Palestine Journal of Mathematics

Vol. 12(1)(2023) , 14–29

On a quotient group $7^4{:}(3 \times 2S_7)$ of a 7-local subgroup of the Monster $\mathbb{M}$

David Mwanzia Musyoka, Lydia Nyambura Njuguna, Abraham Love Prins and Lucy Chikamai

Communicated by Ayman Badawi

MSC 2010 Classifications: Primary: 20C15; Secondary: 20C40.

Keywords and phrases: split extension, extra-special p-group, inertia factor group, fusion map, Fischer-Clifford matrices.

The first author thanks God for His blessings in guiding the research towards the writing of this paper and is also grateful to his supervisors for their invaluable and timely input. May God bless you all abundantly. This research is partially supported by the Faculty of Science at Nelson Mandela University [cost centre:GB85 and account number:4920].

Abstract The largest Sporadic simple group, the Monster $\mathbb{M}$, has a maximal-7-local subgroup $7_+^{1+4}{:}(3\times 2S_7)$ of order $508243680 = 2^5.3^3.5.7^6$. In this paper, the Fischer-Clifford matrices and associated ordinary character table of the quotient group $\overline{G} = \frac{7_+^{1+4}:(3\times 2S_7)}{7_+^{1+4}} \cong 7^4{:}(3\times 2S_7)$ will be computed. We have few, if any, examples in the literature, where the Fischer-Clifford matrices technique is applied to an extension with the kernel being an elementary abelian 7-group. There are quite a number of examples with the kernel of the extension group an elementary abelian 2, 3 or 5-group.

1 Introduction

A split extension of the form $S = 7_+^{1+4}{:}(3\times 2S_7)$ sits maximally inside the Sporadic simple Monster group $\mathbb{M}$ and has order $508243680 = 2^5.3^3.5.7^6$ [4]. The group S is the normalizer $N_{\mathbb{M}}(7B)$ in $\mathbb{M}$ of a group of order 7 with generators inside the conjugacy class $7B$ of $\mathbb{M}$. The normal subgroup $N_1 = 7_+^{1+4}$ of S is an extra-special 7-group with exponent 7. The center $Z(N_1) = 7$ is a cyclic group of order 7 and is a characteristic subgroup of N_1 and therefore normal in G. Therefore, S and $\frac{S}{N_1}$ have the structures $7^{\cdot}(7^4{:}(3 \times 2S_7))$ and $7^4{:}(3 \times 2S_7)$, respectively. The quotient $\frac{S}{N_1} \cong 7^4{:}(3 \times 2S_7)$ is isomorphic to a split extension $\overline{G} = 7^4{:}(3 \times 2S_7)$ of an elementary abelian 7-group $N = 7^4$ by a group $G = 3 \times 2S_7$. In this paper, we will construct the ordinary character table of $\overline{G}$ by the Fischer-Clifford matrices technique [5]. It is worthwhile to mention that the group $\overline{G}$ will be one of the few examples, if any, in the literature where the Fischer-Clifford matrices technique is applied to an extension group with the kernel an elementary abelian 7-group. There are numerous other examples in the literature where the said technique is applied to with the kernels of the extensions either an elementary abelian 2, 3 or 5-group (see for example the papers [2], [11], [19] and most recently [1], [13], [14], [15],[16], [17] and [18]).

In the sections that follow, we will discuss the construction of the groups $\overline{G}$ and G, the action of G on N and Irr(N), the Fischer-Clifford matrices of $\overline{G}$ and the construction of the ordinary irreducible character table of $\overline{G}$. Most of the computations in this paper are carried out using computer algebra systems MAGMA [3] and GAP [6]. Notation from the ATLAS [4] is mostly followed.

2 On the construction of the groups $\overline{G}$ and G

Using a six-dimensional matrix representation of S over the field $GF(7)$ found in the online ATLAS [22], the group S is generated in GAP [6]. Next, we construct a copy of the quotient group $\overline{G} = \frac{S}{N_1} \cong 7^4{:}(3 \times 2S_7)$ as a permutation group on 2401 points. We then use this permutation representation in MAGMA to construct a four-dimensional matrix representation of $G = 3 \times S_7$

over the field $GF(7)$. The MAGMA commands "$m := GModule(\overline{G}, N)$" and "$m{:}Maximal$" are used to construct the matrix group $G =< g_1, g_2 >$, where N is an absolutely irreducible module for G. The generators g_1 and g_2 for G (see Figure 1), have orders $o(g_1) = 4$, $o(g_2) = 6$ and $o(g_1 g_2) = 21$.

$$g_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 6 & 0 \end{pmatrix}, \quad g_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 6 & 1 & 0 \\ 2 & 2 & 0 & 0 \\ 4 & 0 & 0 & 4 \end{pmatrix}.$$

Figure 1. Generators of G

Since $G =< g_1, g_2 >$ acts absolutely irreducibly on $N = V_4(7)$, where N is regarded as a vector space of dimension 4 over $GF(7)$, an isomorphic copy $\overline{S}$ of the group S can be constructed as a subgroup of the general linear group $GL_5(7)$. The generators s_1, s_2 and s_3 for $\overline{S}$ (see Figure 2) have orders of 4, 6 and 7, respectively. It can easily be verified in GAP or MAGMA that $S \cong \overline{S}$.

$$s_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad s_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 6 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 4 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad s_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Figure 2. Generators of $\overline{S}$

3 Action of G on N and Irr(N)

The matrix group $G =< g_1, g_2 >$ has three orbits of lengths 1, 720 and 1680 on N with corresponding point stabilizers $P_1 = 3 \times 2S_7$, $P_2 = 7{:}6$ and $P_3 = 3 \times S_3$, respectively. By Brauer's theorem (see Lemma 5.2 in [7]), the action of G on the set Irr(N) of linear characters of N also has three orbits of lengths 1, 720 and 1680, where the structures of the corresponding stabilizers, known as inertia factor groups H_i, are identified with the help of GAP as $H_1 = G$, $H_2 = 7{:}6$ and $H_3 = 3 \times S_3$.

Using similar techniques as in [10], the permutation character $\chi(G|7^4)$ of G on the conjugacy classes of $N = 7^4$ is computed as

$$\chi(G|7^4) = \sum_{i=1}^{3} I_{P_i}^G = 1aaaef + 6ab + 8abbcc + 14abbccdgghhiijjkl + 15aaabccdd+$$
$$20aaabbccfffggghhiijkllmmnnoo + 21aacccdddef + 28aabc+$$
$$35abbbccddeeefff + 36aaabbbcccdddeeefff.$$

Note that $\chi(G|7^4)$ is the sum of the identity characters $I_{P_i}^G$ of the point stabilizers P_i of the orbits of G on N which are induced to G and it is also written in terms of the ordinary irreducible characters of G. For an element g in a conjugacy class $[g]$ of G, it is required that $\chi(G|7^4)(g) = 7^n$, for some $n \in \{0, 1, 2, 3, 4\}$. The value $k = \chi(G|7^4)(g)$ gives the number of elements of N which is fixed by an element $g \in G$ (by conjugation) and it is also the number k of orbits of N on a coset Ng (see column 2 of Table 2).

The inertia factors $H_2 =< \alpha_1, \alpha_2 >$ and $H_3 =< \alpha_3, \alpha_4 >$ are generated from elements $\alpha_1 \in 6N$, $\alpha_2 \in 6L$, $\alpha_3 \in 6P$, $\alpha_4 \in 6P$ (see Figures 3 and 4) in the conjugacy classes $6N$, $6L$

and $6P$ of G.

$$\alpha_1 = \begin{pmatrix} 6 & 3 & 0 & 4 \\ 0 & 3 & 6 & 6 \\ 0 & 5 & 4 & 2 \\ 0 & 2 & 4 & 6 \end{pmatrix} \quad \alpha_2 = \begin{pmatrix} 6 & 3 & 4 & 3 \\ 0 & 5 & 1 & 1 \\ 0 & 2 & 4 & 0 \\ 0 & 5 & 4 & 1 \end{pmatrix}.$$

Figure 3. Generators of H_2

$$\alpha_3 = \begin{pmatrix} 6 & 1 & 4 & 2 \\ 6 & 4 & 4 & 6 \\ 5 & 5 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \alpha_4 = \begin{pmatrix} 0 & 2 & 2 & 2 \\ 3 & 1 & 0 & 1 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Figure 4. Generators of H_3

The fusion maps of the inertia factor groups H_2 and H_3 into G are found in Table 1.

Table 1. The fusion maps of H_2 and H_3 into $G = 3 \times S_7$

$[h]_{H_2}$	$\longrightarrow [g]_{3\times S_7}$	$[h]_{H_2}$	$\longrightarrow [g]_{3\times S_7}$	$[h]_{H_2}$	$\longrightarrow [g]_{3\times S_7}$
$1A$	$1A$	$3B$	$3F$	$6B$	$6L$
$2A$	$2B$	$6A$	$6N$	$7A$	$7A$
$3A$	$3F$				
$[h]_{H_3}$	$\longrightarrow [g]_{3\times S_7}$	$[h]_{H_3}$	$\longrightarrow [g]_{3\times S_7}$	$[h]_{H_3}$	$\longrightarrow [g]_{3\times S_7}$
$1A$	$1A$	$3B$	$3D$	$3E$	$3H$
$2A$	$2B$	$3C$	$3E$	$6A$	$6O$
$3A$	$3G$	$3D$	$3F$	$6B$	$6K$

4 The Conjugacy Classes of $\overline{G} = 7^4{:}(3 \times 2S_7)$

In this section, the conjugacy classes $[x_j]$, for $j \in \{1, 2, ..., c(g)\}$, of $\overline{G}$, which have their images as a conjugacy class $[g]$ of G under the natural homomorphism $f{:}\overline{G} \longrightarrow G$, will be determined. Let $X(g) = \{x_1, x_2, \cdots, x_{c(g)}\}$ be the set of representatives of these conjugacy classes $[x_j]$ of $\overline{G}$ from a conjugacy class $[g]$ of G. A GAP routine (labelled as Programme A in [20]), which is based on the method of coset-analysis (see [9],[10] or [12]), is used to compute the conjugacy classes of $\overline{G}$. This GAP routine is written for a split extension $p^n{:}Q$ of an elementary abelian p-group p^n by a linear matrix group Q of dimension n over the field $GF(p)$. The group p^n (regarded as a vector space $V_n(p)$ of dimension n over the finite field $GF(p)$ for p a prime) is a Q-module where upon the matrix group Q acts naturally.

For the group $\overline{G} = 7^4{:}(3 \times 2S_7)$, we take $G = 3 \times 2S_7$ as a right transversal for $N = 7^4$ in $\overline{G}$. A coset Ng is considered for each conjugacy class $[g]$ with representative g in G. Consider the action (by conjugation) of the stabilizer $C_g = 7^4{:}C_G(g) = \{x \in \overline{G} | x(Ng)x^{-1} = Ng\}$ of the coset Ng in $\overline{G}$ on the elements of Ng. Since C_g is a split extension we will first act N on Ng to form k orbits $Q_1, Q_2, ..., Q_k$, with each orbit Q_i containing $|N|/k$ elements. Under the action of the centralizer $C_G(g)$ of g in G, f_j of the k orbits Q_i fuse together to form an orbit O_j. The orbit O_j contains the elements from the coset Ng which belong to a conjugacy class $[x_j]$ of $\overline{G}$ with class representative x_j. Note that $\sum f_j = k$. The order of the centralizer $|C_{\overline{G}}(x_j)|$ of the class representative x_j is then computed by $|C_{\overline{G}}(x_j)| = \frac{k|C_G(g)|}{f_j}$. In this manner, the conjugacy classes of $\overline{G}$, with class representatives $X(g) = \{x_1, x_2, ..., x_{c(g)}\}$ coming from the coset Ng, are obtained. Let the order of $g \in G$ be given by m. Since N is an elementary abelian 7-group, the order of elements in the classes $[x_j]$ coming from a coset Ng will be either m or $7m$. For the purpose of computing the orders of classes of $\overline{G}$, we use a GAP routine, Programme B, in [20]. Both Programme A and Programme B are used to compute the $p-$power maps of the classes of $\overline{G}$. The parameter $m_j = \frac{f_j|N|}{k}$ is also computed and is useful in determining the entries of a Fischer-Clifford matrix $M(g)$. Table 2 contains all the information pertaining to the conjugacy classes of $\overline{G}$.

Table 2. The Conjugacy Classes of $\overline{G} = 7^4{:}(3 \times 2S_7)$

$[g]_G$	k	f_j	m_j	$[x]_{\overline{G}}$	$\lvert C_{\overline{G}}(x)\rvert$	2	3	5	7	$[g]_G$	k	f_j	m_j	$[x]_{\overline{G}}$	$\lvert C_{\overline{G}}(x)\rvert$	2	3	5	7
1A	2401	1	1	1A	72606240					2A	1	1	2401	2A	30240	1A			
		720	720	7A	100842				1A										
		1680	1680	7B	43218				1A										
2B	49	1	49	2B	7056	1A				3A	1	1	2401	3A	30240		1A		
		24	1176	14A	294	7B			2B										
		24	1176	14B	294	7A			2B										
3B	1	1	2401	3B	30240		1A			3C	1	1	2401	3C	432		1A		
3D	49	1	49	3D	21168		1A			3E	49	1	49	3E	21168		1A		
		48	2352	21A	441		7B		3C			48	2352	21B	441		7B		3E
3F	49	1	49	3F	5292		1A			3G	7	1	343	3G	756		1A		
		12	588	21C	441		7B		3F			6	2058	21F	126		7B		3G
		18	882	21D	294		7A		3F										
		18	882	21E	294		7A		3F										
3H	7	1	343	3H	756		1A			4A	1	1	2401	4A	720	2A			
		6	2058	21G	126		7B		3G										
4B	1	1	2401	4B	144	2A				4A	1	1	2401	4A	720	2A			
6A	1	1	2401	6A	30240	3B	2A			6B	1	1	2401	6B	30240	3A	2A		
6C	1	1	2401	6C	144	3A	2B			6D	1	1	2401	6D	144	3B	2B		
6E	1	1	2401	6E	432	3E	2A			6F	1	1	2401	6F	432	3D	2A		
6G	1	1	2401	6G	432	3C	2A			6H	1	1	2401	6H	108	3F	2A		
6I	1	1	2401	6I	108	3H	2A			6J	1	1	2401	6J	108	3G	2A		
6K	7	1	343	6K	252	3F	2B			6L	7	1	343	6L	252	3F	2B		
		6	2058	42A	42	21D	14A		6K			6	2058	42B	42	21D	14A		6K
6M	1	1	2401	6M	36	3F	2B			6N	7	1	343	6N	252	3H	2B		
												6	2058	42C	42	21G	14A		6N
6O	7	1	343	6O	252	3F	2B			6P	1	1	2401	6P	36	3F	2B		
		6	2058	42D	42	21D	14A		6K			6							
7A	7	1	343	7C	294				1A	8A	1	1	2401	8A	72	4B			
		6	2058	7D	49				1A	8B	1	1	2401	8B	24	4B			
10A	1	1	2401	10A	60	5A		2A		12A	1	1	2401	12A	720	6B	4A		
12B	1	1	2401	12B	720	6A	4A			12C	1	1	2401	12C	144	6B	4B		
12D	1	1	2401	12D	144	6A	4B			12E	1	1	2401	12E	36	6G	4A		
12F	1	1	2401	12F	36	6F	4A			12G	1	1	2401	12G	36	6E	4A		
12H	1	1	2401	12H	72	6F	4B			12I	1	1	2401	12I	72	6E	4B		
12J	1	1	2401	12J	72	6G	4B			14A	1	1	2401	14C	42	7C			2A
15A	1	1	2401	15A	60		5A	3B		15B	1	1	2401	15B	60		5A	3B	
20A	1	1	2401	20A	60	10A		4A		20B	1	1	2401	20B	60	10A		4A	
21A	1	1	2401	21H	42		7C		3B	21B	1	1	2401	21I	42		7C		3A
24A	1	1	2401	24A	72	12C	8A			24B	1	1	2401	24B	72	12D	8A		
24C	1	1	2401	24C	24	12D	8B			24D	1	1	2401	24D	24	12C	8B		
24E	1	1	2401	24E	72	12I	8A			24F	1	1	2401	24F	72	12I	8A		
24G	1	1	2401	24G	72	12H	8A			24H	1	1	2401	24H	72	12H	8A		
24I	1	1	2401	24I	72	12J	8A			24J	1	1	2401	24J	72	12J	8A		
30A	1	1	2401	30A	60	15B	10A	6A		30B	1	1	2401	30B	60	15A	10A	6B	
42A	1	1	2401	42E	42	21I	14A		6O	42B	1	1	2401	42F	42	21I	14C		6A
60A	1	1	2401	60A	60	30A	20A	12B		60B	1	1	2401	60B	60	30A	20B	12B	
60C	1	1	2401	60C	60	30B	20A	12A		60D	1	1	2401	60D	60	30B	20B	12A	

5 Fischer-Clifford Matrices of $\overline{G} = 7^4{:}(3 \times 2S_7)$

In this section, the Fischer-Clifford matrices of $\overline{G} = 7^4{:}(3 \times 2S_7)$ will be computed. For a more detailed treatment on Fischer-Clifford matrices the reader is referred to [10], [12] or [21].

As $\overline{G} = 7^4{:}(3\times 2S_7)$ acts on Irr(7^4), the linear characters of $N = 7^4$ are partitioned into three orbits O_1, O_2 and O_3. The sizes of the orbits are $|O_1| = 1$, $|O_2| = 720$ and $|O_3| = 1680$, with corresponding inertia groups $\overline{H}_1 = 7^4{:}(3 \times 2S_7)$, $\overline{H}_2 = 7^4{:}(7{:}6)$ and $\overline{H}_3 = 7^4{:}(3 \times S_3)$ in $\overline{G}$. The inertia subgroups $7^4{:}H_i$, $i = 1, 2, 3$, of $\overline{G}$ are defined as $\overline{H}_i = N{:}H_i = \{x \in \overline{G} | \theta_i^x = \theta_i\}$, $i = 1, 2, 3$, where $\theta_i \in O_i$ are representatives of the orbits O_i of $\overline{G}$ on Irr(7^4). Since 7^4 is elementary abelian, by Mackey's Theorem (see Theorem 5.1.15 in [12]) each θ_i extends to a $\psi_i \in$ Irr$(\overline{H}_i)$, i.e. $\psi_i \downarrow_N = \theta_i$. By Theorem 5.1.7, Remark 5.1.8 and Theorem 5.1.19 in [12], an ordinary irreducible character $\chi = (\psi_i\overline{\beta})^{\overline{G}}$ of $\overline{G}$ is obtained by induction of $\psi_i\overline{\beta} \in$ Irr$(\overline{H}_i)$ to $\overline{G}$, where N is contained in the kernel ker$(\overline{\beta})$ of $\overline{\beta} \in$ Irr$(\overline{H}_i)$. Note that $\overline{\beta} \in$ Irr$(\overline{H}_i)$ is a lifting of $\beta \in$ Irr(H_i) into $\overline{H}_i$. Therefore,

$$\text{Irr}(\overline{G}) = \bigcup_{i=1}^{3} \{(\psi_i\overline{\beta})^{\overline{G}} | \overline{\beta} \in \text{Irr}(\overline{H}_i), N \subseteq \ker(\overline{\beta})\} = \bigcup_{i=1}^{3} \{(\psi_i\overline{\beta})^{\overline{G}} | \beta \in \text{Irr}(H_i)\}.$$

Hence the set Irr$(\overline{G})$ is partitioned into 3 blocks B_i with each block B_i corresponding to an inertia subgroup $\overline{H}_i$ of $\overline{G}$. Observe that $|\text{Irr}(\overline{G})| = |\text{Irr}(H_1)| + |\text{Irr}(H_2)| + |\text{Irr}(H_3)| = 69 + 7 + 9 = 85$.

We take $\overline{H_1} = \overline{G}$ and $H_1 = G$. We define the set

$$R(g) = \{(i, y_k) \,|\, 1 \leq i \leq 3, H_i \cap [g] \neq \emptyset, 1 \leq k \leq r\},$$

where y_k, $k = 1, 2, ..., r$, are representatives of conjugacy classes $[y_k]$ of H_i that fuse into a class $[g]$ of $H_1 = G$. Let y_{l_k} be representatives of the conjugacy classes of $\overline{H}_i$, where each y_{l_k} has y_k as an image under the homomorphism $\overline{H}_i \longrightarrow H_i$ whose kernel is 7^4. Then for $x_j \in X(g)$ as defined in Section 4, we have

Lemma 5.1.

$$(\psi_i\overline{\beta})^{\overline{G}}(x_j) = \sum_{y_k:(i,y_k)\in R(g)} \left[\sum_l{}' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{l_k})|}\psi_i(y_{l_k})\right] \beta(y_k)$$

Proof. See [21] □

The Fischer-Clifford matrix $M(g) = \left(a^j_{(i,y_k)}\right)$ is then defined as

$$\left(a^j_{(i,y_k)}\right) = \left(\sum_l{}' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{l_k})|}\psi_i(y_{l_k})\right),$$

with columns indexed by $X(g)$ and rows indexed by $R(g)$ and where $\sum_l'$ is the summation over all l for which y_{l_k} is conjugate to x_j in $\overline{G}$. Since $\overline{G} = \overline{H_1}$ and 7^4 is elementary abelian, it follows that $a^j_{(1,g)} = 1$ for all $j = \{1, 2, .., c(g)\}$ and $a^1_{(i,y_k)} = \frac{|C_G(g)|}{|C_{H_i}(y_k)|}$. Hence a Fischer-Clifford matrix $M(g)$ of $\overline{G} = 7^4{:}(3 \times 2S_7)$ has the form as depicted in Figure 5.

The Fischer-Clifford matrix $M(g)$ (see Figure 5) is partitioned row-wise into blocks $M_i(g)$, where each block corresponds to an inertia group $\overline{H}_i$. We write $|C_{\overline{G}}(x_j)|$, for each $x_j \in X(g)$, at the top of the columns of $M(g)$ and at the bottom we write $m_j \in \mathbb{N}$, where we define $m_j = |N|\frac{|C_G(g)|}{|C_{\overline{G}}(x_j)|}$. On the left of each row we write $|C_{H_i}(y_k)|$, where the conjugacy classes $[y_k]$, $k = 1, 2, ..., r$, of an inertia factor H_i fuse into the conjugacy class $[g]$ of G. Note that $|X(g)| = |R(g)|$ and therefore $M(g)$ is a square matrix of size $c(g)$. In practice it is difficult to compute the elements y_{l_k} or the ordinary irreducible character tables of the inertia groups $\overline{H}_i$, since the sets Irr($\overline{H}_i$) of ordinary irreducible characters of the $\overline{H}_i$'s are in general much larger and more complicated to compute than the one for $\overline{G}$. Instead of using the above formal definition of a Fischer-Clifford matrix $M(g)$, the arithmetical properties of a Fischer-Clifford matrix $M(g)$ [10] are used to complete the entries of a matrix $M(g)$ of $\overline{G} = 7^4{:}(3 \times 2S_7)$.

$$
\begin{array}{c} \\ |C_G(g)| \\ |C_{H_2}(y_1)| \\ |C_{H_2}(y_2)| \\ \vdots \\ |C_{H_3}(y_1)| \\ |C_{H_3}(y_2)| \\ \vdots \\ \\ \end{array}
\text{M(g)=}
\begin{array}{cccc}
|C_{\overline{G}}(x_1)| & |C_{\overline{G}}(x_2)| & \cdots & |C_{\overline{G}}(x_{c(g)})| \\
1 & 1 & \cdots & 1 \\ \hline
\frac{|C_G(g)|}{|C_{H_2}(y_1)|} & a^2_{(2,y_1)} & \cdots & a^{c(g)}_{(2,y_1)} \\
\frac{|C_G(g)|}{|C_{H_2}(y_2)|} & a^2_{(2,y_2)} & \cdots & a^{c(g)}_{(2,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\ \hline
\frac{|C_G(g)|}{|C_{H_3}(y_1)|} & a^2_{(3,y_1)} & \cdots & a^{c(g)}_{(3,y_1)} \\
\frac{|C_G(g)|}{|C_{H_3}(y_2)|} & a^2_{(3,y_2)} & \cdots & a^{c(g)}_{(3,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\ \hline
m_1 & m_2 & \cdots & m_{c(g)}
\end{array}
$$

Figure 5. The Fischer-Clifford Matrix $M(g)$

As an example, we choose the conjugacy class $3F$ of G. Using the information of the conjugacy classes of $\overline{G}$ obtained from the class $3F$ of G in Table 2, the centralizer orders of the classes of the inertia factors H_i that fuse into the conjugacy class $3F$ of G, Theorem 5.2.4 and property (e) in [12], the Fischer-Clifford matrix $M(3F)$ takes the following form with corresponding weights attached to the rows and columns,

$$
\begin{array}{r|cccc}
 & |C_{\overline{G}}(3F)| & |C_{\overline{G}}(21C)| & |C_{\overline{G}}(21D)| & |C_{\overline{G}}(21E)| \\
 & 5292 & 441 & 294 & 294 \\
|C_G(3F)| = 108 & 1 & 1 & 1 & 1 \\
|C_{H_2}(3A)| = 6 & 18 & a & d & g \\
|C_{H_2}(3B)| = 6 & 18 & b & e & h \\
|C_{H_3}(3D)| = 9 & 12 & c & f & i \\
m_j & 49 & 588 & 882 & 882
\end{array}
$$

Using the row and column orthogonality properties of Fischer-Clifford matrices found in [12], we form the following system of equations, $2a^2+2b^2+3c^2=111$, $2d^2+2e^2+3f^2=62$, $2g^2+2h^2+3i^2=62$, $a+b+c=-1$, $d+e+f=-1$, $g+h+i=-1$, $2ad+2be+3cf=-36$, $2ag+2bh+3ci=-36$, $2dg+2eh+3fi=-36$, $2a^2+3d^2+3g^2=144$, $2b^2+3e^2+3h^2=144$, $2c^2+3f^2+3i^2=96$, $2a+3d+3g=3$, $2b+3e+3h=3$, $2c+3f+3i=2$, $2ab+3de+3gh=-54$, $2ac+3df+3gi=-36$ and $2bc+3ef+3hi=-36$. Solving this system of equations, we have that, $a=-3$, $b=-3$, $c=5$, $d=-3$ or 4, $e=-3$ or 4, $f=-2$, $i=-2$, $g=-3$ or 4, and $h=-3$ or 4. Taking into consideration the fact that $\chi(7B) \equiv \chi(21D)$(mod 3) and $\chi(7A) \equiv \chi(21E)$(mod 3) it turns out that $d=4$, $e=-3$, $f=-2$, $i=-2$, $g=-3$, and $h=4$. Hence the unique Fischer-Clifford matrix $M(3F)$ of $\overline{G}$ is obtained (see Figure 6). The complete list of all the Fischer-Clifford matrices of $\overline{G} = 7^4{:}(3 \times 2S_7)$ are given in Table 3.

$$
M(3F) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 18 & -3 & 4 & -3 \\ 18 & -3 & -3 & 4 \\ 12 & 5 & -2 & -2 \end{pmatrix}
$$

Figure 6. Fischer-Clifford matrix $M(3F)$

Table 3. The Fischer-Clifford Matrices of $\overline{G} = 7^4{:}(3 \times 2S_7)$

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 \\ 1680 & -35 & 14 \\ 720 & 34 & -15 \end{pmatrix}$	$M(2B) = \begin{pmatrix} 1 & 1 & 1 \\ 24 & -4 & 3 \\ 24 & 3 & -4 \end{pmatrix}$
$M(3D) = \begin{pmatrix} 1 & 1 \\ 48 & -1 \end{pmatrix}$	$M(3E) = \begin{pmatrix} 1 & 1 \\ 48 & -1 \end{pmatrix}$
$M(3F) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 18 & -3 & 4 & -3 \\ 18 & -3 & -3 & 4 \\ 12 & 5 & -2 & -2 \end{pmatrix}$	$M(g_i) = \begin{pmatrix} 1 & 1 \\ 6 & -1 \end{pmatrix}, \forall g_i \in \{3G, 3H, 6K, 6L, 6N, 6O, 7A\}$
$M(g_i) = \begin{pmatrix} 1 \end{pmatrix}, \forall g_i \notin \{1A, 2B, 3D, 3E, 3F, 3G, 3H, 6K, 6L, 6N, 6O, 7A\}$	

6 The Ordinary Character Table of $\overline{G}$

The partial character table of $\overline{G}$ on the $c(g)$ classes, which are obtained from the coset Ng and with class representatives $\{x_1, x_2, \cdots, x_{c(g)}\}$, is given by $\begin{bmatrix} C_1(g)\, M_1(g) \\ C_2(g)\, M_2(g) \\ C_3(g)\, M_3(g) \end{bmatrix}$, where the Fischer-Clifford matrix $M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ M_3(g) \end{bmatrix}$ is divided into blocks $M_i(g)$. Each block corresponds to an inertia group $\overline{H}_i$ and $C_i(g)$ is the partial character table of H_i consisting of the columns corresponding to the classes that fuse into $[g]$ in G. Note that if there is no class fusion of H_i into $g \in G$, then the block $M_i(g)$ corresponding to H_i is omitted from $M(g)$ and therefore the entries of the submatrix $C_i(g)M_i(g)$ will be all zeroes. Hence the full ordinary character table of $\overline{G}$ will be $\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix}$, where $\Delta_i = [C_i(1)M_i(1)|C_i(g_2)M_i(g_2)|...|C_i(g_{69})M_i(g_{69})]$ with $\{1, g_1, g_2, ..., g_{69}\}$ the representatives of conjugacy classes of $G = 3{\times}2S_7$. Table 4 gives the ordinary character table of $\overline{G}$, which is a 85×85 complex-valued square matrix and is partitioned row-wise into three blocks $\Delta_1 = \{\chi_i | 1 \leq i \leq 69\}$, $\Delta_2 = \{\chi_i | 70 \leq i \leq 78\}$ and $\Delta_3 = \{\chi_i | 79 \leq i \leq 85\}$, where $\chi_i \in \mathrm{Irr}(\overline{G})$. Notice that the faithful characters of $\overline{G}$ appear in the blocks Δ_2 and Δ_3. Checks for consistency and accuracy of Table 4 have been carried out with the GAP routine, Programme E in [20].

Table 4. The Character Table of $7^4{:}(3 \times 2S_7)$

$[g]_G$	1A			2A	2B			3A	3B	3C	3D		3E		3F				3G	
$[x]_{\overline{G}}$	1A	7A	7B	2A	2B	14A	14B	3A	3B	3C	3D	21A	3E	21B	3F	21C	21D	21E	3G	21F
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	1	1	-1	-1	-1	A	$\overline{A}$	1	$\overline{A}$	$\overline{A}$	A	A	1	1	1	1	A	A
χ_4	1	1	1	1	-1	-1	-1	$\overline{A}$	A	1	A	A	$\overline{A}$	$\overline{A}$	1	1	1	1	$\overline{A}$	$\overline{A}$
χ_5	1	1	1	1	1	1	1	A	$\overline{A}$	1	$\overline{A}$	$\overline{A}$	A	A	1	1	1	1	A	A
χ_6	1	1	1	1	1	1	1	$\overline{A}$	A	1	A	A	$\overline{A}$	$\overline{A}$	1	1	1	1	$\overline{A}$	$\overline{A}$
χ_7	6	6	6	6	0	0	0	6	6	3	3	3	3	3	0	0	0	0	0	0
χ_8	6	6	6	6	0	0	0	6	6	3	3	3	3	3	0	0	0	0	0	0
χ_9	6	6	6	6	0	0	0	B	$\overline{B}$	3	N	N	$\overline{N}$	$\overline{N}$	0	0	0	0	0	0
χ_{10}	6	6	6	6	0	0	0	$\overline{B}$	B	3	$\overline{N}$	$\overline{N}$	N	N	0	0	0	0	0	0
χ_{11}	6	6	6	6	0	0	0	B	$\overline{B}$	3	N	N	$\overline{N}$	$\overline{N}$	0	0	0	0	0	0
χ_{12}	6	6	6	6	0	0	0	$\overline{B}$	B	3	$\overline{N}$	$\overline{N}$	N	N	0	0	0	0	0	0
χ_{13}	8	8	8	-8	0	0	0	8	8	-4	-4	-4	-4	-4	2	2	2	2	2	2
χ_{14}	8	8	8	-8	0	0	0	C	$\overline{C}$	-4	$\overline{K}$	$\overline{K}$	K	K	2	2	2	2	M	M
χ_{15}	8	8	8	-8	0	0	0	$\overline{C}$	C	-4	K	K	$\overline{K}$	$\overline{K}$	2	2	2	2	$\overline{M}$	$\overline{M}$
χ_{16}	14	14	14	14	-2	-2	-2	14	14	2	2	2	2	2	-1	-1	-1	-1	-1	-1
χ_{17}	14	14	14	14	0	0	0	14	14	-1	-1	-1	-1	-1	2	2	2	2	2	2
χ_{18}	14	14	14	14	0	0	0	14	14	-1	-1	-1	-1	-1	2	2	2	2	2	2
χ_{19}	14	14	14	14	2	2	2	14	14	2	2	2	2	2	-1	-1	-1	-1	-1	-1
χ_{20}	14	14	14	14	-2	-2	-2	D	$\overline{D}$	2	$\overline{M}$	$\overline{M}$	M	M	-1	-1	-1	-1	-A	-A
χ_{21}	14	14	14	14	-2	-2	-2	$\overline{D}$	D	2	M	M	$\overline{M}$	$\overline{M}$	-1	-1	-1	-1	-$\overline{A}$	-$\overline{A}$
χ_{22}	14	14	14	14	0	0	0	D	$\overline{D}$	-1	-$\overline{A}$	-$\overline{A}$	-A	-A	2	2	2	2	M	M
χ_{23}	14	14	14	14	0	0	0	$\overline{D}$	D	-1	-A	-A	-$\overline{A}$	-$\overline{A}$	2	2	2	2	$\overline{M}$	$\overline{M}$
χ_{24}	14	14	14	14	0	0	0	D	$\overline{D}$	-1	-$\overline{A}$	-$\overline{A}$	-A	-A	2	2	2	2	M	M
χ_{25}	14	14	14	14	0	0	0	$\overline{D}$	D	-1	-A	-A	-$\overline{A}$	-$\overline{A}$	2	2	2	2	$\overline{M}$	$\overline{M}$
χ_{26}	14	14	14	14	2	2	2	D	$\overline{D}$	2	$\overline{M}$	$\overline{M}$	M	M	-1	-1	-1	-1	-A	-A
χ_{27}	14	14	14	14	2	2	2	$\overline{D}$	D	2	M	M	$\overline{M}$	$\overline{M}$	-1	-1	-1	-1	-$\overline{A}$	-$\overline{A}$
χ_{28}	15	15	15	15	3	3	3	15	15	3	3	3	3	3	0	0	0	0	0	0
χ_{29}	15	15	15	15	-3	-3	-3	15	15	3	3	3	3	3	0	0	0	0	0	0
χ_{30}	15	15	15	15	3	3	3	E	$\overline{E}$	3	N	N	$\overline{N}$	$\overline{N}$	0	0	0	0	0	0
χ_{31}	15	15	15	15	3	3	3	$\overline{E}$	E	3	$\overline{N}$	$\overline{N}$	N	N	0	0	0	0	0	0
χ_{32}	15	15	15	15	-3	-3	-3	E	$\overline{E}$	3	N	N	$\overline{N}$	$\overline{N}$	0	0	0	0	0	0
χ_{33}	15	15	15	15	-3	-3	-3	$\overline{E}$	E	3	$\overline{N}$	$\overline{N}$	N	N	0	0	0	0	0	0
χ_{34}	20	20	20	20	0	0	0	20	20	2	2	2	2	2	2	2	2	2	2	2
χ_{35}	20	20	20	20	0	0	0	F	$\overline{F}$	2	$\overline{M}$	$\overline{M}$	M	M	2	2	2	2	M	M
χ_{36}	20	20	20	20	0	0	0	$\overline{F}$	F	2	M	M	$\overline{M}$	$\overline{M}$	2	2	2	2	$\overline{M}$	$\overline{M}$
χ_{37}	20	20	20	-20	0	0	0	20	20	-4	-4	-4	-4	-4	-1	-1	-1	-1	-1	-1
χ_{38}	20	20	20	-20	0	0	0	20	20	-4	-4	-4	-4	-4	-1	-1	-1	-1	-1	-1
χ_{39}	20	20	20	-20	0	0	0	20	20	2	2	2	2	2	2	2	2	2	2	2
χ_{40}	20	20	20	-20	0	0	0	20	20	2	2	2	2	2	2	2	2	2	2	2
χ_{41}	20	20	20	-20	0	0	0	F	$\overline{F}$	-4	$\overline{K}$	$\overline{K}$	K	K	-1	-1	-1	-1	-A	-A
χ_{42}	20	20	20	-20	0	0	0	$\overline{F}$	F	-4	K	K	$\overline{K}$	$\overline{K}$	-1	-1	-1	-1	-$\overline{A}$	-$\overline{A}$

where A = $\frac{-1-\sqrt{3}i}{2}$, B = $-3-3\sqrt{3}i$, C = $-4-4\sqrt{3}i$,
D = $-7-7\sqrt{3}i$, E = $\frac{-15-15\sqrt{3}i}{2}$, F = $-10-10\sqrt{3}i$,
K = $2+2\sqrt{3}i$, M = $-1-\sqrt{3}i$, N = $\frac{-3+3\sqrt{3}i}{2}$

Table 4. The Character Table of $7^4{:}(3 \times 2S_7)$ (continued)

$[g]_G$	1A			2A	2B			3A	3B	3C	3D		3E		3F				3G	
$[x]_{\overline{G}}$	1A	7A	7B	2A	2B	14A	14B	3A	3B	3C	3D	21A	3E	21B	3F	21C	21D	21E	3G	21F
χ_{43}	20	20	20	-20	0	0	0	F	$\bar{F}$	-4	$\bar{K}$	$\bar{K}$	K	K	-1	-1	-1	-1	-A	-A
χ_{44}	20	20	20	-20	0	0	0	$\bar{F}$	F	-4	K	K	$\bar{K}$	$\bar{K}$	-1	-1	-1	-1	-$\bar{A}$	-$\bar{A}$
χ_{45}	20	20	20	-20	0	0	0	F	$\bar{F}$	2	$\bar{M}$	$\bar{M}$	M	M	2	2	2	2	M	M
χ_{46}	20	20	20	-20	0	0	0	F	$\bar{F}$	2	$\bar{M}$	$\bar{M}$	M	M	2	2	2	2	M	M
χ_{47}	20	20	20	-20	0	0	0	$\bar{F}$	F	2	M	M	$\bar{M}$	$\bar{M}$	2	2	2	2	$\bar{M}$	$\bar{M}$
χ_{48}	20	20	20	-20	0	0	0	$\bar{F}$	F	2	M	M	$\bar{M}$	$\bar{M}$	2	2	2	2	$\bar{M}$	$\bar{M}$
χ_{49}	21	21	21	21	3	3	3	21	21	-3	-3	-3	-3	-3	0	0	0	0	0	0
χ_{50}	21	21	21	21	-3	-3	-3	21	21	-3	-3	-3	-3	-3	0	0	0	0	0	0
χ_{51}	21	21	21	21	3	3	3	G	$\bar{G}$	-3	-N	-N	-$\bar{N}$	-$\bar{N}$	0	0	0	0	0	0
χ_{52}	21	21	21	21	3	3	3	$\bar{G}$	G	-3	-$\bar{N}$	-$\bar{N}$	-N	-N	0	0	0	0	0	0
χ_{53}	21	21	21	21	-3	-3	-3	G	$\bar{G}$	-3	-N	-N	-$\bar{N}$	-$\bar{N}$	0	0	0	0	0	0
χ_{54}	21	21	21	21	-3	-3	-3	$\bar{G}$	G	-3	-$\bar{N}$	-$\bar{N}$	-N	-N	0	0	0	0	0	0
χ_{55}	28	28	28	-28	0	0	0	28	28	4	4	4	4	4	-2	-2	-2	-2	-2	-2
χ_{56}	28	28	28	-28	0	0	0	H	$\bar{H}$	4	-$\bar{K}$	-$\bar{K}$	-K	-K	-2	-2	-2	-2	-M	-M
χ_{57}	28	28	28	-28	0	0	0	$\bar{H}$	H	4	-K	-K	-$\bar{K}$	-$\bar{K}$	-2	-2	-2	-2	-$\bar{M}$	-$\bar{M}$
χ_{58}	35	35	35	35	-1	-1	-1	35	35	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{59}	35	35	35	35	1	1	1	35	35	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{60}	35	35	35	35	-1	-1	-1	I	$\bar{I}$	-1	-$\bar{A}$	-$\bar{A}$	-A	-A	-1	-1	-1	-1	-A	-A
χ_{61}	35	35	35	35	-1	-1	-1	$\bar{I}$	I	-1	-A	-A	-$\bar{A}$	-$\bar{A}$	-1	-1	-1	-1	-$\bar{A}$	-$\bar{A}$
χ_{62}	35	35	35	35	1	1	1	I	$\bar{I}$	-1	-$\bar{A}$	-$\bar{A}$	-A	-A	-1	-1	-1	-1	-A	-A
χ_{63}	35	35	35	35	1	1	1	$\bar{I}$	I	-1	-A	-A	-$\bar{A}$	-$\bar{A}$	-1	-1	-1	-1	-$\bar{A}$	-$\bar{A}$
χ_{64}	36	36	36	-36	0	0	0	36	36	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	36	36	36	-36	0	0	0	36	36	0	0	0	0	0	0	0	0	0	0	0
χ_{66}	36	36	36	-36	0	0	0	J	$\bar{J}$	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	36	36	36	-36	0	0	0	J	$\bar{J}$	0	0	0	0	0	0	0	0	0	0	0
χ_{68}	36	36	36	-36	0	0	0	$\bar{J}$	J	0	0	0	0	0	0	0	0	0	0	0
χ_{69}	36	36	36	-36	0	0	0	$\bar{J}$	J	0	0	0	0	0	0	0	0	0	0	0
χ_{70}	720	34	-15	0	24	3	-4	0	0	0	0	0	0	0	36	-6	1	1	0	0
χ_{71}	720	34	-15	0	-24	-3	4	0	0	0	0	0	0	0	36	-6	1	1	0	0
χ_{72}	720	34	-15	0	-24	-3	4	0	0	0	0	0	0	0	-18	3	P	$\bar{P}$	0	0
χ_{73}	720	34	-15	0	-24	-3	4	0	0	0	0	0	0	0	-18	3	$\bar{P}$	P	0	0
χ_{74}	720	34	-15	0	24	3	-4	0	0	0	0	0	0	0	-18	3	P	$\bar{P}$	0	0
χ_{75}	720	34	-15	0	24	3	-4	0	0	0	0	0	0	0	-18	3	$\bar{P}$	P	0	0
χ_{76}	4320	204	-90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	1680	-35	14	0	24	-4	3	0	0	0	48	-1	48	-1	12	5	-2	-2	6	-1
χ_{78}	1680	-35	14	0	-24	4	-3	0	0	0	48	-1	48	-1	12	5	-2	-2	6	-1
χ_{79}	1680	-35	14	0	-24	4	-3	0	0	0	O	-A	$\bar{O}$	-$\bar{A}$	12	5	-2	-2	$\bar{B}$	-$\bar{A}$
χ_{80}	1680	-35	14	0	-24	4	-3	0	0	0	$\bar{O}$	-$\bar{A}$	O	-A	12	5	-2	-2	B	-A
χ_{81}	1680	-35	14	0	24	-4	3	0	0	0	O	-A	$\bar{O}$	-$\bar{A}$	12	5	-2	-2	$\bar{B}$	-$\bar{A}$
χ_{82}	1680	-35	14	0	24	-4	3	0	0	0	$\bar{O}$	-$\bar{A}$	O	-A	12	5	-2	-2	B	-A
χ_{83}	3360	-70	28	0	0	0	0	0	0	0	-48	1	-48	1	-12	-5	2	2	12	-2
χ_{84}	3360	-70	28	0	0	0	0	0	0	0	-$\bar{O}$	$\bar{A}$	-O	A	-12	-5	2	2	Q	-M
χ_{85}	3360	-70	28	0	0	0	0	0	0	0	-O	A	-$\bar{O}$	$\bar{A}$	-12	-5	2	2	$\bar{Q}$	-$\bar{M}$

where A = $\frac{-1-\sqrt{3}i}{2}$, B = $-3-3\sqrt{3}i$, F = $-10-10\sqrt{3}i$,
G = $\frac{-21-21\sqrt{3}i}{2}$, H = $-14-14\sqrt{3}i$, I = $\frac{-35-35\sqrt{3}i}{2}$,
J = $-18-18\sqrt{3}i$, K = $2+2\sqrt{3}i$, M = $-1-\sqrt{3}i$,
N = $\frac{-3+3\sqrt{3}i}{2}$, O = $-24-24\sqrt{3}i$, P = $\frac{-1+7\sqrt{3}i}{2}$,
Q = $-6-6\sqrt{3}i$

Table 4. The Character Table of $7^4{:}(3 \times 2S_7)$ (continued)

$[g]_G$	3H		4A	4B	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K		6L		6M	6N		6O		6P
$[x]_{\overline{G}}$	3H	21G	4A	4B	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	42A	6L	42B	6M	6N	42C	6O	42D	6P
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	1	1	1	1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_3	$\overline{A}$	$\overline{A}$	-1	1	1	A	$\overline{A}$	-$\overline{A}$	-A	$\overline{A}$	1	A	1	A	$\overline{A}$	-1	-1	-1	-1	-A	-A	-A	-$\overline{A}$	-$\overline{A}$	-$\overline{A}$
χ_4	A	A	-1	1	1	$\overline{A}$	A	-A	-$\overline{A}$	A	1	$\overline{A}$	1	$\overline{A}$	A	-1	-1	-1	-1	-$\overline{A}$	-$\overline{A}$	-$\overline{A}$	-A	-A	-A
χ_5	$\overline{A}$	$\overline{A}$	1	1	1	A	$\overline{A}$	$\overline{A}$	A	$\overline{A}$	1	A	1	A	$\overline{A}$	1	1	1	1	A	A	A	$\overline{A}$	$\overline{A}$	$\overline{A}$
χ_6	A	A	1	1	1	$\overline{A}$	A	A	$\overline{A}$	A	1	$\overline{A}$	1	$\overline{A}$	A	1	1	1	1	$\overline{A}$	$\overline{A}$	$\overline{A}$	A	A	A
χ_7	0	0	-4	2	1	6	6	0	0	3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_8	0	0	4	2	1	6	6	0	0	3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_9	0	0	-4	2	1	B	$\overline{B}$	0	0	N	3	$\overline{N}$	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{10}	0	0	-4	2	1	$\overline{B}$	B	0	0	$\overline{N}$	3	N	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{11}	0	0	4	2	1	B	$\overline{B}$	0	0	N	3	$\overline{N}$	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{12}	0	0	4	2	1	$\overline{B}$	B	0	0	$\overline{N}$	3	N	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{13}	2	2	0	0	-2	-8	-8	0	0	4	4	4	-2	-2	-2	0	0	0	0	0	0	0	0	0	0
χ_{14}	$\overline{M}$	$\overline{M}$	0	0	-2	-C	-$\overline{C}$	0	0	-$\overline{K}$	4	-K	-2	-M	-$\overline{M}$	0	0	0	0	0	0	0	0	0	0
χ_{15}	M	M	0	0	-2	-$\overline{C}$	-C	0	0	-K	4	-$\overline{K}$	-2	-$\overline{M}$	-M	0	0	0	0	0	0	0	0	0	0
χ_{16}	-1	-1	-6	2	-1	14	14	-2	-2	2	2	2	-1	-1	-1	1	1	1	1	1	1	1	1	1	1
χ_{17}	2	2	-4	2	-1	14	14	0	0	-1	-1	-1	2	2	2	0	0	0	0	0	0	0	0	0	0
χ_{18}	2	2	4	2	-1	14	14	0	0	-1	-1	-1	2	2	2	0	0	0	0	0	0	0	0	0	0
χ_{19}	-1	-1	6	2	-1	14	14	2	2	2	2	2	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{20}	-$\overline{A}$	-$\overline{A}$	-6	2	-1	D	$\overline{D}$	-$\overline{M}$	-M	$\overline{M}$	2	M	-1	-A	-$\overline{A}$	1	1	1	1	A	A	A	$\overline{A}$	$\overline{A}$	$\overline{A}$
χ_{21}	-A	-A	-6	2	-1	$\overline{D}$	D	-M	-$\overline{M}$	M	2	$\overline{M}$	-1	-$\overline{A}$	-A	1	1	1	1	$\overline{A}$	$\overline{A}$	$\overline{A}$	A	A	A
χ_{22}	$\overline{M}$	$\overline{M}$	-4	2	-1	D	$\overline{D}$	0	0	-$\overline{A}$	-1	-A	2	M	$\overline{M}$	0	0	0	0	0	0	0	0	0	0
χ_{23}	M	M	-4	2	-1	$\overline{D}$	D	0	0	-A	-1	-$\overline{A}$	2	$\overline{M}$	M	0	0	0	0	0	0	0	0	0	0
χ_{24}	$\overline{M}$	$\overline{M}$	4	2	-1	D	$\overline{D}$	0	0	-$\overline{A}$	-1	-A	2	M	$\overline{M}$	0	0	0	0	0	0	0	0	0	0
χ_{25}	M	M	4	2	-1	$\overline{D}$	D	0	0	-A	-1	-$\overline{A}$	2	$\overline{M}$	M	0	0	0	0	0	0	0	0	0	0
χ_{26}	-$\overline{A}$	-$\overline{A}$	6	2	-1	D	$\overline{D}$	$\overline{M}$	M	$\overline{M}$	2	M	-1	-A	-$\overline{A}$	-1	-1	-1	-1	-A	-A	-A	-$\overline{A}$	-$\overline{A}$	-$\overline{A}$
χ_{27}	-A	-A	6	2	-1	$\overline{D}$	D	M	$\overline{M}$	M	2	$\overline{M}$	-1	-$\overline{A}$	-A	-1	-1	-1	-1	-$\overline{A}$	-$\overline{A}$	-$\overline{A}$	-A	-A	-A
χ_{28}	0	0	-5	-1	0	15	15	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}	0	0	5	-1	0	15	15	-3	-3	3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	0	0	-5	-1	0	E	$\overline{E}$	N	$\overline{N}$	N	3	$\overline{N}$	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{31}	0	0	-5	-1	0	$\overline{E}$	E	$\overline{N}$	N	$\overline{N}$	3	N	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	0	0	5	-1	0	E	$\overline{E}$	-N	-$\overline{N}$	N	3	$\overline{N}$	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	0	5	-1	0	$\overline{E}$	E	-$\overline{N}$	-N	$\overline{N}$	3	N	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	2	2	0	-4	0	20	20	0	0	2	2	2	2	2	2	0	0	0	0	0	0	0	0	0	0
χ_{35}	$\overline{M}$	$\overline{M}$	0	-4	0	F	$\overline{F}$	0	0	$\overline{M}$	2	M	2	M	$\overline{M}$	0	0	0	0	0	0	0	0	0	0
χ_{36}	M	M	0	-4	0	$\overline{F}$	F	0	0	M	2	$\overline{M}$	2	$\overline{M}$	M	0	0	0	0	0	0	0	0	0	0
χ_{37}	-1	-1	0	0	0	-20	-20	0	0	4	4	4	1	1	1	V	V	-V	-V	V	-V	-V	V	V	-V
χ_{38}	-1	-1	0	0	0	-20	-20	0	0	4	4	4	1	1	1	-V	-V	V	V	-V	V	V	-V	-V	V
χ_{39}	2	2	0	0	0	-20	-20	0	0	-2	-2	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0
χ_{40}	2	2	0	0	0	-20	-20	0	0	-2	-2	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0
χ_{41}	-$\overline{A}$	-$\overline{A}$	0	0	0	-F	-$\overline{F}$	0	0	-$\overline{K}$	4	-K	1	A	$\overline{A}$	V	V	-V	-V	W	-W	-W	-$\overline{W}$	-$\overline{W}$	$\overline{W}$
χ_{42}	-A	-A	0	0	0	-$\overline{F}$	-F	0	0	-K	4	-$\overline{K}$	1	$\overline{A}$	A	-V	-V	V	V	$\overline{W}$	-$\overline{W}$	-$\overline{W}$	-W	-W	W

where A = $\frac{-1-\sqrt{3}i}{2}$, B = $-3-3\sqrt{3}i$, C = $-4-4\sqrt{3}i$, D = $-7-7\sqrt{3}i$, E = $\frac{-15-15\sqrt{3}i}{2}$, F = $-10-10\sqrt{3}i$, G = $\frac{-21-21\sqrt{3}i}{2}$, H = $-14-14\sqrt{3}i$, I = $\frac{-35-35\sqrt{3}i}{2}$, J = $-18-18\sqrt{3}i$, K = $2+2\sqrt{3}i$, M = $-1-\sqrt{3}i$, N = $\frac{-3+3\sqrt{3}i}{2}$, V = $-\sqrt{3}i$, W = $\frac{-3+\sqrt{3}i}{2}$

Table 4. The Character Table of $7^4{:}(3 \times 2S_7)$ (continued)

$[g]_G$	3H		4A	4B	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K		6L		6M	6N		6O		6P
$[x]_{\overline{G}}$	3H	21G	4A	4B	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	42A	6L	42B	6M	6N	42C	6O	42D	6P
χ_{43}	$-\overline{A}$	$-\overline{A}$	0	0	0	-F	$-\overline{F}$	0	0	$-\overline{K}$	4	-K	1	A	$\overline{A}$	-V	-V	V	V	-W	W	W	$\overline{W}$	$\overline{W}$	$-\overline{W}$
χ_{44}	-A	-A	0	0	0	$-\overline{F}$	-F	0	0	-K	4	$-\overline{K}$	1	$\overline{A}$	A	V	V	-V	-V	$-\overline{W}$	$\overline{W}$	$\overline{W}$	W	W	-W
χ_{45}	$\overline{M}$	$\overline{M}$	0	0	0	-F	$-\overline{F}$	0	0	$-\overline{M}$	-2	-M	-2	-M	$-\overline{M}$	0	0	0	0	0	0	0	0	0	0
χ_{46}	$\overline{M}$	$\overline{M}$	0	0	0	-F	$-\overline{F}$	0	0	$-\overline{M}$	-2	-M	-2	-M	$-\overline{M}$	0	0	0	0	0	0	0	0	0	0
χ_{47}	M	M	0	0	0	$-\overline{F}$	-F	0	0	-M	-2	$-\overline{M}$	-2	$-\overline{M}$	-M	0	0	0	0	0	0	0	0	0	0
χ_{48}	M	M	0	0	0	$-\overline{F}$	-F	0	0	-M	-2	$-\overline{M}$	-2	$-\overline{M}$	-M	0	0	0	0	0	0	0	0	0	0
χ_{49}	0	0	-1	1	1	21	21	3	3	-3	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{50}	0	0	1	1	1	21	21	-3	-3	-3	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{51}	0	0	-1	1	1	G	$\overline{G}$	N	$\overline{N}$	-N	-3	$-\overline{N}$	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	0	-1	1	1	$\overline{G}$	G	$\overline{N}$	N	$-\overline{N}$	-3	-N	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	0	0	1	1	1	G	$\overline{G}$	-N	$-\overline{N}$	-N	-3	$-\overline{N}$	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	0	0	1	1	1	$\overline{G}$	G	$-\overline{N}$	-N	$-\overline{N}$	-3	-N	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{55}	-2	-2	0	0	-2	-28	-28	0	0	-4	-4	-4	2	2	2	0	0	0	0	0	0	0	0	0	0
χ_{56}	$-\overline{M}$	$-\overline{M}$	0	0	-2	-H	$-\overline{H}$	0	0	$\overline{K}$	-4	K	2	M	$\overline{M}$	0	0	0	0	0	0	0	0	0	0
χ_{57}	-M	-M	0	0	-2	$-\overline{H}$	-H	0	0	K	-4	$\overline{K}$	2	$\overline{M}$	M	0	0	0	0	0	0	0	0	0	0
χ_{58}	-1	-1	-5	-1	0	35	35	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{59}	-1	-1	5	-1	0	35	35	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1
χ_{60}	$-\overline{A}$	$-\overline{A}$	-5	-1	0	I	$\overline{I}$	$-\overline{A}$	-A	$-\overline{A}$	-1	-A	-1	-A	$-\overline{A}$	-1	-1	-1	-1	-A	-A	-A	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$
χ_{61}	-A	-A	-5	-1	0	$\overline{I}$	I	-A	$-\overline{A}$	-A	-1	$-\overline{A}$	-1	$-\overline{A}$	-A	-1	-1	-1	-1	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	-A	-A	-A
χ_{62}	$-\overline{A}$	$-\overline{A}$	5	-1	0	I	$\overline{I}$	$\overline{A}$	A	$-\overline{A}$	-1	-A	-1	-A	$-\overline{A}$	1	1	1	1	A	A	A	$\overline{A}$	$\overline{A}$	$\overline{A}$
χ_{63}	-A	-A	5	-1	0	$\overline{I}$	I	A	$\overline{A}$	-A	-1	$-\overline{A}$	-1	$-\overline{A}$	-A	1	1	1	1	$\overline{A}$	$\overline{A}$	$\overline{A}$	A	A	A
χ_{64}	0	0	0	0	1	-36	-36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	0	0	0	0	1	-36	-36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	0	1	-J	$-\overline{J}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	0	0	0	0	1	-J	$-\overline{J}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{68}	0	0	0	0	1	$-\overline{J}$	-J	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{69}	0	0	0	0	1	$-\overline{J}$	-J	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	-1	6	-1	0	0	0	0	0	0
χ_{71}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	1	-6	1	0	0	0	0	0	0
χ_{72}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\overline{B}$	$\overline{A}$	-B	A	0	0	0	0	0	0
χ_{73}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-B	A	$-\overline{B}$	$\overline{A}$	0	0	0	0	0	0
χ_{74}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{B}$	$-\overline{A}$	B	-A	0	0	0	0	0	0
χ_{75}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	B	-A	$\overline{B}$	$-\overline{A}$	0	0	0	0	0	0
χ_{76}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	6	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	-1	6	-1	0
χ_{78}	6	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	1	-6	1	0
χ_{79}	B	-A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\overline{B}$	$\overline{A}$	-B	A	0
χ_{80}	$\overline{B}$	$-\overline{A}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-B	A	$-\overline{B}$	$\overline{A}$	0
χ_{81}	B	-A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{B}$	$-\overline{A}$	B	-A	0
χ_{82}	$\overline{B}$	$-\overline{A}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	B	-A	$\overline{B}$	$-\overline{A}$	0
χ_{83}	12	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{84}	$\overline{Q}$	$-\overline{M}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{85}	Q	-M	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

where A = $\frac{-1-\sqrt{3}i}{2}$, B = $-3-3\sqrt{3}i$, C = $-4-4\sqrt{3}i$, D = $-7-7\sqrt{3}i$, E = $\frac{-15-15\sqrt{3}i}{2}$, F = $-10-10\sqrt{3}i$, G = $\frac{-21-21\sqrt{3}i}{2}$, H = $-14-14\sqrt{3}i$, I = $\frac{-35-35\sqrt{3}i}{2}$, J = $-18-18\sqrt{3}i$, K = $2+2\sqrt{3}i$, M = $-1-\sqrt{3}i$, N = $\frac{-3+3\sqrt{3}i}{2}$, V = $-\sqrt{3}i$, W = $\frac{-3+\sqrt{3}i}{2}$

Table 4. The Character Table of $7^4{:}(3 \times 2S_7)$ (continued)

$[g]_G$	7A		8A	8B	10A	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	14A	15A	15B	20A	20B
$[x]_{\overline{G}}$	7C	7D	8A	8B	10A	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	14C	15A	15B	20A	20B
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	1	1	1	1	1	-1	-1
χ_3	1	1	-1	1	1	-A	-$\overline{A}$	A	$\overline{A}$	-$\overline{A}$	-1	-A	1	A	$\overline{A}$	1	$\overline{A}$	A	-1	-1
χ_4	1	1	-1	1	1	-$\overline{A}$	-A	$\overline{A}$	A	-A	-1	-$\overline{A}$	1	$\overline{A}$	A	1	A	$\overline{A}$	-1	-1
χ_5	1	1	1	1	1	A	$\overline{A}$	A	$\overline{A}$	$\overline{A}$	1	A	1	A	$\overline{A}$	1	$\overline{A}$	A	1	1
χ_6	1	1	1	1	1	$\overline{A}$	A	$\overline{A}$	A	A	1	$\overline{A}$	1	$\overline{A}$	A	1	A	$\overline{A}$	1	1
χ_7	-1	-1	-2	0	1	-4	-4	2	2	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_8	-1	-1	2	0	1	4	4	2	2	1	1	1	-1	-1	-1	-1	1	1	-1	-1
χ_9	-1	-1	-2	0	1	K	$\overline{K}$	M	$\overline{M}$	-$\overline{A}$	-1	-A	-1	-A	-$\overline{A}$	-1	$\overline{A}$	A	1	1
χ_{10}	-1	-1	-2	0	1	$\overline{K}$	K	$\overline{M}$	M	-A	-1	-$\overline{A}$	-1	-$\overline{A}$	-A	-1	A	$\overline{A}$	1	1
χ_{11}	-1	-1	2	0	1	-K	-$\overline{K}$	M	$\overline{M}$	$\overline{A}$	1	A	-1	-A	-$\overline{A}$	-1	$\overline{A}$	A	-1	-1
χ_{12}	-1	-1	2	0	1	-$\overline{K}$	-K	$\overline{M}$	M	A	1	$\overline{A}$	-1	-$\overline{A}$	-A	-1	A	$\overline{A}$	-1	-1
χ_{13}	1	1	0	0	2	0	0	0	0	0	0	0	0	0	0	-1	-2	-2	0	0
χ_{14}	1	1	0	0	2	0	0	0	0	0	0	0	0	0	0	-1	-$\overline{M}$	-M	0	0
χ_{15}	1	1	0	0	2	0	0	0	0	0	0	0	0	0	0	-1	-M	-$\overline{M}$	0	0
χ_{16}	0	0	0	0	-1	-6	-6	2	2	0	0	0	2	2	2	0	-1	-1	-1	-1
χ_{17}	0	0	2	0	-1	-4	-4	2	2	-1	-1	-1	-1	-1	-1	0	-1	-1	1	1
χ_{18}	0	0	-2	0	-1	4	4	2	2	1	1	1	-1	-1	-1	0	-1	-1	-1	-1
χ_{19}	0	0	0	0	-1	6	6	2	2	0	0	0	2	2	2	0	-1	-1	1	1
χ_{20}	0	0	0	0	-1	-B	-$\overline{B}$	M	$\overline{M}$	0	0	0	2	M	$\overline{M}$	0	-$\overline{A}$	-A	-1	-1
χ_{21}	0	0	0	0	-1	-$\overline{B}$	-B	$\overline{M}$	M	0	0	0	2	$\overline{M}$	M	0	-A	-$\overline{A}$	-1	-1
χ_{22}	0	0	2	0	-1	K	$\overline{K}$	M	$\overline{M}$	-$\overline{A}$	-1	-A	-1	-A	-$\overline{A}$	0	-$\overline{A}$	-A	1	1
χ_{23}	0	0	2	0	-1	$\overline{K}$	K	$\overline{M}$	M	-A	-1	-$\overline{A}$	-1	-$\overline{A}$	-A	0	-A	-$\overline{A}$	1	1
χ_{24}	0	0	-2	0	-1	-K	-$\overline{K}$	M	$\overline{M}$	$\overline{A}$	1	A	-1	-A	-$\overline{A}$	0	-$\overline{A}$	-A	-1	-1
χ_{25}	0	0	-2	0	-1	-$\overline{K}$	-K	$\overline{M}$	M	A	1	$\overline{A}$	-1	-$\overline{A}$	-A	0	-A	-$\overline{A}$	-1	-1
χ_{26}	0	0	0	0	-1	B	$\overline{B}$	M	$\overline{M}$	0	0	0	2	M	$\overline{M}$	0	-$\overline{A}$	-A	1	1
χ_{27}	0	0	0	0	-1	$\overline{B}$	B	$\overline{M}$	M	0	0	0	2	$\overline{M}$	M	0	-A	-$\overline{A}$	1	1
χ_{28}	1	1	-1	-1	0	-5	-5	-1	-1	1	1	1	-1	-1	-1	1	0	0	0	0
χ_{29}	1	1	1	-1	0	5	5	-1	-1	-1	-1	-1	-1	-1	-1	1	0	0	0	0
χ_{30}	1	1	-1	-1	0	L	$\overline{L}$	-A	-$\overline{A}$	$\overline{A}$	1	A	-1	-A	-$\overline{A}$	1	0	0	0	0
χ_{31}	1	1	-1	-1	0	$\overline{L}$	L	-$\overline{A}$	-A	A	1	A	-1	-$\overline{A}$	-A	1	0	0	0	0
χ_{32}	1	1	1	-1	0	-L	-$\overline{L}$	-A	-$\overline{A}$	-$\overline{A}$	-1	-A	-1	-A	-$\overline{A}$	1	0	0	0	0
χ_{33}	1	1	1	-1	0	-$\overline{L}$	-L	-$\overline{A}$	-A	-A	-1	-$\overline{A}$	-1	-$\overline{A}$	-A	1	0	0	0	0
χ_{34}	-1	-1	0	0	0	0	0	-4	-4	0	0	0	2	2	2	-1	0	0	0	0
χ_{35}	-1	-1	0	0	0	0	0	K	$\overline{K}$	0	0	0	2	M	$\overline{M}$	-1	0	0	0	0
χ_{36}	-1	-1	0	0	0	0	0	$\overline{K}$	K	0	0	0	2	$\overline{M}$	M	-1	0	0	0	0
χ_{37}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
χ_{38}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
χ_{39}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
χ_{40}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
χ_{41}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
χ_{42}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0

where A = $\frac{-1-\sqrt{3}i}{2}$, B = $-3-3\sqrt{3}i$, K = $2+2\sqrt{3}i$,
L = $\frac{5+5\sqrt{3}i}{2}$, M = $-1-\sqrt{3}i$, T = $-\sqrt{5}i$

Table 4. The Character Table of $7^4{:}(3 \times 2S_7)$ (continued)

$[g]_G$	7A		8A	8B	10A	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	14A	15A	15B	20A	20B
$[x]_{\overline{G}}$	7C	7D	8A	8B	10A	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	14C	15A	15B	20A	20B
χ_{43}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
χ_{44}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
χ_{45}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
χ_{46}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
χ_{47}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
χ_{48}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
χ_{49}	0	0	1	-1	1	-1	-1	1	1	-1	-1	-1	1	1	1	0	1	1	-1	-1
χ_{50}	0	0	-1	-1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
χ_{51}	0	0	1	-1	1	-A	$-\overline{A}$	A	$\overline{A}$	$-\overline{A}$	-1	-A	1	A	$\overline{A}$	0	$\overline{A}$	A	-1	-1
χ_{52}	0	0	1	-1	1	$-\overline{A}$	-A	$\overline{A}$	A	-A	-1	$-\overline{A}$	1	$\overline{A}$	A	0	A	$\overline{A}$	-1	-1
χ_{53}	0	0	-1	-1	1	A	$\overline{A}$	A	$\overline{A}$	$\overline{A}$	1	A	1	A	$\overline{A}$	0	$\overline{A}$	A	1	1
χ_{54}	0	0	-1	-1	1	$\overline{A}$	A	$\overline{A}$	A	A	1	$\overline{A}$	1	$\overline{A}$	A	0	A	$\overline{A}$	1	1
χ_{55}	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	-2	-2	0	0
χ_{56}	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	$-\overline{M}$	-M	0	0
χ_{57}	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	-M	$-\overline{M}$	0	0
χ_{58}	0	0	1	1	0	-5	-5	-1	-1	1	1	1	-1	-1	-1	0	0	0	0	0
χ_{59}	0	0	-1	1	0	5	5	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0
χ_{60}	0	0	1	1	0	L	$\overline{L}$	-A	$-\overline{A}$	$\overline{A}$	1	A	-1	-A	$-\overline{A}$	0	0	0	0	0
χ_{61}	0	0	1	1	0	$\overline{L}$	L	$-\overline{A}$	-A	A	1	$\overline{A}$	-1	$-\overline{A}$	-A	0	0	0	0	0
χ_{62}	0	0	-1	1	0	-L	$-\overline{L}$	-A	$-\overline{A}$	$-\overline{A}$	-1	-A	-1	-A	$-\overline{A}$	0	0	0	0	0
χ_{63}	0	0	-1	1	0	$-\overline{L}$	-L	$-\overline{A}$	-A	-A	-1	$-\overline{A}$	-1	$-\overline{A}$	-A	0	0	0	0	0
χ_{64}	1	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	-1	1	1	T	-T
χ_{65}	1	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	-1	1	1	-T	T
χ_{66}	1	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	-1	$\overline{A}$	A	T	-T
χ_{67}	1	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	-1	$\overline{A}$	A	-T	T
χ_{68}	1	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	-1	A	$\overline{A}$	T	-T
χ_{69}	1	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	-1	A	$\overline{A}$	-T	T
χ_{70}	6	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	6	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{72}	6	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{73}	6	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	6	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{75}	6	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{76}	-6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{82}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{84}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{85}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

where A = $\frac{-1-\sqrt{3}i}{2}$, B = $-3-3\sqrt{3}i$, K = $2+2\sqrt{3}i$,
L = $\frac{5+5\sqrt{3}i}{2}$, M = $-1-\sqrt{3}i$, T = $-\sqrt{5}i$

Table 4. The Character Table of $7^4{:}(3 \times 2S_7)$ (continued)

$[g]_G$	21A	21B	24A	24B	24C	24D	24E	24F	24G	24H	24I	24J	30A	30B	42A	42B	60A	60B	60C	60D
$[x]_{\overline{G}}$	21H	21I	24A	24B	24C	24D	24E	24F	24G	24H	24I	24J	30A	30B	42E	42F	60A	60B	60C	60D
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
χ_3	$\overline{A}$	A	$-\overline{A}$	-A	A	$\overline{A}$	$-\overline{A}$	$-\overline{A}$	-1	-1	-A	-A	$\overline{A}$	A	$\overline{A}$	A	-A	-A	$-\overline{A}$	$-\overline{A}$
χ_4	A	$\overline{A}$	-A	$-\overline{A}$	$\overline{A}$	A	-A	-A	-1	-1	$-\overline{A}$	$-\overline{A}$	A	$\overline{A}$	A	$\overline{A}$	$-\overline{A}$	$-\overline{A}$	-A	-A
χ_5	$\overline{A}$	A	$\overline{A}$	A	A	$\overline{A}$	$\overline{A}$	$\overline{A}$	1	1	A	A	$\overline{A}$	A	$\overline{A}$	A	A	A	$\overline{A}$	$\overline{A}$
χ_6	A	$\overline{A}$	A	$\overline{A}$	$\overline{A}$	A	A	A	1	1	$\overline{A}$	$\overline{A}$	A	$\overline{A}$	A	$\overline{A}$	$\overline{A}$	$\overline{A}$	A	A
χ_7	-1	-1	-2	-2	0	0	1	1	1	1	1	1	1	1	-1	-1	1	1	1	1
χ_8	-1	-1	2	2	0	0	-1	-1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	-1
χ_9	$-\overline{A}$	-A	$-\overline{M}$	-M	0	0	$\overline{A}$	$\overline{A}$	1	1	A	A	$\overline{A}$	A	$-\overline{A}$	-A	A	A	$\overline{A}$	$\overline{A}$
χ_{10}	-A	$-\overline{A}$	-M	$-\overline{M}$	0	0	A	A	1	1	$\overline{A}$	$\overline{A}$	A	$\overline{A}$	-A	$-\overline{A}$	$\overline{A}$	$\overline{A}$	A	A
χ_{11}	$-\overline{A}$	-A	$\overline{M}$	M	0	0	$-\overline{A}$	$-\overline{A}$	-1	-1	-A	-A	$\overline{A}$	A	$-\overline{A}$	-A	-A	-A	$-\overline{A}$	$-\overline{A}$
χ_{12}	-A	$-\overline{A}$	M	$\overline{M}$	0	0	-A	-A	-1	-1	$-\overline{A}$	$-\overline{A}$	A	$\overline{A}$	-A	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	-A	-A
χ_{13}	1	1	0	0	0	0	0	0	0	0	0	0	2	2	-1	-1	0	0	0	0
χ_{14}	$\overline{A}$	A	0	0	0	0	0	0	0	0	0	0	$\overline{M}$	M	$-\overline{A}$	-A	0	0	0	0
χ_{15}	A	$\overline{A}$	0	0	0	0	0	0	0	0	0	0	M	$\overline{M}$	-A	$-\overline{A}$	0	0	0	0
χ_{16}	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	-1	-1	-1	-1
χ_{17}	0	0	2	2	0	0	-1	-1	-1	-1	-1	-1	-1	-1	0	0	1	1	1	1
χ_{18}	0	0	-2	-2	0	0	1	1	1	1	1	1	-1	-1	0	0	-1	-1	-1	-1
χ_{19}	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	1	1	1	1
χ_{20}	0	0	0	0	0	0	0	0	0	0	0	0	$-\overline{A}$	-A	0	0	-A	-A	$-\overline{A}$	$-\overline{A}$
χ_{21}	0	0	0	0	0	0	0	0	0	0	0	0	-A	$-\overline{A}$	0	0	$-\overline{A}$	$-\overline{A}$	-A	-A
χ_{22}	0	0	$\overline{M}$	M	0	0	$-\overline{A}$	$-\overline{A}$	-1	-1	-A	-A	$-\overline{A}$	-A	0	0	A	A	$\overline{A}$	$\overline{A}$
χ_{23}	0	0	M	$\overline{M}$	0	0	-A	-A	-1	-1	$-\overline{A}$	$-\overline{A}$	-A	$-\overline{A}$	0	0	$\overline{A}$	$\overline{A}$	A	A
χ_{24}	0	0	$-\overline{M}$	-M	0	0	$\overline{A}$	$\overline{A}$	1	1	A	A	$-\overline{A}$	-A	0	0	-A	-A	$-\overline{A}$	$-\overline{A}$
χ_{25}	0	0	-M	$-\overline{M}$	0	0	A	A	1	1	$\overline{A}$	$\overline{A}$	-A	$-\overline{A}$	0	0	$-\overline{A}$	$-\overline{A}$	-A	-A
χ_{26}	0	0	0	0	0	0	0	0	0	0	0	0	$-\overline{A}$	-A	0	0	A	A	$\overline{A}$	$\overline{A}$
χ_{27}	0	0	0	0	0	0	0	0	0	0	0	0	-A	$-\overline{A}$	0	0	$\overline{A}$	$\overline{A}$	A	A
χ_{28}	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	1	1	0	0	0	0
χ_{39}	1	1	1	1	-1	-1	1	1	1	1	1	1	0	0	1	1	0	0	0	0
χ_{30}	$\overline{A}$	A	$-\overline{A}$	-A	-A	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	-1	-1	-A	-A	0	0	$\overline{A}$	A	0	0	0	0
χ_{31}	A	$\overline{A}$	-A	$-\overline{A}$	$-\overline{A}$	-A	-A	-A	-1	-1	$-\overline{A}$	$-\overline{A}$	0	0	A	$\overline{A}$	0	0	0	0
χ_{32}	$\overline{A}$	A	$\overline{A}$	A	-A	$-\overline{A}$	$\overline{A}$	$\overline{A}$	1	1	A	A	0	0	$\overline{A}$	A	0	0	0	0
χ_{33}	A	$\overline{A}$	A	$\overline{A}$	$-\overline{A}$	-A	A	A	1	1	$\overline{A}$	$\overline{A}$	0	0	A	$\overline{A}$	0	0	0	0
χ_{34}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0
χ_{35}	$-\overline{A}$	-A	0	0	0	0	0	0	0	0	0	0	0	0	$-\overline{A}$	-A	0	0	0	0
χ_{36}	-A	$-\overline{A}$	0	0	0	0	0	0	0	0	0	0	0	0	-A	$-\overline{A}$	0	0	0	0
χ_{37}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
χ_{38}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
χ_{39}	-1	-1	0	0	0	0	R	-R	R	-R	R	-R	0	0	1	1	0	0	0	0
χ_{40}	-1	-1	0	0	0	0	-R	R	-R	R	-R	R	0	0	1	1	0	0	0	0
χ_{41}	$-\overline{A}$	-A	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{A}$	A	0	0	0	0
χ_{42}	-A	$-\overline{A}$	0	0	0	0	0	0	0	0	0	0	0	0	A	$\overline{A}$	0	0	0	0

where A = $\frac{-1-\sqrt{3}i}{2}$, M = $-1-\sqrt{3}i$, R = $-\sqrt{6}i$,
S = $-2\mathrm{E}(24) + \mathrm{E}(24)^{11} - \mathrm{E}(24)^{17} + 2\mathrm{E}(24)^{19}$,
T = $-\sqrt{5}i$, U = $-\mathrm{E}(60)^7 + \mathrm{E}(60)^{19} + \mathrm{E}(60)^{31} - \mathrm{E}(60)^{43}$

Table 4. The Character Table of $7^4{:}(3 \times 2S_7)$ (continued)

$[g]_G$	21A	21B	24A	24B	24C	24D	24E	24F	24G	24H	24I	24J	30A	30B	42A	42B	60A	60B	60C	60D
$[x]_{\overline{G}}$	21H	21I	24A	24B	24C	24D	24E	24F	24G	24H	24I	24J	30A	30B	42E	42F	60A	60B	60C	60D
χ_{43}	-$\overline{A}$	-A	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{A}$	A	0	0	0	0
χ_{44}	-A	-$\overline{A}$	0	0	0	0	0	0	0	0	0	0	0	0	A	$\overline{A}$	0	0	0	0
χ_{45}	-$\overline{A}$	-A	0	0	0	0	S	-S	-R	R	-$\overline{S}$	$\overline{S}$	0	0	$\overline{A}$	A	0	0	0	0
χ_{46}	-$\overline{A}$	-A	0	0	0	0	-S	S	R	-R	$\overline{S}$	-$\overline{S}$	0	0	$\overline{A}$	A	0	0	0	0
χ_{47}	-A	-$\overline{A}$	0	0	0	0	$\overline{S}$	-$\overline{S}$	R	-R	-S	S	0	0	A	$\overline{A}$	0	0	0	0
χ_{48}	-A	-$\overline{A}$	0	0	0	0	-$\overline{S}$	$\overline{S}$	-R	R	S	-S	0	0	A	$\overline{A}$	0	0	0	0
χ_{49}	0	0	1	1	-1	-1	1	1	1	1	1	1	1	1	0	0	-1	-1	-1	-1
χ_{50}	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	0	0	1	1	1	1
χ_{51}	0	0	$\overline{A}$	A	-A	-$\overline{A}$	$\overline{A}$	$\overline{A}$	1	1	A	A	$\overline{A}$	A	0	0	-A	-A	-$\overline{A}$	-$\overline{A}$
χ_{52}	0	0	A	$\overline{A}$	-$\overline{A}$	-A	A	A	1	1	$\overline{A}$	$\overline{A}$	A	$\overline{A}$	0	0	-$\overline{A}$	-$\overline{A}$	-A	-A
χ_{53}	0	0	-$\overline{A}$	-A	-A	-$\overline{A}$	-$\overline{A}$	-$\overline{A}$	-1	-1	-A	-A	$\overline{A}$	A	0	0	A	A	$\overline{A}$	$\overline{A}$
χ_{54}	0	0	-A	-$\overline{A}$	-$\overline{A}$	-A	-A	-A	-1	-1	-$\overline{A}$	-$\overline{A}$	A	$\overline{A}$	0	0	$\overline{A}$	$\overline{A}$	A	A
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0
χ_{56}	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{M}$	M	0	0	0	0	0	0
χ_{57}	0	0	0	0	0	0	0	0	0	0	0	0	M	$\overline{M}$	0	0	0	0	0	0
χ_{58}	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
χ_{59}	0	0	-1	-1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0
χ_{60}	0	0	$\overline{A}$	A	A	$\overline{A}$	$\overline{A}$	$\overline{A}$	1	1	A	A	0	0	0	0	0	0	0	0
χ_{61}	0	0	A	$\overline{A}$	$\overline{A}$	A	A	A	1	1	$\overline{A}$	$\overline{A}$	0	0	0	0	0	0	0	0
χ_{62}	0	0	-$\overline{A}$	-A	A	$\overline{A}$	-$\overline{A}$	-$\overline{A}$	-1	-1	-A	-A	0	0	0	0	0	0	0	0
χ_{63}	0	0	-A	-$\overline{A}$	$\overline{A}$	A	-A	-A	-1	-1	-$\overline{A}$	-$\overline{A}$	0	0	0	0	0	0	0	0
χ_{64}	1	1	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	T	-T	T	-T
χ_{65}	1	1	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-T	T	-T	T
χ_{66}	$\overline{A}$	A	0	0	0	0	0	0	0	0	0	0	-$\overline{A}$	-A	-$\overline{A}$	-A	U	-U	-$\overline{U}$	$\overline{U}$
χ_{67}	$\overline{A}$	A	0	0	0	0	0	0	0	0	0	0	-$\overline{A}$	-A	-$\overline{A}$	-A	-U	U	$\overline{U}$	-$\overline{U}$
χ_{68}	A	$\overline{A}$	0	0	0	0	0	0	0	0	0	0	-A	-$\overline{A}$	-A	-$\overline{A}$	-$\overline{U}$	$\overline{U}$	U	-U
χ_{69}	A	$\overline{A}$	0	0	0	0	0	0	0	0	0	0	-A	-$\overline{A}$	-A	-$\overline{A}$	$\overline{U}$	-$\overline{U}$	-U	U
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{72}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{73}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{75}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{76}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{82}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{84}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{85}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

where A = $\frac{-1-\sqrt{3}i}{2}$, M = $-1-\sqrt{3}i$, R = $-\sqrt{6}i$,
S = $-2\mathrm{E}(24)+\mathrm{E}(24)^{11}-\mathrm{E}(24)^{17}+2\mathrm{E}(24)^{19}$,
T = $-\sqrt{5}i$, U = $-\mathrm{E}(60)^{7}+\mathrm{E}(60)^{19}+\mathrm{E}(60)^{31}-\mathrm{E}(60)^{43}$

References

[1] A. B. M. Basheer and J. Moori, *On a Maximal Subgroup of the Affine General Linear Group of $GL(6,2)$*, Adv. Group Theory Appl., **11** (2021), 1-30.

[2] A. B. M. Basheer and J. Moori, *A survey on Clifford-Fischer theory*, London Mathematical Society Lecture Notes Series, **422**, Cambridge University Press (2015), 160–172.

[3] W. Bosma and J.J. Canon, *Handbook of Magma Functions*, Department of Mathematics, University of Sydney, November 1994.

[4] J.H. Conway, R.T. Curtis, S.P. Norton, R.A. Parker, and R.A. Wilson, *Atlas of Finite Groups*, Oxford University Press, Oxford, 1985.

[5] B. Fischer, *Clifford-matrices*, Progr. Math. **95**, Michler G.O. and Ringel C.(eds), Birkhauser, Basel (1991), 1 - 16.

[6] The GAP Group, *GAP --Groups, Algorithms, and Programming*, Version 4.11.0, 2020. (http://www.gap-system.org).

[7] D. Gorenstein, *Finite Groups*, Harper and Row Publishers, New York, 1968.

[8] G. Karpilovsky, *Group Representations: Introduction to Group Representations and Characters*, Vol 1 Part B, North - Holland Mathematics Studies 175, Amsterdam, 1992.

[9] J. Moori, *On certain groups associated with the smallest Fischer group*, J. London Math.Soc., **2** (1981), 61-67.

[10] J. Moori and Z.E. Mpono, *The Fischer-Clifford matrices of the group* $2^6{:}SP_6(2)$, Quaest. Math., **22** (1999), 257-298.

[11] J. Moori and T. Seretlo, *On the Fischer-Clifford matrices of a maximal subgroup of the Lyons group* Ly, Bull. Iranian Math. Soc., *39*(5) (2013), 1037–1052.

[12] Z. Mpono, *Fischer-Clifford Theory and Character Tables of Group Extensions*, PhD Thesis, University of Natal, Pietermaritzburg, 1998.

[13] D. M. Musyoka, L. N. Njuguna, A. L. Prins and L. Chikamai, *On a maximal subgroup* $\overline{G} = 5^4{:}((3 \times 2L_2(25)){:}2_2)$ *of the Monster* $\mathbb{M}$, Italian Journal of Pure and Applied Mathematics, accepted for publication.

[14] D. M. Musyoka, L. N. Njuguna, A. L. Prins and L. Chikamai, *On a maximal subgroup of the orthogonal group* $O_8^+(3)$, Proyecciones, **41**(1) (2022), 161-185.

[15] A.L. Prins, *On a two-fold cover* $2.(2^{6\cdot}G_2(2))$ *of a maximal subgroup of Rudvalis group* Ru, Proyecciones, **40**(4) (2021), 1011-1029.

[16] A.L. Prins, *A maximal subgroup* $2^{4+6}{:}(A_5 \times 3)$ *of* $G_2(4)$ *treated as a non-split extension* $\overline{G} = 2^{6\cdot}(2^4{:}(A_5 \times 3))$, Adv. Group Theory Appl., **10** (2020), 43-66.

[17] A.L. Prins, R.L. Monaledi and R.L. Fray, *On a maximal subgroup* $(2^9{:}L_3(4)){:}3$ *of the automorphism group* $U_6(2){:}3$ *of* $U_6(2)$, Afr. Mat., **31** (2020), 1311-1336.

[18] A.L. Prins, *Computing the conjugacy classes and character table of a non-split extension* $2^{6\cdot}(2^5{:}S_6)$ *from a split extension* $2^6{:}(2^5{:}S_6)$, AIMS Math., **5**(3) (2020), 2113-2125.

[19] A.L. Prins, *Fischer-Clifford theory applied to a non-split extension group* $2^{5\cdot}GL_4(2)$, Palest. J. Math., **5**(2) (2016) , 71-82.

[20] T.T. Seretlo *Fischer Clifford Matrices and Character Tables of Certain Groups Associated with Simple Groups* $O_{10}^+(2)$, HS *and* Ly, PhD Thesis, University of KwaZulu Natal, 2011.

[21] N.S. Whitley, *Fischer Matrices and Character Tables of Group Extensions*, MSc Thesis, University of Natal, Pietermaritzburg, 1994.

[22] R.A. Wilson, P. Walsh, J. Tripp, I. Suleiman, S. Rogers, R. Parker, S. Norton, S. Nickerson, S. Linton, J. Bray and R. Abbot, *ATLAS of Finite Group Representations*, http://brauer.maths.qmul.ac.uk/Atlas/v3/.

Author information

David Mwanzia Musyoka, Department of Mathematics and Actuarial Science, Kenyatta University, PO Box 43844 - 00100, Nairobi, Kenya.
E-mail: davidmusyoka21@yahoo.com

Lydia Nyambura Njuguna, Department of Mathematics and Actuarial Science, Kenyatta University, PO Box 43844 - 00100, Nairobi, Kenya.
E-mail: njuguna.lydia@ku.ac.ke or lydiahnjuguna@yahoo.com

Abraham Love Prins, Department of Mathematics and Applied Mathematics, Nelson Mandela University, PO Box 77000, Gqeberha, 6031, South Africa.
E-mail: abraham.prins@mandela.ac.za or abrahamprinsie@yahoo.com

Lucy Chikamai, Department of Mathematics and Actuarial Science, Kibabii University, PO Box 1699 - 50200, Bungoma, Kenya.
E-mail: chikamail@kibu.ac.ke or lucychikamai@gmail.com

Received: January 3, 2022.
Accepted: April 7, 2022.