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SOME THEOREMS ON SS-ELEMENTS OF A RING

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Abstract: A characterization of a class of unit elements, using SS-elements of a ring, is obtained. As an immediate consequence, an analogue of familiar Hua identity for SS-elements, is obtained besides some theorems on SS-rings. It is also proved that there exists rings containing an infinitely many SS-elements. Further more, the notion of SBSS-ring(suitable for building SS-elements) is initiated and proved that the matrix ring $M_n(Z_2)$,$n \geq$2 is an SBSS-ring. Examples are provided for justification as well.

1 Introduction

In the theory of rings, it is quite natural and interesting to observe that there are elements of a ring satisfying the condition $a^2 = a + a$. These elements are called the SS-elements of the ring. The concept of SS-element of a ring was first initiated by W.B.V.K. Swamy[12]. In a ring with unity, we can always find two SS-elements 0(additive identity) and 2(=1+1), which are the trivial SS-elements of the ring. An SS-element of the ring other than 0 and 2 is said to be a non-trivial SS-element. A ring is called an SS-ring if it contains at least one non-trivial SS-element. Examples of SS-rings can naturally be found in the literature. In [9], we have obtained some characterizations and applications of SS-elements of a ring.

In this paper, we shall present a characterization of a class of unit elements of a ring using SS-elements. We shall also present, as an immediate consequence, an analogue of the familiar Hua identity for SS-elements besides some theorems on SS-rings. It is natural and reasonable to raise a question that, does there exist an SS-ring containing an infinitely many SS-elements? we shall prove that the answer in affirmative. Further, we shall also initiate the notion of SBSS-ring(suitable to build SS-elements), which is an analogue of SBI ring (suitable to build idempotent elements, the terminology is due to Kaplansky.I[6]) and prove that the matrix ring $M_n(Z_2)$ is an SBSS-ring. Consequently, there are infinitely many SBSS-rings.

In section-2, we shall give definitions, propositions, theorems and in section-3, we shall present our main results and in section-4 we shall provide examples for justification. For basic definitions, fundamental concepts and elementary results, the reader can refer to Jacobson[6,7], Serge Lang[5] and T. Srinivas and A.K.S. Chandrasekhar Rao[9].
We begin with the following;

2 Section

Definition 2.1. [7] An element $0 \neq a$ in a ring with unity is said to be a unit if there exists an element b in R such that $ab = ba = 1$. If $b = a$, in other words, $a^2 = 1$, then a is unit having its own inverse.

Definition 2.2. [6] An element e in a ring R with unity is said to be an idempotent if $e^2 = e$.

Definition 2.3. [6] If e and f are idempotents in a ring R with unity then we say that e and f are orthogonal if and only if ef=0=fe.

Definition 2.4. [7] An element z in a ring R with unity is called nilpotent if $z^n = 0$ for some $n \in Z^+$.

Theorem 2.5. [7]*(Hua identity) Let a and b be elements of a ring with unity such that a, b and $ab-1$ are units. Then $a–b^{-1}$ and $(a–b^{-1})^{-1}-a^{-1}$ are units and the following identity holds: $((a–b^{-1})^{-1}-a^{-1})^{-1}$=$aba-a$.*

Remark 2.6. [7] If the characteristic of a ring R with unity is $k>0$ then $ka=(k1)a=0$, for all a in R. Clearly, k is the smallest positive integer having this property.

Theorem 2.7. [12]*An SS-ring contains(always) non-zero divisors of zero.*

Theorem 2.8. [3] *If a is a nontrivial SS-element of a ring R then it should necessarily satisfy the condition $a^2 \neq a$.*

Remark 2.9. [3] The idempotent elements of a ring can not be non-trivial SS-elements and a Boolean algebra regarded as a ring (Boolean ring) is not an SS-ring.

Definition 2.10. [8] Let $\sum_2$ be the set of all infinite sequences of $0's$ and $1's$. This set is called the sequence space of 0 and 1 or the symbol space of 0 and 1. More precisely, $\sum_2 = \{(s_0, s_1, ...)/s_i = 0 \text{ or } 1\}$. We will often refer to elements of $\sum_2$ as points in $\sum_2$.

3 Main Results

Theorem 3.1. *Let R be a ring with unity and of characteristic $\neq 2$ and $0 \neq a$ be an element of R. Then $1-a$ is an SS-element of R if and only if a is a unit having its own inverse.*

Proof. Let $0 \neq a$ be an element of R, our assertion is that $1-a$ is an SS-element if and only if $a^2=1$. First assume that $1-a$ is an SS-element of R. Then we have $1-a+1-a = (1-a)(1-a) = 1-a-a+a^2$. In view of cancellation laws and commutative axiom under addition, we get $a^2=1$. Therefore, a is a unit having its own inverse that means $a^2=1$.
On the other hand,
$a^2=1 \Rightarrow a^2-a-a+1=1-a-a+1$
$\Rightarrow (1-a)(1-a)=(1-a)+(1-a)$
It is obvious that $(1-a)$ is an SS-element of R. □

Theorem 3.2. *(Analogue of Hua identity) In a commutative ring with unity and of characteristic $\neq 2$, if $0 \neq a$,$0 \neq b$ be any two elements of R such that $1-a, 1-b, 2-ab$ are SS-elements then $1-(a-b)$ is an SS-element and the identity $a-b=aba$ holds.*

Proof. Since $1-a, 1-b$ are SS-elements, we have, in view of theorem(3.1), $a^2=1$, $b^2=1$. Also, $2-ab$ is an SS-element. $\Rightarrow (ab-1)^2=1 \Rightarrow a^2b^2+1-2ab=1$
$\Rightarrow 1+1-2ab=1$.
$\Rightarrow 2ab=1$......(3.2a)
Next to prove $1-(a-b)$ is an SS-element, in view of the theorem 2.5, it is sufficient to show that $(a-b)^2=1$. For, $(a-b)^2=a^2-2ab+b^2=1-1+1=1$ (in view of 3.2a).

Therefore $(a-b)^2=1$. Further, we write $a-b=a-b^{-1}$.
Next, $-b=a-b-a=a-b^{-1}-a^{-1}=(a-b^{-1})^{-1}-a^{-1}$
$\Rightarrow b^2=((a-b^{-1})^{-1}-a^{-1})^2$
$\Rightarrow ((a-b^{-1})^{-1}-a^{-1})^2=1$.
Hence $1-((a-b^{-1})^{-1}-a^{-1})$ is an SS-element.
From the above discussion we have
$1-a$ is an SS-element$\Leftrightarrow a^{-1}=a$;
$1-b$ is an SS-element$\Leftrightarrow b^{-1}=b$;
$1-(a-b)$ is an SS-element$\Leftrightarrow (a-b)^{-1}=a-b$;
$1-((a-b^{-1})^{-1}-a^{-1})$ is an SS-element$\Leftrightarrow ((a-b^{-1})^{-1}-a^{-1})^{-1}=(a-b^{-1})^{-1}-a^{-1})$
Let us recall Hua identity (Theorem 2.5),
$((a–b^{-1})^{-1}-a^{-1})^{-1}=aba-a$
$\Rightarrow (a–b^{-1})^{-1}-a^{-1}=aba-a$
$\Rightarrow a-b^{-1}-a=aba-a$
$\Rightarrow a-b-a=aba-a$
$\Rightarrow a-b=aba$. This completes the proof. □

Next, we present further results on SS-elements of a ring.

Theorem 3.3. *Let R be a commutative ring with unity and of characteristic $\neq 2$ and e, f be two non zero SS-elements of R. Then $e + f$ is non-zero SS-element of R if and only if $ef = 0$.*

Proof. Let e, f be two non-zero SS-elements of a commutative ring R with unity and of characteristic $\neq 2$. Then, we have $e + e = e^2$; $f + f = f^2$.
First assume that $e + f$ is an SS-element of R.
Then $e + f + e + f = (e + f)(e + f) = e^2 + ef + fe + f^2$.
$\Rightarrow ef + ef = 0 \Rightarrow ef = 0$ as R is a ring of characteristic $\neq 2$.
In view of (definition 2.3), we have e and f are orthogonal.
On the other hand, assume that $ef = 0 \Rightarrow ef + ef = 0$
$\Rightarrow e + f + ef + fe + e + f = e + f + e$+f
$\Rightarrow (e + f)(e + f) = (e + f) + (e + f)$. Hence $e + f$ is an SS-element.
This completes the proof. □

Theorem 3.4. *Let R be a ring with unity and of characteristic $\neq 2$, and $0 \neq u$ be an SS-element and unit then*

(3.4a) : $u = 2$
(3.4b) : $u + u = 1$.

Proof. Let $0 \neq u$ be an SS-element and a unit in a ring R with unit and of characteristic $\neq 2$. Then, in view of definition 2.1, there exists an element ν in R such that $u\nu = \nu u = 1$.
We have $u + u = u^2 \Rightarrow u\nu + u\nu = uu\nu$
$\Rightarrow 1 + 1 = u \Rightarrow u = 2$
Next, it is readily seen that $u\nu u = u \; and \; \nu u^2\nu$=1
Now, $\nu u^2\nu$=1$\Rightarrow \nu(u + u)\nu$=1
$\Rightarrow (\nu u + \nu u)\nu = 1$
$\Rightarrow \nu u\nu + \nu u\nu = 1$
$\Rightarrow u + u = 1$. This is completes the proof. □

Theorem 3.5. *Let R and $R^{'}$ be two rings with unity 1 and $1^{'}$ respectively. Let $\phi : R \to R^{'}$ be a monomorphism. Then*
(3.5a) : $a \in R$ is an SS-element if and only if $\phi(a)$ is an SS element of $R^{'}$
(3.5b) : If the ring R is of characteristic 2 and $0 \neq a \in R$ is an SS-element of R then ring $R^{'}$ contains nilpotents.

Proof. First we prove (3.5a).
Let $a \in$ R be an SS-element $\Rightarrow a + a = a^2 = aa$
$\Rightarrow (a + a) = \phi(aa)$
$\Rightarrow \phi(a) + \phi(a) = \phi(a)\phi(a)$
Therefore, $\phi(a)$ is an SS-element of $R^{'}$.
On the other hand, $\phi(x) \in R^{'}$ is an SS-element of $R^{'}$
$\Rightarrow \phi(x) + \phi(x)$=$\phi(x)\phi(x)$
$\Rightarrow \phi(x + x) = \phi(xx)$
$\Rightarrow x + x = x^2$ as ϕ is a monomorphism.
$\Rightarrow x$ is an SS-element of R.
This completes the proof of (3.5a).
Next, we prove (3.5b). Let the ring R be of characteristic 2 and $0 \neq a$ be an SS-element of R. Then in view of (3.5a), we have $\phi(a)$ is an SS-elements of $R^{'}$. So, $\phi(a) + \phi(a) = \phi(a)^2$.
$\Rightarrow \phi(a + a)$=$\phi(a)^2$
$\Rightarrow \phi(o) = \phi(a)^2$
$\Rightarrow \phi(a)^2$=$o^{'}$
$\Rightarrow \phi(a)$ is nilpotent, where $0^{'}$ is additive identity in $R^{'}$.

This completes the proof of (3.5b). □

Corollary 3.6. *The homomorphic image of an SS-ring is an SS-ring.*

Proof. Obvious. □

Lemma 3.7. *In a ring R with unity and of characteristic 2, $0 \neq a \in R$ is an SS-element if and only if $a^2 = 0$ (a is nilpotent).*

Proof. Obvious. □

Theorem 3.8. *Let R be a ring with unity and of characteristic 2 and z be a non zero SS-element of R. If $0 \neq w$ be any element of R then zwz is an SS-element of R.*

Proof. In view of the above lemma 3.7, we have z^2=0. Let $0 \neq w \in R$.
Now, $zwz + zwz = 0$ as $zwz \in$ R and characteristic of R is 2.
Next, $(zwz)(zwz) = (zw)z^2(wz) = 0$. so, $zwz + zwz = (zwz)(zwz)$.
Hence, zwz is an SS-element of R. □

Theorem 3.9. *Let R be a commutative ring with unity and of characteristic 2 and z be a non zero SS-element of R, If w is a non zero element of R then zw is an SS-element of R.*

Proof. In view of the lemma 3.7, we have z^2=0. Let $0 \neq w \in R$.
Now, $zw + zw$=0 as $zw \in R$. Next, $(zw)(zw) = (wz)(zw) = (wz^2)w$=0
Therefore, zw is an SS-element of R □

Theorem 3.10. *Let R be a ring with unity and of characteristic 2 and a be a non-zero SS-element of R. Then, there does't exists a non zero element x in R such that a^2 is a factor of x.*

Proof. In a contrary way, assume that there exists a nonzero element x in R such that $x = a^2y$. In view of the lemma 3.7 we have x =0, which is not possible. This completes the proof. □

Theorem 3.11. *Let R be a commutative ring with unity and of characteristic 2 and $a_1, a_2, ..., a_n, n \geq 2$ be non zero elements of R. If for some $i(1 \leq i \leq n)a_i$ is an SS-element then the product $\prod_j a_j$ is an SS-element.*

Proof. In view of lemma 3.7, ${a_i}^2 = 0$. It readily seen that $\prod_j a_j + \prod_j a_j = 0$; Using mathematical induction and applying commutative law repeatedly, we can easily see that $(\prod_j a_j)\ (\prod_j a_j) = 0$.
Hence,$\prod_j a_j$ is an SS-element of R.
This completes the proof. □

Definition 3.12. Let R be a ring with unity and S(R) be a ring with unity over the ring R, (the elements of the ring S(R) are formed with elements of R). The ring S(R) is called an SBSS-ring (suitable for building SS-elements) if and only if
(3.12a) the characteristic of the ring R is 2.
(3.12b) S(R) contains non zero elements a such that $a^2 = 0$.

Theorem 3.13. *The matrix ring $M_n(Z_2)$, $n \geq 2$ is an SBSS-ring.*

Proof. It is readily seen that the characteristic of the field $Z_2 = \{0, 1\}$ is 2. Therefore the matrix ring $M_n(Z_2)$ satisfies (3.12a).
Next, choose a matrix $A = \begin{bmatrix} a_{11} & a_{12} & \cdot & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & \cdot & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdot & \cdot & \cdot & a_{nn} \end{bmatrix}$ in $M_n(Z_2)$ such that the $(i, j)^{th}$ entry element $p_{ij} = a_{i1}a_{1j} + a_{i2}a_{2j} + ... + a_{in}a_{nj} = 0$ in the matrix

$$A^2 = \begin{bmatrix} \sum a_{1k}a_{k1} & \sum a_{1k}a_{k2} & \cdot & \cdot & \cdot & \sum a_{1k}a_{kn} \\ \sum a_{2k}a_{k1} & \sum a_{2k}a_{k2} & \cdot & \cdot & \cdot & \sum a_{2k}a_{kn} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sum a_{nk}a_{k1} & \sum a_{nk}a_{k2} & \cdot & \cdot & \cdot & \sum a_{nk}a_{kn} \end{bmatrix}, \text{ i=1,2,...,n, j=1,2,...,n and k=1,2,...,n.}$$

Care must be taken while choosing the entries in the matrix A.
For t=1,2,...,n., $p_{ij} = \sum a_{it}a_{tj} = 0$ if $a_{it} = 0; a_{tj} = 0$ or $a_{it} = 1; a_{tj} = 0$ or $a_{it} = 0; a_{tj} = 1$ or $a_{it} = 1; a_{tj} = 1$ such that summation $\sum a_{iz}a_{zj}$ contains an even number of $1^{'}s$.

We apply the principle of mathematical induction. If n=2, then $\left[\begin{smallmatrix}0&0\\1&0\end{smallmatrix}\right]$,$\left[\begin{smallmatrix}0&1\\0&0\end{smallmatrix}\right]$,$\left[\begin{smallmatrix}1&1\\1&1\end{smallmatrix}\right]$ in$M_2(Z_2)$ are nilpotent and hence $M_2(Z_2)$ is SBSS-ring.
Next, let B_{n-1} be a nilpotent in $M_{n-1}(Z_2)$ then augment one row and one column containing all entries equal to zero to the matrix B_{n-1}. Consequently, we get a matrix B_n in $M_n(Z_2)$, which is nilpotent. Hence $M_n(Z_2)$, $n \geq 2$ is an SBSS-ring. This completes the proof. □

Corollary 3.14. *There exists an infinity of SBSS-rings.*

Proof. In view of the theorem 3.13, for n ≥2, the matrix ring$M_n(Z_2)$ is an SBSS-ring. Since $n \geq 2$, is arbitrary, there are infinitely many SBSS-rings. □

Theorem 3.15. *The polynomial ring $P(Z_2)$ is an SBSS-ring. Further we can find infinity many SS-elements in polynomial ring $P(Z_2)$.*

Proof. It is readily seen that the polynomial ring $P(Z_2)$ is a ring with unity and the field $Z_2 = \{0, 1\}$ is of characteristic 2. Now, choose the polynomial $f(x)$ consisting of an even number of terms with coefficients 1. We can easily see that $f(x).f(x) = 0(x)$, where $0(x)$is the zero polynomial in the ring $P(Z_2)$. Therefore, the ring$P(Z_2)$ is an SBSS-ring. Further more, it can readily be seen that there are infinity many polynomials consisting of an even number of terms with coefficients 1. Hence, we can find infinity of SS-elements in the polynomial ring $P(Z_2)$. □

Corollary 3.16. *The sequence space $\sum_2$ of 0 and 1 is an SBSS-ring. Further, we can find infinitely many SS-elements in the sequence space $\sum_2$.*

Proof. In view of the definition 2.10, the sequence space $\sum_2 = \{(s_0, s_1, ...)/s_i = 0 \text{ or } 1\}$ can be viewed as the set $P(Z_2)$ of polynomials over the field $Z_2 = \{0, 1\}$. It can be readily seen that $\sum_2 = P(Z_2)$ is a polynomial ring with unity. In view of the theorem 15, the sequence space $\sum_2$ is an SBSS-ring consisting of infinitely many SS-elements. □

Corollary 3.17. *The set $Z_2 \times Z_2 \times ....$ is an SBSS-ring consisting of infinitely many SS-elements.*

Proof. In view of the corollary 3.16, the proof is obvious. □

4 Examples

Example 4.1. Let us consider the ring $R = \{0, 1, 2\}$ with addition and multiplication defined by the following tables :

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

.	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

It is obvious that R is a ring with unity and of characteristic $\neq 2$. From the tables, we can see that $1^2 = 1$, then 1-1=1+2=0 which is an SS-element, next, $2^2 = 1$, then 1-2=1+1=2 which is an SS-element.
This verification illustrates theorem 3.1.
Next, take a=1, b=2, it is obvious that a, b satisfy the hypothesis of theorem 3.2. Now $a - b$=1-2=2, also $aba = 2$. Therefore $a-b = aba$, this justifies theorem(3.2a). Further more this example justifies theorem 3.4.

In [9], we have given the following example.

Example 4.2. Let G=$\{e, a\}$ be a cyclic group of order 2 and $Z_2 = \{0, 1\}$ be a field of characteristic 2. $Z_2(G)$=$\{0, a, e, e + a\}$ is a group algebra with respect to the operations defined by following tables :

$+$	0	a	e	$e+a$
0	0	a	e	$e+a$
a	a	0	$e+a$	e
e	e	$e+a$	0	a
$e+a$	$e+a$	e	a	0

$.$	0	a	e	$e+a$
0	0	0	0	0
a	0	e	e	$e+a$
e	0	a	e	$e+a$
$e+a$	0	$e+a$	$e+a$	0

This example illustrates lemma 3.7, theorem 3.8 and theorem 3.9.

Example 4.3. Let $Z_3 = \{0, 1, 2\}$ be a prime field of characteristic 3 and let $G = \{g : g^2 = 1\}$ be a group. $Z_3(G) = \{0, 1, 2, g, 2g, 1+g, 1+2g, 2+2g\}$ is a group algebra with respect to the operations defined by the following tables

+	0	1	2	g	2g	1+g	2+g	1+2g	2+2g
0	0	1	2	g	2g	1+g	2+g	1+2g	2+2g
1	1	2	0	1+g	1+2g	2+g	g	2+2g	2g
2	2	0	1	2+g	2+2g	g	1+g	2g	1+2g
g	g	1+g	2+g	2g	0	1+2g	2+2g	1	2
2g	2g	1+2g	2+2g	0	g	1	2	1+g	2+g
1+g	1+g	2+g	g	1+2g	1	2+2g	2g	2	0
2+g	2+g	g	1+g	2+2g	2	2g	1+2g	0	1
1+2g	1+2g	2+2g	2g	1	1+g	2	0	2+g	g
2+2g	2+2g	2g	1+2g	2	2+g	0	1	g	1+g

.	0	1	2	g	2g	1+g	2+g	1+2g	2+2g
0	0	0	0	0	0	0	0	0	0
1	0	1	2	g	2g	1+g	2+g	1+2g	2+2g
2	0	2	1	2g	g	2+2g	1+2g	2+g	1+g
g	0	g	2g	1	2	1+g	1+2g	2+g	2+2g
2g	0	2g	g	2	1	2+2g	2+g	1+2g	1+g
1+g	0	1+g	2+2g	1+g	2+2g	2+2g	0	0	1+g
2+g	0	2+g	1+2g	1+2g	2+g	0	2+g	1+2g	0
1+2g	0	1+2g	2+g	2+g	1+2g	0	1+2g	2+g	0
2+2g	0	2+2g	1+g	2+2g	1+g	1+g	0	0	2+2g

It is obvious that $1^2 = 1$ and 1-1=1+2=0. Therefore, 1-1 is an SS-element.
Next, 2^2=1, and 1-2=1+1=2. Therefore 1-2 is an SS-elements.
Next, $g^2 = 1$ and $1 - g = 1 + 2g$. Therefore $1 - g$ is an SS-element as $1 + 2g$ is an SS-element.
Next,$(2g)^2 = 1$ and $1-2g = 1+g$. Therefore $1-2g$ is an SS-element as $1+g$ is an SS-element.
Hence the example 3 illustrates theorem 3.1.
To illustrate theorem 3.2. Take $a = g, b = 2g$ then $1 - a = 1 - g = 1 + 2g; 1 - b = 1 - 2g = 1 + g, 2 - ab = 2 - g(2g) = 2 - 2 = 2 + 1 = 0$ are obviously SS-elements.
Now $1 - (a - b) = 1 - (g - 2g) = 1 - (g + g) = 1 - 2g = 1 + g$ is an SS-element. Next, $g - 2g = g + g = 2g; g(2g)(g) = 2g$ Therefore $a - b = g - 2g = g(2g)(g) = aba$. Further more, the example 3, illustrates theorem 3.4.

Example 4.4. To illustrate theorem 3.13 we provide the following;
The matrices $\begin{bmatrix} 0&0&1 \\ 0&0&1 \\ 0&0&0 \end{bmatrix}$, $\begin{bmatrix} 0&0&1 \\ 0&0&0 \\ 0&0&0 \end{bmatrix}$, $\begin{bmatrix} 0&0&0 \\ 0&0&0 \\ 1&0&0 \end{bmatrix}$, $\begin{bmatrix} 0&0&0 \\ 1&0&0 \\ 1&0&0 \end{bmatrix}$ etc, are SS-elements in the matrix ring $M_3(Z_2)$.
Next, the matrices $\begin{bmatrix} 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \end{bmatrix}$, $\begin{bmatrix} 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}$, $\begin{bmatrix} 0&0&0&1 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}$, $\begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 1&0&0&0 \end{bmatrix}$, $\begin{bmatrix} 1&1&1&1 \\ 1&1&1&1 \\ 1&1&1&1 \\ 1&1&1&1 \end{bmatrix}$ etc, are SS-elements in the matrix ring $M_4(Z_2)$.

Likewise $\begin{bmatrix} 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{bmatrix}$, $\begin{bmatrix} 0&0&0&0&0&1\\ 0&0&0&0&0&1\\ 0&0&0&0&0&1\\ 0&0&0&0&0&1\\ 0&0&0&0&0&1\\ 0&0&0&0&0&0 \end{bmatrix}$, $\begin{bmatrix} 0&0&0&0&0&1\\ 0&0&0&0&0&1\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{bmatrix}$, $\begin{bmatrix} 0&0&0&0&0&1\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{bmatrix}$, $\begin{bmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 1&0&0&0&0&0 \end{bmatrix}$, $\begin{bmatrix} 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1 \end{bmatrix}$ etc, are SS-elements in the matrix ring $M_6(Z_2)$.

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