_v\diamondsuit$ and $w_\diamondsuit = a_w\diamondsuit b_w\diamondsuit c_w$. The product of $\Gamma_{3,1}$ of $v_\diamondsuit w_\diamondsuit = a_v\diamondsuit a_w\diamondsuit b_w\diamondsuit c_w$.

(vi) $Case$ $c_{3,2}$

For the same $v_\diamondsuit$,$w_\diamondsuit$ mentioned in $c_{3,1}$, define $\widetilde{\Gamma}_{3,2} = rev(a_v)\diamondsuit rev(a_w)\diamondsuit rev(b_w)\diamondsuit rev(c_w)$.

Proposition 3.1. Let $v_\diamondsuit = pqrpq\diamondsuit$ and $W_\diamondsuit = \{w_\diamondsuit | w_\diamondsuit = (pqr)^{(l/2)}\}$. Then the partial words $v_\diamondsuit$ and $w_\diamondsuit$ are 3-periodic.

Proof. Given $v_\diamondsuit \subset (pqr)^3$ and $w_\diamondsuit \subset (pqr)^{(l/2)}$ and by the definition of periodicity the partial words are 3-periodic. □

Proposition 3.2. Let $\Gamma_{i,j}$ and $\widetilde{\Gamma}_{i,j}$; $1 \leq i \leq 3$, $1 \leq j \leq 2$ are all of length $6 + l$ from the above constructed cases of partial words .

Proof. The evidence is obvious from the cases as all the factors are exactly from $v_\Diamond, w_\Diamond$ and $h(\Gamma_{i,j}) = h(\widetilde{\Gamma}_{i,j}) = h(v) + h(w) = h(v) + h(w) \forall\, i, j$. □

Lemma 3.1. *Let $(v_\Diamond, w_\Diamond)$ be non-empty partial words and if $gcd(v_\Diamond, w_\Diamond) = 1$ from all the cases defined in section 3, then Γ and $\widetilde{\Gamma}$ are (k,l)-special.*

Proof. We know that $|v_\Diamond| = 6$ and $|w_\Diamond|$=l where l varies i.e., $w \subset (pqr)^{l/2}$. If $gcd(v_\Diamond, w_\Diamond) = 1$ then we have $l = 6(s+1)$ (for some positive integer s). Hence the sequence is ordered as:
$seq_{(k,l)}(0) : 0, 6, 12, \cdots, 6(s+1), 5, 11, \cdots, 6(s+1)-1, 4, 10, \cdots, 6(s+1)-2, 3, \cdots, 6(s+1)-3, 2, \cdots, 6(s+1)-4, 1, \cdots, 6(s+1)-5, 0.$
Hence in this single sequence all the elements of $\Gamma, \widetilde{\Gamma}$ are placed. All the construction of Γ has atleast two holes and satisfies the basic definition of (k,l)-special. Thus, $\Gamma, \widetilde{\Gamma}$ are special. □

Lemma 3.2. *Let $gcd(v_\Diamond, w_\Diamond) = 2$ then from lemma 1 it shows that Γ and $\widetilde{\Gamma}$ are (k,l)-special.*

Proof. The proof is consistent to the previously mentioned lemma, $|k| = 6$ and $|l| = 6(s+1)$ (for some positive integer s). Two sequences are constructed as:
$seq_{(k,l)}(0) : 0, 6, 12, \cdots, 6(s+1), 4, 10, \cdots, 6(s+1)-2, 2, 8, \cdots, 6(s+1)-4, 0.$
$seq_{(k,l)}(1) : 1, 7, 13, \cdots, 6(s+1)+1, 5, 11, \cdots, 6(s+1)-1, 3, \cdots, 6(s+1)-3, 1.$
Hence the sequences satisfies the basic definition of (k,l)-special. Hence $\Gamma, \widetilde{\Gamma}$ are special. □

Lemma 3.3. *Let $v_\Diamond, w_\Diamond$ be non-empty partial words and if $gcd(v_\Diamond, w_\Diamond) = 3$ then $\Gamma, \widetilde{\Gamma}$ are not (k,l)-special.*

Proof. The cardinality of k is 6 and l is $6(s+1)$ (for some positive integer s). The proof is obvious as the above lemma. Here the sequences are constructed as:
$seq_{(k,l)}(0) : 0, 6, 12, \cdots, 6(s+1), 3, 9, \cdots, 6(s+1)-3, 0.$
$seq_{(k,l)}(1) : 1, 7, 13, \cdots, 6(s+1)+1, 4, 10, \cdots, 6(s+1)-2, 1.$
$seq_{(k,l)}(2) : 2, 8, 14, \cdots, 6(s+1)+2, 5, 11, \cdots, 6(s+1)-1, 2.$
These are the sequence constructed from the basic definition of (k,l)-special. Hence $\Gamma, \widetilde{\Gamma}$ are not (k,l)-special. Since it does not satisfy the condition as per definition. □

Lemma 3.4. *If $gcd(v_\Diamond, w_\Diamond) = 6$ then $\Gamma, \widetilde{\Gamma}$ are not (k,l)-special.*

Proof. From the above lemma, the proof is evident. The constructed sequences are:
$seq_{(k,l)}(0) : 0, 6, 12, \cdots, 6(s+1), 0.$
$seq_{(k,l)}(1) : 1, 7, 13, \cdots, 6(s+1)+1, 1.$
$seq_{(k,l)}(2) : 2, 8, 14, \cdots, 6(s+1)+2, 2.$
$seq_{(k,l)}(3) : 3, 9, 15, \cdots, 6(s+1)+3, 3.$
$seq_{(k,l)}(4) : 4, 10, 16, \cdots, 6(s+1)+4, 4.$
$seq_{(k,l)}(5) : 5, 11, 17, \cdots, 6(s+1)+5, 5.$
It clearly shows that Γ and $\widetilde{\Gamma}$ are not special. Since it does not satisfy the condition as per definition of (k,l)-special. □

For example: Let $v_\Diamond = pqrpq\Diamond$ and $w_\Diamond = \Diamond qrp \Diamond rpqr$ be a partial words of length $k = 6$ and $l = 9$ respectively. Then the concatenation of $v_\Diamond$, $w_\Diamond$ is $\Gamma = v_\Diamond w_\Diamond = pqrpq\Diamond\Diamond qrp\Diamond rpqr$. We generate the requisite sequences:

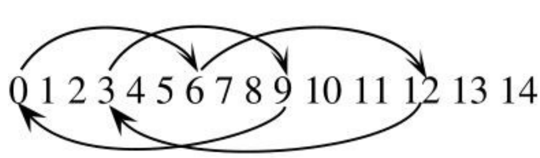

Figure 1

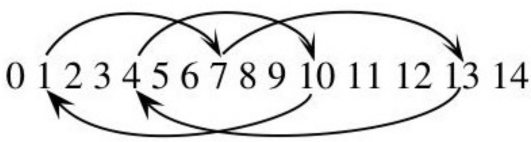

Figure 2

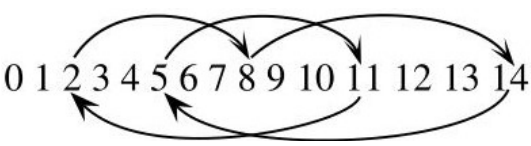

Figure 3

From the figures$(1,2,3)$ we obtain, $seq_{(6,9)}(0) = (0,6,12,3,9,0)$, $seq_{(6,9)}(1) = (1,7,13,4,10,1)$ and $seq_{(6,9)}(2) = (2,8,14,5,11,2)$. $seq_{(6,9)}(0) = p\Diamond pppp$, this sequence contain a hole and it is 1-periodic. $seq_{(6,9)}(1) = qqqq\Diamond q$, this sequence contain one hole and it is 1-periodic. $seq_{(6,9)}(2) = rrr\Diamond rr$, it does not satisfy the conditions of $\{k,l\}$-special. Hence Γ is (6,9)-not special partial word.

Theorem 3.5. *Let $v_\Diamond$,$w_\Diamond$ be a non-compatible partial words with three holes such that $l \geq 6$. Then from the $c_{1,1}$ and $c_{1,2}$ as defined in section 3, the periodicity for each $gcd(6,l)$ is generalized.*

Proof. Let $\Gamma = v_\Diamond w_\Diamond$ be a non-compatible partial word where $v_\Diamond \subset (pqr)^3$ and $w_\Diamond \subset (pqr)^{(l/2)}$ is of length l. We can check whether Γ is (k,l)-special or not if there exists $0 \leq i < gcd(k,l)$ such that the sequences constructed contains atleast two positions that are holes while $\Gamma(i_0)\Gamma(i_1)\cdots\Gamma(i_{(n+1)})$ is not 1-periodic. Once the sequence satisfies the conditions then Γ can be declared (k,l)-special. However, it is necessary to calculate all sequences in order to classify Γ as not (k,l)-special. The periodicity for each $gcd(6,l)$. From the case $c_{1,1}$ as follows:

gcd(1)

$\Gamma = v_\Diamond w_\Diamond$=(6,l)= $\begin{cases} l-(3m+2), l+1, l+4, \Gamma & \text{when l=3n+2 n is odd} \\ l-(3m+1), l+2, l+5, \Gamma & \text{when l=3n+1 n is even} \end{cases}$
where $m \in N$

$\widetilde{\Gamma} = v_\Diamond w_\Diamond = (6,l) = (l-3, l, l+3, \Gamma)$

gcd(2)

$\Gamma = v_\Diamond w_\Diamond$=(6,l)= $\begin{cases} l-(3m+2), l+1, l+4, \Gamma & \text{when l=3n+1 n is odd} \\ l-(3m+1), l+2, l+5, \Gamma & \text{when l=3n+2 n is even} \end{cases}$
where $m \in N$

$\widetilde{\Gamma} = v_\Diamond w_\Diamond = (6,l) = (l-3, l, l+3, \Gamma)$
Here gcd(1) and gcd(2) are (k,l)-special. The periodicity for gcd(3),gcd(6) are same and not (k,l)-special.

gcd(3)

$\Gamma = v_\Diamond w_\Diamond = (6,l) = (l-3m, l+3, \Gamma)$

$\widetilde{\Gamma} = v_\Diamond w_\Diamond = (6, l) = (l - 3m, l + 3, \Gamma)$

where $m \in N$

gcd(6)

$\Gamma = v_\Diamond w_\Diamond = (6, l) = (l - 3m, l + 3, \Gamma)$
$\widetilde{\Gamma} = v_\Diamond w_\Diamond = (6, l) = (l - 3m, l + 3, \Gamma)$

where $m \in N$

□

Theorem 3.6. *Let $v_\Diamond$,$w_\Diamond$ be a non-empty partial words such that $l \geq 6$. The periodicity is generalized for each $gcd(6, l)$ from the $c_{2,1}$ and $c_{2,2}$. If $v_\Diamond \uparrow w_\Diamond$ then there exists a word Γ such that $v_\Diamond \subset (pqr)^3$ and $w_\Diamond \subset (pqr)^{(l/2)}$ is of length l.*

Proof. Let $\Gamma = v_\Diamond w_\Diamond$ be a partial word with three holes. The periodicity for each $gcd(6, l)$. From the constructions $c_{2,1}$ and $c_{2,2}$ the periodicity of Γ and $\widetilde{\Gamma}$ are

gcd(1)

$\Gamma = v_\Diamond w_\Diamond$=(6,l)= $\begin{cases} l - (3m + 1), l + 2, l + 5, \Gamma & \text{when l=3n+1 n is even} \\ l - (3m + 2), l + 1, l + 4, \Gamma & \text{when l=3n+2 n is odd} \end{cases}$
where $m \in N$

$\widetilde{\Gamma} = v_\Diamond w_\Diamond = (6, l) = (l - 3, l, l + 3, \Gamma)$

gcd(2)

$\Gamma = v_\Diamond w_\Diamond$=(6,l)= $\begin{cases} l - (3m + 1), l + 2, l + 5, \Gamma & \text{when l=3n+1 n is even} \\ l - (3m + 2), l + 1, l + 4, \Gamma & \text{when l=3n+2 n is odd} \end{cases}$
where $m \in N$

$\widetilde{\Gamma} = v_\Diamond w_\Diamond = (6, l) = (l - 3, l, l + 3, \Gamma)$
Here gcd(1) and gcd(2) are (k, l)-special. The periodicity for gcd(3),gcd(6) are same and not (k, l)-special.

gcd(3)

$\Gamma = v_\Diamond w_\Diamond = (6, l) = (l - 3m, l + 3, \Gamma)$
$\widetilde{\Gamma} = v_\Diamond w_\Diamond = (6, l) = (l - 3m, l + 3, \Gamma)$

where $m \in N$

gcd(6)

$\Gamma = v_\Diamond w_\Diamond = (6, l) = (l - 3m, l + 3, \Gamma)$
$\widetilde{\Gamma} = v_\Diamond w_\Diamond = (6, l) = (l - 3m, l + 3, \Gamma)$

where $m \in N$

□

Theorem 3.7. *Let $v_\Diamond$,$w_\Diamond$ be a non-empty partial words such that $l \geq 6$. The periodicity is generalized for each $gcd(6, l)$ from the $c_{3,1}$ and $c_{3,2}$. If $v_\Diamond \uparrow w_\Diamond$ then there exists a word Γ such that $v_\Diamond \subset (pqr)^3$ and $w_\Diamond \subset (pqr)^{(l/2)}$ is of length l.*

Proof. Assume $v_\Diamond \subset (pqr)^3$ and $w_\Diamond \subset (pqr)^{(l/2)}$ which is of length l. To check whether a partial word is $\{k, l\}$-special the sequence is constructed and if it satisfies the conditions then Γ can be declared (k, l)-special. However, it is necessary to calculate all sequences in order to classify Γ as not (k, l)-special. We can note that the reverse of every partial word which are constructed from the cases $c_{3,1}$ and $c_{3,2}$ has the same periodicity.

gcd(1)

$$\Gamma = v_{\Diamond}w_{\Diamond}=(6,l)= \begin{cases} l-(3m+2), l+1, l+4, \Gamma & \text{when l=3n+2 n is odd} \\ l-(3m+1), l+2, l+5, \Gamma & \text{when l=3n+1 n is even} \end{cases}$$

where $m \in N$

$$\widetilde{\Gamma} = v_{\Diamond}w_{\Diamond} = (6, l) = (l+2, l+5, \Gamma)$$

gcd(2)

$$\Gamma = v_{\Diamond}w_{\Diamond}=(6,l)= \begin{cases} l-(3m+2), l+1, l+4, \Gamma & \text{when l=3n+2 n is even} \\ l-(3m+1), l+2, l+5, \Gamma & \text{when l=3n+1 n is odd} \end{cases}$$

where $m \in N$

$$\widetilde{\Gamma} = v_{\Diamond}w_{\Diamond} = (6, l) = (l+2, l+5, \Gamma)$$

gcd(3)

$$\Gamma = v_{\Diamond}w_{\Diamond} = (6, l) = (l-3m, l+3, \Gamma)$$
$$\widetilde{\Gamma} = v_{\Diamond}w_{\Diamond} = (6, l) = (l+2, l+5, \Gamma)$$

where $m \in N$

gcd(6)

$$\Gamma = v_{\Diamond}w_{\Diamond} = (6, l) = (l-3m, l+3, \Gamma)$$
$$\widetilde{\Gamma} = v_{\Diamond}w_{\Diamond} = (6, l) = (l+2, l+5, \Gamma)$$

where $m \in N$

□

4 Conclusion

In this paper, we generalize the periodicity for non-empty partial words of $gcd(6, l)$. Further this work can be done by increasing the number of 'do not know' symbols and a bipartite graph can be constructed from this generalization.

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