Palestine Journal of Mathematics

Vol. 12(1)(2023) , 75–80

Radio Labeling of Certain Networks

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Communicated by V. Lokesha

MSC 2010 Classifications: 05C78, 05C15.

Keywords and phrases: Radio labeling, Radio mean labeling, Honeycomb network, Honeycomb torus network.

Abstract For a connected graph G, an injective function $\pi : V(G) \to N$ such that for every distinct vertices u and v of G, $d(u,v) + |\pi(u) - \pi(v)| \geq 1 + diam(G)$ is called a radio labeling of G. The radio number of π, $rn(\pi)$ is the highest number allotted to any vertex of G. The radio number of G, $rn(G)$ is the minimum value of $rn(\pi)$ taken over all radio labeling π of G. Let G be a connected graph and $\pi : V(G) \to N$ is an injective function. Then π is called a radio mean labeling if for every distinct vertex u and v of G, $d(u,v) + \lceil \frac{\pi(u)+\pi(v)}{2} \rceil \geq 1 + diam(G)$. The highest number allotted to any vertex of G is called radio mean number of π and is denoted by $rmn(\pi)$. The least value of $rmn(\pi)$ taken over all radio mean labeling π of G is called radio mean number of G and is denoted by $rmn(G)$. The upper bound of radio number and radio mean number for honeycomb and honeycomb torus networks are found.

1 Introduction

Radio labeling is inspired by Hale's problem of channel assignment. If we are given a set of radio stations, then the job will be to assign a channel (non-negative integer) to each station (or transmitter) to avoid interference. The interference is intricately linked to the station's geographical position, the closer the stations are uncommon the greater the interference that may occur. Also the packing coloring problem arise from the restrictions concerning the assignment of broadcast frequencies to radio stations, which is similar to radio labeling [1, 2, 3]. The division of channels allocated to the neighbouring stations must be wide enough to prevent interference. To model this problem, the researchers have constructed a graph such that each station is represented by a vertex and two vertices are adjacent when their respective stations are close. The ultimate objective is to find a valid label that reduces the period (range) of the channels used [4, 5] . The radio labeling of different graphs are discussed in [1, 4, 5, 6, 7, 10, 11, 12].

An interconnection network can be modelled by a simple graph whose vertices mean components of network and whose edges mean the connections between them. This idea has been broadly perceived and utilized by computer researchers and engineers. Graph theory is a effective mathematical technique for the design and analysis of topological interconnection network structures. A system's interconnection network logically offers a basic means of linking all device components. For example, hypercubes, butterfly networks, benes networks, honeycomb networks, honeycomb torus networks and grids are some interconnection networks [6, 7, 8, 13].

The definitions of radio number and radio mean number are taken from [5] and [10]. In this paper, the upper bounds of radio numbers and radio mean numbers for honeycomb and honeycomb torus networks are determined.

2 Honeycomb Network

Definition 2.1. An n-dimensional honeycomb network is denoted as $HC_n$where n is the number of hexagons between central and boundary of hexagon. Honeycomb network HC_n is constructed from HC_{n-1} by adding a layer of hexagons around the boundary of HC_{n-1}. The number of vertices in the honeycomb network HC_n are $6n^2$ and the number of edges is $9n^2 - 3n$. The diameter of the honeycomb network is $4n - 1$ [8, 13].

Theorem 2.2. *For,* $HC_n, n \geq 2$, $rn(HC_n) \leq 24n^3 - 16n^2 - 2n + 3$.

Proof. The vertices of HC_n are labeled as we see in $Figure$ 1. We know that $diam(HC_n) = 4n - 1$.

Define a mapping $\pi : V(HC_n) \to N$ as follows
$\pi(v_i) = 4n(i-1) - 2i + 3, \quad 1 \leq i \leq 6n^2 - n$
$\pi(u_i) = 4n(i-1) - 2i + 4, \quad 1 \leq i \leq n.$

Claim: The mapping π is a radio labeling and need to prove that $d(u, v) + |\pi(u) - \pi(v)| \geq 1 + diam(HC_n) = 4n$ holds for all pairs of vertices (u, v), where $u \neq v$.

Case I: Suppose $u = v_k$ and $v = v_l$, $1 \leq k \neq l \leq 6n^2 - n$, $|k - l| > 1$.
Then,
$\pi(u) = 4n(k-1) - 2k + 3$
$\pi(v) = 4n(l-1) - 2l + 3$ and $d(u, v) \geq 1$.
Hence, $d(u, v) + |\pi(u) - \pi(v)| \geq 1 + |(4n-2)(k-l)| \geq 4n$.

Case II: Suppose $u = u_k$ and $v = u_l$, $1 \leq k \neq l \leq n$.
Then,
$\pi(u) = 4n(k-1) - 2k + 4$
$\pi(v) = 4n(l-1) - 2l + 4$ and $d(u, v) \geq 2$.
Hence, $d(u, v) + |\pi(u) - \pi(v)| \geq 2 + |(4n-2)(k-l)| \geq 4n$.

Case III: Suppose $u = u_k$ and $v = v_l$, $1 \leq k, l \leq n$.
Then,
$\pi(u) = 4n(k-1) - 2k + 4$
$\pi(v) = 4n(l-1) - 2l + 3$ and $d(u, v) \geq 4n - 1$.
Hence, $d(u, v) + |\pi(u) - \pi(v)| \geq 4n - 1 + |(4n-2)(k-l) + 1| \geq 4n$.

Case IV: Suppose $u = u_k$ and $v = v_l$, $1 \leq k \leq n$, $n + 1 \leq l \leq 6n^2 - n$, $|k - l| > 1$.
Then,
$\pi(u) = 4n(k-1) - 2k + 4$
$\pi(v) = 4n(l-1) - 2l + 3$ and $d(u, v) \geq 1$.
Hence, $d(u, v) + |\pi(u) - \pi(v)| \geq 1 + |(4n-2)(k-l) + 1| \geq 4n$.

Case V: Suppose $u = v_k$ and $v = v_l$, $1 \leq k \neq l \leq 6n^2 - n$, $|k - l| = 1$.
Then,
$\pi(u) = 4n(k-1) - 2k + 3$
$\pi(v) = 4n(l-1) - 2l + 3$ and $d(u, v) \geq 2$.
Hence, $d(u, v) + |\pi(u) - \pi(v)| \geq 2 + |(4n-2)(k-l)| \geq 4n$.

Thus $d(u, v) + |\pi(u) - \pi(v)| \geq 4n$ for all $u, v \in V(HC_n)$, $n \geq 2$. These five cases establish the claim that π is a radio labeling of HC_n. Since the vertex v_{6n^2-n} receives the maximum label, the radio number of honeycomb HC_n satisfies $rn(HC_n) \leq 24n^3 - 16n^2 - 2n + 3$. □

Theorem 2.3. *For,* $HC_n, n \geq 2$, $rmn(HC_n) \leq 6n^2 + 4n - 6$.

Proof. The vertices of HC_n are labeled as we see in $Figure$ 2. We know that $diam(HC_n) = 4n - 1$.

Define a mapping $\pi : V(HC_n) \to N$ as follows

$$\pi(v_i) = \begin{cases} 4(n-1) + i, & 1 \leq i \leq 3n^2 \\ 4, & i = 3n^2 + 1 \\ 4(n-1) + (i-1), & 3n^2 + 2 \leq i \leq 6n^2 - 1 \\ 1, & i = 6n^2. \end{cases}$$

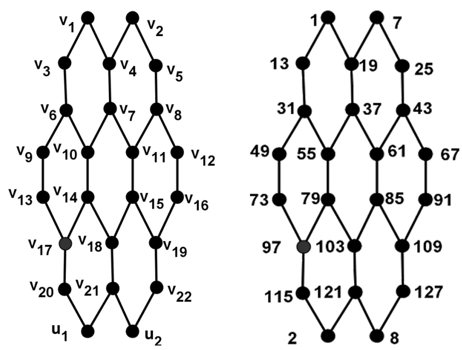

Figure 1. Radio Labeling of HC_2

Claim: The mapping π is a radio labeling and need to prove that $d(u,v) + \lceil\frac{\pi(u)+\pi(v)}{2}\rceil \geq 1 + diam(HC_n) = 4n$ is true for every pair of vertices (u,v), where $u \neq v$.

Case I: Suppose $u = v_{6n^2}$ and $v = v_{3n^2+1}$.
Then,
$\pi(u) = 1$
$\pi(v) = 4$ and $d(u,v) = 4n-3$.
Hence, $d(u,v) + \lceil\frac{\pi(u)+\pi(v)}{2}\rceil \geq 4n-3+\lceil\frac{5}{2}\rceil = 4n$.

Case II: Suppose $u = v_k$ and $v = v_l,\ 1 \leq k \neq l \leq 3n^2,\ 3n^2+2 \leq k \neq l \leq 6n^2-1$.
Then,
$\pi(u) = 4(n-1)+k$
$\pi(v) = 4(n-1)+(l-1)$ and $d(u,v) \geq 1$.
Hence, $d(u,v) + \lceil\frac{\pi(u)+\pi(v)}{2}\rceil \geq 1+\lceil\frac{8n-9+k+l}{2}\rceil \geq 4n$.

Case III: Suppose $u = v_k$ and $v = v_l,\ 1 \leq k \neq l \leq 3n^2$.
Then,
$\pi(u) = 4(n-1)+k$
$\pi(v) = 4(n-1)+l$ and $d(u,v) \geq 1$.
Hence, $d(u,v) + \lceil\frac{\pi(u)+\pi(v)}{2}\rceil \geq 1+\lceil\frac{8n-8+k+l}{2}\rceil \geq 4n$.

Case IV: Suppose $u = v_k$ and $v = v_l,\ 3n^2+2 \leq k \neq l \leq 6n^2-1$.
Then,
$\pi(u) = 4(n-1)+(k-1)$
$\pi(v) = 4(n-1)+(l-1)$ and $d(u,v) \geq 1$.
Hence, $d(u,v) + \lceil\frac{\pi(u)+\pi(v)}{2}\rceil \geq 1+\lceil\frac{8n-10+k+l}{2}\rceil \geq 4n$.

Case V: Suppose $u = v_{3n^2+1}$ and $v = v_l,\ 1 \leq l \leq 3n^2$.
Then,
$\pi(u) = 4$
$\pi(v) = 4(n-1)+l$ and $d(u,v) \geq 1$.
Hence, $d(u,v) + \lceil\frac{\pi(u)+\pi(v)}{2}\rceil \geq 1+\lceil\frac{4n+l}{2}\rceil \geq 4n$.

Case VI: Suppose $u = v_{3n^2+1}$ and $v = v_l,\ 3n^2+2 \leq l \leq 6n^2-1$.
Then,
$\pi(u) = 4$
$\pi(v) = 4(n-1)+(l-1)$ and $d(u,v) \geq 1$.
Hence, $d(u,v) + \lceil\frac{\pi(u)+\pi(v)}{2}\rceil \geq 1+\lceil\frac{4n+l-1}{2}\rceil \geq 4n$.

Therefore, for all $u,v \in V(HC_n)$, $n \geq 2$, $d(u,v) + \lceil\frac{\pi(u)+\pi(v)}{2}\rceil \geq 4n$. Hence these cases verify the claim that π is a radio mean labeling of HC_n. Since the vertex v_{6n^2-1} receives the maximum label, the radio number of honeycombs HC_n satisfies $rmn(HC_n) \leq 6n^2+4n-6$. □

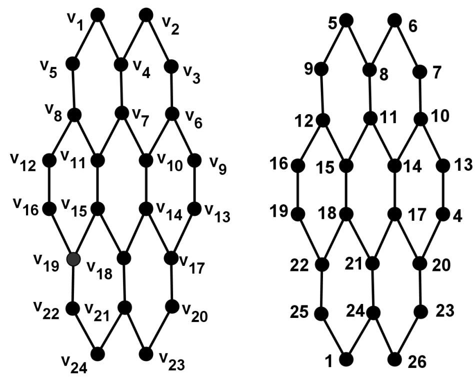

Figure 2. Radio Mean Labeling of HC_2

3 Honeycomb Torus Network

Definition 3.1. The honeycomb torus network is created by linking pairs of honeycomb mesh nodes of degree two. $HCT(r)$ is obtained by adding a layer of hexagons around the boundary of $HC(r-1)$, with wraparound edges. The number of vertices and edges of $HCT(r)$ are $6r^2$ and $9r^2$ respectively [9].

Theorem 3.2. *For,* $HCT_n, n \geq 2$, $rn(HCT_n) \leq 12n^3 - 6n^2 - 2n + 2$.

Proof. The vertices of HCT_n are labelled as we see in $Figure$ 3. We know that $diam(HCT_n) = 2n$.

Define a mapping $\pi : V(HCT_n) \to N$ as follows,
$\pi(v_i) = 2n(i-1) - i + 2, \quad 1 \leq i \leq 6n^2.$

Claim: The mapping π is a radio labeling and we need to prove that $d(u,v)+|\pi(u)-\pi(v)| \geq 1 + diam(HCT_n) = 2n+1$ is true for every pair of vertices (u,v), where $u \neq v$.

Case I: Suppose $u = v_k$ and $v = v_l$, $1 \leq k \neq l \leq 6n^2$, $|k-l| > 1$.
Then,
$\pi(u) = 2n(k-1) - k + 2$
$\pi(v) = 2n(l-1) - l + 2$ and $d(u,v) \geq 1$.
Hence, $d(u,v) + |\pi(u) - \pi(v)| \geq 1 + |(2n-1)(k-l)| \geq 2n+1$.

Case II: Suppose $u = v_1$ and $v = v_l$, $2 \leq l \leq 6n^2$.
Then,
$\pi(u) = 2n(1-1) - k + 2 = 1$
$\pi(v) = 2n(l-1) - l + 2$ and $d(u,v) \geq 1$.
Hence, $d(u,v) + |\pi(u) - \pi(v)| \geq 1 + |(2n-1)(1-l)| \geq 2n+1$.

Case III: Suppose $u = v_k$ and $v = v_l$, $1 \leq k \neq l \leq 6n^2$, $|k-l| = 1$.
Then,
$\pi(u) = 2n(k-1) - k + 2$
$\pi(v) = 2n(l-1) - l + 2$ and $d(u,v) \geq 2$.
Hence, $d(u,v) + |\pi(u) - \pi(v)| \geq 2 + |(2n-1)(k-l)| \geq 2n+1$.

Thus for all $(u,v) \in V(HCT_n)$, $n \geq 2$, $d(u,v) + |\pi(u) - \pi(v)| \geq 2n+1$. Here all the three cases confirm the claim that π is a radio labeling of HCT_n. Since the vertex v_{6n^2} accepts the maximum label, the radio number of honeycomb torus satisfies $rn(HCT_n) \leq 12n^3 - 6n^2 - 2n + 2$. □

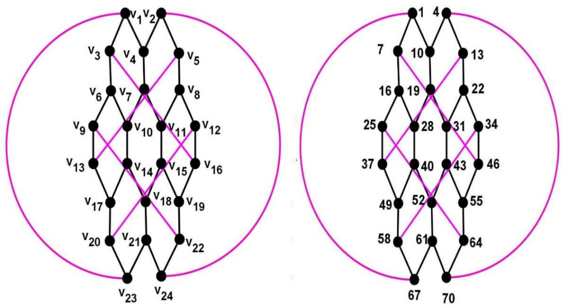

Figure 3. Radio Labeling of HCT_2

Theorem 3.3. *For,* $HCT_n, n \geq 2$, $rmn(HCT_n) \leq 6n^2 + n - 2$.

Proof. The proof is closely connected to the Theorem 2.3. Radio mean labeling of honeycomb torus HCT_2 is shown in $Figure$ 4. □

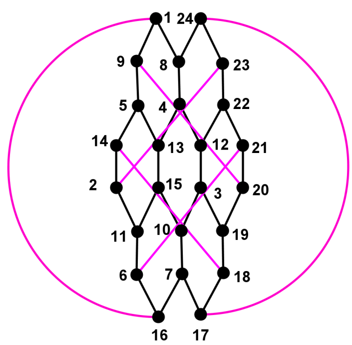

Figure 4. Radio Mean Labeling of HCT_2

Conclusion

We have found the upper bounds of radio numbers and radio mean numbers for honeycomb and honeycomb torus networks. The radio and radio mean number of Benes network is under investigation.

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Received: July 19, 2021.
Accepted: December 28, 2021.