Palestine Journal of Mathematics

Vol. 12(1)(2023) , 836–847

REPRESENTATION OF FUZZY HYPERSOFT SET IN GRAPHS

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Communicated by Thabet Abdeljawad

MSC 2010 Classifications: 05C72; 03E72; 05C62.

Keywords and phrases: Soft set, fuzzy soft set, hypersoft set, soft graph, fuzzy soft graph, hypersoft graph.

Abstract In this study, a novel framework of fuzzy hypersoft graphs (FHS-graphs) is developed which is more flexible as compared to fuzzy soft graphs (FS-graphs) in the sense that it not only generalizes FS-graphs but also tackles its insufficiency for multi-argument approximate function. Some fundamentals, aggregation operations and results are investigated and elaborated with appropriate examples and their graphical depictions.

1 Introduction

The traditional logic (i.e. Boolean logic) is not always pertinent in real-world scenarios, where the available data is vague or imprecise. To deal with such kind of situations, a particular class of sets known as fuzzy sets (F-sets) which were proposed by Zadeh [40] is considered appropriate. In these sets, every member of universe is specified by a membership grade in unit closed interval. However, to tackle with scenarios having more complexity and uncertainty, it was observed that F-Sets depicted some sort of limitation regarding the validation for some parameterization tool. To address this scarcity, Molodtsov [19] characterized soft sets (S-sets) as a new mathematical parameterized model. In S-sets, every parameter in a set of parameters maps to power set of universe of discourse while defining single-argument approximate function. To hybridize the characteristics of F-sets, fuzzy soft sets (FS-sets) [18, 8] were conceptualized. Ali et al. [3], Li et al. [16], Maji et al. [17], Pei et al. [20] and Sezgin et al. [37] discussed the rudiments of S-sets with numerical examples. Babitha et al. [6, 7] presented the notions of relations, functions and orders under soft set environment. The researcher [1, 4, 5, 10] expanded the concept of S-sets and applied it in various fields.
In certain real-world scenarios the classification of attributes into sub-attributive values in the form of sets is necessary. The existing concept of S-sets is not sufficient and incompatible with such scenarios so Smarandache [38] introduced the concept of hypersoft sets (HS-sets) to address the insufficiency of S-sets and to cope up the situations with multi-argument approximate function. The rudiments and elementary axioms of HS-sets have been discussed in [29] and elaborated with numerical examples. The validity of HS-sets for the entitlement of multi-decisive opinions under expert set environment has been discussed in [11, 13, 14, 15]. Rahman et al. [21, 22, 23, 24, 25, 26, 27] investigated the hybridized properties of HS-sets under the environments of complex set, convexity and concavity, parameterization, rough set and bijection. They employed decision-making algorithmic approaches to solve real-world problems. Debnath [9] conceptualized fuzzy hypersoft set by combining fuzzy set with hypersoft set and discussed its properties, operations and results. Saeed et al. [29, 30, 31, 32, 33, 34, 35] developed the theories of neutrosophic hypersoft mappings, complex multi-fuzzy HS-set and neutrosophic hypersoft graphs with applications in decision-making and clinical diagnosis. Ahsan et al. [2] studied the HIV diagnosis using complex fuzzy hypersoft mapping and proposed appropriate treatment. Kamacı [12] discussed the hybridized structure of HS-sets with rough set with some of its important properties, operations and results.
Thumbakara et al. [39] initiated the structure of soft graphs (S-graphs) as the proposal of representation of S-sets in graph theory. Later on, Samanta et al. [36] described the structure of

FS-graphs by combining S-graphs with F-sets. Both S-graphs and FS-graphs are insufficient to tackle the scenario where each attribute is further classified into respective attribute-valued disjoint sets. In order to deal their insufficiency, the concept of FS-graphs is presented in this study which considers multi-argument approximate function for the entitlement of further partitioning of parameters into sub-parametric values in the form of disjoint sets. The rest of the paper is structured as: some elementary definitions like fuzzy set, soft set, hypersoft set etc., are recalled for proper understanding of main results in second section. Fuzzy hypersoft graphs and some of its fundamentals are characterized with the help of pictorial depiction and illustrated examples in third section. Lastly, paper is summarized systematically with more description on future scope and directions.

2 Preliminaries

This portion of the paper presents some elementary terms and definitions by reviewing the existing literature for vivid understanding of the proposed study.

Definition 2.1. (Fuzzy Set) [40]
An F-*set* $\mathfrak{P}$ defined as

$$\mathfrak{P} = \{(\mathfrak{x}, \mathfrak{A}_{\mathfrak{P}}(\mathfrak{x}))|\mathfrak{x} \in \mathfrak{X}\}$$

such that $\mathfrak{A}_{\mathfrak{P}} : \mathfrak{X} \to \mathbb{I}\,(unit\,closed\,interval)$ where $\mathfrak{A}_{\mathfrak{P}}(\mathfrak{x})$ describes the membership value of $\mathfrak{x} \in \mathfrak{P}$.

$\mathfrak{A}_{\mathfrak{P}}(\mathfrak{x})$ has just two membership values in the crisp set: 0 ('false') and 1 ('true'). Instead of primarily analyzing the two truth values, an F-set can calculate the value of $\mathfrak{A}_{\mathfrak{P}}(\mathfrak{x})$ between 1('true') and 0 ('false') for a better interpretation.

Definition 2.2. (Soft Set) [19]
Let $\mathfrak{X}$ be an initial universe then the pair $(\mathfrak{F}_{\mathfrak{S}}, \mathfrak{G})$ is called S-*set* over $\mathfrak{X}$ if $\mathfrak{F}_{\mathfrak{S}} : \mathfrak{G} \to \mathfrak{P}(\mathfrak{X})$ where $\mathfrak{G} \subseteq \mathfrak{E}$ (a set of parameters) and $\mathfrak{P}(\mathfrak{X})$ is the power set of $\mathfrak{X}$. The function $\mathfrak{F}_{\mathfrak{S}}$ is known as approximate function of S-set and $\mathfrak{F}_{\mathfrak{S}}(g)$ is its approximate element for all $g \in \mathfrak{G}$.

Definition 2.3. (Union of two Soft Sets) [17]
Union of two S-sets $(\mathfrak{M}_{\mathfrak{S}_1}, \mathfrak{Z}_1)$ and $(\mathfrak{M}_{\mathfrak{S}_2}, \mathfrak{Z}_2)$ is an S-set $(\mathfrak{M}_{\mathfrak{S}_3}, \mathfrak{Z}_3)$ with $\mathfrak{Z}_3 = \mathfrak{Z}_1 \cup \mathfrak{Z}_2$ and for $\mathfrak{z} \in \mathfrak{Z}_3$,

$$\mathfrak{M}_{\mathfrak{S}_3}(\mathfrak{z}) = \begin{cases} \mathfrak{M}_{\mathfrak{S}_1}(\mathfrak{z}) \cup \mathfrak{M}_{\mathfrak{S}_2}(\mathfrak{z}) & \mathfrak{z} \in (\mathfrak{Z}_1 \cap \mathfrak{Z}_2) \\ \mathfrak{M}_{\mathfrak{S}_1}(\mathfrak{z}) & \mathfrak{z} \in (\mathfrak{Z}_1 \setminus \mathfrak{Z}_2) \\ \mathfrak{M}_{\mathfrak{S}_2}(\mathfrak{z}) & \mathfrak{z} \in (\mathfrak{Z}_2 \setminus \mathfrak{Z}_1) \end{cases}$$

Definition 2.4. (Intersection of two Soft Set) [17]
Intersection of two S-sets $(\mathfrak{M}_{\mathfrak{S}_1}, \mathfrak{Z}_1)$ and $(\mathfrak{M}_{\mathfrak{S}_2}, \mathfrak{Z}_2)$ is an S-set $(\mathfrak{M}_{\mathfrak{S}_3}, \mathfrak{Z}_3)$ with $\mathfrak{Z}_3 = \mathfrak{Z}_1 \cap \mathfrak{Z}_2$ and for $\omega \in \mathfrak{Z}_3$,

$$\mathfrak{M}_{\mathfrak{S}_3}(\omega) = \mathfrak{M}_{\mathfrak{S}_1}(\omega) \cap \mathfrak{M}_{\mathfrak{S}_2}(\omega)$$

More details on soft set and its operations can be seen in [17, 18].

Definition 2.5. (Hypersoft set) [38]
A HS-*set* over an initial universe $\mathfrak{X}$ is a pair $(\mathcal{W}, \mathcal{H})$, where $\mathcal{H}$ is the cartesian product of $\mathcal{H}^i, i = 1, 2, 3, ..., n, \mathcal{H}^i \cap \mathcal{H}^j = \emptyset$ for all $i \neq j$ having attribute values of attributes $\hat{h}^i, i = 1, 2, 3, ..., n, \hat{h}^i \neq \hat{h}^j, i \neq j$ respectively and $\mathcal{W} : \mathcal{H} \to \mathfrak{P}(\mathfrak{X})$ where $\mathfrak{P}(\mathfrak{X})$ is the power set of $\mathfrak{X}$.

Definition 2.6. (Fuzzy Hypersoft Set) [38]
A HS-set $(\mathcal{W}, \mathcal{H})$ is called fuzzy HS-set if $\mathfrak{P}(\mathfrak{X})$ in $\mathcal{W} : \mathfrak{X} \to \mathfrak{P}(\mathfrak{X})$ is replaced with $\mathbb{F}(\mathfrak{X})$ (a collection of fuzzy subsets).

Definition 2.7. (Fuzzy Soft Graph) [36]
Deliberate a non empty set $\mathfrak{X} = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3, ..., \mathfrak{x}_n\}$, and $\mathfrak{A} \subseteq \mathfrak{E}$ (a set of parameters). And let

(i) $\rho : \mathfrak{A} \to \mathbb{F}(\mathfrak{X})$ (Assortment of all f-subsets in $\mathfrak{X}$)

$$\mathfrak{e} \mapsto \rho(\mathfrak{e}) = \rho_{\mathfrak{e}}$$

and

$$\rho(\mathfrak{e}) : \mathfrak{X} \rightarrow [0,1]$$

$$\mathfrak{x}_i \mapsto \rho_{\mathfrak{e}}(\mathfrak{x}_i)$$

$(\mathfrak{A}, \rho)$: $\mathbb{F}$-soft vertex.

(ii) $\beta : A \rightarrow \mathbb{F}(\mathfrak{X} \times \mathfrak{X})$ (a collection of all $\mathbb{F}$-subsets in $\mathfrak{X} \times \mathfrak{X}$)

$$\mathfrak{e} \mapsto \beta(\mathfrak{e}) = \beta_{\mathfrak{e}}$$

and

$$\beta(\mathfrak{e}) : \mathfrak{X} \times \mathfrak{X} \rightarrow [0,1]$$

$$(\mathfrak{x}_i, \mathfrak{x}_j) \mapsto \beta_{\mathfrak{e}}(\mathfrak{x}_i, \mathfrak{x}_j)$$

$(\mathfrak{A}, \beta)$: $\mathbb{F}$-soft edge.

Now $((\mathfrak{A}, \rho), (\mathfrak{A}, \beta))$ is termed as $\mathbb{F}$-soft graph iff $\beta_{\mathfrak{e}}(\mathfrak{x}_i, \mathfrak{x}_j) \leq \rho_e(\mathfrak{x}_i) \wedge \rho_{\mathfrak{e}}(\hat{x}_j)$ for all $\mathfrak{e} \in \mathfrak{A}$ and $\forall \quad i \,\&\, j = 1, 2, 3, ..., n$ and symbolized as $\hat{\mathcal{G}}_{\mathfrak{A},\mathfrak{X}}$.

Definition 2.8. (Fuzzy Soft Subgraph) [36]
The $\mathbb{F}$-soft graph $\mathfrak{H}_{\mathfrak{A},\mathfrak{X}} = ((\mathfrak{Z}, \lambda), (\mathfrak{Z}, \alpha))$ is termed as a $\mathbb{F}$-soft subgraph of $\ddot{\bar{\mathcal{G}}}_{\mathfrak{A},\mathfrak{X}} = ((\mathfrak{Z}, \rho), (\mathfrak{Z}, \beta))$ if $\rho_{\mathfrak{e}}(\mathfrak{x}_i \geq \lambda_{\mathfrak{e}}(\mathfrak{x}_i))$ for all $\mathfrak{x}_i \in \mathfrak{X}$, $\mathfrak{e} \in \mathfrak{A}$ and $\alpha_{\mathfrak{e}}(\mathfrak{x}_i, \mathfrak{x}_j) \leq \beta_{\mathfrak{e}}(\mathfrak{x}_i, \mathfrak{x}_j)$ for all $\mathfrak{x}_i, \mathfrak{x}_j \in \mathfrak{X}, \mathfrak{e} \in \mathfrak{A}$.

Definition 2.9. (Spanning Fuzzy Soft Subgraph) [36]
The $\mathbb{F}$-soft graph $\mathfrak{H}_{\mathfrak{A},\mathfrak{X}} = ((\mathfrak{Z}, \lambda), (\mathfrak{Z}, \alpha))$ is termed as a spanning fuzzy soft subgraph (SF-soft subgraph) of $\ddot{\bar{\mathcal{G}}}_{\mathfrak{A},\mathfrak{X}} = ((\mathfrak{Z}, \rho), (\mathfrak{Z}, \beta))$ if $\rho_{\mathfrak{e}}(\mathfrak{x}_i \geq \lambda_{\mathfrak{e}}(\mathfrak{x}_i))$ for all $\mathfrak{x}_i \in \mathfrak{X}$, $\mathfrak{e} \in \mathfrak{A}$. The two $\mathbb{F}$-soft graphs in this scenario share the similar $\mathbb{F}$-soft vertex set; the only difference is in the arc weights.

3 Fuzzy Hypersoft Graph

In this segment we call several important definitions about fuzzy hypersoft graph, Also describe union and intersection of two $\mathbb{F}$-soft graphs.

Definition 3.1. (Fuzzy Hypersoft Graph)
Deliberate a universe discourse set $\mathfrak{X} = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3, ..., \mathfrak{x}_n\}$, $\mathfrak{E}$(parameters set) and $\mathfrak{K}_i \subseteq \mathfrak{E}, i = 1, 2, 3, ..., n$. Also let

[1] $\rho : \mathfrak{M} \rightarrow \mathfrak{F}(\mathfrak{X})$ (Assortment of all $\mathbb{F}$-subsets in $\mathfrak{X}$) where

$$\mathfrak{M} = \mathfrak{K}_1 \times \mathfrak{K}_2 \times ... \times \mathfrak{K}_n$$

$$\underline{\mathfrak{e}} \mapsto \rho(\underline{\mathfrak{e}}) = \rho_{\underline{\mathfrak{e}}}$$

where

$$\underline{\mathfrak{e}} = (\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, ..., \mathfrak{k}_n)$$

and

$$\rho(\underline{\mathfrak{e}}) : \mathcal{V} \rightarrow [0,1]$$

$$\mathfrak{x}_i \mapsto \rho_{\underline{\mathfrak{e}}}(\mathfrak{x}_i)$$

(M, ρ) : $\mathbb{F}$-hypersoft vertex.

[2] $\beta : \mathfrak{M} \rightarrow \hat{\mathfrak{F}}(\mathfrak{X} \times \mathfrak{X})$ (Assortment of all $\mathbb{F}$-subsets in $\mathfrak{X} \times \mathfrak{X}$) where

$$\mathfrak{M} = \mathfrak{K}_1 \times \mathfrak{k}_2 \times ... \times \mathfrak{K}_n$$

$$\underline{\mathfrak{e}} \mapsto \beta(\underline{\mathfrak{e}}) = \beta_{\underline{\mathfrak{e}}}$$

where

$$\underline{\mathfrak{e}} = (\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, ..., \mathfrak{k}_n)$$

and

$$\beta(\underline{\mathfrak{e}}) : \mathfrak{X} \times \mathfrak{X} \to [0,1]$$

$$(\mathfrak{x}_i, \mathfrak{x}_j) \mapsto \beta_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j)$$

(M, β) : $\mathbb{F}$-hypersoft edge.
Then $((\mathfrak{M}, \rho), (\mathfrak{M}, \beta))$ is called $\mathbb{F}$-hypersoft graph iff

$$\beta_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) \leq \rho_{\underline{\mathfrak{e}}}(\mathfrak{x}_i) \wedge \rho_{\underline{\mathfrak{e}}}(\mathfrak{x}_j) \quad \forall \quad \underline{\mathfrak{e}} \in \mathfrak{K}_i$$

and $\forall \quad i \,\&\, j = 1, 2, 3, ..., n$ and symbolized as $\hat{\mathcal{G}}_{\mathfrak{M},\hat{\mathfrak{V}}}$.

Example 3.2. Deliberate a $\mathbb{F}$-hypersoft graph $\hat{\mathcal{G}}_{\mathfrak{M},\hat{\mathfrak{V}}}$, here $\hat{\mathfrak{X}} = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3, \mathfrak{x}_4\}$, $\mathfrak{K}_1 = \{\mathfrak{k}_{11}, \mathfrak{k}_{12}\}$, $\mathfrak{K}_2 = \{\mathfrak{k}_{21}, \mathfrak{k}_{22}\}$ and $\mathfrak{K}_3 = \{\mathfrak{k}_{31}\}$. Here

$$\mathfrak{M} = \mathfrak{K}_1 \times \mathfrak{K}_2 \times \mathfrak{K}_3 = \{(\mathfrak{k}_{11}, \mathfrak{k}_{21}, \mathfrak{k}_{31}), (\mathfrak{k}_{12}, \mathfrak{k}_{22}, \mathfrak{k}_{31}), (\mathfrak{k}_{12}, \mathfrak{k}_{21}, \mathfrak{k}_{31}), (\mathfrak{k}_{11}, \mathfrak{k}_{22}, \mathfrak{k}_{31})\}$$

$\mathfrak{M} = \{\hat{\mathfrak{E}}_1, \hat{\mathfrak{E}}_2, \hat{\mathfrak{E}}_3, \hat{\mathfrak{E}}_4\}$
$\hat{\mathcal{G}}_{\mathfrak{M},\hat{\mathfrak{V}}}$ is elaborated by Tables 1, 2, Figure 1 and $\beta_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) = 0$ for all

$$(\mathfrak{x}_i, \mathfrak{x}_j) \in \mathfrak{X} \times \mathfrak{X} \setminus \{(\mathfrak{x}_1, \mathfrak{x}_4), (\mathfrak{x}_1, \mathfrak{x}_2), (\mathfrak{x}_2, \mathfrak{x}_4), (\mathfrak{x}_2, \mathfrak{x}_3)\}$$

and $\forall\ \underline{\mathfrak{e}} \in \mathfrak{K}_i$.

Table 1. Table of Weighted Vertices

ρ	$\mathfrak{x}_1$	$\mathfrak{x}_2$	$\mathfrak{x}_3$	$\mathfrak{x}_4$
$\underline{\mathfrak{e}}_1$	0.6	0.3	0	0
$\underline{\mathfrak{e}}_2$	0.4	0.1	0.3	0.7
$\underline{\mathfrak{e}}_3$	0.1	0.3	0.5	0.2
$\underline{\mathfrak{e}}_4$	0.3	0.2	0.4	0.1

Table 2. Table of Weighted Edges

β	$(\mathfrak{x}_2, \mathfrak{x}_3)$	$(\mathfrak{x}_1, \mathfrak{x}_2)$	$(\mathfrak{x}_1, \mathfrak{x}_4)$	$(\mathfrak{x}_2, \mathfrak{x}_4)$
$\underline{\mathfrak{e}}_1$	0	0.3	0	0
$\underline{\mathfrak{e}}_2$	0.1	0.1	0.4	0.3
$\underline{\mathfrak{e}}_3$	0.3	0.1	0.1	0
$\underline{\mathfrak{e}}_4$	0.2	0.2	0.1	0.1

Definition 3.3. (Fuzzy Hypersoft Subgraph)
The $\mathbb{F}$-hypersoft graph $\hat{\mathcal{H}}_{\mathfrak{M},\mathfrak{X}} = ((\mathfrak{M}, \lambda), (\mathfrak{M}, \alpha))$ is called a $\mathbb{F}$-hypersoft subgraph of $\hat{\mathcal{G}}_{\mathfrak{M},\hat{\mathfrak{X}}} = ((\mathfrak{M}, \rho), (\mathfrak{M}, \beta))$ if $\rho_{\underline{\mathfrak{x}}}(\mathfrak{x}_i) \geq \lambda_{\underline{\mathfrak{e}}}(\mathfrak{x}_i)\ \forall\, \mathfrak{x}_i \in \mathfrak{X}$, $\underline{\mathfrak{e}} \in \mathfrak{K}_i$ where

$$\mathfrak{M} = \mathfrak{K}_1 \times \mathfrak{K}_2 \times ... \times \mathfrak{K}_n$$

$$\underline{\mathfrak{e}} = (\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, ..., \mathfrak{k}_n)$$

and

$$\alpha_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) \leq \beta_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) \quad \forall \quad \mathfrak{x}_i, \mathfrak{x}_j \in \mathfrak{X}, \quad \underline{\mathfrak{e}} \in \mathfrak{K}_i$$

Example 3.4. Let $\mathfrak{X} = \{\mathfrak{x}, \mathfrak{x}_2, \mathfrak{x}\}$, $\mathfrak{K}_1 = \{\mathfrak{k}_{11}, \mathfrak{k}_{12}\}$ and $\mathfrak{K}_2 = \{\mathfrak{k}_{21}, \mathfrak{k}_{22}\}$. Here

$$\mathfrak{M} = \mathfrak{K}_1 \times \mathfrak{K}_2 = \{(\mathfrak{k}_{11}, \mathfrak{k}_{21}), (\mathfrak{k}_{11}, \mathfrak{k}_{22}), (\mathfrak{k}_{12}, \mathfrak{k}_{21}), (\mathfrak{k}_{12}, \mathfrak{k}_{22})\}$$

$\mathfrak{M} = \{\hat{\mathfrak{E}}_1, \hat{\mathfrak{E}}_2, \hat{\mathfrak{E}}_3, \hat{\mathfrak{E}}_4\}$
$\hat{\mathcal{H}}_{\mathfrak{M},\hat{\mathfrak{X}}}$ is described by Tables 3, 4, Figure 2 and $\alpha_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) = 0$

$$(\mathfrak{x}_i, \mathfrak{x}_j) \in \mathfrak{X} \times \mathfrak{X} \setminus \{(\mathfrak{x}_1, \mathfrak{x}_3), (\mathfrak{x}_2, \mathfrak{x}_3), (\mathfrak{x}_1, \mathfrak{x}_2)\}$$

and $\forall\ \underline{\mathfrak{e}} \in \mathfrak{K}_i$. $\hat{\mathcal{H}}_{\mathfrak{M},\mathfrak{X}}$ is a $\mathbb{F}$-hypersoft subgraph of $\hat{\mathcal{G}}_{\mathfrak{M},\mathfrak{X}}$, where $\hat{\mathcal{G}}_{\mathfrak{M},\mathfrak{X}}$ is given in the Example 3.2.

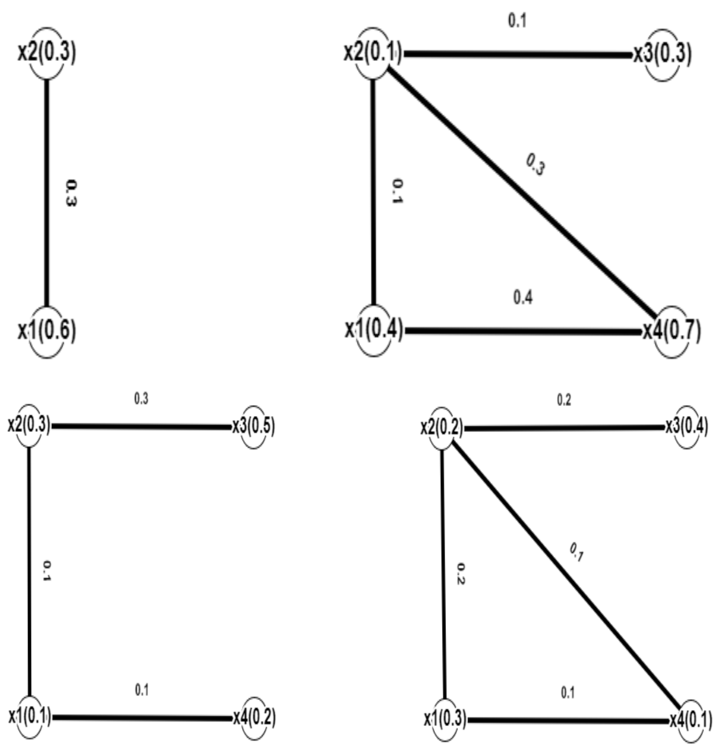

Figure 1. Representation of $\mathbb{F}$-hypersoft Graph

Table 3. Table of Weighted Vertices

λ	$\mathfrak{x}_1$	$\mathfrak{x}_2$	$\mathfrak{x}_3$
$\underline{\mathfrak{e}}_1$	0.3	0.1	0
$\underline{\mathfrak{e}}_2$	0.2	0.4	0.3
$\underline{\mathfrak{e}}_3$	0.5	0.3	0.2
$\underline{\mathfrak{e}}_4$	0.3	0.4	0.2

Table 4. Table of Weighted Edges

α	$(\mathfrak{x}_1, \mathfrak{x}_2)$	$(\mathfrak{x}_1, \mathfrak{x}_3)$	$(\mathfrak{x}_2, \mathfrak{x}_3)$
$\underline{\mathfrak{e}}_1$	0.1	0	0
$\underline{\mathfrak{e}}_2$	0.2	0	0.3
$\underline{\mathfrak{e}}_3$	0.3	0	0.2
$\underline{\mathfrak{e}}_4$	0.3	0.2	0

Definition 3.5. (Spanning Fuzzy Hypersoft Subgraph) The $\mathbb{F}$-hypersoft graph $\hat{\mathcal{H}}_{\mathfrak{M},\mathfrak{X}} = ((\mathfrak{M}, \lambda), (\mathfrak{M}, \alpha))$ is said to be SF-hypersoft subgraph of $\hat{\mathcal{G}}_{\mathfrak{M},\mathfrak{X}} = ((\mathfrak{M}, \rho), (\mathfrak{M}, \beta))$ if $\rho_{\underline{\mathfrak{e}}}(\mathfrak{x}_i = \lambda_{\underline{\mathfrak{e}}}(\mathfrak{x}_i))$ for all $\mathfrak{x}_i \in \hat{\mathcal{V}}$, $\underline{\mathfrak{e}} \in \mathfrak{K}_i$ where

$$M = \mathfrak{K}_1 \times \mathfrak{K}_2 \times ... \times \mathfrak{K}_n$$

$$\underline{\mathfrak{e}} = (\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, ..., \mathfrak{k}_n)$$

The two $\mathbb{F}$-hypersoft graphs in this scenario share the similar $\mathbb{F}$-hypersoft vertex set; the only difference is in the arc weights.

Example 3.6. Let a $\mathbb{F}$-hypersoft graph $\hat{\mathcal{H}}_{\mathfrak{M},\mathfrak{X}}$, where $\mathfrak{X} = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3, \mathfrak{x}_4\}$, $\mathfrak{K}_1 = \{\mathfrak{k}_{11}, \mathfrak{k}_{12}\}$, $\mathfrak{K}_2 = \{\mathfrak{k}_{21}, \mathfrak{k}_{22}\}$ and $\mathfrak{K}_3 = \{\mathfrak{k}_{31}\}$. Here

$$\mathfrak{M} = \mathfrak{K}_1 \times \mathfrak{K}_2 \times \mathfrak{K}_3 = \{(\mathfrak{k}_{11}, \mathfrak{k}_{21}, \mathfrak{k}_{31}), (\mathfrak{k}_{12}, \mathfrak{k}_{21}, \mathfrak{k}_{31}), (\mathfrak{k}_{11}, \mathfrak{k}_{22}, \mathfrak{k}_{31}), (\mathfrak{k}_{12}, \mathfrak{k}_{22}, \mathfrak{k}_{31})\}$$

$$\mathfrak{M} = \{\hat{\mathfrak{E}}_1, \hat{\mathfrak{E}}_2, \hat{\mathfrak{E}}_3, \hat{\mathfrak{E}}_4\}$$

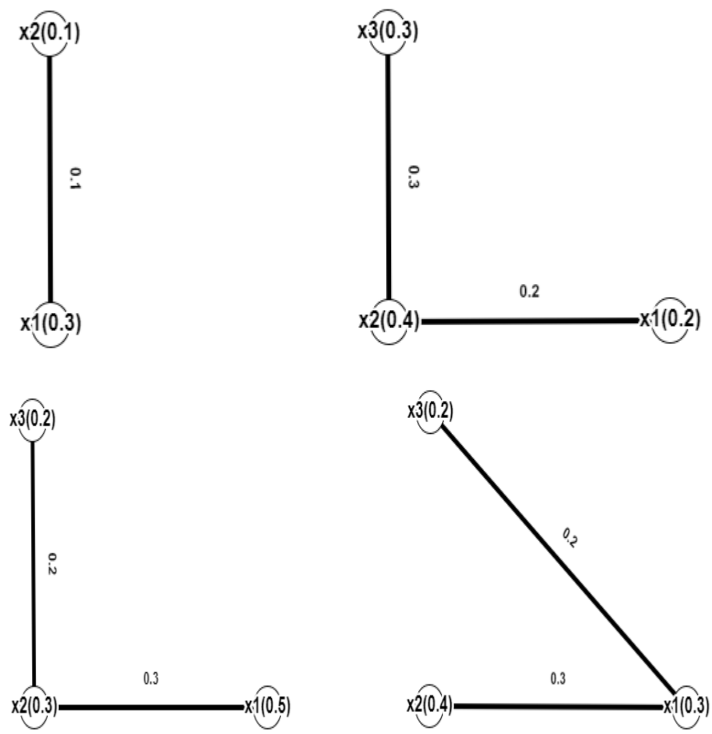

Figure 2. Representation of $\mathbb{F}$-hpersoft Subgraph

$\hat{\mathcal{H}}_{\mathfrak{M},\mathfrak{X}}$ is elaborated by Tables 5, 6, Figure 3 and $\alpha_{\underline{\mathfrak{e}}}(\mathfrak{x}_i,\mathfrak{x}_j)=0$ for all

$$(\mathfrak{x}_i,\mathfrak{x}_j)\in\mathfrak{X}\times\mathfrak{X}\setminus\{(\mathfrak{x}_1,\mathfrak{x}_2),(\mathfrak{x}_2,\mathfrak{x}_4),(\mathfrak{x}_1,\mathfrak{x}_4),(\mathfrak{x}_2,\mathfrak{x}_3)\}$$

and for all $\underline{\mathfrak{e}}\in\mathfrak{K}_i$.

$\hat{\mathcal{H}}_{\mathfrak{M},\mathfrak{X}}$ is a SF-hypersoft subgraph of $\hat{\mathcal{G}}_{\mathfrak{M},\mathfrak{X}}$, where $\hat{\mathcal{G}}_{\mathfrak{M},\mathfrak{X}}$ is given in the Example 3.2.

Table 5. table of weighted vertices.

λ	$\mathfrak{x}_1$	$\mathfrak{x}_2$	$\mathfrak{x}_3$	$\mathfrak{x}_4$
$\underline{\mathfrak{e}}_1$	0.6	0.3	0	0
$\underline{\mathfrak{e}}_2$	0.4	0.1	0.3	0.7
$\underline{\mathfrak{e}}_3$	0.1	0.3	0.5	0.2
$\underline{\mathfrak{e}}_4$	0.3	0.2	0.4	0.1

Table 6. table of weighted edges.

α	$(\mathfrak{x}_1,\mathfrak{x}_2)$	$(\mathfrak{x}_2,\mathfrak{x}_4)$	$(\mathfrak{x}_1,\mathfrak{x}_4)$	$(\mathfrak{x}_2,\mathfrak{x}_3)$
$\underline{\mathfrak{e}}_1$	0.3	0	0	0
$\underline{\mathfrak{e}}_2$	0.1	0	0.4	0.1
$\underline{\mathfrak{e}}_3$	0.1	0	0.1	0.3
$\underline{\mathfrak{e}}_4$	0	0.2	0.1	0.2

Definition 3.7. (Union of two Fuzzy Hypersoft Graphs)
Let $\mathfrak{X}_1,\mathfrak{X}_2\subset\mathfrak{X}$ and $\mathfrak{M},\hat{\mathfrak{P}}\subset\hat{\mathfrak{E}}$. And the union of two $\mathbb{F}$-hypersoft graphs $\hat{\mathcal{G}}^1_{\mathfrak{M},\mathfrak{X}^1}=((\mathfrak{M},\rho^1_{\underline{\mathfrak{e}}}),(\mathfrak{M},\beta^1_{\underline{\mathfrak{e}}}))$ and $\hat{\mathcal{G}}^2_{\hat{\mathfrak{P}},\mathfrak{X}^2}=((\hat{\mathfrak{P}},\rho^2_{\underline{\mathfrak{e}}}),(\hat{\mathfrak{P}},\beta^2_{\underline{\mathfrak{e}}}))$ is defined to be $G^1_{Q,\mathfrak{X}^3}=((\hat{\mathcal{Q}},\rho^3_{\underline{\mathfrak{e}}}),(\hat{\mathcal{Q}},\beta^3_{\underline{\mathfrak{e}}}))$ (say), where $Q=\mathfrak{M}\cup\hat{\mathfrak{P}},\mathfrak{X}_3=\mathfrak{X}_1\cup\mathfrak{X}_2$

$$\mathfrak{M}=\mathfrak{K}_1\times\mathfrak{K}_2\times\ldots\times\mathfrak{K}_n$$

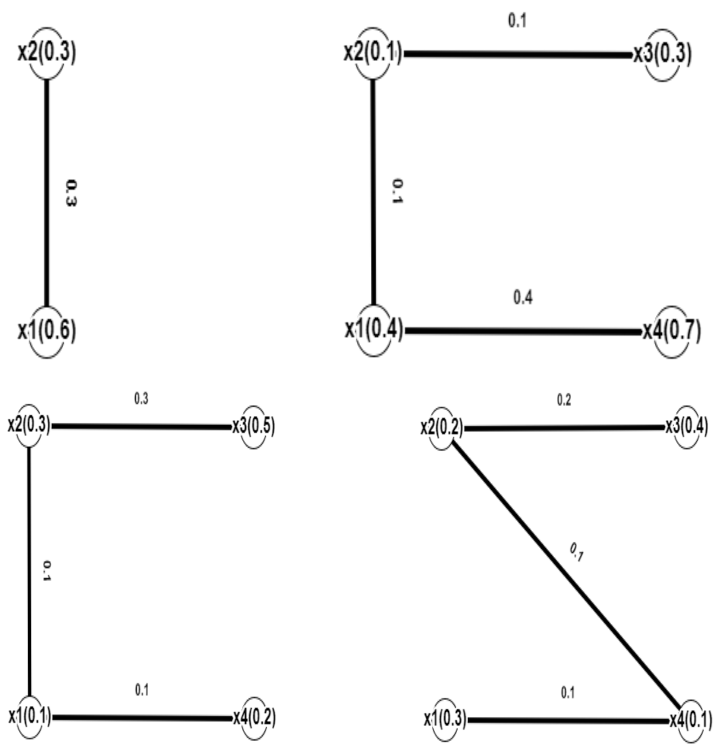

Figure 3. Representation of SF-hypersoft Subgraph

$$\hat{\mathfrak{P}} = \hat{\mathcal{L}}_1 \times \hat{\mathcal{L}}_2 \times ... \times \hat{\mathcal{L}}_n$$
$$\underline{\mathfrak{e}} = (\hat{c}_1, \hat{c}_2, \hat{c}_3, ..., \hat{c}_n)$$

and

$$\rho^3_{\underline{\mathfrak{e}}} = \begin{cases} \rho^1_{\underline{\mathfrak{e}}} & \forall \quad \underline{\mathfrak{e}} \in \mathfrak{M} \setminus \hat{\mathfrak{P}} \quad ; \quad \mathfrak{x}_i \in \mathfrak{X}_1 \cap \mathfrak{X}_2 \\ \rho^2_{\underline{\mathfrak{e}}} & \forall \quad \underline{\mathfrak{e}} \in \hat{\mathfrak{P}} \setminus \mathfrak{M} \quad ; \quad \mathfrak{x}_i \in \mathfrak{X}_2 \setminus \mathfrak{X}_1 \\ \rho^1_{\underline{\mathfrak{e}}} & \forall \quad \underline{\mathfrak{e}} \in \mathfrak{M} \setminus \hat{\mathfrak{P}} \quad ; \quad \mathfrak{x}_i \in \mathfrak{X}_1 \setminus \mathfrak{X}_2 \\ 0 & \forall \quad \underline{\mathfrak{e}} \in \mathfrak{M} \setminus \hat{\mathfrak{P}} \quad ; \quad \mathfrak{x}_i \in \mathfrak{X}_2 \setminus \mathfrak{X}_1 \\ \rho^1_{\underline{\mathfrak{e}}} \vee \rho^2_{\underline{\mathfrak{e}}} & \forall \quad \underline{\mathfrak{e}} \in \mathfrak{M} \cap \hat{\mathfrak{P}} \quad ; \quad \mathfrak{x}_i \in \mathfrak{X}_1 \cap \mathfrak{X}_2 \\ \rho^1_{\underline{\mathfrak{e}}} & \forall \quad \underline{\mathfrak{e}} \in \mathfrak{M} \cap \hat{\mathfrak{P}} \quad ; \quad \mathfrak{x}_i \in \mathfrak{X}_1 \setminus \mathfrak{X}_2 \\ 0 & \forall \quad \underline{\mathfrak{e}} \in \hat{\mathfrak{P}} \setminus \mathfrak{M} \quad ; \quad \mathfrak{x}_i \in \mathfrak{X}_1 \setminus \mathfrak{X}_2 \\ \rho^2_{\underline{\mathfrak{e}}} & \forall \quad \underline{\mathfrak{e}} \in \hat{\mathfrak{P}} \setminus \mathfrak{M} \quad ; \quad \mathfrak{x}_i \in \mathfrak{X}_1 \cap \mathfrak{X}_2 \\ \rho^2_{\underline{\mathfrak{e}}} & \forall \quad \underline{\mathfrak{e}} \in \mathfrak{M} \cap \hat{\mathfrak{P}} \quad ; \quad \mathfrak{x}_i \in \mathfrak{X}_2 \setminus \mathfrak{X}_1 \end{cases}$$

and

$$\beta^3_e(\mathfrak{x}_i) = \begin{cases} \beta^1_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) & \forall \quad \underline{\mathfrak{e}} \in \mathfrak{M} \setminus \hat{\mathfrak{P}} \quad ; \quad (\mathfrak{x}_i, \mathfrak{x}_j) \in (\mathfrak{X}_1 \times \mathfrak{X}_1) \setminus (\mathfrak{X}_2 \times \mathfrak{X}_2) \\ \beta^1_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) & \forall \quad \underline{\mathfrak{e}} \in \mathfrak{M} \setminus \hat{\mathfrak{P}} \quad ; \quad (\mathfrak{x}_i, \mathfrak{x}_j) \in (\mathfrak{X}_1 \times \mathfrak{X}_1) \cap (\mathfrak{X}_2 \times \mathfrak{X}_2) \\ \beta^2_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) & \forall \quad \underline{\mathfrak{e}} \in \hat{\mathfrak{P}} \setminus \mathfrak{M} \quad ; \quad (\mathfrak{x}_i, \mathfrak{x}_j) \in (\mathfrak{X}_2 \times \mathfrak{X}_2) \times (\mathfrak{X}_1 \times \mathfrak{X}_1) \\ 0 & \forall \quad \underline{\mathfrak{e}} \in \mathfrak{M} \setminus \hat{\mathfrak{P}} \quad ; \quad (\mathfrak{x}_i, \mathfrak{x}_j) \in (\mathfrak{X}_2 \times \mathfrak{X}_2) \times (\mathfrak{X}_1 \times \mathfrak{X}_1) \\ \beta^1_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) \vee \beta^2_{\underline{\mathfrak{e}}}(\hat{x}_i, \hat{x}_j) & if \quad \underline{\mathfrak{e}} \in \mathfrak{M} \cap \hat{\mathfrak{P}} \quad ; \quad (\mathfrak{x}_i, \mathfrak{x}_j) \in (\mathfrak{X}_1 \times \mathfrak{X}_1) \cap (\mathfrak{X}_2 \times \mathfrak{X}_2) \\ \beta^1_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) & if \quad \underline{\mathfrak{e}} \in \mathfrak{M} \cap \hat{\mathfrak{P}} \quad ; \quad (\mathfrak{x}_i, \mathfrak{x}_j) \in (\mathfrak{X}_1 \times \mathfrak{X}_1) \setminus (\mathfrak{X}_2 \times \mathfrak{X}_2) \\ 0 & \forall \quad \underline{\mathfrak{e}} \in \hat{\mathfrak{P}} \setminus \mathfrak{M} \quad ; \quad (\mathfrak{x}_i, \mathfrak{x}_j) \in (\mathfrak{X}_1 \times \mathfrak{X}_1) \times (\mathfrak{X}_2 \times \mathfrak{X}_2) \\ \beta^2_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) & \forall \quad \underline{\mathfrak{e}} \in \hat{\mathfrak{P}} \setminus \mathfrak{M} \quad ; \quad (\mathfrak{x}_i, \mathfrak{x}_j) \in (\mathfrak{X}_1 \times \mathfrak{X}_1) \times (\mathfrak{X}_2 \times \mathfrak{X}_2) \\ \beta^2_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) & if \quad \underline{\mathfrak{e}} \in \mathfrak{M} \cap \hat{\mathfrak{P}} \quad ; \quad (\mathfrak{x}_i, \mathfrak{x}_j) \in (\mathfrak{X}_2 \times \mathfrak{X}_2) \setminus (\mathfrak{X}_1 \times \mathfrak{X}_1) \end{cases}$$

Example 3.8. Let $\mathfrak{X} = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3, \mathfrak{x}_4, \mathfrak{x}_5, \mathfrak{x}_6\}$ and $\mathfrak{X}_1 = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3, \mathfrak{x}_4\}$ $\mathfrak{K}_1 = \{\mathfrak{k}_{11}, \mathfrak{k}_{12}\}$, $\mathfrak{K}_2 = \{\mathfrak{k}_{21}, \mathfrak{k}_{22}\}$ and $\mathfrak{K}_3 = \{\mathfrak{k}_{31}\}$. Here

$$\mathfrak{M} = \mathfrak{K}_1 \times \mathfrak{K}_2 \times \mathfrak{K}_3 = \{(\mathfrak{k}_{11}, \mathfrak{k}_{21}, \mathfrak{k}_{31}), (\mathfrak{k}_{12}, \mathfrak{k}_{22}, \mathfrak{k}_{31}), (\mathfrak{k}_{11}, \mathfrak{k}_{22}, \mathfrak{k}_{31}), (\mathfrak{k}_{12}, \mathfrak{k}_{21}, \mathfrak{k}_{31})\}$$

$\mathfrak{M} = \{\hat{\mathfrak{E}}_1, \hat{\mathfrak{E}}_2, \hat{\mathfrak{E}}_3, \hat{\mathfrak{E}}_4\}$
$\hat{\mathcal{G}}_{\mathfrak{M},\mathfrak{X}_1}$ is elaborated by Tables 1, 2 and $\beta_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) = 0$ for all

$$(\mathfrak{x}, \mathfrak{x}_j) \in \mathfrak{X}_1 \times \mathfrak{X}_1 \setminus \{(\mathfrak{x}_1, \mathfrak{x}_2), (\mathfrak{x}_2, \mathfrak{x}_3), (\mathfrak{x}_1, \mathfrak{x}_4), (\mathfrak{x}_2, \mathfrak{x}_4))\}$$

and $\forall\ \underline{\mathfrak{e}} \in \mathfrak{K}_i$. $\hat{\mathcal{L}}_2 = \{\mathfrak{x}_2, \mathfrak{x}_3, \mathfrak{x}_5\}$, $\hat{\mathcal{L}}_1 = \{\hat{l}_{11}, \hat{l}_{12}\}$ and $\hat{\mathcal{L}}_2 = \{\hat{l}_{21}, \hat{l}_{22}\}$. Here

$$\hat{\mathfrak{P}} = \hat{\mathcal{L}}_1 \times \hat{\mathcal{L}}_2 = \{(\hat{l}_{11}, \hat{l}_{21}), (\hat{l}_{11}, \hat{l}_{22}), (\hat{l}_{12}, \hat{l}_{21}), (\hat{l}_{12}, \hat{l}_{22})\}$$

$\hat{\mathfrak{P}} = \{\hat{\mathfrak{E}}_1, \hat{\mathfrak{E}}_2, \hat{\mathfrak{E}}_3, \hat{\mathfrak{E}}_4\}$
$\hat{\mathcal{H}}_{\hat{\mathfrak{P}},\hat{\mathcal{V}}_2}$ is elaborated by Tables 7, 8 and $\alpha_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) = 0$ for all

$$(\mathfrak{x}_i, \mathfrak{x}_j) \in \mathfrak{X}_2 \times \mathfrak{X}_2 \setminus \{(\mathfrak{x}_2, \mathfrak{x}_3), (\mathfrak{x}_3, \mathfrak{x}_5), (\mathfrak{x}_2, \mathfrak{x}_5)\}$$

and for all $\underline{\mathfrak{e}} \in \hat{\mathcal{L}}_i$.

Then $\hat{\mathcal{G}}_{\hat{\mathcal{Q}},\mathfrak{X}_3}$ is a union of $\hat{\mathcal{G}}_{\mathfrak{M},\mathfrak{X}_1}$ and $\hat{\mathcal{G}}_{\hat{\mathfrak{P}},\mathfrak{X}_2}$ and the union is elaborated by Tables 9, 10, here $\hat{\mathcal{Q}} = \mathfrak{M} \cup \hat{\mathfrak{P}}$ and $\gamma_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) = 0$ for all $(\mathfrak{x}_i, \mathfrak{x}_j) \in \mathfrak{X}_3 \times \mathfrak{X}_3 \setminus \{\mathfrak{x}_1, \mathfrak{x}_2), (\mathfrak{x}_2, \mathfrak{x}_5), (\mathfrak{x}_5, \mathfrak{x}_6), (\mathfrak{x}_2, \mathfrak{x}_3), (\mathfrak{x}_3, \mathfrak{x}_4),\}$ and $\forall\ \underline{\mathfrak{e}} \in \mathfrak{K}_i \cup \hat{\mathcal{L}}_i$.

Table 7. table of weighted vertices of graph H.

λ	$\mathfrak{x}_2$	$\mathfrak{x}_3$	$\mathfrak{x}_5$
$\underline{\mathfrak{e}}_2$	0.1	0.3	0.6
$\underline{\mathfrak{e}}_4$	0.2	0.4	0.7
$\underline{\mathfrak{e}}_5$	0.3	0.1	0

Table 8. table of weighted edges of graph H.

α	$(\mathfrak{x}_2, \mathfrak{x}_3)$	$(\mathfrak{x}_2, \mathfrak{x}_5)$	$(\mathfrak{x}_3, \mathfrak{x}_5)$
$\underline{\mathfrak{e}}_2$	0.3	0	0.3
$\underline{\mathfrak{e}}_3$	0.2	0.2	0.4
$\underline{\mathfrak{e}}_4$	0.1	0	0

Table 9. table of weighted vertices of Union.

γ	$\mathfrak{x}_1$	$\mathfrak{x}_2$	$\mathfrak{x}_3$	$\mathfrak{x}_4$	$\mathfrak{x}_5$
$\underline{\mathfrak{e}}_1$	0.5	0.3	0	0	0
$\underline{\mathfrak{e}}_2$	0.4	0.1	0.3	0.7	0.6
$\underline{\mathfrak{e}}_3$	0.1	0.3	0.5	0.2	0.7
$\underline{\mathfrak{e}}_4$	0.3	0.2	0.4	0.1	0

Table 10. table of weighted edges of Union.

ω	$(\mathfrak{x}_1, \mathfrak{x}_2)$	$(\mathfrak{x}_2, \mathfrak{x}_3)$	$(\mathfrak{x}_1, \mathfrak{x}_4)$	$(\mathfrak{x}_2, \mathfrak{x}_4)$	$(\mathfrak{x}_2, \mathfrak{x}_5)$	$(\mathfrak{x}_3, \mathfrak{x}_5)$
$\underline{\mathfrak{e}}_1$	0.3	0	0	0	0	0
$\underline{\mathfrak{e}}_2$	0.1	0.1	0.4	0.3	0	0.3
$\underline{\mathfrak{e}}_3$	0.1	0.3	0.1	0	0	0
$\underline{\mathfrak{e}}_4$	0.2	0.2	0.1	0.1	0.2	0.4
$\underline{\mathfrak{e}}_5$	0	0.1	0	0	0	0

Definition 3.9. (Intersection of two Fuzzy Hypersoft Graphs)
Let $\mathfrak{X}_1, \mathfrak{X}_2 \subset \mathfrak{X}$ and $\mathfrak{M}, \hat{\mathfrak{P}} \subset \hat{\mathfrak{E}}$. Then the intersection of $\mathbb{F}$-hypersoft graphs $\hat{\mathcal{G}}^1_{\mathfrak{M},\mathfrak{X}^1} = ((\mathfrak{M}, \rho^1_{\underline{\mathfrak{e}}}), (\mathfrak{M}, \beta^1_{\underline{\mathfrak{e}}}))$ and $\hat{\mathcal{G}}^2_{\hat{\mathfrak{P}},\mathfrak{X}^2} = ((\hat{\mathfrak{P}}, \rho^2_{\underline{\mathfrak{e}}}), (\hat{\mathfrak{P}}, \beta^2_{\underline{\mathfrak{e}}}))$ is defined to be $\hat{\mathcal{G}}^1_{\hat{\mathcal{Q}},\mathfrak{X}^3} = ((\hat{\mathcal{Q}}, \rho^3_{\underline{\mathfrak{e}}}), (\hat{\mathcal{Q}}, \beta^3_{\underline{\mathfrak{e}}}))$ (say), where $\hat{\mathcal{Q}} = \mathfrak{M} \cap \hat{\mathfrak{P}}, \mathfrak{X}_3 = \mathfrak{X}_1 \cap \mathfrak{X}_2$

$$\hat{\mathcal{M}} = \mathfrak{K}_1 \times \mathfrak{K}_2 \times ... \times \mathfrak{K}_n$$

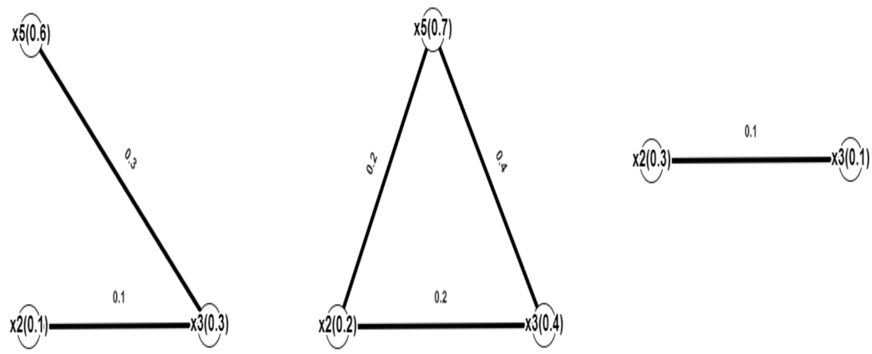

Figure 4. Representation of FH-Graph H

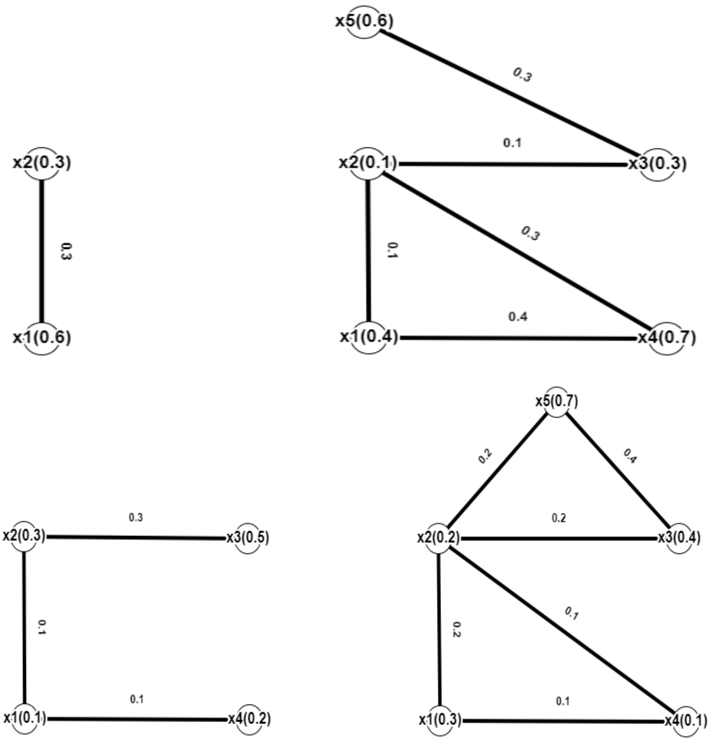

Figure 5. Representation of Union of Two FH-Graphs

$$\hat{\mathcal{P}} = \hat{\mathcal{L}}_1 \times \hat{\mathcal{L}}_2 \times ... \times \hat{\mathcal{L}}_n$$

$$\underline{\mathfrak{e}} = (\hat{c}_1, \hat{c}_2, \hat{c}_3, ..., \hat{c}_n)$$

$\rho^3_{\underline{\mathfrak{e}}} = \rho^1_{\underline{\mathfrak{e}}} \wedge \rho^2_{\underline{\mathfrak{e}}}$ for all $\mathfrak{x}_i \in \mathfrak{X}_3$ and $\underline{\mathfrak{e}} \in \hat{\mathcal{Q}}$ and $\beta^3_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) = \beta^1_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) \wedge \beta^2_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j)$ if $\mathfrak{x}_i, \mathfrak{x}_j \in \mathfrak{X}_3$ and $\underline{\mathfrak{e}} \in \hat{\mathcal{Q}}$.

Example 3.10. Let $\mathfrak{X} = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3, \mathfrak{x}_4, \mathfrak{x}_5, \mathfrak{x}_6\}$ and $\mathfrak{X}_1 = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3, \mathfrak{x}_4\}$ $\mathfrak{K}_1 = \{\mathfrak{k}_{11}, \mathfrak{k}_{12}\}$, $\mathfrak{K}_2 = \{\mathfrak{k}_{21}, \mathfrak{k}_{22}\}$ and $\mathfrak{K}_3 = \{\mathfrak{k}_{31}\}$. Here

$$\mathfrak{M} = \mathfrak{K}_1 \times \mathfrak{K}_2 \times \mathfrak{K}_3 = \{(\mathfrak{k}_{11}, \mathfrak{k}_{21}, \mathfrak{k}_{31}), (\mathfrak{k}_{12}, \mathfrak{k}_{22}, \mathfrak{k}_{31}), (\mathfrak{k}_{11}, \mathfrak{k}_{22}, \mathfrak{k}_{31}), (\mathfrak{k}_{12}, \mathfrak{k}_{21}, \mathfrak{k}_{31})\}$$

$\mathfrak{M} = \{\hat{\mathfrak{E}}_1, \hat{\mathfrak{E}}_2, \hat{\mathfrak{E}}_3, \hat{\mathfrak{E}}_4\}$
$\hat{\mathcal{G}}_{\mathfrak{M}, \mathfrak{X}_1}$ is elaborated by Tables 1, 2 and $\beta_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) = 0$ for all

$$(\mathfrak{x}_i, \mathfrak{x}_j) \in \mathfrak{X}_1 \times \mathfrak{X}_1 \setminus \{(\mathfrak{x}_1, \mathfrak{x}_2), (\mathfrak{x}_2, \mathfrak{x}_4), (\mathfrak{x}_1, \mathfrak{x}_3), (\mathfrak{x}_2, \mathfrak{x}_3), (\mathfrak{x}_1, \mathfrak{x}_4)\}$$

and $\forall\ \underline{\mathfrak{e}} \in \hat{\mathfrak{A}}_i$. $\mathfrak{X}_2 = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3\}$, $\hat{\mathcal{L}}_1 = \{\hat{l}_{11}, \hat{l}_{12}\}$ and $\hat{\mathcal{L}}_2 = \{\hat{l}_{21}, \hat{l}_{22}\}$. Here

$$\hat{\mathfrak{P}} = \hat{\mathcal{L}}_1 \times \hat{\mathcal{L}}_2 = \{(\hat{l}_{11}, \hat{l}_{21}), (\hat{l}_{11}, \hat{l}_{22}), (\hat{l}_{12}, \hat{l}_{21}), (\hat{l}_{12}, \hat{l}_{22})\}$$

$\hat{\mathfrak{P}} = \{\hat{\mathfrak{E}}_1, \hat{\mathfrak{E}}_2, \hat{\mathfrak{E}}_3, \hat{\mathfrak{E}}_4\}$
$\hat{\mathcal{H}}_{\hat{\mathfrak{P}}, \hat{\mathcal{V}}_2}$ is described by Tables 7, 8 and $\alpha_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) = 0$

$$(\mathfrak{x}_i, \mathfrak{x}_j) \in \mathfrak{X}_2 \times \mathfrak{X}_2 \setminus \{(\mathfrak{x}_1, \mathfrak{x}_2), (\mathfrak{x}_1, \mathfrak{x}_3), (\mathfrak{x}_2, \mathfrak{x}_3)\}$$

and $\forall\ \underline{\mathfrak{e}} \in \hat{\mathcal{L}}_i$.

Then the intersection of $\hat{\mathcal{G}}_{\mathfrak{M}, \mathfrak{X}_1}$ and $\hat{\mathcal{G}}_{\hat{\mathfrak{P}}, \mathfrak{X}_2}$ is $\hat{\mathcal{G}}_{\hat{\mathcal{Q}}, \mathfrak{X}_3}$ is given by Tables 11, 12, here $\hat{\mathcal{Q}} = \mathfrak{M} \cap \hat{\mathfrak{P}}$ and $\gamma_{\underline{\mathfrak{e}}}(\mathfrak{x}_i, \mathfrak{x}_j) = 0\ \forall\ (\mathfrak{x}_i, \mathfrak{x}_j) \in \mathfrak{X}_3 \times \mathfrak{X}_3 \setminus \{(\mathfrak{x}_2, \mathfrak{x}_2), (\mathfrak{x}_3, \mathfrak{x}_3), (\hat{x}_3, \hat{x}_2)\}$ and $\forall\ \underline{\mathfrak{e}} \in \mathfrak{K}_i \cap \hat{\mathcal{L}}_i$.

Table 11. table of weighted vertices.

γ	$\mathfrak{x}_2$	$\mathfrak{x}_3$
$\underline{\mathfrak{e}}_2$	0.1	0.3
$\underline{\mathfrak{e}}_4$	0.2	0.4

Table 12. table of weighted edges.

ω	$(\mathfrak{x}_2, \mathfrak{x}_3)$
$\underline{\mathfrak{e}}_2$	0.1
$\underline{\mathfrak{e}}_4$	0.2

Figure 6. Representation of Intersection of Two FH-Graph

4 Conclusion

In this study, the existing models i.e. fuzzy graphs and fuzzy soft graphs, have been made adequate with the development of fuzzy hypersoft graph that tackles the scenario of multi-argument approximate function which considers its domain as cartesian product of sub-parametric valued sets. Some of its essential elementary properties, set-theoretic operations and results have been discussed briefly with the description of illustrated examples and graphical exploration. The study provides a conceptual framework for researchers working on graph theory under uncertain environments. It not only tackles the limitations of relevant existing models but also validates all of their features and properties. As this study has made deep focus on attributes and their sub-attribute values therefore it will increase the validity and reliability of decision-making process. Future work may include:

(i) the computation of certain kinds of graph products under FHS-environment,

(ii) the investigation on the various types of FHS-graphs

(iii) the characterization of various kinds of graph matrices under FHS-environment,

(iv) the employment of FHS-graphs in solving decision-making problems,

(v) the calculation of dominations and spectrums of FHS-graphs

and many others.

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Received: October 29th, 2021
Accepted: February 8th, 2022