Palestine Journal of Mathematics

Vol. 12(1)(2023) , 608–619

Using Markov Chains Models to Predict Water Supply and Demand and their Behavior: Case Study from Tulkarm City

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Communicated by Thabet Abdeljawad
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AMS Classification: 60J22, 62M20

Keywords and phrases: Stochastic Processes, Ergodic Markov Chains, Markov Model in Prediction.

Acknowledgment: The authors would like to thank Tulkarm Municipality for their cooperation in providing the necessary data for water demand and supply and Palestine Technical University for their cooperation and support. Special thank goes to Prof. Dr. Thabet Abdeljawad and Dr. Hadi Khalilia for useful discussions and the reviewers for their valuable comments.

Abstract This study aimed to predict the amount of water supply and demand in Tulkarm municipality using Markov chains models. The model is mainly based on the monthly data for the water demand and supply in Tulkarm municipality within the period from January 2010 to December 2019. The variability in water supply and demand, in term of increasing, decreasing or stability, were studied. The results showed that the supply chain stabilizes after 24 months where a rise in the amount of water demand at the rate 55.8% has been reached. While the rate of decrease was 36.7% and the rate of stability was 7.5%. Referring to water demand chain, the probability transition matrix stabilizes after 16 months, in which a rise in the amount of water demand has been reached nearly at the same rate, while the rate of decrease was 34.46% and the rate of stability was 10.08%. The predicted results for supply and demands scenarios emphasize the necessity for implementing an integrated water resources management plan in the district to ensure sustainable water for the local community on the future.

1 Introduction

Stochastic processes (Random phenomena) are the processes that randomly change with time and include many forms, where one of the most used forms is the usual Markov forms which belong to the statistical forms. The Markov processes perform has a huge importance in analyzing the stochastic processes, this rank reinforces the variety of practical applications in our daily life, as well as its applications in the statistical and engineering forms, so it became a focal subject for many organizations and researchers dealing with this topic [10].

It's been a usual practice to use time series and regression analysis when performing the future predictions in general and predicting the water future demand/supply for the upcoming years in particular [1], [3, 4, 5, 6], [9, 10] and [12]. In addition to the previous two methods, Markov chains model is a powerful tool for prediction [6, 7, 8] and [11].

This study as part of master thesis work of Al-Mallak, Ghadeer [2] aimed to predict the amount of water supply and demand in Tulkarm municipality, based on monthly data for the amount of water demand and supply in Tulkarm municipality within the period from January 2010 to December 2019. The matrix of transition probabilities was formed and then predicted values based on the probability transition matrix were obtained by using the Mat Lab statistical program, for both supply and demand. The study also tried to compare statistical models (Markov model, time series model, and simple linear regression model) to predict the future values of both series. This has been done through using the forecast accuracy scale to find the most suitable model for analyzing the data in the study.

In this paper the Markov model has been used to predict different scenarios of water supply and demand based on the prediction outcomes [2].

2 Method

2.1 Markov Chains

A Markov Process $\{X_t : t \in T\}$ is a stochastic process with the property that: given the value of X_t, the values of X_s with $s > t$ are not influenced by the values of X_u with $u < t$ That is: The probability of any particular future behavior of the process when the current is known exactly is not altered by additional knowledge concerning the past behavior [10].

A discrete time Markov chain is a Markov process whose state space S (the range of possible values for the random variables X_t) is a finite or countable set and whose time index set is $T = \{0, 1, 2, \dots\}$ [10].

The Markov property is:

$$p(X_{n+1} = j|X_0 = i_0, X_1 = i_1, \dots, X_{n-1} = i_{n-1}, X_n = i)$$

$= p\left(X_{n+1} = j|X_n = i\right) = p_{\text{ij}}^{n,n+1}$ (2.1) (the one step transition probability).

When the one step transition probabilities are independent of time n, we say the Markov chain has stationary transition probabilities or homogeneous, that is $p_{\text{ij}}^{n,n+1} = p_{\text{ij}}$ (2.2).

The Markov matrix or transition probability matrix $P = [p_{\text{ij}}]_{i,j \in S}$ (2.3) of the process satisfies $p_{\text{ij}} \geq 0,\ \forall i, j \in S$ (2.4) and $\sum_j p_{\text{ij}} = 1$ (2.5).

A Markov process is completely defined once its matrix and initial state X_0 are specified $p(X_0 = i) = p_i$ (2.6).

The classifications of the states can be found in [10].

When the state space is finite and $\exists n,\ p_{\text{ij}}^{(n)} > 0,\ \forall i, j \in S$ (2.7) then all state are positive recurrent and aperiodic. That is, it is Ergodic Markov chain.

For an Ergodic Markov chain, there exists a probability vector $\pi = (\pi_j)_{j \in S}$ (2.8) such that:

1) $\pi_j > 0,\ \forall j$ 2) $\sum_{j \in S} \pi_j = 1$ 3) $\pi_j = \sum_{i \in S} \pi_i p_{\text{ij}}$ 4) $\pi_j = \lim_{n \to \infty} p_{\text{ij}}^{(n)}$ (2.9).

Using the Markov Model in Prediction

There are four important features that are necessary to use the Markov model in prediction [1]:

a) *Mean:* $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} x_i$ (2.10).

b) Standard Deviation: $S = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left(x_i - \overline{X}\right)^2}$ (2.11).

c) Twisting Coefficient:

$g = \frac{n \sum_{i=1}^{n} \left(x_i - \overline{X}\right)^3}{(n-1)(n-2)S}$ (2.12).

d) Correlation Coefficient:

$r_k = \frac{n \sum_{i=1}^{n-k} (x_i - \overline{X})(x_{i+k} - \overline{X})}{(n-k)S^2}$ (2.13).

The monthly prediction Markov model developed by (Thomas and Fiering), has taken the following formula:

$q_{\text{ij}} = \overline{q_j} + b_j(q_{i-1,j-1} - \overline{q}_{j-1}) + S_j t_{\text{ij}} \sqrt{(1 - r_j^2)}$ (2.14)

i : The month symbol of the generated series and takes values from 1 to the length of the series

j : Month symbol in the series (From January to December)

$\overline{q_j}$: The mean of values the month (jth) (From January to December)

b_j : Regression coefficient for values in two months j and (j-1) and it is calculated as follows: $b_j = \frac{r_j s_j}{s_{j-1}}$ (2.15).

$q_{i-1,j-1}$: Value at the month immediately before it.

$\overline{q}_{j-1}$: The mean of values the month before jth.

S_j : Standard deviation of values per month (From January to December).

t_{ij} : The random generated number follows the standard natural distribution.

r_j : Linear correlation coefficient for two months j and $(j-1)$.

2.2 Data Collection and Processing

Data (amount of supply and demand on water in Tulkarm city monthly) were collected from the Tulkarm Municipality as the main water service provider in the district. The monthly supply and demand data for the years 2018, 2019, and only five months of 2020 were obtained (Table 1). However, due to covid-19 contingency, additional data from other institutes were not collected appropriately.

Therefore, and due to the lack of sufficient data (29 values), and in order to be able to implement the three models "time series, regression analysis, and the Markov chains" extra monthly data values from year 2010 to year 2017 (96 additional values) were synthetically generated, by which total values of 125 records has been reached; of which 120 records were used to generate the three previously mentioned models and the other 5 from year 2020 were used to validate the estimated forms (Table 2&3).

Table (1): Represents data from the municipality of Tulkarm - Department of Water - in the period (2018-2020)

Month	Demand			Supply		
	2020	2019	2018	2020	2019	2018
Jan	342116	325261	355628	552465	595379	585333
Feb	329008	335053	325052	548691	568120	588242
Mar	367220	367566	366762	598768	619416	612514
Apr	372342	377204	374223	637289	642554	641932
May	416814	379944	412416	702951	727326	717864
Jun		392458	401228		751413	744319
Jul		405507	443773		767656	762135
Aug		458690	427765		758925	762183
Sep		496451	424208		752563	759489
Oct		471374	411882		737737	742549
Nov		450015	404332		720426	718011
Dec		358512	384651		658395	611725

The additional synthetic monthly data for the year 2010 to year 2017 (96 values) were generated using Microsoft Excel and according to the following formula:

$y = 6 * (1.27x + x)$ (2.16).

y = Annual value of the amount of water (Supply /demand).

x = Amount of water supply/demand in winter.

Table (2): Represents amount of water supply in the period (2010-2020)

Month	Supply										
	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010
Jan	552465	595379	585333	704846	756241	690162	484582	352423	455213	381791	308370
Feb	548691	568120	588242	710589	778959	701025	536532	359360	468287	395356	265396
Mar	598768	619416	612514	744060	813692	734705	544416	368014	482480	433635	339034
Apr	637289	642554	641932	778198	824657	770247	577179	398014	494512	456955	368588
May	702951	727326	717864	805049	867905	851445	585349	420004	536278	464844	385417
Jun	.	751413	744319	864928	929148	868626	609802	420142	551539	478006	388766
Jul	.	767656	762135	895155	960426	876505	615419	447577	578121	484875	391629
Aug	.	758925	762183	898219	961316	898322	649195	495486	602118	509311	413148

Month					Supply						
Sep	.	752563	759489	881662	945954	840802	653307	491972	592118	525985	435835
Oct	.	737737	742549	844589	913692	800774	585219	444391	560416	483071	403230
Nov	.	720426	718011	798219	850504	792046	514679	427118	492568	420041	351312
Dec	.	658395	611725	745136	815978	735505	496307	398821	432908	408529	323230

Table (3): Represents amount of water demand in the period (2010-2020)

Month					Demand						
	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010
Jan	342116	325261	355628	474074	496296	570370	323054	391791	264317	242291	205580
Feb	329008	335053	325052	482136	499642	586104	333214	390471	265715	237007	224407
Mar	367220	367566	366762	483498	512257	607105	352675	412375	276771	260385	235017
Apr	372342	377204	374223	491967	526192	624305	370798	422464	289870	263366	234212
May	416814	379944	412416	552274	595481	698371	391146	448811	304135	280533	245456
Jun	.	392458	401228	576705	619596	705798	409768	452683	329866	301345	255055
Jul	.	405507	443773	592593	620370	712963	410279	484876	335682	307709	261086
Aug	.	458690	427765	602669	628764	731346	433555	502410	360790	362629	281991
Sep	.	496451	424208	604372	614308	730403	447273	522947	381180	385586	305262
Oct	.	471374	411882	586740	607506	728614	434996	512282	329588	370533	291991
Nov	.	450015	404332	575180	541964	654671	408136	456255	296781	301287	283950
Dec	.	358512	384651	477491	487549	614718	356378	417443	271122	275482	230215

3 Model Processing and Results

3.1 Behavior of the Water Supply/Demand in the Long Run

In order to investigate the behavior of the water supply/demand at long run, the following steps were followed:

Water supplied/demanded data for the domestic sector in Tulkarm municipality and samples of (120) months has been processed (Table 4&5).

Table (4): Represents the amount of water supply from 2010-2019

Month	supply	Month	supply	Month	supply	Month	supply	Month	supply
10-Jan	308370	12-Jan	455213	14-Jan	484582	16-Jan	756241	18-Jan	585333
10-Feb	265396	12-Feb	468287	14-Feb	536532	16-Feb	778959	18-Feb	588242
10-Mar	339034	12-Mar	482480	14-Mar	544416	16-Mar	813692	18-Mar	612514
10-Apr	368588	12-Apr	494512	14-Apr	577179	16-Apr	824657	18-Apr	641932
10-May	385417	12-May	536278	14-May	585349	16-May	867905	18-May	717864
10-Jun	388766	12-Jun	551539	14-Jun	609802	16-Jun	929148	18-Jun	744319
10-Jul	391629	12-Jul	578121	14-Jul	615419	16-Jul	960426	18-Jul	762135
10-Aug	413148	12-Aug	602118	14-Aug	649195	16-Aug	961316	18-Aug	762183
10-Sep	435835	12-Sep	592118	14-Sep	653307	16-Sep	945954	18-Sep	759489
10-Oct	403230	12-Oct	560416	14-Oct	585219	16-Oct	913692	18-Oct	742549
10-Nov	351312	12-Nov	492568	14-Nov	514679	16-Nov	850504	18-Nov	718011
10-Dec	323230	12-Dec	432908	14-Dec	496307	16-Dec	815978	18-Dec	611725

Month	supply	Month	supply	Month	supply	Month	supply	Month	supply
11-Jan	381791	13-Jan	352423	15-Jan	690162	17-Jan	704846	19-Jan	595379
11-Feb	395356	13-Feb	359360	15-Feb	701025	17-Feb	710589	19-Feb	568120
11-Mar	433635	13-Mar	368014	15-Mar	734705	17-Mar	744060	19-Mar	619416
11-Apr	456955	13-Apr	398014	15-Apr	770247	17-Apr	778198	19-Apr	642554
11-May	464844	13-May	420004	15-May	851445	17-May	805049	19-May	727326
11-Jun	478006	13-Jun	420142	15-Jun	868626	17-Jun	864928	19-Jun	751413
11-Jul	484875	13-Jul	447577	15-Jul	876505	17-Jul	895155	19-Jul	767656
11-Aug	509311	13-Aug	495486	15-Aug	898322	17-Aug	898219	19-Aug	758925
11-Sep	525985	13-Sep	491972	15-Sep	840802	17-Sep	881662	19-Sep	752563
11-Oct	483071	13-Oct	444391	15-Oct	800774	17-Oct	844589	19-Oct	737737
11-Nov	420041	13-Nov	427118	15-Nov	792046	17-Nov	798219	19-Nov	720426
11-Dec	408529	13-Dec	398821	15-Dec	735505	17-Dec	745136	19-Dec	658395

Table (5): Represents the amount of water demand from 2010-2019

Month	demand	Month	Demand	month	demand	Month	demand	month	demand
10-Jan	205580	12-Jan	264317	14-Jan	323054	16-Jan	496296	18-Jan	355628
10-Feb	224407	12-Feb	265715	14-Feb	333214	16-Feb	499642	18-Feb	325052
10-Mar	235017	12-Mar	276771	14-Mar	352675	16-Mar	512257	18-Mar	366762
10-Apr	234212	12-Apr	289870	14-Apr	370798	16-Apr	526192	18-Apr	374223
10-May	245456	12-May	304135	14-May	391146	16-May	595481	18-May	412416
10-Jun	255055	12-Jun	329866	14-Jun	409768	16-Jun	619596	18-Jun	401228
10-Jul	261086	12-Jul	335682	14-Jul	410279	16-Jul	620370	18-Jul	443773
10-Aug	281991	12-Aug	360790	14-Aug	433555	16-Aug	628764	18-Aug	427765
10-Sep	305262	12-Sep	381180	14-Sep	447273	16-Sep	614308	18-Sep	424208
10-Oct	291991	12-Oct	329588	14-Oct	434996	16-Oct	607506	18-Oct	411882
10-Nov	283950	12-Nov	296781	14-Nov	408136	16-Nov	541964	18-Nov	404332
10-Dec	230215	12-Dec	271122	14-Dec	356378	16-Dec	487549	18-Dec	384651
11-Jan	242291	13-Jan	391791	15-Jan	570370	17-Jan	474074	19-Jan	325261
11-Feb	237007	13-Feb	390471	15-Feb	586104	17-Feb	482136	19-Feb	335053
11-Mar	260385	13-Mar	412375	15-Mar	607105	17-Mar	483498	19-Mar	367566
11-Apr	263366	13-Apr	422464	15-Apr	624305	17-Apr	491967	19-Apr	377204
11-May	280533	13-May	448811	15-May	698371	17-May	552274	19-May	379944
11-Jun	301345	13-Jun	452683	15-Jun	705798	17-Jun	576705	19-Jun	392458
11-Jul	307709	13-Jul	484876	15-Jul	712963	17-Jul	592593	19-Jul	405507
11-Aug	362629	13-Aug	502410	15-Aug	731346	17-Aug	602669	19-Aug	458690
11-Sep	385586	13-Sep	522947	15-Sep	730403	17-Sep	604372	19-Sep	496451
11-Oct	370533	13-Oct	512282	15-Oct	728614	17-Oct	586740	19-Oct	471374
11-Nov	301287	13-Nov	456255	15-Nov	654671	17-Nov	575180	19-Nov	450015
11-Dec	275482	13-Dec	417443	15-Dec	614718	17-Dec	477491	19-Dec	358512

The transition matrix reflects changes in the amount of water supply/demand (increase +, stability 0, decrease -) respectively.

The transition probability matrix and the initial probability vector for the supply case were obtained from table 4:

$$P = \begin{bmatrix} 56/69 & 6/69 & 7/69 \\ 4/9 & 0 & 5/9 \\ 8/41 & 3/41 & 30/41 \end{bmatrix} = \begin{bmatrix} 0.8116 & 0.0870 & 0.1014 \\ 0.4444 & 0 & 0.5556 \\ 0.1951 & 0.0732 & 0.7317 \end{bmatrix} \text{ (3.1).}$$

$\pi_0 = (\frac{69}{119} \quad \frac{9}{119} \quad \frac{41}{119}) \;= (0.5798 \quad 0.0756 \quad 0.3446)$ (3.2)

To reach the stability of the matrix, Matlab was used. The results are described below:

$\lim_{n\to\infty} P^n = \pi$ (3.3).

$$P^{24} = \begin{bmatrix} 0.558 & 0.075 & 0.367 \\ 0.558 & 0.075 & 0.367 \\ 0.558 & 0.075 & 0.367 \end{bmatrix} \text{ (3.4).}$$

Equation (3.4) indicates a steady state (which is reached after 24 month).

The same result is obtained by solving the linear system of equations (2.9). Thus:

$\pi_1 = 0.558 = 55.8\%$, $\pi_2 = 0.075 = 7.5\%$, $\pi_3 = 0.367 = 36.7\%$ (3.5).

According to (3.5), we conclude that the rate of increase in the amount of water supply is (55.8%), the rate of decrease is 36.7% and the rate of stability situation is (7.5%).

Similarly, data from table (5) were obtain to generate the transition probability matrix and the initial probability vector for the demand case:

$$P = \begin{bmatrix} \frac{48}{66} & \frac{9}{66} & \frac{9}{66} \\ \frac{8}{12} & 0 & \frac{4}{12} \\ \frac{10}{41} & \frac{3}{41} & \frac{28}{41} \end{bmatrix} = \begin{bmatrix} 0.7272 & 0.1364 & 0.1364 \\ 0.6667 & 0 & 0.3333 \\ 0.2439 & 0.0732 & 0.6829 \end{bmatrix} \text{ (3.6).}$$

$\pi_0 = (\frac{66}{119} \quad \frac{12}{119} \quad \frac{41}{119}) \;= (0.5546 \quad 0.1008 \quad 0.3446)$ (3.7).

$$P_{\text{ij}}^{16} = \begin{bmatrix} 0.5546 & 0.1008 & 0.3446 \\ 0.5546 & 0.1008 & 0.3446 \\ 0.5546 & 0.1008 & 0.3446 \end{bmatrix} \text{ (3.8).}$$

$\pi_1 = 0.5546 = 55.46\%$, $\pi_2 = 0.1008 = 10.08\%$, $\pi_3 = 0.3446 = 34.46\%$ (3.9).

According to (3.9), we conclude that the rate of the increase in water demand is 55.46%, whereas, the rate of decrease is 34.46% and the rate of stability situation is 10.08%. Equation (3.8) indicates a steady state (which is reached after 16 month).

3.2 Markov Chains Model Construction Stages

According to Markov chains model Thomas-Fiering, the monthly data of Markov chain marks from 2010 to 2019 is shown in table 6 and 7 below:

Table (6): Monthly data of Markov chain marks for water supply amount from 2010 to 2019

	$\overline{q_j}$	s_j	s_j^2	r_j	b_j	$\overline{q_{j-1}}$
JAN	531434	158113.5	2.5E+10	0.991843	0.991843	562653.4
FEB	537186.6	166428.1	2.77E+10	0.98857	1.040555	531434
MAR	569196.6	164014.5	2.69E+10	0.992655	0.978259	537186.6
APR	595283.6	163644.7	2.68E+10	0.998595	0.996344	569196.6
MAY	636148.1	181066.7	3.28E+10	0.989977	1.095373	595283.6
JUN	660668.9	197745	3.91E+10	0.997811	1.089721	636148.1
JUL	677949.8	202137	4.09E+10	0.998719	1.020901	660668.9
AUG	694822.3	190500.4	3.63E+10	0.997736	0.940299	677949.8
SEP	687968.7	174907.2	3.06E+10	0.996448	0.914885	694822.3
OCT	651566.8	179390.1	3.22E+10	0.996632	1.022176	687968.7
NOV	608492.4	185464.3	3.44E+10	0.992017	1.025607	651566.8
DEC	562653.4	172588.2	2.98E+10	0.990514	0.921746	608492.4

Table (7): Monthly data of Markov chain marks for water demand amount from 2010 to 2019

	$\overline{q_j}$	s_j	s_j^2	r_j	b_j	$\overline{q_{j-1}}$
JAN	364866.2	118352.6	1.4E+10	0.990163	0.990163	387356.1
FEB	367880.1	120559.5	1.45E+10	0.993345	1.011868	364866.2
MAR	387441.1	118947	1.41E+10	0.995394	0.98208	367880.1
APR	397460.1	122680.8	1.51E+10	0.999268	1.030635	387441.1
MAY	430856.7	145780.5	2.13E+10	0.996375	1.183984	397460.1
JUN	444450.2	145979.9	2.13E+10	0.996825	0.998188	430856.7
JUL	457483.8	145969.7	2.13E+10	0.995481	0.995412	444450.2
AUG	479060.9	138796.8	1.93E+10	0.990653	0.941972	457483.8
SEP	491199	129299.9	1.67E +10	0.995479	0.927365	479060.9
OCT	474550.6	136177.7	1.85E+10	0.995985	1.048964	491199
NOV	437257.1	125359.6	1.57E+10	0.98293	0.904845	474550.6
DEC	387356.1	117037.5	1.37E+10	0.978291	0.913347	437257.1

In this stage random numbers have been generated using Microsoft excel, through the command RAND (), to get the random variable (T) with mean =0 and standard deviation =1, inverse error function (erf^{-1}) is used according to the following formula:

$\text{erf}^{-1}(z) = \frac{1}{2}$ (3.10).

While the value Z can be found through cumulative distribution function for logarithm distribution as follows:

$\text{CDF} = \text{erf}(z) = z = [\text{RAND}() - 0.5] * 2$ (3.11).

The mean value of the normal logarithm of the random figures equals its standard deviation, i.e. $\mu = \sigma = 1$, then:

$\text{erf}\left[\frac{\ln x - 1}{\sqrt{2}}\right] = z$ (3.12)

$\text{erf}^{-1}(z) = \frac{\ln x - 1}{\sqrt{2}} \Rightarrow lnx - 1 = \sqrt{2}\,\text{erf}^{-1}(z)$ (3.13)

$t = \ln x = \sqrt{2}\,\text{erf}^{-1}(z) + 1$ (3.14).

The random numbers in table (8) were used by Abu Libda,2018) [1], these values give better results when applied on the data used in this study.

Table (8): Represents the steps followed to generate random numbers

	Rand	Z	erf^{-1}	t_{ij}
1	0.699645	0.399289	0.370085	1.523379
2	0.45481	-0.090379	-0.08027	0.886483
3	0.63732	-0.872536	-1.0558	-0.49313
4	0.224711	-0.50577	-0.53482	0.243657
5	0.236038	-0.527923	-0.50840	0.280915
6	0.471912	-0.56176	-0.04983	0.929536
7	0.999341	0.998683	1.443813	3.041859
8	0.533139	0.066278	0.58805	1.083163
9	0.095672	-0.808656	-0.91763	-0.29772
10	0044676	-0.910651	-1.15355	-0.63136
11	0.997494	0.994989	1.429319	3.021363
12	0.407816	-0.184368	-0.1687	0.766834

The Markov model consists of two parts; the first one deterministic part, which considers the effect of previous value in the model. The other part is random part which represents the random model. By combining of the two parts the monthly model of Markov's for prediction is constructed according to equation (2.14).

Tables 9 and 10 show the prediction of water supply/demand for 2019, depending on which month of the year can be predicted based on condition where the previous values for this month is known.

Table (9): Markov model construction for prediction of water supply amount for 2019

$q_{i-1,j-1}$	$\overline{q_j}+b_j(q_{i-1,j-1}-\overline{q_{j-1}})$	t_{ij}	$t_{ij}s_j(1-r_j^2)^{1/2}$	q_{ij}
611725	580105.3	1.523379	30702.26	610807.6
585333	593271.5	0.886483	22243.2	615514.7
588242	619142	-0.49313	-9784.95	609357
612514	638442.6	0.243657	2112.548	640555.2
641932	687245.5	0.280915	7183.508	694429
717864	749716.4	0.929536	12154.6	761871
744319	763348.3	3.041859	31111.69	794460
762135	773981.5	1.083163	13876.02	787857.5
762183	749596	-0.29772	-4385.11	745210.9
759489	724673.2	-0.63136	-9287.44	715385.7
742549	701804.4	3.021363	70662.08	772466.5
718011	663601.7	0.766834	18186.36	681788.1

Table (10): Markov model construction for prediction of water demand amount for 2019

$q_{i-1,j-1}$	$\overline{q_j}+b_j(q_{i-1,j-1}-\overline{q_{j-1}})$	t_{ij}	$t_{ij}s_j(1-r_j^2)^{1/2}$	q_{ij}
355628	362187.7	1.523379	30702.26	392890
325052	358532.3	0.886483	22243.2	380775.5
366762	345380.5	-0.49313	-9784.95	335595.5
374223	376147.5	0.243657	2112.548	378260
412416	403344.4	0.280915	7183.508	410527.9
401228	426042.9	0.929536	12154.6	438197.5
443773	414459.9	3.041859	31111.69	445571.6
427765	466145.7	1.083163	13876.02	480021.7
424208	443629	-0.29772	-4385.11	439243.9
411882	404279.5	-0.63136	-9287.44	394992
404332	380551.7	3.021363	70662.08	451213.8
384651	357284.1	0.766834	18186.36	375470.4

In the same manner, the amount of water (supply/demand) can be predicted for 2020 year, as shown in the tables (11) & (12):

Table (11): Markov model construction for prediction of water supply amount for 2020

$q_{i-1,j-1}$	$\overline{q_j}+b_j(q_{i-1,j-1}-\overline{q_{j-1}})$	t_{ij}	$t_{ij}s_j(1-r_j^2)^{1/2}$	q_{ij}
658395	626394.6	1.523379	30702.26	657096.9
595379	603724.9	0.886483	22243.2	625968.1
568120	599457.5	-0.49313	-9784.95	589672.5
619416	645319.4	0.243657	2112.548	647432

$q_{i-1,j-1}$	$\overline{q_j}+b_j(q_{i-1,j-1}-\overline{q_{j-1}})$	t_{ij}	$t_{ij}s_j(1-r_j^2)^{1/2}$	q_{ij}
642554	687926.8	0.280915	7183.508	695110.3
727326	760027.3	0.929536	12154.6	772181.9
751413	770590.6	3.041859	31111.69	801702.3
767656	779172.9	1.083163	13876.02	793048.9
758925	746615.3	-0.29772	-4385.11	742230.2
752563	717593.6	-0.63136	-9287.44	708306.1
737737	696869.2	3.021363	70662.08	767531.2
720426	665827.7	0.766834	18186.36	684014.1

Table (12): Markov model construction for prediction of water demand amount for 2020

$q_{i-1,j-1}$	$\overline{q_j}+b_j(q_{i-1,j-1}-\overline{q_{j-1}})$	t_{ij}	$t_{ij}s_j(1-r_j^2)^{1/2}$	q_{ij}
358512	336305.8	1.523379	30702.26	367008.1
325261	327804.9	0.886483	22243.2	350048.1
335053	355202.3	-0.49313	-9784.95	345417.3
367566	376976.1	0.243657	2112.548	379088.7
377204	406873.8	0.280915	7183.508	414057.3
379944	393629.8	0.929536	12154.6	405784.4
392458	405730.2	3.041859	31111.69	436841.9
405507	430100.2	1.083163	13876.02	443976.2
458690	472307.7	-0.29772	-4385.11	467922.6
496451	480059.8	-0.63136	-9287.44	470772.3
471374	434382.8	3.021363	70662.08	505044.8
450015	399008.5	0.766834	18186.36	417194.8

Table (13): The predictive value (supply) of Markov model

Month	y_t	$\widehat{y_t}$	error
JAN(1/2020)	552465	657096.9	0.18939=18.9%
FEB(2/2020)	548691	625968.1	0.14084=14%
MAR(3/2020)	598768	589672.5	0.01519=1.5%
APR(4/2020)	637289	647432	0.01592=1.6%
MAY(5/2020)	702951	695110.3	0.011154=1.1%

Table (14): The predictive value (demand) of Markov model

Month	y_t	$\widehat{y_t}$	error
JAN(1/2020)	325261	367008.1062	0.12835=12.8%
FEB(2/2020)	335053	350048.0712	0.04475=4.5%
MAR(3/2020)	367566	345417.3019	0.060258=6%
APR(4/2020)	377204	379088.6677	0.005=0.5%
MAY(5/2020)	379944	414057.3141	0.08979=8.9%

Prediction accuracy criteria MAE, MAPE, RMSE are considered an indication of how efficient it is in prediction as explained in table (15):

Table (15): Represents the results of the test of statistical criteria (MAE) (MAPE) (RMSE)

Criteria	RMSE	MAPE	MAE
Supply	11960.58	0.310408	1741.568
Demand	5496.595941	0.273453	957.4071438

Table (16): Represents the Predicted values of the amount of water supply during the years (2020-2022-2024-2026-2028-2030) by Markov prediction model.

Month			Supply			
	2020	**2022**	**2024**	**2026**	**2028**	**2030**
JAN	657096.9	725571.8	688073.1	746066.1	733998	709123.8
FEB	625968.1	716629.6	732726.3	753556.5	800957.2	775308.7
MAR	589672.5	709087.6	778790.4	740619.3	799652.2	787367.6
APR	647432	674180.7	762546.9	778236.1	798539	844739.6
MAY	695110.3	667992.4	798318.2	874389.6	832730.9	897157.5
JUN	772181.9	742898.4	774827.1	880305.3	899032.8	923267.3
JUL	801702.3	787065.4	756896.8	901883.9	986513.2	940167.9
AUG	793048.9	844999.8	816889.1	847539	948793.1	966770.5
SEP	742230.2	802738.5	790146.9	764193.9	888921.2	961724.8
OCT	708306.1	729655.9	778238.9	751950.6	780613.7	875303.8
NOV	767531.2	726514.3	789948.2	776747.8	749540	880298

Table (17): Represents the Predicted values of the amount of water demand during the years (2020-2022-2024-2026-2028-2030) by Markov prediction model.

MONTH			Demand			
	2020	**2022**	**2024**	**2026**	**2028**	**2030**
JAN	367008.1	383799.7	449900.2	461683.6	497046.1	478572.3
FEB	350048.1	392290.6	500945.1	464314.8	498154.5	525554.2
MAR	345417.3	360143.7	508825.9	484002	495711.7	530852.6
APR	379088.7	356261.4	431393.1	524171.9	487096	521347.3
MAY	414057.3	416288.8	457854.5	588661.9	558370.4	572659.2
JUN	405784.4	439835.8	433022.5	503878.6	613528.1	569710.4
JUL	436841.9	450107.1	459365.1	527519.6	657490.9	627393
AUG	443976.2	473492.8	520005.2	511528.1	577966.3	680779.2
SEP	467922.6	454277.6	522847.3	501325.5	560861.9	674398.7
OCT	470772.3	440847.1	467402.1	500492.8	492246.5	556875.8
NOV	505044.8	504500.4	490452.3	529554.5	509127	565636
DEC	417194.8	467456.1	430398.6	464173.6	491521	484706

4 Discussion and Recommendations

This study aims at predicting the amount of water (supply and demand) at long run in Tulkarm city, based on the actual data. For this purpose, Markov chain model has been chosen and by which the best and valid model to undergoes such prediction was tested. The used time series data has been generated for the amount of water (supply, demand), based on the actual municipality data obtained for the period from (JAN, 2010) to (DEC, 2019). Based on the processing of the above mentioned data, the following remarks can be concluded:

- The results indicated that the amount of water supply will stabilize at n=24, where the probability of increase state is 0.558, decrease state is 0.367, and stability state is 0.075.
- The water demand series showed stability at n=16, where the probability of increase state is 0.5546, decrease state is 0.3446 and stability state is 0.1008. This incremental change in water demand followed many factors including population growth and the yearly variation in rainfall amount.

The study also aims to choose the best model for the water (supply, demand) in Tulkarm city as case study, three statistical models have been tested (Markov chains - time series model (ARIMA) - simple regression model), where these models were tested on water supply and demand data. The comparative study for the three models reflected the following:

- Time series for water supply and demand represents an unstable time series.
- The amount of water supply series showed an unstable trend, however, taking the first difference and the first seasonal component, has made the series more stable. This means that the model is susceptible for several factors that may affect its stability such as seasonal variation and fluctuation in supply.
- The amount of water demand series showed an unstable trend, however, taking the first difference and the first seasonal component, has made the series also stable. The same as for supply, this also means that the model is susceptible for several factors that may affect its stability such as seasonal variation and fluctuation in demand.

After examining several models and through the evaluation criteria for comparing the proposed ARIMA models, it was found that the best amount of water supply series was by using ARIMA (2,1,2) (1,1,1), and the best amount of water supply series model was by using ARIMA (2,1.2) (1,1,1).

ARIMA (2,1,2) (0,1,1) & (ARIMA) (2,1,2) (1,1,1)) was found to be the best tool for the proposed models because they have less value of the evaluation criteria (AIC)(BIC), which exceed all other models by passing all tests and residual check properties of the model.

It was also found that the simple linear regression analysis model ($\widehat{Y}_i = 309941.918 + 1953.360X$) was the most suitable model for the water demand series than the water supply series, where the most appropriate model for this purpose was the ($\widehat{Y}_i = 309941.918 + 1953.360X$) model.

The results of the models were also compared according to the predictive accuracy criterion (RAMS) between the three used models (Markov model, the model (ARIMA) and the regression model). The comparison results showed that the Markov model outperformed the other models in demand series. In addition, results showed that the (ARIMA) model outperformed the other models in supply series.

As a result, researchers can conclude that the Markov model is the best applied tool to predict the time series for the amount of water demand in 2021-2025-2030 in Tulkarm city. While, the time series ARIMA (2,1,2) (0,1,1) found to be the best applied model to predict the time series for the amount of water supply in 2021-2025-2030 in Tulkarm city.

Palestinian Water Authority PWA is invited to continue the application of these mathematical models, that can give significant results and information about the changes in supply and demand, in order to predict its changes with incremental population growth and climate change conditions, thus to take the proper action that can help the PWA in better management for the available water resources to ensure its sustainability for the next generations. It has to take into account the amount of increase in water demand that faced by the decrease in water supply, specially that the results showed more decrease in water supply in a shorter period that reached 16 months while the increase on demand take relatively longer period of 24 months.

Researchers are also encouraged to conduct studies that aim to compare Markov models with other advanced statistical methods in many fields.

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Received: September 2nd, 2021
Accepted: December 12th, 2021