

Analysis of Noise Induced Allelopathic Prey-Predator System with Ergodic Behavior

C Sreerag, B S N Murthy, J Venkateswara Rao and M N Srinivas

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Abstract In this paper, we argue that in terms of spatiotemporal fluctuations non population density, the genuine allelopathic prey-predator system is typically inhomogeneous. With this in mind, we add noise to the system to simulate the effect of environmental heterogeneity on prey-predator interactions. The noise analysis of the system suggests the impression of the environmental pollution on the dynamical behavior of solutions. We mainly focused on the importance of stochastic coefficient, diffusion coefficient and its possible ergodic behavior in a time dependent and time independent prey population density problem is taken under consideration in the study.

1 Introduction

The interactions between populations and the natural environment are what define ecological systems. Predation is a significant sort of ecological interaction that happens across a wide range of stochastic, diffusion, and ergodic domains. In nature, there are large classes of marine species other than the fish species such as Phytoplankton, Algae, Bacteria and other plants which release toxic substances. The toxin released by one species may affect that species and/or affect the growth of the other species in the biological system. The result may be either positive or negative effect on the growth of the species. For example, Green algae and *Chlorella vulgaris* in a marine environment release a toxic substance which controls the growth of its own population and prevents the growth of the plankton algae *Bacillareae*. On the other side, two competing Phytoplankton species, (*Enteromorpha linza* and *Enteromorpha*) where one species produces auxin and has a positive response on the growth of the other phytoplankton species. Many studies [1-6] [8-18][21-24] have looked at the allelopathic effects of algae on other algae and the effect of one plant on the growth of another by releasing chemical compounds (allele chemicals) in biological species, particularly marine species. In the context of toxicity, Chattopadhyay [20] presented a two-species competition model for phytoplankton communities. Maynard smith [19] studied the effects of toxicant in a competitive system by assuming each of the species releasing a toxic substance in the presence of the other species. Kar and Chaudhuri [36] described a competition fish species model subjected to a combined harvesting effort in the presence of toxicity. Same phenomenon also exists in prey-predator models. Purohit et al. [7] believe that allelopathic and other genetic chemical effects will eventually have to be included in all relevant, functional ecological models. In population dynamics, prey-predator models are critical.

Some scholars have highlighted the impact of noise on ecological dynamics [32-34]. Noise can come from stochastic disorders in the external environment, as well as from noise itself; species dynamics appear to be constantly stochastic. In a prey-predator system, Xu et.al. [33-35] looked at the colour of the noise and population extinction. They also looked into noise with a white variance spectrum and no temporal autocorrelation, which shows population dynamics are random. Banerjee [30] studied the impact of colour noise on predator and prey biomass spatiotemporal behaviour. We were inspired by a few researchers to calculate the population intensities of white noise fluctuations. It's also crucial to look into how white noise affects one-dimensional oscillations in partial differential equation systems.

In numerous natural systems, the species viable may scatter spatially just as advancing as expected. This spatial dispersal or dissemination emerges from the propensity of specific species to move towards regions of lower populace density, predominantly because of asset impediment. In locales of high populace thickness, food will turn out to be scant, and people will in general move to areas of lower populace thickness. As of late, incredible consideration has been paid with the impact of scattering of a populace in a limited living space, and in this circumstance the overseeing conditions for the populace densities are depicted by an arrangement of response dispersion conditions. An environmental intriguing and numerically testing issue is to be decide under what condition the time subordinate arrangement merges to a positive consistent state arrangement, and what role is played by the impact of diffusion[25-27].

Species may travel in the marine system due to currents, turbulent diffusion, or other factors. The spatial variation of prey-predator population densities is a feature of prey-predator populations. Many writers have used diffusion to model the geographical variation of population in the prey-predator system [28-31]. They indicated that diffusion may cause turning instability in the prey-predator system when compared to labour without spatial variation. We will investigate the impact of spatial variation on model (2.1) performance in this research. There are numerous elements that can influence the spatial variation of the prey-predator system in the marine environment, but we cannot consider all of them at this time. We only consider the prey-predator system’s physical diffusion. Petersen, K.E[27] observes the fundamental and physical application and application of statistical techniques in his article and Hassler [15] investigates ergodic for mean in his work.

2 Noise induced Allelopathic formulation

In this section, a two-species prey-predator model in which both the species having densities p, z is proposed. The two species are harvested by different agencies with harvesting efforts e_1, e_2 respectively and the corresponding catch-ability coefficients of the species being a_4, b_4 . In the proposed dynamical model, the two species obey law of logistic growth with intrinsic growth rates a_1, b_1 and have carrying capacities Γ_1, Γ_2 respectively. Both the species release toxic substances which affect the other, the toxic coefficients of prey-predator are a_3, b_3 . The terms a_3p^2z (b_3pz^2) measure the effect of the toxic substance released by the predator (prey) on the prey (predator). b_2 represents the net rate of interaction on predator. The rate of decrease of prey is not only by the predation but also by the toxin released by the predator. The predator population increases by having prey as its food source and decreases by the effect of toxic substance by the predator. Here it is assumed that the rate of predation is more than the rate of effect of toxin release by prey.

$$\left. \begin{aligned} Dp &= p \left(a_1 - \frac{a_1p}{\Gamma_1} - a_2z - a_3pz - a_4e_1 \right) + \alpha_1\rho_1(t) \\ Dz &= z \left(b_1 - \frac{b_1z}{\Gamma_2} + b_2p - b_3pz - b_4e_2 \right) + \alpha_2\rho_2(t) \end{aligned} \right\} \tag{2.1}$$

where α_1, α_2 are real constants and $\rho(t) = [\rho_1(t), \rho_2(t)]$ is a 2 dimensional Gaussian white noise process satisfying $E[\rho_i(t)] = 0; i=1,2$ and $E[\rho_i(t)\rho_j(t)] = \delta_{ij} \cdot \delta(t-t^1); i=1,2$. Here δ_{ij} is the Kronecker symbol and δ is the Dirac function. Let $p(t) = \xi_1(t) + u^*$; and $z(t) = \xi_2(t) + v^*$ be the perturbed variables of the system (1). The linearized system (2.1) is

$$\left. \begin{aligned} D\xi_1 &= -\frac{a_1}{\Gamma_1}\xi_1u^* - a_2\xi_2u^* + \alpha_1\rho_1 \\ D\xi_2 &= b_2\xi_1v^* - \frac{b_1}{\Gamma_2}\xi_2v^* + \alpha_2\rho_2 \end{aligned} \right\} \tag{2.2}$$

We separate the individual direct facsimile. By apply Fourier change on the straight representation, we get a numerical system

$$\tilde{\rho}(\omega) = N(\omega) \tilde{\xi}(\omega) \tag{2.3}$$

where

$$N(\omega) = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix}; \tilde{\xi}(\omega) = \begin{bmatrix} \tilde{\xi}_1(\omega) \\ \tilde{\xi}_2(\omega) \end{bmatrix}; \tilde{\rho}(\omega) = \begin{bmatrix} \alpha_1\tilde{\rho}_1(\omega) \\ \alpha_2\tilde{\rho}_2(\omega) \end{bmatrix}$$

$$n_{11} = i\omega + \left(\frac{a_1}{\Gamma_1}\right) u^*; n_{12} = 0; n_{21} = 0; n_{22} = i\omega + \left(\frac{b_1}{\Gamma_2}\right) v^*$$

Condition (2.3) can likewise be composed as

$$\tilde{\xi}(\omega) = [N(\omega)]^{-1} \tilde{\rho}(\omega) = M(\omega)\tilde{\rho}(\omega) \tag{2.4}$$

where

$$M(\omega) = [N(\omega)]^{-1} = \begin{bmatrix} \frac{n_{22}}{|N(\omega)|} & -\frac{n_{12}}{|N(\omega)|} \\ -\frac{n_{21}}{|N(\omega)|} & \frac{n_{11}}{|N(\omega)|} \end{bmatrix}, \text{ and } |N(\omega)|^2 = \Omega^2 + \Theta^2$$

where $\Omega = \frac{a_1 b_1 u^* v^*}{\Gamma_1 \Gamma_2} - \omega^2$ and $\Theta = \frac{\omega a_1 u^*}{\Gamma_1} + \frac{\omega b_1 v^*}{\Gamma_2}$

The segments of the arrangement (2.4) are given by

$$\tilde{\xi}_\tau(\omega) = \sum_{v=1}^2 M_{\tau v}(\omega) \alpha_v \tilde{\rho}_v(\omega); \tau = 1, 2 \tag{2.5}$$

where $M_{\tau v}(\omega)$ are the components of the matrix $M(\omega)$ and $\tilde{\xi}_\tau(\omega)$ are the mean estimations of the populaces. The intensities of fluctuations of $\xi_\tau; \tau = 1, 2$ are given by

$$\mu_{\xi_\tau}^2 = \frac{1}{2\pi} \sum_{v=1}^2 \int_{-\infty}^{\infty} \alpha_v |M_{\tau v}(\omega)|^2 d\omega; \tau = 1, 2 \tag{2.6}$$

where $M_{\tau v}(\omega) = \frac{G_{\tau v}(\omega)}{|N(\omega)|}; \tau, v = 1, 2$

Utilizing (2.4), we have the populace differences of the model (2.1) as follows

$$\mu_{\xi_\tau}^2 = \frac{1}{2\pi} \sum_{v=1}^2 \int_{-\infty}^{\infty} \alpha_v |M_{\tau v}(\omega)|^2 d\omega; \tau = 1, 2 \tag{2.7}$$

where

$$G_{\tau v} = \chi_{\tau v} + i\delta_{\tau v}; \tau, v = 1, 2$$

$$\chi_{11} = \frac{b_1}{\Gamma_2} v^*; \delta_{11} = \omega; \chi_{12} = 0; \delta_{12} = 0; \chi_{21} = 0; \delta_{21} = 0; \chi_{22} = \frac{a_1}{\Gamma_1} u^*; \delta_{22} = \omega;$$

Using (2.6) and (2.7), we get

$$\mu_{\xi_1}^2 = \frac{1}{2\pi} \left\{ \int_{-\infty}^{\infty} \frac{1}{\Omega^2(\omega) + \Theta^2(\omega)} \left[\alpha_1 \left\{ \omega^2 + \left(\frac{a_1 u^*}{\Gamma_1}\right)^2 \right\} + \alpha_2 (0)^2 \right] d\omega \right\} \tag{2.8}$$

$$\mu_{\xi_2}^2 = \frac{1}{2\pi} \left\{ \int_{-\infty}^{\infty} \frac{1}{\Omega^2(\omega) + \Theta^2(\omega)} \left[\alpha_1 (0)^2 + \alpha_2 \left\{ \omega^2 + \left(\frac{b_1 v^*}{\Gamma_2}\right)^2 \right\} \right] d\omega \right\} \tag{2.9}$$

On the off chance that we are keen on the elements of the system (2.1) with either $\alpha_1 = 0$ or $\alpha_2 = 0$. When $\alpha_1 = 0, \mu_{\xi_1}^2 = 0$

$$\mu_{\xi_2}^2 = \frac{\alpha_2}{2\pi} \left\{ \int_{-\infty}^{\infty} \frac{1}{\Omega^2(\omega) + \Theta^2(\omega)} \left[\omega^2 + \left(\frac{b_1 v^*}{\Gamma_2}\right)^2 \right] d\omega \right\} \tag{2.10}$$

When $\alpha_2 = 0, \mu_{\xi_2}^2 = 0$

$$\mu_{\xi_1}^2 = \frac{\alpha_1}{2\pi} \left\{ \int_{-\infty}^{\infty} \frac{1}{\Omega^2(\omega) + \Theta^2(\omega)} \left[\omega^2 + \left(\frac{a_1 u^*}{\Gamma_1}\right)^2 \right] d\omega \right\} \tag{2.11}$$

Example 1: For the following values of parameters $a_1 = 4; b_1 = 5; \Gamma_1 = 500; \Gamma_2 = 400; a_2 = 0.1; b_2 = 0.9; a_3 = 0.008; b_3 = 0.005; a_4 = 0.2; b_4 = 0.1; e_1 = 30; e_2 = 40;$

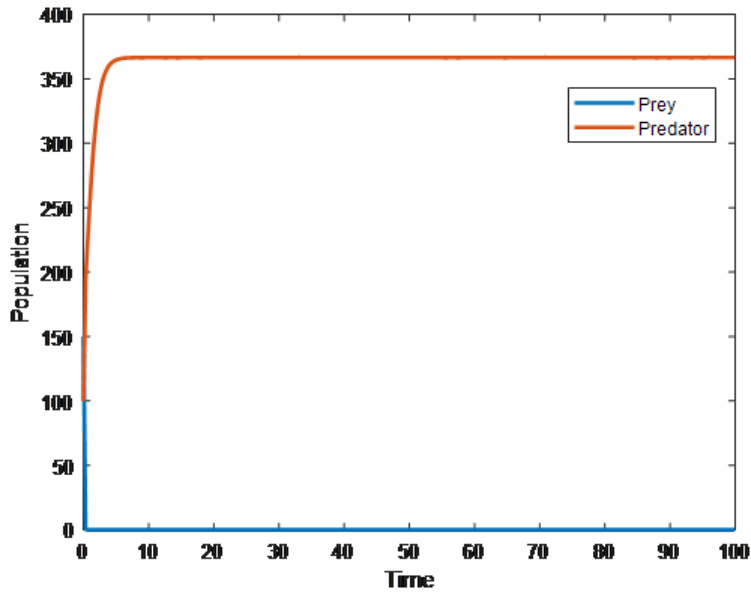


Figure 1

Figure 1. represents the time series evaluation for the attributes of Example 1 with noise intensities $\alpha_1 = 0.01$; $\alpha_2 = 0.02$.

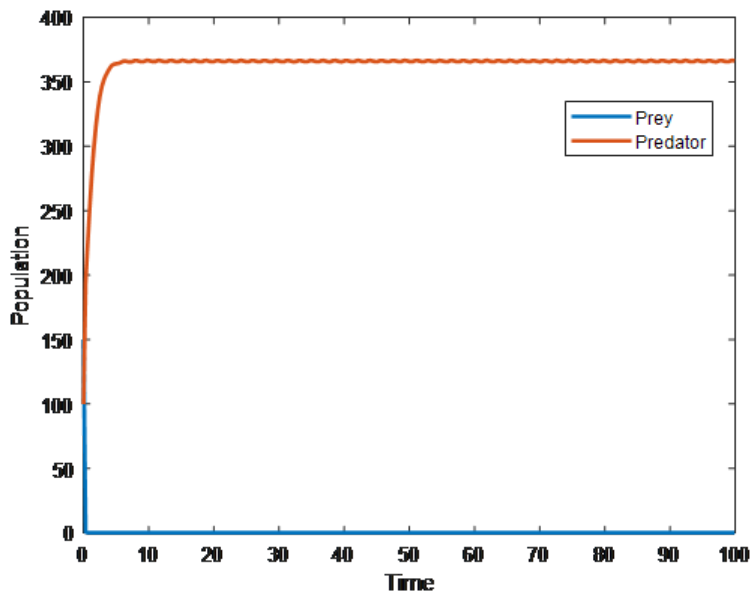


Figure 2

Figure 2. represents the time series evaluation for the attributes of Example 1 with noise intensities $\alpha_1 = 0.1$; $\alpha_2 = 0.2$.

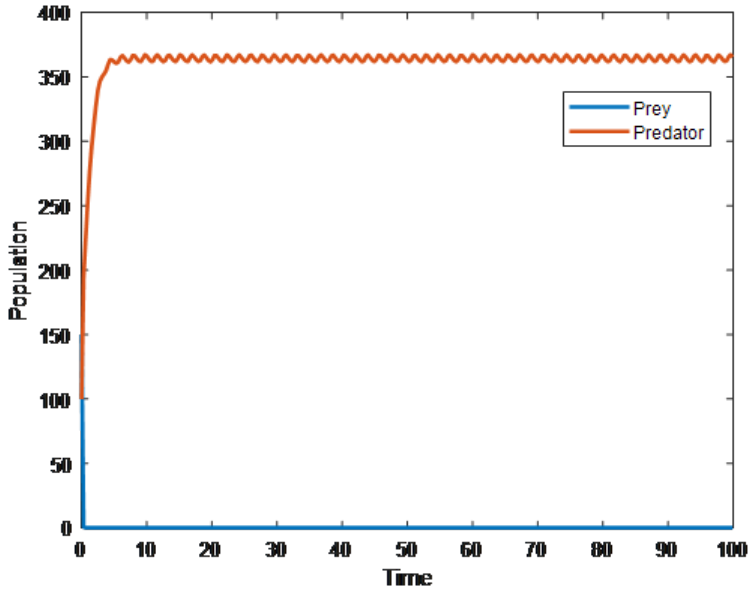


Figure 3

Figure 3. represents the time series evaluation for the attributes of Example 1 with noise intensities $\alpha_1 = 0.5$; $\alpha_2 = 0.6$.

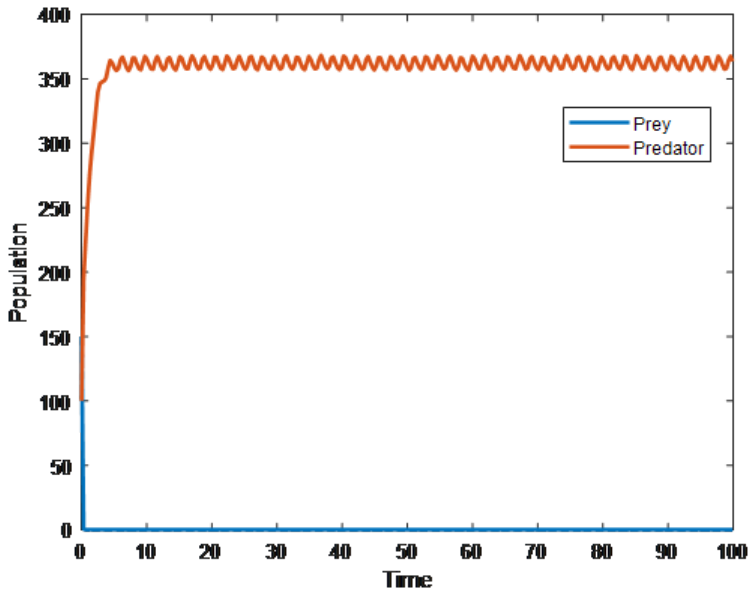


Figure 4

Figure 4. represents the time series evaluation for the attributes of Example 1 with noise intensities $\alpha_1 = 1$; $\alpha_2 = 2$.

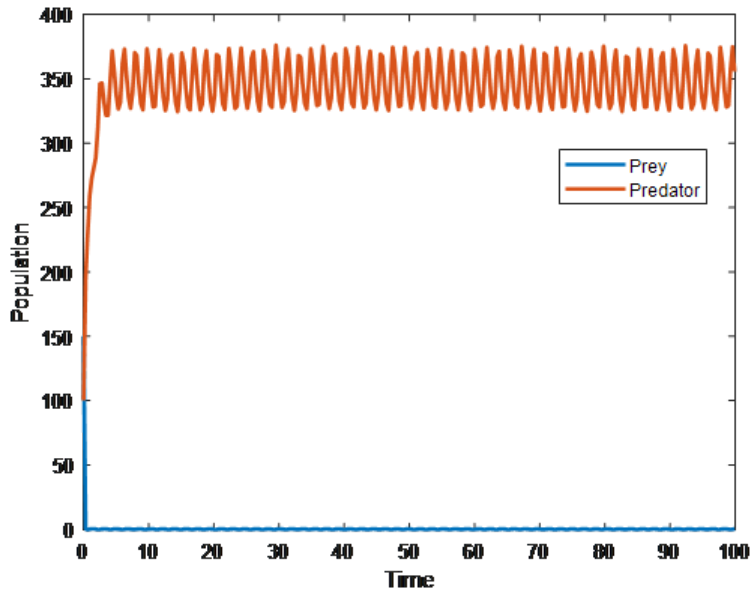


Figure 5

Figure 5. represents the time series evaluation for the attributes of Example 1 with noise intensities $\alpha_1 = 5$; $\alpha_2 = 6$.

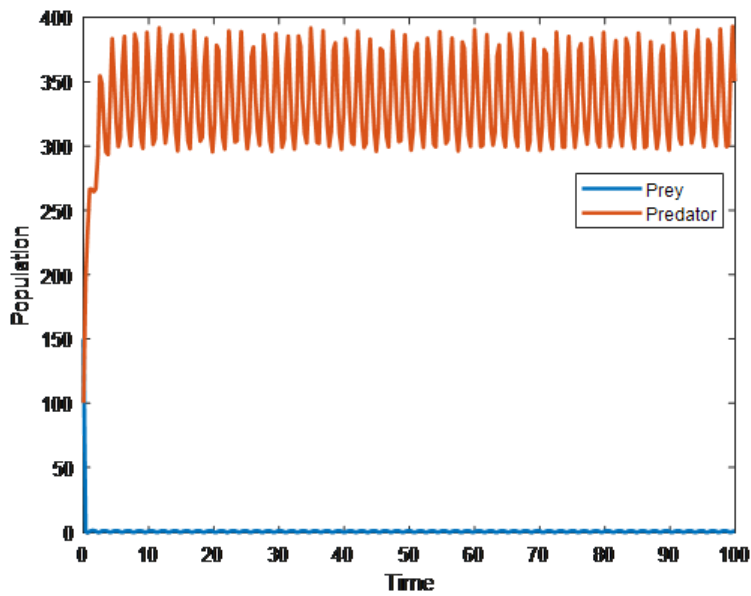


Figure 6

Figure 6. represents the time series evaluation for the attributes of Example 1 with noise intensities $\alpha_1 = 10$; $\alpha_2 = 20$.

3 Diffusion Induced Allelopathic Formulation

We consider the reaction-diffusion prey-predator model, which looks like this:

$$\left. \begin{aligned} p_t &= D_p p_{tt} + Dp \\ z_t &= D_q z_{tt} + Dz \end{aligned} \right\} \tag{3.1}$$

with the accompanying states of the population

$$\left. \begin{aligned} p(\Phi, t) \text{ and } z(\Phi, t) &> 0 \text{ in } 0 \leq \Phi \leq M, M > 0 \\ p_t(0, t) = p_t(M, t) = z_t(0, t) = z_t(M, t) &= 0 \end{aligned} \right\} \tag{3.2}$$

3.1 Diffusion coefficient and possibility for ergodic behavior

Diffusion coefficient can be observed as a time dependent dynamic function of population density (equation, 4), hence, the sample space in the total ecological environment under consideration (refer figure 4) will have a particular diffusion coefficient D_i with a population density of the prey μ_i

$$D_i = f(\mu_i, \phi, t) \tag{3.3}$$

In this section the fundamental objective is to study the possibility of ergodic behavior by mean for the colonized diffusion coefficient over time. Where, ϕ is the random variable, t is the instantaneous time. In order to define the instantaneous time as a fraction of the total time $T, (t/T)$ is defined. By assuming the equation (3.3) as a linear dynamic function with a constant K , we get equation (3.4),

$$D_i = f(\mu_i, \phi, t) = K\mu_i \left[\frac{t}{T} \right]^\phi \tag{3.4}$$

The random variable ϕ is determines the randomized behaviour of the time dependency of the diffusion coefficient. The range value of ϕ is from -1 to 1, $-1 \leq \phi \leq 1$. The negative values bring the possibility of the decreasing diffusion coefficient over time and positive values notify the increasing diffusion coefficient over time (refer Figure 8). Time independence is when $\phi = 0$

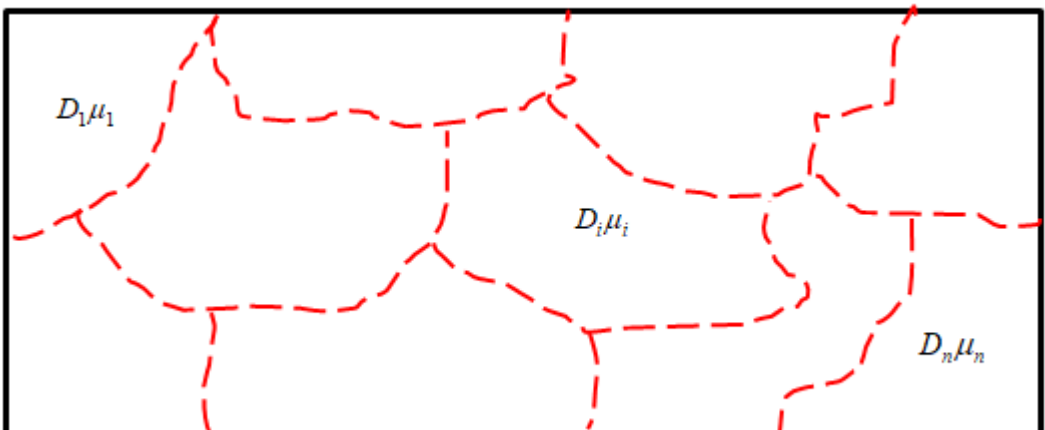


Figure 7

Figure 7. represents sample spaces in the total ecological environment

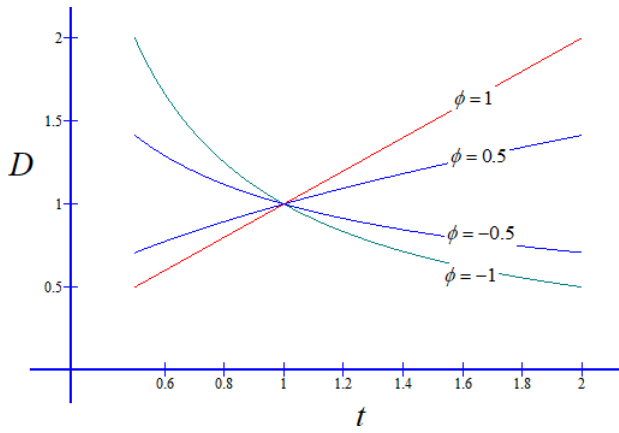


Figure 8

Figure 8. represents the rate of change of diffusion coefficient in accordance with the possible random variable ϕ .

The probability distribution function of the random variable $P(\phi)$ is considered to be uniform. Therefore,

$$P(\phi) = \begin{cases} \frac{1}{2}, & -1 \leq \phi \leq 1 \\ 0, & \phi \leq -1 \\ 0, & \phi \geq 1 \end{cases} \tag{3.5}$$

3.1.1 Necessary condition for ergodic behavior

The system is said to be ergodic, when the mean diffusion coefficient of the sample space possesses the tendency to become the mean diffusion coefficient of the entire ecological system over time ($T \rightarrow \infty$). In order to obtain the necessary condition for ergodic by mean, the ensemble average and total time average of the diffusion equation must be equal, therefore,

$$\int_{-1}^1 f\left(\mu, \phi, \frac{t}{T}\right) P(\phi) d\phi = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f\left(\mu, \phi, \frac{t}{T}\right) dt \tag{3.6}$$

Lemma1: The ensemble average for the diffusion coefficient function

$$\int_{-1}^1 f\left(\mu_i, \phi, \frac{t}{T}\right) P(\phi) d\phi = \frac{1}{2 \ln [t/T]} K \mu_i \left(\frac{t^2 - T^2}{t}\right) \tag{3.7}$$

Lemma2: The total time average for the diffusion coefficient function

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f\left(\mu_i, \phi, \frac{t}{T}\right) dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} K \mu_i [t/T]^\phi dt \tag{3.8}$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f\left(\mu_i, \phi, \frac{t}{T}\right) dt = \frac{1}{\phi + 1} K \mu_i \tag{3.9}$$

Lemma 3: Time dependent population density with constant growth rate r ($\mu_i^1(t) = r$)

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f\left(\mu_i(t), \phi, \frac{t}{T}\right) dt = \frac{k}{\phi + 1} (\mu_i(t) - r) \tag{3.10}$$

Case 1: Necessary condition for ergodic by mean for the diffusion coefficient, if population density is independent of time (3.7) and (3.9) in associate to equation (3.6), we have

$$\frac{1}{2 \ln [t/T]} \left[\frac{t^2 - T^2}{t} \right] = \frac{1}{\phi + 1}$$

Case 2: Necessary condition for ergodic by mean for the diffusion coefficient, if population density is dependent of time. By combining equation (3.7) and (3.10) in associate to equation (3.6), we have

$$\left[\frac{1}{\phi + 1} - \frac{1}{2 \ln (t/T)} \left(\frac{t^2 - T^2}{t} \right) \right] \mu_i(t) = \frac{r}{\phi + 1}$$

Since population growth rate is constant, $\mu_i^1(t) = r$, therefore $\mu_i(t) = rt$

$$\frac{1}{2 \ln [t/T]} [t^2 - T^2] = \frac{1}{\phi + 1} (t - 1) \tag{3.11}$$

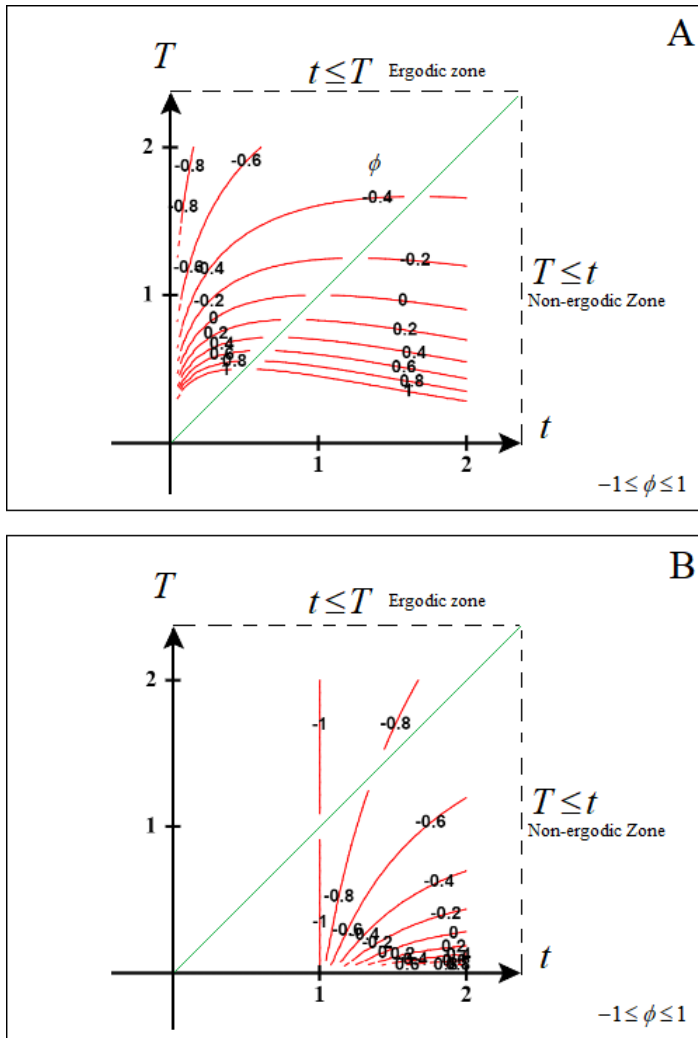


Figure 9(A) & 9(B)

9(A) represents the feasible t-T plot (equation 3.10) for different possible random variable ϕ , when population density is independent of time. 9(B) represents the feasible t-T plot (equation 3.11) for different possible random variable ϕ , when population density is dependent of time.

In order to identify the real solutions, we must consider some boundary conditions, this includes, and the instantaneous time and the total time must be a non-negative quantity, $0 \leq t, T$. And the instantaneous time shall be lesser in comparison with the total time $t \leq T$, this leads to a conclusion to the time ratio already defined (t/T) , the quantity (t/T) will always have a non-zero positive fraction less than unit in a real system, $0 < (t/T) < 1$.

In the figure 9(A), for a time independent population density, it is observable that the possible feasible solution of the necessary condition from case 1 (marked as red lines in the figure 9(A-B)) is distributed across both sides of the line $(t/T) = 1$ (marked as green in colour in figure 9(A-B)). Since, for real solutions, we have $0 < (t/T) < 1$, which can be categorized as the ergodic zone. Here for time independent population density the diffusion coefficient shows more ergodic behavior for all values of the random variable ϕ . This indicates over time the diffusion coefficient of the sample space might become the diffusion coefficient of the total ecological system.

Similarly, in the case 2, when the population density is time dependent, majority of the feasible solution (from equation 3.11, figure 9(A)) lie on the non ergodic zone, which implies, if the population density possess a time dependency, there is less chance for diffusion coefficient to become ergodic by its mean. That is, over time the diffusion coefficient of the sample space might not become the diffusion coefficient of the total ecological system.

4. Conclusions and Remarks

In this study, we argued that the real allelopathic prey-predator system is typically inhomogeneous in terms of spatiotemporal variations and non-population density. In order to model the effect of environmental heterogeneity on prey-predator interactions, we added noise to the system. The following are some of the observations:

- (i) The system's noise analysis reveals that environmental pollution has an impact on the dynamical behaviour of solutions.
- (ii) The paper focuses on the significance of stochastic coefficients, diffusion coefficients, and their likely ergodic behavior in a time dependent and time independent prey population density problem.
- (iii) The ergodic behavior for the diffusion coefficient is analyzed and the feasible solution t-T plot is notified as well, and seen that the time dependency of the population density which is directly influencing the diffusion coefficient bears significant role in determining the process to be ergodic or non-ergodic.
- (iv) This will have the implication of understanding and adjusting the localized, globalized harvesting zones in a prey-predator ecological environment.
- (v) Moreover, briefly alarms the possibility of having an influence on the global ecological system by proving actions localized sample spaces in the ecological environment.
- (vi) From figures (3-4), it is observed that both the species exhibit low dynamics as low oscillatory for lower values of noise densities. In biological point of view, as environmental noise are at low densities, both the species undergone some disturbance which results imbalance for a short period of time and may come to stable after noise vibrations vanished.
- (vii) From figures (5-6), it is observed that both the species exhibit high dynamics as highly oscillatory for higher values of noise densities. At higher values of noise densities ($\alpha_1 = 5$; $\alpha_2 = 6$.) and ($\alpha_1 = 10$; $\alpha_2 = 20$.) both the species are highly effected and exhibits good dynamics at these critical values. In biological point of view, as environmental noise reaches high densities, both the species are effected quantitatively as highly oscillatory in their zones which results in imbalance in the ecosystem against time.

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Conflict of Interest

The authors have no conflict so that the publication of the manuscript can be interrupted.

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Author information

C Sreerag, Department of Industrial Engineering and Engineering Management, National Tsing Hua University, Hsinchu, Taiwan.

E-mail: sreeragc1347@gmail.com

B S N Murthy, Department of Mathematics, Aditya College of Engineering and Technology, Surampalem, Andhrapradesh, India.

E-mail: bsn3213@gmail.com

J Venkateswara Rao, Department of Mathematics, School of Science & Technology, United States International University, Nairobi, Kenya.

E-mail: jvrhao@usiu.ac.ke; drjvenkateswararao@gmail.com

M N Srinivas, Department of Mathematics, School of School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamilnadu, India.

E-mail: mnsrinivaselr@gmail.com

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