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Wavelet based Numerical solution of Allen-Cahn and Newell-Whitehead-Segel equations by Lifting schemes

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Abstract Many of the phenomena that arise in mathematical physics and engineering fields can be described by partial differential equations (PDEs). In this paper, we present wavelet-based lifting schemes for the numerical solution of Allen-Cahn and Newell-Whitehead-Segel equations using different wavelet filter coefficients. The numerical solutions obtained by these schemes are compared with the exact solution to demonstrate the accuracy and convergence in low computational time as compared with the existing scheme. Some test problems are taken to exhibit the applicability and accuracy of the schemes.

1 Introduction

Partial differential equations (PDEs) occur in many branches of applied mathematics, for example, in quantum mechanics, hydrodynamics, elasticity, and electromagnetic theory. The analytical behavior of these equations is a rather involved process and requires applications of advanced mathematical methods. On the other hand, in recent years, greater importance has been shifted from analytical techniques to numerical methods. The principal attraction of numerical methods is that solutions could be obtained for many problems which are not ready for analytical treatment. The accessibility of modern digital computers paved the way for the development of efficient methods for solving PDEs [1].
In general, PDEs with or without reaction terms are used as a fundamental tool to model a wide class of phenomena occurring in the physical and biological sciences. Some of the equations of this type are:

a) The **Newell-Whitehead equation** describes the dynamical behavior near the bifurcation point for the Rayleigh -Benard convection of binary fluid mixtures.

b) The **Allen-Cahn equation** is a reaction–diffusion equation of mathematical physics which describes the process of phase separation in multi-component alloy systems, as well as order-disorder transitions.

c) Newell-Whitehead-Segel equation is one of the most important of amplitude equations, which describes the appearance of the stripe pattern in two-dimensional systems.
Recently, many authors are proposed for the analytical and numerical solution of these equations; some of them are: Adomian decomposition method [2], differential transformation method [3], homotopy perturbation method, and homotopy analysis method [4], Haar wavelet method [5] etc.
For finding the solutions to PDEs, for most cases, it is necessary to employ discretization methods to reduce the sets of PDEs to systems of algebraic equations, and such types of equations are solved by using direct methods. Direct methods are theoretically producing the exact solution to the system in a finite number of steps. In practice, of course, the solution obtained will be contaminated by the round-off error. To minimize such round-off error, iterative methods are frequently used for solving linear systems. For large systems, these methods are inefficient in terms of both computer storage and computational cost.
The applications of wavelet theory in numerical methods for solving differential equations are

roughly 25 years old. In the early nineties people were very optimistic because it seemed that many nice properties of wavelets would automatically lead to efficient numerical methods for differential equations. The reason for this optimism was the fact that many partial differential equations (PDEs) have solution containing local phenomena (e.g. formation of shock, hurricanes) and interactions between several scales (e.g. turbulence particularly atmospheric turbulence because there is motion on a continuous range of length scales). Such solutions can be well represented in wavelet basis because of its nice properties; few of them are like compact support (locality in space) and vanishing moment (locality in scale). Wavelets permit the perfect representation of a variety of functions and operators. Moreover, wavelets establish a connection with fast numerical algorithms [6].
In recent times, some of the work on wavelet-based methods is the discrete wavelet transforms (DWT) and the full approximation scheme (FAS) were introduced in [7 - 8]. The wavelet based full approximation scheme (WFAS) has been shown to be a very efficient and favorable method for numerous problems related to computational science and engineering fields [9]. These methods can be either used as an iterative solver or as a preconditioning technique, offering in many cases better performance than some of the most innovative and existing FAS algorithms. Due to the efficiency and potentiality of WFAS, further have been carried out for its enrichment. In order to realize this task, work was built that is orthogonal/Biorthogonal discrete wavelet transform using lifting scheme [10]. Wavelet-based lifting techniques were introduced by Sweldens [11], which permit some improvements on the properties of existing wavelet transforms. Wavelet-based numerical solution of elasto-hydrodynamic lubrication problems via lifting scheme was introduced by Shiralashetti et al. [12]. The technique has some numerical benefits as a reduced number of operations which are fundamental in the context of the iterative solvers. Evidently, all attempts to simplify wavelet solutions for PDE are welcome. In PDE, matrices arising from systems are dense with non-smooth diagonal and smooth away from the diagonal. This smoothness of the matrix transforms into smallness using wavelet transform and it leads to design the effective wavelets based lifting scheme.
The lifting scheme is a new approach to construct the so-called second-generation wavelets that are not necessarily translations and dilations of one function. The latter we refer to as first-generation wavelets or classical methods. The lifting scheme has some additional advantages in comparison to classical wavelets. This transform works for signals of an arbitrary size with correct treatment of boundaries. Another feature of the lifting scheme is that all constructions are derived in the spatial domain. This is in contrast to the traditional approach, which relies heavily on the frequency domain.
The lifting scheme starts with a set of well-known filters; thereafter lifting steps are used, in an attempt to improve (lift) the properties of a corresponding wavelet decomposition. This procedure has some mathematical benefits as a reduced number of operations which are essential in the context of iterative solvers. In addition to this, the present paper illustrates the application of the lifting scheme for the numerical solution of Allen-Cahn and Newell-Whitehead-Segel equations.
The present paper is organized as follows: in section 2, preliminaries of wavelet filter coefficients and lifting scheme are given. The method of solution is explained in section 3, Numerical results of the problems are presented in section 4 and finally, conclusion of the proposed work is given in section 5.

2 Preliminaries of wavelet filter coefficients and Lifting scheme

The lifting scheme starts with a set of well-known filters; thereafter lifting steps are used in an attempt to improve the properties of the corresponding wavelet decomposition [13].
Now, we have discussed about different wavelet filters as follows:

a) Haar wavelet filter coefficients

We know that low pass filter coefficients $[a_0,\ a_1]^T = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]^T$ and high pass filter coefficients $[b_0,\ b_1]^T = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]^T$ play an important role in decomposition.

b) Daubechies wavelet filter coefficients

Daubechies introduced scaling functions having the shortest possible support. The scaling function ϕ_N has support$[0,\ N-1]$, while the corresponding wavelet ψ_N has support in the interval $\left[1\ -\ \frac{N}{2},\ \frac{N}{2}\right]$. We have low pass filter coefficients $[a_0,\ a_1,\ \ a_2,\ a_3]^T = \left[\frac{1+\sqrt{3}}{4\sqrt{2}},\ \ \frac{3+\sqrt{3}}{4\sqrt{2}},\ \ \frac{3-\sqrt{3}}{4\sqrt{2}},\ \ \frac{1-\sqrt{3}}{4\sqrt{2}}\right]^T$ and high pass filter coefficients $[b_0,\ b_1,\ b_2,\ b_3]^T = \left[\frac{1-\sqrt{3}}{4\sqrt{2}},\ \ -\frac{3-\sqrt{3}}{4\sqrt{2}},\ \ \frac{3+\sqrt{3}}{4\sqrt{2}},\ \ -\frac{1+\sqrt{3}}{4\sqrt{2}}\right]^T$

c) Biorthogonal (CDF (2,2)) wavelets

Let's consider the (5, 3) biorthogonal spline wavelet filter pair, the low pass filter pair are $(\tilde{a}_{-1}, \tilde{a}_0, \tilde{a}_1) = \left(\frac{1}{2\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$ and $(a_{-2}, a_{-1}, a_0, a_1, a_2) = \left(\frac{-1}{4\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{-1}{4\sqrt{2}}\right)$.
But, we have $b_k = (-1)^k \tilde{a}_{1-k}$ and $\tilde{b}_k = (-1)^k a_{1-k}$, the high pass filter pair are $b_0 = \frac{1}{2\sqrt{2}},\ \ b_1 = \frac{-1}{\sqrt{2}},\ \ b_2 = \frac{1}{2\sqrt{2}}$ & $\tilde{b}_{-1} = \frac{1}{4\sqrt{2}},\ \ \tilde{b}_0 = \frac{1}{2\sqrt{2}},\ \ \tilde{b}_1 = \frac{-3}{2\sqrt{2}},\ \ \tilde{b}_2 = \frac{1}{2\sqrt{2}},\ \ \tilde{b}_3 = \frac{1}{4\sqrt{2}}$

Foundations of lifting scheme:

Consider numbers *a, b* as two neighboring samples of a sequence, and then these have some correlation which we would like to take advantage. The simple linear transform which replaces *a* and *b* by average ***s*** and difference *d* i.e.

$$s = \frac{a\ +\ b}{2}\ \ \&\ \ d\ \ =\ \ b\ -\ a$$

The idea is that if *a* and *b* are highly correlated, the expected absolute value of their difference d will be small and can be represented with fever bits. In the case that *a* = *b*, the difference is simply zero. We have not lost any information because we can always recover *a* and *b* from the given *s* and *d* as:

$$a = s\ -\ \frac{d}{2}\ \ \&\ \ b\ =\ s\ +\ \frac{d}{2}$$

Finally, a wavelet transforms built through lifting consists of three steps: split. Predict and update as given in Figure 1 [14].

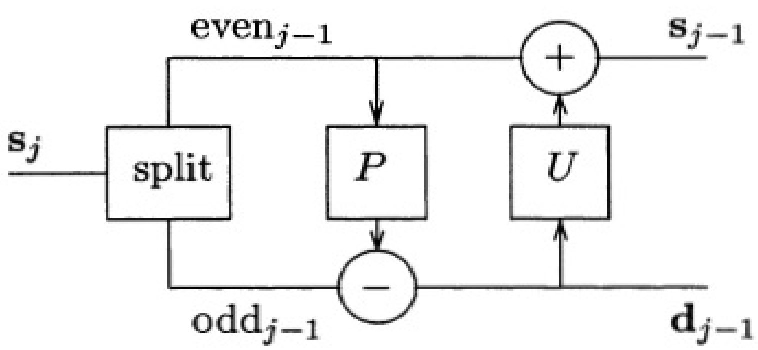


Fig. 1. Steps in lifting scheme

Split: Splitting the signal into two disjoint sets of samples.
Predict: If the signal contains some structure, then we can expect a correlation between a sample and its nearest neighbors. i.e. $d_{j-1} = \text{odd}_{j-1} - \text{P}(\text{even}_{j-1})$
Update: Given an even entry, we have predicted that the next odd entry has the same value, and stored the difference. We then update our even entry to reflect our knowledge of the signal. i.e. $s_{j-1} = \text{even}_{j-1} + \text{U}(d_{j-1})$
The general lifting stages for decomposition and reconstruction of a signal are given in Figure 2.

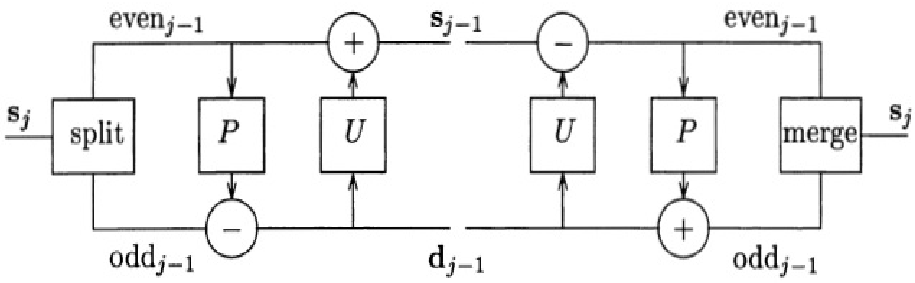


Fig. 2. Lifting wavelet algorithm

The detailed algorithm using different wavelets is given in the next section.

3 Method of solution

Consider that the general nonlinear parabolic PDE is of the form,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + au + bu^n, \; 0 \le x \le 1 \; \& \; t > 0 \tag{3.1}$$

Where a, b are constants and n is positive integer [15].

a) For $b = -b'$ & $n = 3$, Eq. (3.1) becomes Newell-Whitehead equation i.e.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + au - b'u^3, \; 0 \le x \le 1 \; \& \, t > 0 \tag{3.2}$$

b) For $a = 1$, $b = -1$ & $n = 3$, Eq. (3.1) becomes Allen-Cahn equation i.e.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u - u^3, \; 0 \le x \le 1 \; \& \, t > 0 \tag{3.3}$$

c) For $a = 1$, $b = -1$ & $n = 4$, Eq. (3.1) becomes Newell-Whitehead-Segel equation i.e.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u - u^4, \; 0 \le x \le 1 \; \& \; t > 0 \tag{3.4}$$

After discretizing the equation (2) or (3) or (4) through the finite difference method (FDM), we get a system of algebraic equations. Through this system we can write the system as

$$Au = b \tag{3.5}$$

where A is $N \times N$ coefficient matrix, b is $N \times N$ matrix and u is $N \times N$ matrix to be determined. Where $N = 2^J$, N is the number of grid points and J is the level of resolution.
Solve Eq. (3.5) through the iterative method, we get an approximate solution. Approximate solution containing some error, therefore required solution equals sum of approximate solution and error. There are many methods to minimize such error to get the accurate solution. Some of them are FAS, WFAS etc. Now we are using the advanced technique based on different wavelets called as lifting scheme. Recently, lifting schemes have been useful in signal analysis and image processing in the area of science and engineering. But currently it extends to approximations in numerical analysis [6]. Here, we are discussing the algorithm of the lifting schemes as follows:

a) Haar wavelet Lifting scheme (HWLS)

In [9], Daubechies and Sweldens have shown that every wavelet filter can be decomposed into lifting steps. More details of the advantages as well as other important structural advantages of the lifting technique can be available in [10]. The representation of Haar wavelet via lifting form presented as;

Decomposition:

Consider approximate solution $S = u$ like as signal and then apply the HWLS decomposition (finer to coarser) procedure as,

$$\left.\begin{array}{l} d^{(1)} = S_{2j} - S_{2j-1}, \quad s^{(1)} = S_{2j-1} + \frac{1}{2}d^{(1)}, \quad S_1 = \sqrt{2}\, s^{(1)} \\ \text{and} \quad D = \frac{1}{\sqrt{2}}d^{(1)} \end{array}\right\} \tag{3.6}$$

In this stage finally, we get new approximation as,

$$S = [S_1 \ \ D] \tag{3.7}$$

Reconstruction:

Now apply the HWLS reconstruction (coarser to finer) procedure as,

$$\left.\begin{array}{l} d^{(1)} = \sqrt{2}\, D, \quad s^{(1)} = \frac{1}{\sqrt{2}} S_1, \quad S_{2j-1} = s^{(1)} - \frac{1}{2}d^{(1)} \\ \text{and} \quad S_{2j} = d^{(1)} + S_{2j-1} \end{array}\right\} \tag{3.8}$$

which is the required solution of the given equation.

b) Daubechies wavelet Lifting scheme (DWLS)

As discussed in the previous section **a**, we follow the same procedure, but we used a different wavelet, i.e. Daubechies 4^{th} order wavelet coefficient. The DWLS procedure is as follows;

Decomposition:

$$\left.\begin{array}{l} s^{(1)} = S_{2j-1} + \sqrt{3}\, S_{2j}\,, \quad d^{(1)} = S_{2j} - \frac{\sqrt{3}}{4}s^{(1)} - \left(\frac{\sqrt{3}-2}{4}\right) s_1^{(j-1)}, \\ s^{(2)} = s^{(1)} - d_1^{(j+1)}, \quad S_1 = \frac{\sqrt{3}-1}{\sqrt{2}}s^{(2)} \ \text{ and} \\ D = \frac{\sqrt{3}+1}{\sqrt{2}}d^{(1)} \end{array}\right\} \tag{3.9}$$

Here, we get new approximation as,

$$S = [S_1 \ \ D] \tag{3.10}$$

Reconstruction:

Now, we apply the DWLS reconstruction (coarser to finer) procedure as,

$$\left.\begin{array}{l} d^{(1)} = \frac{\sqrt{2}}{\sqrt{3}+1}D, \\ s^{(2)} = \frac{\sqrt{2}}{\sqrt{3}-1}\, S_1, \\ s_1^{(j)} = s^{(2)} + d_1^{(j+1)}, \\ S_{2j} = d^{(1)} + \frac{\sqrt{3}}{4}s_1^{(j)} + \frac{\sqrt{3}-2}{4}s_1^{(j-1)} \ \text{ and} \\ S_{2j-1} = s^{(1)} - \sqrt{3}\, S_{2j} \end{array}\right\} \tag{3.11}$$

which is the required solution of the given equation.

c) Biorthogonal wavelet Lifting scheme (BWLS)
As discussed in the previous sections **a** and **b**, we follow the same procedure here we used another wavelet, i.e. a biorthogonal wavelet (CDF (2,2)). The BWLS procedure is as follows;
Decomposition:

$$\left.\begin{array}{l} d^{(1)} = S_{2j} - \frac{1}{2}\left[S_{2j-1} + S_{2j+2}\right], \\ s^{(1)} = S_{2j-1} + \frac{1}{4}\left[d^{(1)}_{j-1} + d^{(1)}\right], \\ D = \frac{1}{\sqrt{2}} d^{(1)}, \\ S_1 = \sqrt{2}\, s^{(1)} \end{array}\right\} \tag{3.12}$$

In this stage finally, we get new signal as,

$$S = [S_1 \;\; D] \tag{3.13}$$

Reconstruction:
Now, we apply the BWLS reconstruction (coarser to finer) procedure as

$$\left.\begin{array}{l} s^{(1)} = \frac{1}{\sqrt{2}} S_1, \\ d^{(1)} = \sqrt{2}\, D, \\ S_{2j-1} = s^{(1)} - \frac{1}{4}\left[d^{(1)}_{j-1} + d^{(1)}\right] \\ S_{2j} = d^{(1)} + \frac{1}{2}\left[S_{2j-1} + S_{2j+2})\right], \end{array}\right\} \tag{3.14}$$

which is the required solution of the given equation.
The coefficients $s_1^{(j)}$ and $d_1^{(j)}$ are the average and detailed coefficients respectively, of the approximate solution u_a. The new approaches are tested through some of the numerical problems and the results are shown in the next section.

4 Numerical examples

In this section, we applied Lifting scheme for the numerical solution of Allen-Cahn and Newell-Whitehead-Segel equations and also show the validity and applicability of HWLS, DWLS and BWLS. The error is computed by

$$E_{max} = \max \; |u_e(x, t) - u_a(x, t)|,$$

where $u_e(x, t)$ and $u_a(x, t)$ are exact and approximate solution respectively.
Problem 4.1: Consider the Newell-Whitehead equation, (In Eq.(3.2); , $a = 3, \; b' = 4$)

$$\text{i.e. } \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 3u - 4u^3, \; 0 \le x \le 1 \;\&\; t > 0 \tag{4.1}$$

subject to the I.C.: $u(x, 0) = \frac{\sqrt{3}}{4}\left[\frac{e^{\sqrt{6}x}}{e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x}}\right]$

and B.C.s: $\left.\begin{array}{l} u(0, t) = \frac{\sqrt{3}}{4}\left[\frac{1}{1 + e^{-\frac{9}{2}t}}\right] \\ u(1, t) = \frac{\sqrt{3}}{4}\left[\frac{e^{\sqrt{6}}}{e^{\sqrt{6}} + e^{\frac{\sqrt{6}}{2} - \frac{9}{2}t}}\right] \end{array}\right\}$

Which has the exact solution $u(x, t) = \frac{\sqrt{3}}{4}\left[\frac{e^{\sqrt{6}x}}{e^{\sqrt{6}x} + e^{\frac{\sqrt{6}}{2}x - \frac{9}{2}t}}\right]$ [16].
Using the methods described in the section 3, we find the numerical solutions and in comparison with the exact solutions are presented in figure 3. The maximum absolute errors with CPU time of the methods are presented in table 1.

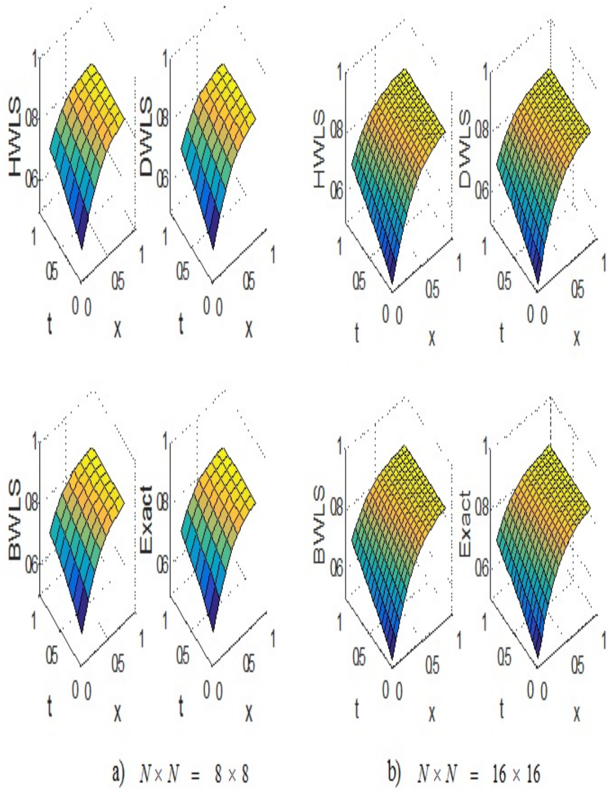


Fig. 3. Comparison of numerical solutions with the exact solution of problem 4.1.

Table 1. Maximum error and CPU time (in seconds) of the methods of problem.4.1.

$N \times N$	Method	$E_{\max}$	Setup time	Running time	Total time
4×4	FDM	1.0812e-02	2.7698	0.0019	2.7717
	HWLS	1.0812e-02	0.0010	0.0028	0.0038
	DWLS	1.0812e-02	0.0003	0.0095	0.0098
	BWLS	1.0812e-02	0.0003	0.0040	0.00043
. 16×16	FDM	3.9352e-03	3.4449	0.0024	3.4473
	HWLS	3.9352e-03	0.0010	0.0032	0.0042
	DWLS	3.9352e-03	0.0003	0.0102	0.0105
	BWLS	3.9352e-03	0.0004	0.0044	0.0048
64×64	FDM	1.0881e-03	6.7603	0.0039	6.7642
	HWLS	1.0881e-03	0.0010	0.0030	0.0040
	DWLS	1.0881e-03	0.0003	0.0097	0.0100
	BWLS	1.0881e-03	0.0003	0.0041	0.0044

Problem 4.2: Next, consider the Allen-Cahn equation, (i.e. Eq.(3.3))

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u - u^3, \quad 0 \leq x \leq 1 \;\&\; t > 0 \tag{4.2}$$

subject to the I.C.: $u\,(x\,,\,0) = \dfrac{\sinh\left(\frac{x}{\sqrt{2}}\right)}{1 + \cosh\left(\frac{x}{\sqrt{2}}\right)}$

and B.C.s: $\left.\begin{aligned} u(0, t) &= 0 \\ u(1, t) &= \frac{e^{\frac{1}{\sqrt{2}}} - e^{-\frac{1}{\sqrt{2}}}}{e^{\frac{1}{\sqrt{2}}} + e^{-\frac{1}{\sqrt{2}}} + 2e^{-\left(\frac{3}{2}\right)t}} \end{aligned}\right\}$

Which has the exact solution $u(x, t) = = \frac{e^{\frac{x}{\sqrt{2}}} - e^{-\frac{x}{\sqrt{2}}}}{e^{\frac{x}{\sqrt{2}}} + e^{-\frac{x}{\sqrt{2}}} + 2e^{-\left(\frac{3}{2}\right)t}}$ [15].

By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solutions are presented in figure 4 and the maximum absolute errors with CPU time of the methods are presented in table 2.

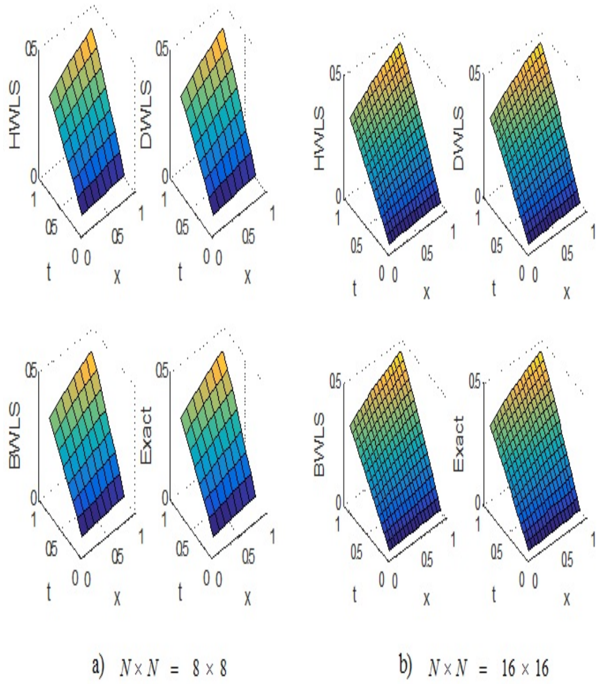


Fig. 4. Comparison of numerical solutions with the exact solution of problem 4.2.

Table 2. Maximum error and CPU time (in seconds) of the methods of problem 4.2.

$N \times N$	Method	$E_{\max}$	Setup time	Running time	Total time
4×4	FDM	1.1747e-03	3.7246	0.0019	3.7265
	HWLS	1.1747e-03	0.0010	0.0029	0.0039
	DWLS	1.1747e-03	0.0003	0.0098	0.0101
	BWLS	1.1747e-03	0.0003	0.0040	0.0043
. 16×16	FDM	3.2847e-04	3.5575	0.0023	3.5598
	HWLS	3.2847e-04	0.0010	0.0029	0.0039
	DWLS	3.2847e-04	0.0003	0.0096	0.0099
	BWLS	3.2847e-04	0.0004	0.0040	0.0044
64×64	FDM	1.0187e-04	7.2244	0.0040	7.2284
	HWLS	1.0187e-04	0.0010	0.0030	0.0040
	DWLS	1.0187e-04	0.0003	0.0097	0.0100
	BWLS	1.0187e-04	0.0004	0.0044	0.0048

Problem 4.3: Next, consider the Newell-Whitehead-Segel equation, (i.e. Eq.(3.4))

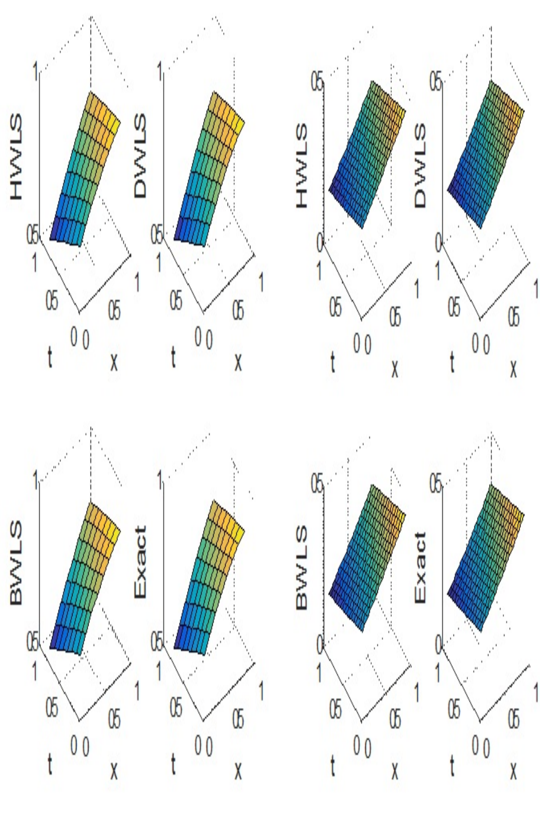
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u - u^4, \; 0 \le x \le 1 \;\&\; t > 0 \tag{4.3}$$

subject to the I.C.: $u\,(x\,,\,0) \;=\; \left[\frac{1}{2}\tanh\left\{-\frac{3\,x}{2\sqrt{10}}\right\} + \frac{1}{2}\right]^{2/3}$

and B.C.s: $$\left.\begin{aligned} u\,(0\,,\,t) &= \left[\frac{1}{2}\tanh\left\{\frac{21}{20}\,t\right\} + \frac{1}{2}\right]^{2/3} \\ u\,(1\,,\,t) &= \left[\frac{1}{2}\tanh\left\{-\frac{3}{2\sqrt{10}}\left(1 - \frac{7}{\sqrt{10}}\,t\right)\right\} + \frac{1}{2}\right] \end{aligned}\right\}$$

Which has the exact solution $u\,(x\,,\,t) = \left[\frac{1}{2}\tanh\left\{-\frac{3}{2\sqrt{10}}\left(x - \frac{7}{\sqrt{10}}\,t\right)\right\} + \frac{1}{2}\right]^{2/3}$ [16].

By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solutions are presented in figure 5. The maximum absolute errors with CPU time of the methods are presented in table 3



a) $N\times N = 8\times 8$ b) $N\times N = 16\times 16$

Fig. 5. Comparison of numerical solutions with the exact solution of problem 4.3.

Table 3 Maximum error and CPU time (in seconds) of the methods of problem 4.3.

$N \times N$	Method	$E_{\max}$	Setup time	Running time	Total time
4 × 4	FDM	3.7098e-03	3.3340	0.0019	3.3359
	HWLS	3.7098e-03	0.0009	0.0029	0.0038
	DWLS	3.7098e-03	0.0003	0.0100	0.0103
	BWLS	3.7098e-03	0.0003	0.0040	0.0043
. 16 × 16	FDM	1.1184e-03	4.2557	0.0025	4.2582
	HWLS	1.1184e-03	0.0010	0.0032	0.0042
	DWLS	1.1184e-03	0.0003	0.0104	0.0107
	BWLS	1.1184e-03	0.0004	0.0043	0.0047
64 × 64	FDM	2.9054e-04	13.3630	0.0051	13.3681
	HWLS	2.9054e-04	0.0010	0.0033	0.0043
	DWLS	2.9054e-04	0.0003	0.0105	0.0108
	BWLS	2.9054e-04	0.0004	0.0045	0.0049

5 Conclusions

In this paper, we find the wavelets-based numerical solution of Allen-Cahn and Newell-Whitehead-Segel equations by Lifting schemes using different wavelet filters. We observe that:

a) From the above figures (3-5), the numerical solutions obtained by different Lifting schemes agree with the exact solution.

b) Also, from the tables, the convergence of the presented schemes, i.e., the error decreases when the level of resolution *N* increases.

c) In addition, the calculations involved in lifting schemes are simple, straight-forward and low computation cost compared to classical method, i.e. FDM.

Hence, the presented Lifting schemes in particular HWLS & BWLS are very effective for solving non-linear partial differential equations.

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