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AN ANALYSIS OF CUSTOMER PREFERENCES OF AIRLINES BY MEANS OF DYNAMIC APPROACH TO LOGARITHMIC SIMILARITY MEASURES FOR PYTHAGOREAN FUZZY SETS

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Abstract The airline industry is currently the world's largest mode of transportation. It focuses on a service approach to attract customers, as providing excellent service is a means to earn customer loyalty. The goal of this research is to find the best airline in terms of service quality, which includes tangibility, trustworthiness, and sensitivity. Pythagorean fuzzy sets (PFSs) proposed by Yager [55, 56] is a significant tool for displaying ambiguous and vague information. The goal of this investigation is to broaden application of logarithmic similarity measures under PFSs. Novel logarithmic similarity measures for PFSs are developed. Numerical computations have been carried out to validate our proposed measures. Application of logarithmic similarity measures have been applied to some real-life decision-making problems of customer satisfaction to airlines in India using PFSs based on PFSs. Moreover, comparison of the result for the proposed measures has been carried out with the existing analogous similarity measures to show the efficacy.

1 Introduction

Transportation services have become a community's basic requirement for both daily activity and travel. Most individuals choose air transportation for long-distance travel since it is more efficient and effective in terms of time. Air transport can reach regions that other forms of transportation, such as land and sea, cannot, as well as move faster and have a straight, sensible path (Setiani [43]). The intuitionistic fuzzy sets (IFSs) presented by Atanassov [1] which is an augmentation of fuzzy set theory suggested by Zadeh (1965) where there is membership degree (δ) and a non-membership degree (ζ) such that $\delta + \zeta \leq 1$. By now, there have been remarkable outcomes on IFSs in both hypothesis and practical (Garg [11]; Hung and Yang [18]; Hwang *et al.* [20, 21]; Ngan *et al.* [32]; Nguyen and Nguyen [33]; Peng *et al.* [37]; Szmidt and Kacprzyk [47]; Thao [50]; Thao, *et al.* [51]; Ye [59, 60]; Zhu and Li [71]). There are instances in IFSs when $\delta + \zeta \geq 1$, Pythagorean fuzzy sets (PFSs) plays a vital role in eradicating this constraint by improving the modelling capacity of $\delta + \zeta \leq 1$ or $\delta + \zeta \geq 1$ such that $\delta^2 + \zeta^2 + \eta^2 = 1$, where η is the degree of indeterminacy.

The possibility of PFSs proposed by Yager [56, 57] is another apparatus to manage ambiguity considering the membership degree, δ and non-membership degree, ζ so that the amount of the square of every one of the membership grades and the non-membership grade is not exactly or equivalent to one, distinct in IFSs. The beginning of PFSs radiated from IFSs of second kind as presented by Atanassov [2]. The idea of PFSs can be utilized to describe vague data more adequately and precisely than IFSs. Garg [12] introduced an improved ranking order interval valued PFSs using TOPSIS technique. Indeed, the hypothesis of PFSs has been widely considered, as demonstrated by various researchers (Garg [13]; Liang and Xu [26]; Peng and Yang [36]). In association with the uses of PFSs, Rahman *et al.* [40] worked on some aggregation operators on interval valued PFSs and utilized it in the decision-making process. Rahman *et al.* [41] proposed a few ways to deal with multi-attribute group decision making. Overall, the possibility of PFSs has pulled in incredible considerations of numerous researchers, and the idea has been applied to a few application regions viz; aggregation operators, multicriteria decision-making, information measures and many more (Gao and Wei [10]; Rahman and Abdullah [42]; Khan *et al.* [23]; Yager and Abbasov [58]; Du *et al.* [6]; Garg [11], Yager [57]; Zhang *et al.* [69], Ejegwa [8]).

The description of similarity/distance measure between two objects is one of the most fascinating issues in PFSs theory. A similarity measure is described to assess the information borne by PFSs. Measures of similarity between PFSs is an essential device for decision making, pattern recognition, machine learning, and image processing in recent times (Bustince *et al.* [3, 4]; Hung and Yang [19]; Li and Cheng [24]; Zhang and Xu [69]; Ejegwa [7]. Some formulae of Pythagorean fuzzy information measures on similarity measures and corresponding transformation relationships were also developed (Peng *et al.* [37, 35]; Li *et al.* [25]). Similarity measures for trigonometric function for FSs, IFSs and PFSs were also proposed (Taruna *et al*. [48]; Shi and Ye [44]; Ye [60, 64]; Tian [49]; Rajarajeswari and Uma [39]; Wei and Wei [53]; Mohd and Abdullah [29]; Immaculate *et al.* [22]; Mondal and Pramanik [30]). The similarity measures of the IFSs and PFSs are widely used in various fields, comparable to the pattern recognition (Peng and Garg [35]; Song *et al*. [46]; Gong [17]; Zhang *et al.* [70]), the clinical finding (Muthukumar and Krishnan [31]; Son and Phong [45]; Wei *et al.* [52]; Hung and Wang [19]; Maoying [28]), decision-making (Ye [61, 62, 63]; Chen *et al*. [5]; Xu [54]; Zhang [67]; Zhang *et al.* [68]; Ejegwa [9]). However, Lu and Ye [27] offered similarity measure of IVFSs on log function.

In this article, we are exploring the resourcefulness of logarithmic similarity measures of PFSs in the application in choose the best airline in India with respect to many crucial attributes based on customer satisfaction. This paper is organized as follows: Section 2 introduces preliminaries of FSs, IFSs and the PFSs. Section 3 comprises of the concept of proposed logarithmic similarity measures of PFSs. We introduce logarithmic similarity measures and weighted similarity measures of the PFSs and its numerical computations to validate our measures. Application of the proposed measures in airline industry through PFSs is demonstrated in Section 4. Section 5 compares the new logarithmic similarity measures with the existing similarity measure by an example. Finally, Section 6 summarizes the article and delivers directions for future experiments.

2 Preliminaries

In this section, we bring in some basic theories related to fuzzy sets, intuitionistic fuzzy sets and Pythagorean fuzzy sets used in the outcome.

Definition 2.1 (Zadeh [65]). A fuzzy set $\mathcal{M}$ in $\mho$ is characterized by a membership function:

$$\mathcal{M} = \{\langle u, \delta_{\mathcal{M}}(u) \mid u \in \mho\} \tag{2.1}$$

where $\delta_{\mathcal{M}}(u) : \mathcal{M} \to [0, 1]$ is a measure of belongingness of degree of participation of an element $u \in \mho$ in $\mathcal{M}$.

Definition 2.2 (Atanassov [1]). An IFS $\mathcal{M}$ in $\mho$ is given by

$$\mathcal{M} = \{\langle u, \delta_{\mathcal{M}}(u), \zeta_{\mathcal{M}}(u)\rangle \mid u \in \mho\}, \tag{2.2}$$

where $\delta_{\mathcal{M}}(u), \zeta_{\mathcal{M}}(u) : \mathcal{M} \to [0, 1]$, and $0 \leq \delta_{\mathcal{M}}(u) + \zeta_{\mathcal{M}}(u) \leq 1, \forall\, u \in \mho$. The number $\delta_{\mathcal{M}}(u)$ and $\zeta_{\mathcal{M}}(u)$ represents, respectively, the participation and non-participation grade of the element u to the set P. For each IFS $\mathcal{M}$ in $\mho$, if

$$\eta_{\mathcal{M}}(u) = 1 - \delta_{\mathcal{M}}(u) - \zeta_{\mathcal{M}}(u), \quad \forall\, u \in \mho. \tag{2.3}$$

Then $\eta_{\mathcal{M}}(x)$ is the degree of indeterminacy of u to $\mho$.

Definition 2.3 (Yager [56]). An IFS $\mathcal{M}$ in $\mho$ is given by

$$\mathcal{M} = \{\langle u, \delta_{\mathcal{M}}(u), \zeta_{\mathcal{M}}(u)\rangle \mid u \in \mho\},$$

where $\delta_{\mathcal{M}}(u), \zeta_{\mathcal{M}}(u) : \mathcal{M} \to [0, 1]$, and with the condition

$$0 \leq \delta^2_{\mathcal{M}}(u) + \zeta^2_{\mathcal{M}}(u) \leq 1, \quad \forall\, u \in \mho \tag{2.4}$$

and the degree of indeterminacy for any PFS $\mathcal{M}$ and $u \in \mho$ is given by

$$\eta_{\mathcal{M}}(u) = \sqrt{1 - \delta^2_{\mathcal{M}}(u) - \zeta^2_{\mathcal{M}}(u)}\,. \tag{2.5}$$

3 Logarithmic Similarity Measures

Firstly, we recall the axiomatic preposition of similarity for Pythagorean fuzzy sets.

Proposition 3.1 (Ejegwa [7]). *Let X be nonempty set and $P,\ Q,\ R \in PFS(X)$. The similarity measure Sim between P and Q is a function Sim : $PFS \times PFS \to [0,1]$ satisfies*

(P1) *Boundedness:* $0 \le Sim(P,Q) \le 1$.

(P2) *Separability:* $Sim(P,Q) = 1 \Leftrightarrow P = Q$.

(P3) *Symmetric:* $Sim(P,Q) = Sim(Q,P)$.

(P4) *Inequality: If R is a PFS in X and $P \subseteq Q \subseteq R$, then* $Sim(P,R) \le Sim(P,Q)$ *and* $Sim(P,R) \le Sim(Q,R)$.

In several circumstances, the weight of the elements $\boldsymbol{x}_i \in X$ must be considered. For instance, in decision making, the attributes usually have distinct significance, and thus ought to be designated unique weights. As a result, we propose two weighted logarithmic similarity measures between P and Q, as follows:
Let $P,Q \in PFS(X)$ such that $X = \{x_1, x_2 \ldots, x_n\}$ then

$$S_{PFSL1}(P,Q) = \frac{1}{n}\sum_{i=1}^{n} \log_2\left[\left\{2 - \frac{(|\delta^2_P(x_i) - \delta^2_Q(x_i)| + |\zeta^2_P(x_i) - \zeta^2_Q(x_i)|)}{2}\right\}\right] \tag{3.1}$$

$$S_{PFSL2}\,(P,Q) = \frac{1}{n}\sum_{i=1}^{n} \log_2\left[\left\{2 - \frac{(|\delta^2_P(x_i) - \delta^2_Q(x_i)| + |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| + |\eta^2_P(x_i) - \eta^2_Q(x_i)|)}{3}\right\}\right] \tag{3.2}$$

$$S_{WPFSL1}\,(P,Q) = \frac{1}{n}\sum_{i=1}^{n} \omega_i\left[\log_2\left\{2 - \frac{(|\delta^2_P(x_i) - \delta^2_Q(x_i)| + |\zeta^2_P(x_i) - \zeta^2_Q(x_i)|)}{2}\right\}\right] \tag{3.3}$$

$$S_{WPFSL2}\,(P,Q) = \frac{1}{n}\sum_{i=1}^{n} \omega_i\left[\log_2\left\{2 - \frac{(|\delta^2_P(x_i) - \delta^2_Q(x_i)| + |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| + |\eta^2_P(x_i) - \eta^2_Q(x_i)|)}{3}\right\}\right] \tag{3.4}$$

where $\eta_P(x_i) = \sqrt{1 - \delta^2_P(x_i) - \zeta^2_P(x_i)}$ and $\eta_Q(x_i) = \sqrt{1 - \delta^2_Q(x_i) - \zeta^2_Q(x_i)}$; $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of x_i $(i = 1, 2, \ldots, n)$, with $\omega_k \in [0,1]$, $k = 1, 2, \ldots, n$, $\sum_{k=1}^{n} \omega_k = 1$. If $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots \frac{1}{n}\right)^T$, then the weighted logarithmic similarity measure reduces to proposed logarithmic similarity measures i.e., if we take $\omega_k = 1$, $k = 1, 2, \ldots, n$, then $S_{WPFSL1}(P,Q) = S_{PFSL1}(P,Q)$. Similarly, it can be verified that $S_{WPFSL2}(P,Q) = S_{PFSL2}(P,Q)$.

Theorem 3.2. *The Pythagorean fuzzy similarity measures $S_{PFSL1}(P,Q)$ and $S_{PFSL2}(P,Q)$ defined in equation (3.1)-(3.4) are valid measures of Pythagorean fuzzy similarity.*

Proof. All the necessary four conditions to be a divergence measure are satisfied by the new divergence measures as follows:

(P1) *Boundedness:* $0 \le S_{PFSL1}(P,Q), S_{PFSL2}(P,Q) \le 1$
For $S_{PFSL1}(P,Q)$: As $0 \le |\delta^2_P(x_i) - \delta^2_Q(x_i)| \le 1$ and $0 \le |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| \le 1$, therefore,

$$0 \le |\delta^2_P(x_i) - \delta^2_Q(x_i)| + |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| \le 2$$

$$\Rightarrow \quad 1 \leq 2 - \frac{|\delta_P^2(x_i) - \delta_Q^2(x_i)| + |\zeta_P^2(x_i) - \zeta_Q^2(x_i)|}{2} \leq 2$$

$$\Rightarrow \quad 0 \leq \frac{1}{n}\sum_{i=1}^{n} \log_2 \left\{ 2 - \frac{|\delta_P^2(x_i) - \delta_Q^2(x_i)| + |\zeta_P^2(x_i) - \zeta_Q^2(x_i)|}{2} \right\} \leq 1$$

Thus, $0 \leq S_{PFSL1}(P,Q) \leq 1$.
Measure $S_{PFSL2}(P,Q)$ can be proved similarly.

(P2) *Separability:* $S_{PFSL1}(P,Q), S_{PFSL2}(P,Q) = 1 \Leftrightarrow P = Q$.
For $S_{PFST1}(P,Q)$: For two PFSs P and Q in $X = \{x_1, x_2 \ldots, x_n\}$, if $P = Q$, then $\delta_P^2(x_i) = \delta_Q^2(x_i)$ and $\zeta_P^2(x_i) = \zeta_Q^2(x_i)$. Thus, $|\delta_P^2(x_i) - \delta_Q^2(x_i)| = 0$ and $|\zeta_P^2(x_i) - \zeta_Q^2(x_i)| = 0$.

$$\Rightarrow \quad |\delta_P^2(x_i) - \delta_Q^2(x_i)| + |\zeta_P^2(x_i) - \zeta_Q^2(x_i)| = 0$$

$$\Rightarrow \quad \log_2 2 = 1$$

$$\Rightarrow \quad \frac{1}{n}\sum_{i=1}^{n} \log_2 2 = 1$$

Therefore, $S_{PFSL1}(P,Q) = 1$.
If $S_{PFST1}(P,Q) = 1$, this implies,

$$\frac{1}{n}\sum_{i=1}^{n} \log_2 \left[\left\{ 2 - \frac{(|\delta_P^2(x_i) - \delta_Q^2(x_i)| + |\zeta_P^2(x_i) - \zeta_Q^2(x_i)|)}{2} \right\} \right] = 1$$

$$\Rightarrow \quad \log_2 \left[\left\{ 2 - \frac{(|\delta_P^2(x_i) - \delta_Q^2(x_i)| + |\zeta_P^2(x_i) - \zeta_Q^2(x_i)|)}{2} \right\} \right] = 1$$

$$\Rightarrow \quad 2 - \frac{(|\delta_P^2(x_i) - \delta_Q^2(x_i)| + |\zeta_P^2(x_i) - \zeta_Q^2(x_i)|)}{2} = 2$$

$$\Rightarrow \quad \frac{(|\delta_P^2(x_i) - \delta_Q^2(x_i)| + |\zeta_P^2(x_i) - \zeta_Q^2(x_i)|)}{2} = 0$$

Either $|\delta_P^2(x_i) - \delta_Q^2(x_i)| = 0$ or $|\zeta_P^2(x_i) - \zeta_Q^2(x_i)| = 0$. Therefore $\delta_P^2(x_i) = \delta_Q^2(x_i)$ and $\zeta_P^2(x_i) = \zeta_Q^2(x_i)$. Hence $P = Q$.
Measure $S_{PFSL2}(P,Q)$ can be proved similarly.

(P3) *Symmetric:* $S_{PFSL1}(P,Q) = S_{PFSL1}(Q,P)$ and $S_{PFSL2}(P,Q) = S_{PFSL2}(Q,P)$
Proofs are self-explanatory and straight forward.

(P4) *Inequality:* If R is a PFS in X and $P \subseteq Q \subseteq R$, then $S_{PFST1}(P,R) \leq S_{PFST1}(P,Q)$; $S_{PFST2}(P,R) \leq S_{PFST2}(Q,R)$ and $S_{PFST2}(P,R) \leq S_{PFST2}(P,Q)$; $S_{PFST2}(P,R) \leq S_{PFST2}(Q,R)$.
For $S_{PFST1}(P,Q)$: If $P \subseteq Q \subseteq R$, then for $x_i \in X$, we have $0 \leq \delta_P(x_i) \leq \delta_Q(x_i) \leq \delta_R(x_i) \leq 1$ and $1 \geq \zeta_P(x_i) \geq \zeta_Q(x_i) \geq \zeta_R(x_i) \geq 0$.
This implies that $0 \leq \delta_P^2(x_i) \leq \delta_Q^2(x_i) \leq \delta_R^2(x_i) \leq 1$ and $1 \geq \zeta_P^2(x_i) \geq \zeta_Q^2(x_i) \geq \zeta_R^2(x_i) \geq 0$.
This we have, $|\delta_P^2(x_i) - \delta_Q^2(x_i)| \leq |\delta_P^2(x_i) - \delta_R^2(x_i)|$; $|\delta_Q^2(x_i) - \delta_R^2(x_i)| \leq |\delta_P^2(x_i) - \delta_R^2(x_i)|$ and $|\zeta_P^2(x_i) - \zeta_Q^2(x_i)| \leq |\zeta_P^2(x_i) - \zeta_R^2(x_i)|$; $|\zeta_Q^2(x_i) - \zeta_R^2(x_i)| \leq |\zeta_P^2(x_i) - \zeta_R^2(x_i)|$.
From the above we can write,

$$\frac{|\delta_P^2(x_i) - \delta_Q^2(x_i)| + |\zeta_P^2(x_i) - \zeta_Q^2(x_i)|}{2} \leq \frac{|\delta_P^2(x_i) - \delta_R^2(x_i)| + |\zeta_P^2(x_i) - \zeta_R^2(x_i)|}{2}$$

$$\Rightarrow \quad 2 - \frac{|\delta_P^2(x_i) - \delta_Q^2(x_i)| + |\zeta_P^2(x_i) - \zeta_Q^2(x_i)|}{2} \geq 2 - \frac{|\delta_P^2(x_i) - \delta_R^2(x_i)| + |\zeta_P^2(x_i) - \zeta_R^2(x_i)|}{2}$$

$$\Rightarrow \quad \frac{1}{n}\sum_{i=1}^{n} \log_2 \left[\left\{ 2 - \frac{|\delta_P^2(x_i) - \delta_Q^2(x_i)| + |\zeta_P^2(x_i) - \zeta_Q^2(x_i)|}{2} \right\} \right]$$
$$\geq \frac{1}{n}\sum_{i=1}^{n} \log_2 \left[\left\{ 2 - \frac{|\delta_P^2(x_i) - \delta_R^2(x_i)| + |\zeta_P^2(x_i) - \zeta_R^2(x_i)|}{2} \right\} \right]$$

$\Rightarrow \quad S_{PFSL1}(P,R) \leq S_{PFSL1}(P,Q).$

Similarly, $S_{PFSL1}(P,R) \leq S_{PFSL1}(Q,R)$.
Similar proofs can be made for $S_{PFSL2}(P,R) \leq S_{PFSL2}(P,Q)$ and $S_{PFSL2}(P,R) \leq S_{PFSL2}(Q,R)$.
Analogous to the proofs done above, we can also validate properties depicted in Proposition 3.1 for weighted similarity measures $S_{WPFSL1}(P,Q)$ and $S_{WPFSL2}(P,Q)$ accordingly.

3.1 Numerical Verification of the Similarity Measures

Based on the parameters suggested by Wei and Wei (2018), we verify whether proposed similarity measures satisfy above four properties:

Example 3.3. Let $P,Q,R \in PFS(X)$ for $X = \{x_1,x_2,x_3\}$. Suppose
$P = \{\langle x_1, 0.6, 0.2\rangle, \langle x_2, 0.4, 0.6\rangle, \langle x_3, 0.5, 0.3\rangle\}$, $Q = \{\langle x_1, 0.8, 0.2\rangle, \langle x_2, 0.7, 0.3\rangle, \langle x_3, 0.6, 0.3\rangle\}$ and $R = \{\langle x_1, 0.9, 0.1\rangle, \langle x_2, 0.8, 0.2\rangle, \langle x_3, 0.7, 0.1\rangle\}$.
Calculating the similarity using proposed similarity measures are as follows:

$$S_{PFSL1}(P,Q) = \frac{1}{3}\left[\log_2\left\{2 - \frac{(|0.6^2 - 0.8^2| + |0.2^2 - 0.2^2|)}{2}\right\} + \log_2\left\{2 - \frac{(|0.4^2 - 0.7^2| + |0.6^2 - 0.3^2|)}{2}\right\} + \log_2\left\{2 - \frac{(|0.5^2 - 0.6^2| + |0.3^2 - 0.3^2|)}{2}\right\}\right]$$

$$S_{PFSL1}(P,Q) = \frac{1}{3}[\log_2(2 - 0.14) + \log_2(2 - 0.3) + \log_2(2 - 0.055)]$$
$$= \frac{1}{3}(2.620604) = 0.873534$$

$$S_{PFSL1}(P,R) = \frac{1}{3}\left[\log_2\left\{2 - \frac{(|0.6^2 - 0.9^2| + |0.2^2 - 0.1^2|)}{2}\right\} + \log_2\left\{2 - \frac{(|0.4^2 - 0.8^2| + |0.6^2 - 0.2^2|)}{2}\right\} + \log_2\left\{2 - \frac{(|0.5^2 - 0.7^2| + |0.3^2 - 0.1^2|)}{2}\right\}\right]$$

$$S_{PFSL1}(P,Q) = \frac{1}{3}[\log_2(2 - 0.24) + \log_2(2 - 0.4) + \log_2(2 - 0.16)]$$
$$= \frac{1}{3}(2.373351) = 0.7911173$$

$$S_{PFSL1}(Q,R) = \frac{1}{3}\left[\log_2\left\{2 - \frac{(|0.8^2 - 0.9^2| + |0.2^2 - 0.1^2|)}{2}\right\} + \log_2\left\{2 - \frac{(|0.7^2 - 0.8^2| + |0.3^2 - 0.2^2|)}{2}\right\} + \log_2\left\{2 - \frac{(|0.6^2 - 0.7^2| + |0.3^2 - 0.1^2|)}{2}\right\}\right]$$

$$S_{PFSL1}(P,Q) = \frac{1}{3}[\log_2(2 - 0.1) + \log_2(2 - 0.1) + \log_2(2 - 0.105)]$$
$$= \frac{1}{3}(2.774195) = 0.9247319.$$

The detailed computation for the proposed measures can be summarized in the following table:

Table 1. Numerical illustration to validate proposed measures

Proposed Measure 1	Numerical Values	Proposed Measure 2	Numerical Values	Proposed Measure 3	Numerical Values	Proposed Measure 4	Numerical Values
$S_{PFSL1}(P,Q)$	0.87353	$S_{PFSL2}(P,Q)$	0.878876	$S_{WPFSL1}(P,Q)$	0.289755	$S_{WPFSL2}(P,Q)$	0.289369
$S_{PFSL1}(P,R)$	0.79111	$S_{PFSL2}(P,R)$	0.797901	$S_{WPFSL1}(P,R)$	0.262383	$S_{WPFSL2}(P,R)$	0.261082
$S_{PFSL1}(Q,R)$	0.92473	$S_{PFSL2}(Q,R)$	0.925976	$S_{WPFSL1}(Q,R)$	0.308413	$S_{WPFSL2}(Q,R)$	0.307646

Numerical Justification: From the above computations, it supports that

P1: $0 \leq S_{PFSLj}(P,Q); 0 \leq S_{WPFSLj}(P,Q) \leq 1; j = 1,2.$

P2: $S_{PFSLj}(P,Q), S_{WPFSLj}(P,Q) = 1 \Leftrightarrow P = Q; j = 1,2.$

P3: It follows that $S_{PFSLj}(P,Q) = S_{PFSLj}(Q,P)$ and $S_{WPFSLj}(P,Q) = S_{WPFSLj}(Q,P)$, $j = 1,2$ (because use of square and absolute value).

P4: $S_{PFSLj}(P,R) \leq S_{PFSTj}(P,Q)$ and $S_{PFSLj}(P,R) \leq S_{PFSLj}(Q,R)$.
Also, $S_{WPFSLj}(P,R) \leq S_{WPFSLj}(P,Q)$ and $S_{WPFSLj}(P,R) \leq S_{WPFSLj}(Q,R)\ \forall\ j = 1,2.$

4 Application of Logarithmic Similarity Measure for Customer Preferences in Airlines

To demonstrate the legitimacy of the logarithmic similarity measures for PFSs proposed in Section 3, a numerical example is presented to illustrate the usage of proposed measures.

India is a rapidly expanding air transport market. Each year, the number of passengers flying for both business and pleasure continues to rise. Flights are becoming more affordable, with costs that are far from extravagant. As a result, there is fierce competition in the airline industry. Airlines are always striving to improve their services, punctuality, and reach to gain a larger share of the market.To know the best Airline in India, the civil aviation administration of India nominates four experts, Virat ($\boldsymbol{E}_1$), Rohit ($\boldsymbol{E}_2$), Jaspreet ($\boldsymbol{E}_3$) and Hardik ($\boldsymbol{E}_4$)to form a committee to assess the six major domestic airlines. The five airlines are Airline_1($\boldsymbol{A}_1$), Airline_2($\boldsymbol{A}_2$), Airline_3($\boldsymbol{A}_3$), Airline_4($\boldsymbol{A}_4$), Airline_5($\boldsymbol{A}_5$) and Airline_6($\boldsymbol{A}_6$). The alternatives are assessed on Comfort($\boldsymbol{C}_1$), On-Time performance($\boldsymbol{C}_2$), Staff behaviour($\boldsymbol{C}_3$), Price($\boldsymbol{C}_4$), Frequency of flights($\boldsymbol{C}_5$) and booking and ticketing service ($\boldsymbol{C}_6$). Weight vector of the attributes is $\omega = (\ 0.20, 0.15, 0.18, 0.22, 0.13, 0.12)$. Experts are required to utilize a PFSs to express their assessments for the above attributes$\boldsymbol{C_j}$ of various airlines$\boldsymbol{A_i}$.Thesix leading airlines are to be evaluated by the decision-maker under the above six criteriain the following steps.

Step 1: We construct a relation between Experts and their attributes in the form of PFSs, which is presented in Table 2.

Table 2. The relation between Experts and their attributes

Relation 1	Comfort ($\boldsymbol{C}_1$)	On-time performance ($\boldsymbol{C}_2$)	Staff behaviour ($\boldsymbol{C}_3$)	Price ($\boldsymbol{C}_4$)	Frequency of flights ($\boldsymbol{C}_5$)	Booking and ticketing service ($\boldsymbol{C}_6$)
Virat($\boldsymbol{E}_1$)	⟨0.8, 0.1⟩	⟨0.6, 0.1⟩	⟨0.2, 0.8⟩	⟨0.6, 0.1⟩	⟨0.1, 0.6⟩	⟨0.5, 0.3⟩
Rohit($\boldsymbol{E}_2$)	⟨0.0, 0.8⟩	⟨0.4, 0.4⟩	⟨0.6, 0.1⟩	⟨0.1, 0.7⟩	⟨0.1, 0.8⟩	⟨0.6, 0.2⟩
Jaspreet($\boldsymbol{E}_3$)	⟨0.6, 0.1⟩	⟨0.5, 0.4⟩	⟨0.3, 0.4⟩	⟨0.7, 0.2⟩	⟨0.3, 0.4⟩	⟨0.1, 0.4⟩
Hardik($\boldsymbol{E}_4$)	⟨0.7, 0.2⟩	⟨0.4, 0.3⟩	⟨0.6, 0.2⟩	⟨0.8, 0.1⟩	⟨0.2, 0.5⟩	⟨0.2, 0.6⟩
Weights	0.20	0.15	0.18	0.22	0.13	0.12

Step 2: A relation between customers preferences on attributes and the leading airlines in the form of PFSs is presented in Table 3.

Step 3: Determine the degree of similarity between A and B using tangent similarity measures (equations 22-25). The obtained measure values are presented in Table 3-Table 6.

Table 3. The relation between Customer preferences and the Airlines

Relation 2	Airline_1 (A_1)	Airline_2(A_2)	Airline_3(A_3)	Airline_4 (A_4)	Airline_5 (A_5)	Airline_6 (A_6)
(C_1)	⟨0.4, 0.0⟩	⟨0.3, 0.5⟩	⟨0.1, 0.7⟩	⟨0.4, 0.3⟩	⟨0.1, 0.6⟩	⟨0.5, 0.3⟩
(C_2)	⟨0.7, 0.1⟩	⟨0.2, 0.6⟩	⟨0.0, 0.9⟩	⟨0.1, 0.8⟩	⟨0.1, 0.8⟩	⟨0.6, 0.2⟩
(C_3)	⟨0.3, 0.3⟩	⟨0.6, 0.1⟩	⟨0.2, 0.7⟩	⟨0.2, 0.6⟩	⟨0.3, 0.4⟩	⟨0.1, 0.4⟩
(C_4)	⟨0.1, 0.8⟩	⟨0.2, 0.4⟩	⟨0.8, 0.0⟩	⟨0.2, 0.7⟩	⟨0.2, 0.5⟩	⟨0.2, 0.6⟩
(C_5)	⟨0.3, 0.4⟩	⟨0.2, 0.3⟩	⟨0.7, 0.4⟩	⟨0.1, 0.5⟩	⟨0.5, 0.4⟩	⟨0.2, 0.7⟩
(C_6)	⟨0.6, 0.2⟩	⟨0.8, 0.3⟩	⟨0.4, 0.5⟩	⟨0.4, 0.6⟩	⟨0.0., 0.3⟩	⟨0.1, 0.4⟩

Table 4. The relation between experts' similarity with the Airlines for $S_{PFSL1}(P, Q)$

Logarithmic Similarity Measure	Airline_1 (A_1)	Airline_2 (A_2)	Airline_3 (A_3)	Airline_4 (A_4)	Airline_5 (A_5)	Airline_6 (A_6)
Virat(E_1)	0.826716138	0.764349501	0.751885315	0.795673374	0.750868194	**0.862047255**
Rohit(E_2)	0.848135135	**0.868166006**	0.711143103	0.807763917	0.823497109	0.843246621
Jaspreet(E_3)	0.852393758	0.82398362	0.806876887	0.847994985	0.847994985	**0.89378884**
Hardik(E_4)	0.789149522	0.815021967	0.77550736	0.803013497	0.780601235	**0.842014066**

Table 5. The relation between experts' similarity with the Airlines for $S_{PFSL2}(P, Q)$

Logarithmic Similarity Measure	Airline_1 (A_1)	Airline_2 (A_2)	Airline_3 (A_3)	Airline_4 (A_4)	Airline_5 (A_5)	Airline_6 (A_6)
Virat(E_1)	0.815521371	0.781511654	0.777749802	0.804256103	0.776625956	**0.856239172**
Rohit(E_2)	0.837350117	0.845187561	0.770974553	0.816528096	0.819131044	**0.852071825**
Jaspreet(E_3)	0.873341386	0.83162419	0.804548506	0.869493356	0.869493356	**0.907605193**
Hardik(E_4)	0.82652795	0.82001004	0.803315224	0.830274273	0.789372855	**0.840650183**

Table 6. The relation between experts' similarity with the Airlines for $S_{WPFSL1}(P, Q)$

Logarithmic Similarity Measure	Airline_1 (A_1)	Airline_2 (A_2)	Airline_3 (A_3)	Airline_4 (A_4)	Airline_5 (A_5)	Airline_6 (A_6)
Virat (E_1)	0.133843879	0.125728966	0.125360829	0.130739503	0.123380201	**0.141179759**
Rohit(E_2)	0.140625672	**0.145141757**	0.117808357	0.135530784	0.139044335	0.139943691
Jaspreet(E_3)	0.139354649	0.136770814	0.13431608	0.13886007	0.13886007	**0.146885452**
Hardik(E_4)	0.12833178	0.135346996	0.12935157	0.129619796	0.127323377	**0.13732312**

Table 7. The relation between experts' similarity with the Airlines for $S_{WPFSL2}(P, Q)$

Logarithmic Similarity Measure	Airline_1 (A_1)	Airline_2 (A_2)	Airline_3 (A_3)	Airline_4 (A_4)	Airline_5 (A_5)	Airline_6 (A_6)
Virat(E_1)	0.132023713	0.129317789	0.129709841	0.132427479	0.128199029	**0.140861244**
Rohit (E_2)	0.138679202	**0.141623646**	0.127832887	0.137130386	0.138035203	0.141543632
Jaspreet(E_3)	0.14376411	0.138757403	0.134838473	0.143322367	0.143322367	**0.149988348**
Hardik (E_4)	0.135375632	0.135992636	0.134915288	0.13558011	0.129676076	**0.138087427**

Observations:

(a) Taking into an account of numerical computations of above tables, it is being determined that for the logarithmic similarity measures $S_{PFSL1}(P,Q)$, $S_{WPFSL1}(P,Q)$, and $S_{WPFSL2}(P,Q)$, Virat (E_1), Jaspreet (E_3) and Hardik (E_4) opted Airline_6; however, Rohit (E_2) prefers Airline_2 (Table 4, 6 and 7).

(b) For the measure $S_{PFSL2}(P,Q)$, it is being noticed that all of them opted for Airline_6 (Table 5).

This analysis is done on the grounds that higher value of the candidates against every similarity measure demonstrates the greater likelihood of having the option to choose the branch.

5 Comparative Study

To demonstrate the dominance of the proposed logarithmic similarity measures, a comparison between the proposed similarity measures and the existing similarity measures is conducted based on the numerical cases suggested. We first demonstratesome existing similarity measures for the sake of comparison as defined in Table 8.

Table 8. Similarity measures proposed by various authors

Authors	Similarity Measures
Peng *et al.* [37]	$Sim^1(P,Q) = 1 - \frac{1}{2n}\sum_{i=1}^{n}\left[\lvert\delta_P^2(x_i) - \delta_Q^2(x_i)\rvert \vee \lvert\zeta_P^2(x_i) - \zeta_Q^2(x_i)\rvert\right]$
Wei and Wei [53]	$Sim^2(P,Q) = \frac{1}{n}\sum_{i=1}^{n}\cos\left[\frac{\pi}{2}\left(\lvert\delta_P^2(x_i) - \delta_Q^2(x_i)\rvert \vee \lvert\zeta_P^2(x_i) - \zeta_Q^2(x_i)\rvert\right)\right]$ $Sim^3(P,Q) = \frac{1}{n}\sum_{i=1}^{n}\cos\left[\frac{\pi}{4}\left(\lvert\delta_P^2(x_i) - \delta_Q^2(x_i)\rvert + \lvert\zeta_P^2(x_i) - \zeta_Q^2(x_i)\rvert\right)\right]$ $Sim^4(P,Q) = \frac{1}{n}\sum_{i=1}^{n}w_i\cos\left[\frac{\pi}{2}\left(\lvert\delta_P^2(x_i) - \delta_Q^2(x_i)\rvert \vee \lvert\zeta_P^2(x_i) - \zeta_Q^2(x_i)\rvert \vee\lvert\eta_P^2(x_i) - \eta_Q^2(x_i)\rvert\right)\right]$ $Sim^5(P,Q) = \frac{1}{n}\sum_{i=1}^{n}w_i\cos\left[\frac{\pi}{4}\left(\lvert\delta_P^2(x_i) - \delta_Q^2(x_i)\rvert + \lvert\zeta_P^2(x_i) - \zeta_Q^2(x_i)\rvert +\lvert\eta_P^2(x_i) - \eta_Q^2(x_i)\rvert\right)\right]$
Ejegwa [7]	$Sim^6(P,Q) = 1 - \frac{1}{2n}\sum_{i=1}^{n}[\lvert\delta_P(x_i) - \delta_Q(x_i)\rvert + \lvert\zeta_P(x_i) - \zeta_Q(x_i)\rvert +\lvert\eta_P(x_i) - \eta_Q(x_i)\rvert]$ $Sim^7(P,Q) = 1 - \left(\frac{1}{2n}\sum_{i=1}^{n}\left[(\delta_P(x_i) - \delta_Q(x_i))^2 + (\zeta_P(x_i) - \zeta_Q(x_i))^2 +(\eta_P(x_i) - \eta_Q(x_i))^2\right]\right)^{\frac{1}{2}}$ $Sim^8(P,Q) = 1 - \frac{1}{2n}\sum_{i=1}^{n}[\lvert\delta_P^2(x_i) - \delta_Q^2(x_i)\rvert + \lvert\zeta_P^2(x_i) - \zeta_Q^2(x_i)\rvert +\lvert\eta_P^2(x_i) - \eta_Q^2(x_i)\rvert]$
Zhang *et al.* [68]	$Sim^9(P,Q) = \frac{1}{n}\sum_{i=1}^{n}\left[2^{1-\left(\lvert\delta_P^2(x_i) - \delta_Q^2(x_i)\rvert \vee \lvert\zeta_P^2(x_i) - \zeta_Q^2(x_i)\rvert\right)} - 1\right]$ $Sim^{10}(P,Q) = \frac{1}{n}\sum_{i=1}^{n}\left[2^{1-\frac{1}{2}\left(\lvert\delta_P^2(x_i) - \delta_Q^2(x_i)\rvert + \lvert\zeta_P^2(x_i) - \zeta_Q^2(x_i)\rvert\right)} - 1\right]$ $Sim^{11}(P,Q) = \frac{1}{n}\sum_{i=1}^{n}\left[2^{1-\left(\lvert\delta_P^2(x_i) - \delta_Q^2(x_i)\rvert \vee \lvert\zeta_P^2(x_i) - \zeta_Q^2(x_i)\rvert \vee \lvert\eta_P^2(x_i) - \eta_Q^2(x_i)\rvert\right)} - 1\right]$ $Sim^{12}(P,Q) = \frac{1}{n}\sum_{i=1}^{n}\left[2^{1-\frac{1}{2}\left(\lvert\delta_P^2(x_i) - \delta_Q^2(x_i)\rvert + \lvert\zeta_P^2(x_i) - \zeta_Q^2(x_i)\rvert + \lvert\eta_P^2(x_i) - \eta_Q^2(x_i)\rvert\right)} - 1\right]$

Table 9 represents a comprehensive evaluation of the logarithmic similarity measures for PFSs on some common data sets displayed in table 2 and table 3. From the numerical results presented in the Tables 9, comparison has been done between the similarity measures proposed by authors shown in table 8 and the results attained using our proposed similarity measures for PFSs. It has been noticed that the results obtained by using our proposed similarity measures are analogous with the existing measures.

Table 9. Comparison of existing measures with the proposed similarity measures

Comparison	*Virat* (E_1)	*Rohit* (E_2)	*Jaspreet* (E_3)	*Hardik* (E_4)
$Sim^1(P,Q)$	Airline_6	Airline_6	Airline_6	Airline_6
$Sim^2(P,Q)$	Airline_6	Airline_6	Airline_6	Airline_6
$Sim^3(P,Q)$	Airline_6	Airline_2	Airline_6	Airline_6
$Sim^4(P,Q)$	Airline_6	Airline_2	Airline_6	Airline_6
$Sim^5(P,Q)$	Airline_6	Airline_2	Airline_6	Airline_6
$Sim^6(P,Q)$	Airline_6	Airline_2	Airline_6	Airline_6
$Sim^7(P,Q)$	Airline_6	Airline_2	Airline_6	Airline_6
$Sim^8(P,Q)$	Airline_6	Airline_6	Airline_6	Airline_6
$Sim^9(P,Q)$	Airline_6	Airline_6	Airline_6	Airline_6
$Sim^{10}(P,Q)$	Airline_6	Airline_2	Airline_6	Airline_6
$Sim^{11}(P,Q)$	Airline_6	Airline_6	Airline_6	Airline_6
$Sim^{12}(P,Q)$	Airline_6	Airline_6	Airline_6	Airline_6
$S_{PFSL1}(P,Q)$	Airline_6	Airline_2	Airline_6	Airline_6
$S_{PFSL2}(P,Q)$	Airline_6	Airline_6	Airline_6	Airline_6
$S_{WPFSL1}(P,Q)$	Airline_6	Airline_2	Airline_6	Airline_6
$S_{WPFSL2}(P,Q)$	Airline_6	Airline_2	Airline_6	Airline_6

6 Conclusion

In recent times, numerous similarity measures have been established for measuring the level of similarity between PFSs. Nevertheless, it appears that there have been no examinations on similarity measures based of logarithmic function for PFSs. In this paper, we have offered some new logarithmic similarity measures and weighted similarity measures which comply with the conventional parameters of PFSs. We confirmed the credibility of the proposed similarity measures through numerical computations as well. Further, we employed these similarity measures for the application to DM problems. Also, comparative analysis of the investigated similarity measures was performed to determine the effectiveness of the proposed measures. Recommended PFSs for similarity measure is a significant device to address the vulnerabilities in the data in a more productive way when contrasted with the other existing sets. These intended measures can be applied to medical diagnosis, complex decision making and risk analysis in the future course of action.

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