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A Study on Avoiding RFI in the Movement of Robots via Radio Resolving Number Problem

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Abstract Let $G = (V, E)$ be a connected graph with diameter d and order n. A non-empty subset X of V is called a resolving set if for each pair $u,\ v \in V$ such that $u \neq v$, then there is a vertex x in X satisfies $d(x, u) \neq d(x, v)$. The minimum cardinality of all such resolving sets is called the *resolving number* of G. Let X be a non-empty minimum cardinality resolving set of G. An injection $\sigma : V\left(G\right) \to N$ is said to be a *radio resolving labelling* if $d\left(u, x\right)+|\sigma\left(u\right) - \sigma(x)| \geq 1 + d,\ \ \forall\, u \in V\backslash X\ ,\ x \in X$, where $d\left(u, x\right)$ is the distance between u and x. The radio resolving number of σ denoted $r\beta(\sigma)$ is the biggest number labelled under the mapping σ. The minimum taken over all $r\beta(\sigma)$ is called *radio resolving number*, denoted by $r\beta\left(G\right)$. If $r\beta\left(G\right) = n$, then G is called radio resolving graceful graph. In this research article, we introduce the concept of radio resolving number and present some results connecting the radio resolving number with resolving number and radio number. Further, we investigate the exact radio resolving number for complete graph, path, cycle, star graph, complete $k-$partite graph, and complete binary tree.

1 Introduction

With the exceptional capabilities of 5G, it virtually connects everything together including objects, communication devices, and machines. Due to the introduction of 5G in the market place, the efficiency of robotic process automation (RPA) has been enhanced in several ways. Today in the world of modern technology, the robotic systems are used in the field of Health care, vehicle automation, and smart industries. In the medical field, RPA plays a major role in urologic, cardiothoracic, neck, head, gynaecologic, and general surgeries [1]. Due to the growing needs of robots in medical field, nowadays the designers and manufacturers of robots are focusing on enhancing the robotic capabilities to meet the same. Robotic surgery systems are computer-controlled, and therefore, sensitive electronic components must be shielded from radio frequency interference (RFI). Radio frequency interference (RFI) or electromagnetic interference (EMI) happens when an electromagnetic field of a natural source or device interferes with another system or device, resulting in both of them getting distorted [2, 3]. These problems make the robots to act unexpectedly such as, moving unintentionally, showing a limited radio frequency range, affecting other nearby robots, and requiring frequent restarts. Hence, the robotic developers must understand the radio frequency interference undoubtedly and take appropriate steps to prevent it. To avoid this frequency interference between the robot, we introduce the concept of radio resolving number problem which helps monitor the robot's positions and avoid the radio frequency interference from other devices.

Samir et al. [4] described the navigation of robots in a graph-structured framework, which can move from a verted to vertex of a graph-space. It can navigate itself by sensing the distances to a set of labelled vertices of resolving set (landmark nodes) in the graph-space. Evidently, if the robot knows its distances to a large set of landmarks, its location on the graph is uniquely determined. A smallest non-empty resolving set which uniquely determines the robot's location is called a minimum resolving set and its cardinality is called the resolving number or minimum metric dimension of a graph. The graph theoretical definition of the above problem was first formulated by Harary et al. [5] as follows: Let $G = (V, E)$ be a connected graph with diameter

d and order n. A non-empty subset X of V is called a resolving set if for each pair $u,\ v \in V$ such that $u \neq v$, then there is a vertex x in X satisfies $d(x,u) \neq d(x,v)$. The minimum cardinality of all such resolving sets is called the *resolving number* of G. It is denoted by dim(G) or $\beta(G)$. This problem and its application have been studied in various fields such as pharmaceutical chemistry [6], combinatorial optimization [7], and robot navigation [4].

Avoiding the radio frequency interferences between the transmitters on assigning the channels to the radio stations was extensively studied in [8, 9] as a radio labelling problem. This radio labelling problem was formulated from the real-life problem of frequency assignment to radio transmitters based on the shortest distance between the radio transmitters. According to the pre-established bandwidth assigned by the federal radio commission, if there is a large network of transmitters spread out in a geographical area, then the channel assignment problem is to assign a frequency (numerical value) to each transmitter without co-channel interference. To avoid such interferences, few separation constraints must be satisfied between the channels allotted to nearby radio transmitters. The main goal of this problem is to maximize the number of channels in a geographical area by minimizing the usage of allotted portion of frequency bandwidth without radio frequency interference. In 2001, Chartrand et al. [8] was fascinated by this frequency assignment problem and introduced a classical graph vertex-labelling problem called radio number which includes the difference between frequencies and the distance between the transmitters as its parameters. A radio labelling σ of a connected graph $G = (V, E)$ with diameter d is a mapping of distinct natural numbers to $V(G)$ satisfying the radio condition $|\,\sigma\,(u)\ \ \sigma\ \ (w)| + d\,(u,w)\ \geq\ 1 + d,\ \forall\ u,\ \ w\ \ \in\ V\,(G)$. The maximum number assigned to any vertex of G under the mapping σ is called the *radio number of* σ and its denoted by $rn\,(\sigma)$. The minimum taken overall labellings of G denoted $rn(G)$, is called the *radio number of* G. This problem is also proved as NP-complete problem [10] and its application to channel allocation in radio networks has been broadly studied in [11, 12].

Using both the concepts of resolving number and the radio number, we are introducing a new vertex labelling technique called radio resolving number as follows: Let $G = (V, E)$ be a graph with order n and diameter d. Let X be a non-empty minimum cardinality resolving set of G. An injection $\sigma : V\,(G) \rightarrow N$ is said to be a *radio resolving labelling* if $d\,(u,x) + |\sigma\,(u) - \sigma(x)| \geq 1 + d,\ \ \forall\, u \in V \backslash X\ ,\ x \in X$, where $d\,(u,x)$ is the distance between u and x. The radio resolving number of σ denoted $r\beta(\sigma)$ is the biggest number labelled under the mapping σ. The minimum taken over all $r\beta(\sigma)$ is called *radio resolving number*. It is denoted by $r\beta\,(G)$. If $r\beta\,(G) = n$, then G is called radio resolving graceful graph. The following example in Figure 1, illustrates the definition of radio resolving number of a graph G:

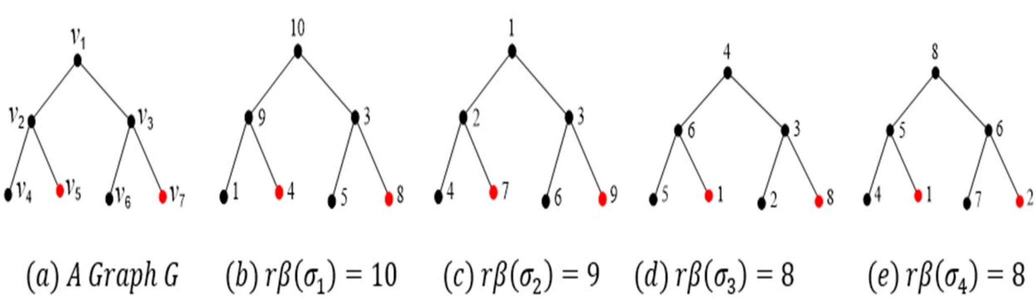

Figure 1. A graph G and its different radio resolving labellings

In Figure 1(a), we have given a graph G with $V = \{v_1, v_2 \ldots v_7\}$ and $X = \{v_5, v_7\}$ as its vertex set and resolving set, respectively. Also, from Figure 1(b) to 1(e) the different radio resolving labelling's for G and their radio resolving number under the mappings are illustrated. From the definition, radio resolving number of G is obtained by finding the minimum value among all $r\beta(\sigma)$. That is, $r\beta(G) = \min\{r\beta(\sigma_1), r\beta(\sigma_2), r\beta(\sigma_3), r\beta(\sigma_1)\} = \{10, 9, 8, 8\} = 8$. For this particular example, $r\beta(G) > n$, where n is the number of vertices. For solving this problem, we first need to find the minimum resolving set. However, by the method of reduction from 3-dimensional matching problem, Garey et al. [13] proved that finding the minimum resolving set for general graphs is NP-complete. Hence, computing the radio resolving number for general graph is a non-trivial problem.

2 Application of Radio Resolving Number

As more medical companies seek robots for increasingly complex tasks of robotic-assisted surgeries, robotic systems developers tend to build these robots with a sophisticated robotic control and tasking software without radio frequency interference. Motion Control software in robotic system enables articulated arms to move through the action of rotating and sliding joints, to move through locomotion and steering. This controlled motion enables these complex tasks to be achieved with whatever end effector is appropriate on the robot [1]. In the motion control, the methodology of obtaining the precise location of the robot or its nearby systems are often posing challenge due to several other factors like global positioning system (GPS) limitations or correlating the positioning information with the neighbouring systems. In either scenario, the received radio signals can be used by a robot to infer positioning information, but the process is challenging because of the radio frequency interference [2, 3]. Since the robotic surgery systems are computer-controlled, the sensitive electronic components must be shielded from radio frequency interference. Otherwise, if it gets to interfere with other natural sources or devices and gets distorted, which may result in poor controlled surgery in turn will give unwanted results of the surgical procedures. These motion control and avoiding radio frequency interference of robots are studied in a graph-space using radio resolving number. Let us consider a robot that is navigating in the graph-space and is modelled by a graph $G = (V, E)$. The main goal is to avoid the radio frequency interference between nearby robots or other natural sources and wants to know its current location. By computing the minimum resolving set for a graph, the robot can find its distance from its current location by sending signals to the vertices of the minimum resolving set or landmarks. This process can be done by finding the minimum resolving set as in the first part of the definition of radio resolving labelling. The next goal of avoiding the radio frequency interference of a robot is taken care by the radio labelling condition technique between any other device or natural source position u in $V \backslash X$ to the vertex (a landmark of the robot's position) x of a minimum resolving set X. It is illustrated diagrammatically in Figure 2.

3 General Results on Radio Resolving Number

In this section, we have proved certain general results of radio resolving number with the aid of known theorems. First, we have listed few theorems proved by Chartrand et al. [8, 14] in minimum metric dimension or minimum resolving set.

Theorem 3.1. *If G is a connected graph of order $n \geqslant 2$ and diameter d, then $1 \leq dim(G) \leqslant n-1$.*

Theorem 3.2. *A connected graph G of order n has dimension 1 if and only if $G = P_n$.*

Theorem 3.3. *A connected graph G of order $n \geqslant 2$ has dimension $n-1$ if and only if $G = \mathrm{K_n}$.*

Theorem 3.4. *For the cycle C_n, $dim(C_n) = 2,\ n \geq 3$.*

Theorem 3.5. *A connected graph G of order has dimension $n-2$ if and only if G is a complete bi-patriate graph.*

Theorem 3.6. *If G is a connected graph of order n and diameter d; then $n \leq rn(G) \leq (n-1)\,d$.*

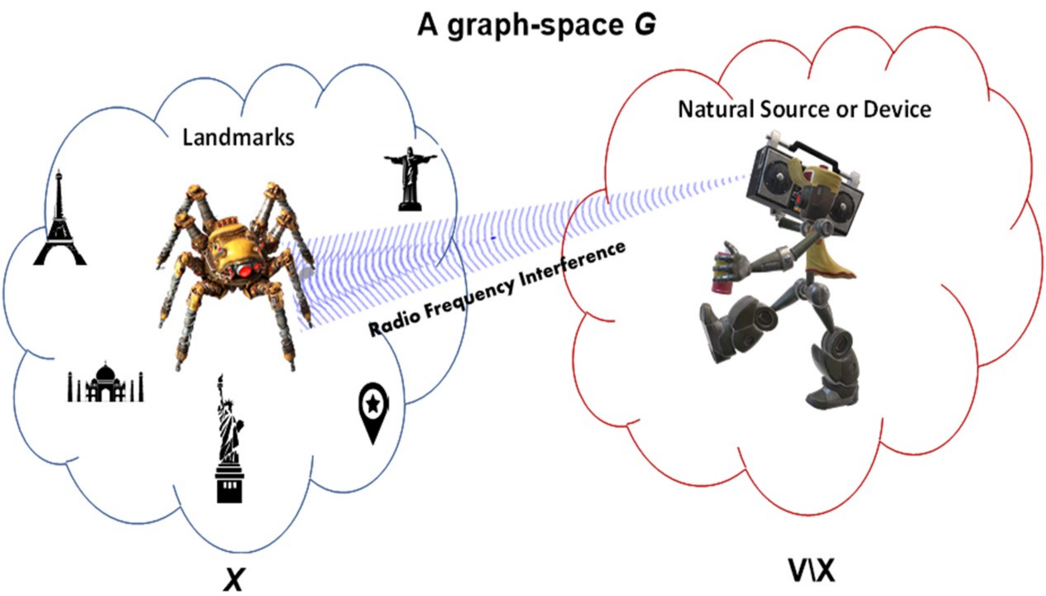

Figure 2. A diagrammatical representation of landmarks and the RFI between the robot and natural source or the device in a graph-space

Theorem 3.7. *If G is a connected graph of diameter d and clique number ω, then $rn(G) \leq 1 + d(\omega - 1)$.*

The following theorem is showed by Saputro et al. [15].

Theorem 3.8. *For a complete $k-$partite graph G of order n, $rn(G) = n + (k-1)$.*

Using the above results, now we are presenting few of the general results connecting the radio number, minimum resolving set and the radio resolving number.

Theorem 3.9. *For any non-trivial connected graph G, $rn(G) \geq r\beta(G)$.*

Proof. We know from the definition of radio labelling, for every distinct pair of vertices in G the radio labelling inequality must be satisfied. But, for radio resolving number the condition is restricted between the vertices of the resolving set X and its complement in G. Hence, $rn(G) \geq r\beta(G)$. □

Theorem 3.10. *A connected graph $G = (V, E)$ of order n satisfies $r\beta(G) \geq n$.*

Proof. As per the definition of radio resolving labelling, it is a mapping from $V(G)$ to the set of distinct natural numbers and hence $r\beta(G) \geq |V(G)| = n$. □

Theorem 3.11. *For any graph G of order n, the radio resolving number is always greater than the resolving number.*

Proof. From the definition of minimum resolving set, it is a proper subset of the vertex set of G. Therefore, the minimum resolving number of G is less than n. Hence, by using Theorem 3.10, the result is proved. □

The next theorem provides an efficient upper bound for the radio resolving number of any graph.

Theorem 3.12. *For any connected graph G of resolving number k and order n, $r\beta(G) \leq n + d - 1$, where d is the diameter of the graph G.*

Proof. Initially, let us assume that the vertices in G are named as $u_1, u_2 \dots u_n$. Let $\{u_{\alpha_1}, u_{\alpha_2} \dots u_{\alpha_k}\} \subseteq \{u_1, u_2 \dots u_n\}$ be a minimum resolving set of G. We now start renaming the $n-k$ vertices in

$\{u_1,\ u_2 \ldots u_n\} \setminus \{u_{\alpha_1},\ u_{\alpha_2} \ldots u_{\alpha_k}\}$ as $v_1,\ v_2 \ldots v_{n-k}$ and the remaining vertices $u_{\alpha_1},\ u_{\alpha_2} \ldots u_{\alpha_k}$ as $v_{n-k+1},\ v_{n-k+2} \ldots v_n$. Define a valid radio resolving labelling σ by assigning first $n-k$ natural numbers to the first $n-k$ vertices as $\sigma(v_i) = i, 1, 2..n-k$. In addition, label the vertex v_{n-k+1} with a difference d, so that it satisfies the radio resolving condition with the vertices in the set $\{v_1,\ v_2 \ldots v_{n-k}\}$. Finally, the vertices $v_{n-k+2},\ v_{n-k+3} \ldots v_n$ are labelled with consecutive numbers so that it trivially satisfies the radio resolving condition. Thus, the maximum number needed to label all the n vertices is at most $n+d-1$. Therefore, we concluded that, $n \leq r\beta(G) \leq\ n+d-1$. □

4 Radio Resolving Number of Certain Graphs

In this section we have investigated the radio resolving number of complete graphs, paths, cycles, complete $k-$partite graphs, star graph and complete binary trees.

Theorem 4.1. *The radio resolving number of path is* $r\beta(P_n) = n,\ n > 1$.

Proof. Let $u_1,\ u_2 \ldots u_n$ be the vertices of the path P_n. It is trivial that $d(u, u_1) \neq d(v, u_1)$ for all $u, v \in \{u_2,\ u_3 \ldots u_n\}$. Using Theorem 3.2, the set $X = \{u_1\}$ becomes a minimum resolving set. Now, we define an injection $\sigma : V(P_n) \to \{1, 2 \ldots n\}$ as $\sigma(u_{i+1}) = i\ ,\ i = 1, 2, ..n-1$ and $\sigma(u_1) = n$. As the diameter of path is $n-1$, we must verify σ is a radio resolving labelling by showing that $d(u, x) + |\sigma(u) - \sigma(x)| \geq n\ \forall\ \ u \in V(P_n),\ x \in X$. Since $|X| = 1$, from the definition of radio resolving number, we can find for each vertex $u \in V \setminus W$ the unique vertex $u_1 \in X$ such that $d(u_{k+1}, x) = k,\ 1 \leq k \leq n-1$ and $|\sigma(u_{k+1}) - \sigma(x)| = |(k-n| = n-k$. Therefore, $d(u_k, x) + |\sigma(u_k) - \sigma(x)| = k+n-k = n$. So, $r\beta(G) \geq n$. But, by Theorem 4.1, $r\beta(G) \geq n$, and hence, we get: $r\beta(P_n) = n,\ n > 1$. □

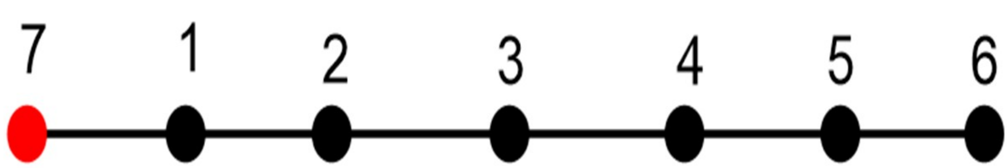

Figure 3. A radio resolving labelling and the minimum radio resolving set of a Path P_7

As the radio resolving number of path is n, we have the following theorem.

Theorem 4.2. *For* $n > 1$*, the path* P_n *is radio resolving graceful.*

Theorem 4.3. *For* $n > 2$*, the complete graph* K_n *is radio resolving graceful.*

Proof. As we know that, the diameter of a complete graph is 1, by using main Theorem, we get $rn(K_n) \leq n+1-1 = n$. Also, by Theorem 3.12, we have, $rn(K_n) \geq n$. Therefore, $rn(K_n) = n$ and hence the complete graph is radio resolving graceful □

Theorem 4.4. *For $n > 3$, the cycle graph C_n is radio resolving graceful.*

Proof. Let us name the vertices of cycle C_n as $w_1,\ w_2 \dots w_n$ in the clockwise sense. By using Theorem 3.4, we can choose the set $X = \{w_1, w_n\}$ as the minimum resolving set for C_n. Now, define an injection $\sigma : V(C_n) \to N$ as $\sigma(w_1) = 1,\ \sigma(w_n) = 2,\ \sigma\left(w_{(1-i)+\lceil \frac{n}{2} \rceil}\right) = 2+i,\ i = 1,2\dots\lceil \frac{n}{2} \rceil - 1,\ \ \sigma\left(w_{\lceil \frac{n}{2} \rceil + i}\right) = \lceil \frac{n}{2} \rceil + 1 + i,\ i = 1,2\dots\lfloor \frac{n}{2} \rfloor - 1$. Since the diameter of cycle is $\lfloor \frac{n}{2} \rfloor$, we must verify that $d(u,x) + |\sigma(u) - \sigma(x)| \geq \lfloor \frac{n}{2} \rfloor + 1\ \forall\ u \in V \backslash X, x \in X$. □

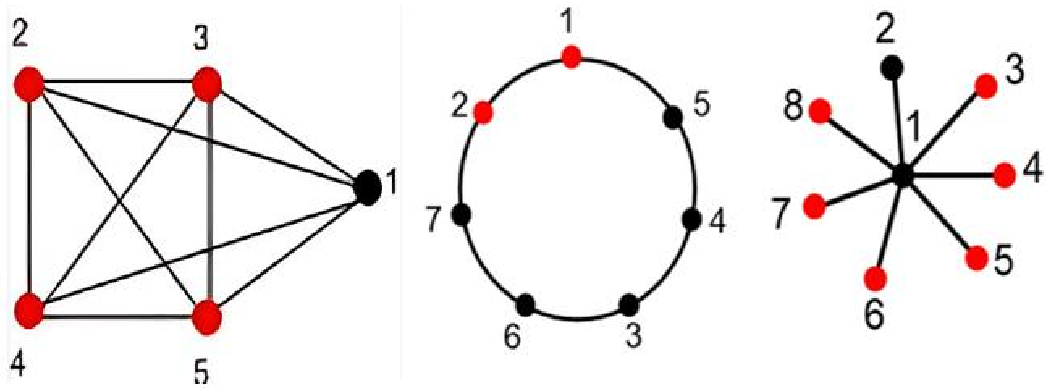

Figure 4. A radio resolving labelling and the minimum radio resolving set a complete graph K_5, a cycle C_7 and a star graph S_9

Case 4.1: Let $u = w_{\lceil \frac{n}{2} \rceil - k+1},\ 1 \leq k \leq \lceil \frac{n}{2} \rceil - 1$ be a vertex in $V \backslash X$. If $x = w_1$, then $d\left(w_1,\ w_{\lceil \frac{n}{2} \rceil - k+1}\right) = \lceil \frac{n}{2} \rceil - k$ and $|\sigma(u) - \sigma(x)| \geq |1 - (2+k)| = k+1$. Also, if $x = w_n$, then $d\left(w_n,\ w_{\lceil \frac{n}{2} \rceil - k+1}\right) = \lfloor \frac{n}{2} \rfloor + 1 - k$ and $|\sigma(u) - \sigma(x)| \geq |2 - (2+k)| = k$. Therefore, in both the possibilities, $d(u,x) + |\sigma(u) - \sigma(x)| \geq \lfloor \frac{n}{2} \rfloor + 1$.
Case 4.2: Assume $u = w_{\lceil \frac{n}{2} \rceil + k}$ be a vertex in $V \backslash X$, where $1 \leq k \leq \lfloor \frac{n}{2} \rfloor - 1$. If we choose w_1 in X, then $d\left(w_1,\ w_{\lceil \frac{n}{2} \rceil + k}\right) = \lceil \frac{n}{2} \rceil - k$ and $|\sigma(u) - \sigma(x)| \geq \left|1 - \left(\lceil \frac{n}{2} \rceil + 1 + k\right)\right| = k + \lceil \frac{n}{2} \rceil$. Again, if $x = w_n$, then $d\left(w_n,\ w_{\lceil \frac{n}{2} \rceil - k+1}\right) = \lfloor \frac{n}{2} \rfloor - k$ and $|\sigma(u) - \sigma(x)| \geq \lfloor 2 - \left(\lceil \frac{n}{2} \rceil + 1 + k\right) \rfloor = k + \lceil \frac{n}{2} \rceil - 1$. Consequently, in both the subcases, $d(u,x) + |\sigma(u) - \sigma(x)| \geq \lfloor \frac{n}{2} \rfloor + 1$. Hence the radio resolving labelling condition is true for any pair of vertices in the cycle C_n.
Thus, $r\beta(C_n) \leq n$. Further, applying Theorem 3,12, we have attained the result, $r\beta(C_n) = n$. Consequently, the cycle C_n is radio resolving graceful.

Theorem 4.5. *For the star graph $S_{n+1} = K_{1,n}$, the radio resolving number is $n+1,\ \ n > 1$.*

Proof. Let w_1 be the vertex of degree n in S_{n+1} and the remaining n vertices of degree one are named as $w_2, w_3 \dots w_{n+1.}$ Choose the minimum resolving set for S_{n+1} as $\{w_3, w_4 \dots w_{n+1.}\} = X$. Define a mapping σ from $\{w_1, w_2 \dots w_{n+1.}\}$ to N as $\sigma(w_i) = i,\ i = 1,2..n+1$. For this mapping it is easy to verify the radio resolving condition. Hence, $r\beta(S_{n+1}) = n+1$ □

Theorem 4.6. *Let $n_1 \leq n_2 \leq \dots \leq n_k$ be the number of vertices in the $k-$partitions of the complete $k-$partite graph $K_{n_1,n_2\dots n_k}$. Then, the graph $K_{n_1,n_2\dots n_k}$ is radio resolving graceful.*

Proof. Let $n = n_1 + n_2 + \dots + n_k$ be the number of vertices in $K_{n_1,n_2\dots n_k}$. First, we name the vertices of $K_{n_1,n_2\dots n_k}$ as $\left\{w_i^j,\ \ i = 1,2\dots n_j,\ \ j = 1,2\dots .k\right\}$. If we apply the same concept as in Theorem 2.8, for the complete $k-$patite graph we can able to find a minimum resolving set with cardinality $n-k$. Let $X = \left\{\left\{w_{i+1}^j \backslash i = 1,\ 2\dots n_j - 1, j = 1,2\dots .k\right\}\right\}$ be the minimum resolving set. Now, we define an injection σ from the vertex set of complete $k-$partite graph to the set of distinct natural numbers is as follows: $\sigma\left(w_1^j\right) = j,\ \ j = 1, 2 \ldots k$ and $\sigma\left(w_{i+1}^j\right) = i + k - j + 1 + \sum_{r=1}^{j-1} n_r,\ \ i = 1, 2 \ldots n_j - 1, j = 1, 2 \ldots .k$. Next, we prove that σ is a radio resolving labelling by verifying the condition $d\,(u, x) + |\sigma\,(u) - \sigma(x)| \geq 3\ \forall\ u \in V\backslash X, x \in X$. Since $n_1 > 1$ and $k > 1$, the graph contains al teast four vertices and so the greatest number labelled in the graph is atleast 4. Suppose that $u \in V\backslash X$, then u is of the form $w_1^t,\ 1 \leq\ t \leq k$. If x is of the form $w_{s+1}^t,\ \ 1 \leq\ s \leq n_t - 1,\ 1 \leq\ t \leq k,$ then the distance between u and x is 2 and$|\sigma\,(u) - \sigma(x)| \geq 1$. Otherwise, since $n \geq 4$ the modulus difference between $\sigma\,(u)$ and $\sigma\,(x)$ is at least 2. Thus, in both the possibilities, the inequality for the radio resolving labelling is satisfied. Hence, $r\beta\,(K_{n_1,n_2\ldots n_k}) = n_1 + n_2 + \cdots + n_k$, whenever $n_1,\ k > 1$. Since the radio resolving number of $K_{n_1,n_2\ldots n_k}$is equal to the cardinality of the vertex set, it is radio resolving graceful. □

Corollary 4.7. *For $m, n > 1$, the radio resolving number of the complete bi-partite graph $K_{m,n}$ is $m + n$.*

Proof. Take $m = n_1$ and $n = n_2$ in the main Theorem, we get $r\beta\,(K_{m,n}) = m + n$. □

Harary et al. [5] proved the following results for the trees.

Theorem 4.8. *Let L be the set of leaf nodes and P be the set of vertices that have degree at least three and that are connected by paths of degree-2 vertices to one or more leaves in a tree T, then the metric dimension is $|L|\ -\ |P|$.*

Further, Harary et al. [5] explained that the minimum resolving set may be obtained by removing one of the leaves from *L* that associated with each vertex in *P.*

A *binary tree* is particular type of tree in which it starts with a single node (vertex) called root node and each node has at most two children. If such children exist, then they are called left and right nodes respectively. If all the leaves in a binary tree will have the same distance from the root vertex, then such a binary tree is called a *complete binary tree.* We denote it by $BT\,(\xi\ )$, where ξ is the distance from the leaf node to the root node.

Remark 4.9. In the complete binary tree, the minimum resolving set can be formed by choosing half of the leaf vertices alternatively. Since, there are 2^ξ leaves in $BT\,(\xi\ )$ is 2^ξ, the resolving number for complete binary tree is equal to $2^{\xi-1}$. See Figure 4.

Remark 4.10. We note that the number of levels in $BT\,(\xi)$ is $\xi + 1$ and the cardinality of $V(BT\,(\xi))$ in k^{th} level is $2^{k-1}, 1 \leq k \leq \xi + 1$. Therefore, the total number of vertices $BT\,(\xi)$ is $1 + 2 + 2^2 + \cdots + 2^\xi = 2^{\xi+1} - 1$. In addition, the diameter of $BT\,(\xi)$ is 2ξ.

Remark 4.11. In this paper, we are naming the vertices of $BT\,(\xi)$ starting from the root vertex as v_1, then the second level from left to right as $v_2,\ v_3$, continue in the same manner, we name the k^{th} level vertices as $v_{2^{k-1}},\ v_{2^{k-1}+1} \ldots v_{2^{k-1}+2^{k-1}-1}$. Finally, the leaf vertices in the $\xi + 1^{th}$ level are named as $v_{2^\xi},\ v_{2^\xi+1} \ldots v_{2^{\xi+1}-1}$. We denote the right and left components formed by the deletion of the root vertex as $LBT\,(\xi)$left and $RBT\,(\xi)$ respectively.

Theorem 4.12. *Let $BT\,(\xi)$ be the complete binary tree, where ξ is the distance from the leaf vertex to the root vertex. If $\xi > 3$, then the radio resolving number of complete binary tree is $r\beta\,(BT\,(\xi)\ \ ) = 2^{\xi+1} - 1$.*

Proof. Let $X = \left\{v_{2^\xi+2j-1}\backslash\ j = 1, 2..\,, 2^{\xi-1}\right\}$ be the minimum resolving set. Define an injection σ from $V\,(BT\,(\xi))$ to N as follows: $\sigma\,(v_j) = j,\ \ \ j = 1, 2 \ldots 2^\xi - 1,\ \ \sigma\left(v_{2^\xi+2(j-1)}\right) = j - 1{+}2^\xi,\ \ j = 1, 2 \ldots 2^{\xi-1},\ \sigma\left(v_{2^\xi+2j+1}\right) = 2^\xi + 2^{\xi-1} + j - 1,\ \ j = 1, 2 \ldots 2^{\xi-1}$. Next, we claim that $d\,(u, x) + |\sigma\,(u) - \sigma\,(x)| \geq 2\xi + 1,\ \forall\ u \in V\backslash X$ and $x \in X$. Let u be an arbitrary vertex in $V\backslash X$. □

Case 4.3: Suppose u lies in $RBT\,(\xi)$, then $\sigma\,(u) \leq 2^\xi - 1$.
Case 4.3.1: For any $x \in X$ in $LBT\,(\xi)$, then the distance between u and x is at least $\xi + 1$ and $\sigma\,(x) \geq 2^\xi{+}2^{\xi-1}$. Therefore, the radio resolving conditions becomes, $d\,(u, x){+}|\sigma\,(u) - \sigma\,(x)| \geq (\xi + 1) + \left|(2^\xi - 1) - (2^{\xi-1} + 2^\xi)\right| \geq 2\xi + 1.$

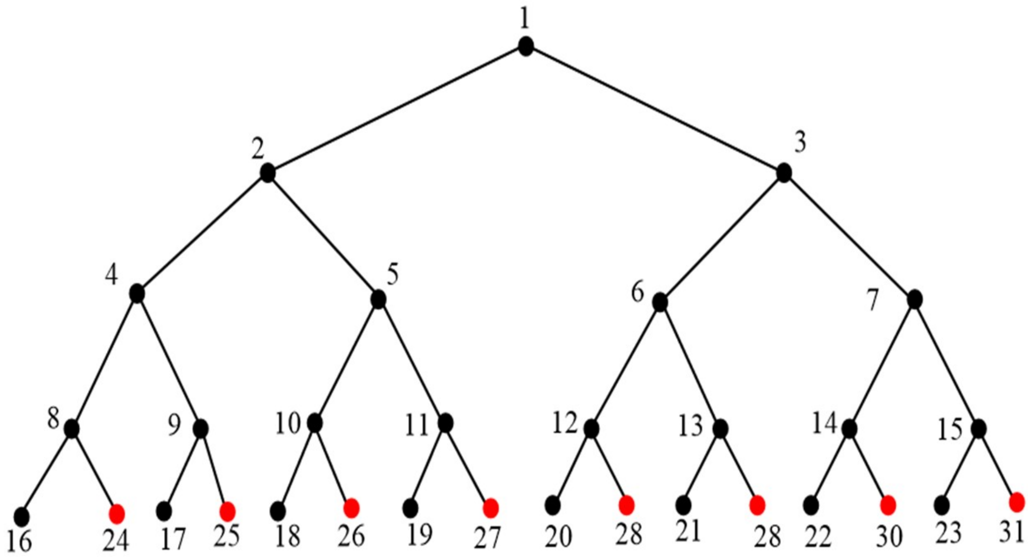

Figure 5. A radio resolving labelling of complete binary tree $BT\,(4)$

Case 4.3.2: If $x \in X$ in $RBT\,(\xi)$, then $d(u,x) \geq 1$ and $\sigma\,(x) \geq 2^{\xi} + 2^{\xi-1} + 2^{\xi-2}$. So, $d\,(u,x) + |\sigma\,(u) - \sigma\,(x)| \geq 1 + \left|2^{\xi} - 1 - \left(2^{\xi} + 2^{\xi-1} + 2^{\xi-2}\right)\right| \geq 2\xi + 1,$ since $\xi > 3$.
Case 4.4: Assume $u \in LBT\,(\xi)$, then $\sigma\,(u) \leq 2^{\xi} - 2^{\xi-2} - 1$.
Case 4.4.1: Let $x \in X$ in $LBT\,(\xi)$, then $\sigma\,(x) \geq 2^{\xi} + 2^{\xi-1}$. If u is a leaf vertex of the form $v_{2^{\xi}+2(k-1)}$, $1 \leq k \leq 2^{\xi-2}$, then $\sigma\,(u) = 2^{\xi} + k - 1$ the distance between u and x is $2\,(k-1)$ and hence $d\,(u,x) + |\sigma\,(u) - \sigma\,(x)| \geq 2\xi + 1$. Otherwise, the condition is obviously true.
Case 4.4.2: If $x \in X$ in $RBT\,(\xi)$, then as in case 1.1, we can easily verify the condition holds.
Case 4.5: If u is the root vertex, then the distance between u and x is $\xi + 1$ and $|\sigma\,(u) - \sigma\,(x)| \geq 2^{\xi} + 2^{\xi-1}$. Hence, the radio resolving labelling condition is verified. Thus, $r\beta\,(BT\,(\xi)\ ) = 2^{\xi+1} - 1,\ \ \xi > 3$.

5 Conclusion

In this paper, we have introduced a new vertex labelling problem called radio resolving number that is used to find the current position of a robot in the graph-space. It also avoids the radio frequency interference between the robot and natural source or any other devices. In addition, certain theorems have been proven connecting the radio number and resolving number for any connected graph G. Moreover, we have completely determined the radio resolving number of certain graphs such as complete graphs, cycles, paths, star graph, complete $k-$partite graphs, and complete binary trees. Furthermore, this new research topic can be extended into the other classes of graphs and interconnection networks which will be helpful in the filed of applications involving robotic process automation with other 5G application areas such as vehicle automation, massive communication systems, mission critical applications, and smart IoT applications.

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